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Short Communication

Gaussian soliton solution to nonlinear Schrodinger equation with log-law nonlinearity

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This paper integrates the nonlinear Schrödinger equation with log-law nonlinearity. This is done by the aid of Lie symmetry approach. The stationary solutions are thus obtained.

Key words: Optical solitons, log-law nonlinearity, lie symmetry.

INTRODUCTION

The nonlinear Schrödinger equation (NLSE) plays a very important role in various areas of physical sciences (Biswas, 2009; Lü et al., 2007, 2008a, 2008b, 2009a, 2009b; Emary and Kareem, 2008; Guerrero et al., 2010; Hadzievski et al., 2002; Khalique and Biswas, 2009; Kudryashov, 2009; Mancic et al., 2006; Misra and Chowdhury, 2006; Wazwaz, 2009, 2006). It appears in nonlinear optics (Lü et al. 2007). Bose–Einstein condensates (Lü et al., 2008) fluid dynamics (Lü et al., 2008) mathematical biology, plasma physics (Lü, et al., 2008) and many more areas. There are various kinds of nonlinearity that are studied in this equation (Khalique and Biswas, 2009). In various contexts that arises from reality. One of the important questions that arise in this aspect is the issue of inte-grability. Most of these nonlinear forms make the NLSE non-integrable. There are a lot of studies that are being carried out globally to address this issue of integrability. In this paper, there is one such nonlinear form of the NLSE that will be studied. A closed form stationary solution of the NLSE, with log-law nonlinearity (Guerrero et al., 2010) will be obtained.

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The NLSE is one useful model among various nonlinear evolution equations (Biswas, 2009; Lü et al., 2007, 2008a, 2008b, 2009a, 2009b; Khalique and Biswas, 2009; Kudryashov, 2009; Mancic et al., 2006; Misra and Chowdhury, 2006; Wazwaz, 2009, 2006). Nowadays, some integrability methods are available to integrate these nonlinear evolution equa-tions (Lü et al., 2007, 2008a, 2008b, 2009a, 2009b). This is a true blessing in the world of such integrable equations. Once upon a time, there was the monopoly of the method of inverse scattering transform (IST) that allowed one to carry out the integration of only a few of these nonlinear evolution equations. Today there are an abundance of modern techniques available to integrate these kinds of equations.

Some of these modern methods of integrability are tanh-coth method, sine-cosine method, *F*-expansion method, Riccati equation method, G'/G -method, exponential function method, Hirota bilinear method, Bäcklund transformation method, Painleve expansion and many more. One must be however, very careful in applying these techniques of integration as it could lead to incorrect results. This fact was first pointed out by Kudryashov in 2009. In this paper, there will be one such method that will be used to carry out the integration of the NLSE log-law nonlinearity. This is the method of Lie symmetry. Although this is not one of the modern methods of integrability, this method still remains one of the most powerful methods of integrability.

MATHEMATICAL FORMULATION

The NLSE studied in this paper is given by

$$iq_t + aq_{tt} + bF(|q|^2|)q = 0. \tag{1}$$

Where; F is a real-valued algebraic function and we need to have the smoothness of the complex function $F(|q|^2)q : C \mapsto C$. Considering the complex plane C as a two-dimensional linear space R^2, the function $F(|q|^2)q$ is k times continuously differentiable so that

$$F(|q|^2)q \in \bigcup_{m,n=1}^{\infty} C^k((-n,n)\times(-m,m);R^2) \tag{2}$$

In equation (1), the first term represents the evolution term, while a stands for the coefficient of group velocity dispersion and b represents the coefficient of nonlinearity. Equation (1) is a nonlinear partial differential equation (PDE) of parabolic type and is not integrable, in general. The special case, $F(s) = s$, also known as the Kerr law of nonlinearity, is integrable by the IST method (Emary and Kareem, 2008).

The IST is the nonlinear analog of Fourier transform that is used for solving the linear PDEs. Schematically, therefore IST and the technique of Fourier transform are similar (Emary and Kareem, 2008).

The general case $F(s) \neq s$ takes (1) away from the IST picture as it is not of Painleve type.

Equation (1) has three integrals of motion (Kudryashov, 2009). They are the energy (E) also known as the wave power, linear momentum (M) and the Hamiltonian (H) that are respectively given by

$$E = \int_{-\infty}^{\infty} |q|^2 dx \tag{3}$$

$$M = ia\int_{-\infty}^{\infty} (q^* q_x - qq_x^*)dx \tag{4}$$

$$H = \int_{-\infty}^{\infty} [a|q_x|^2 - f(I))]dx \tag{5}$$

Where;

$$f(I) = \int_0^1 F(\xi)\,d\xi \tag{6}$$

and the intensity I is given by $I = |q|^2$. One can see that (1) can therefore be written in a canonical form as

$$iq_t = \frac{\delta H}{\delta q^*} \tag{7}$$

$$iq_t^* = -\frac{\delta H}{\delta q}. \tag{8}$$

This defines a Hamiltonian dynamical system on an infinite dimensional phase space. It can be analyzed using the theory of Hamiltonian systems. This means that a behavior of the solution is defined, to a large extent, by the singular points of the system, namely the stationary solutions of (1) and depends on the nature of these points as determined by the stability of its stationary solutions (Khalique and Biswas, 2009).

LOG-LAW NONLINEARITY

The NLSE with log-law nonlinearity studied in this paper is given by (Guerrero et al., 2010).

$$iq_t + aq_{xx} + bq\log|q|^2 = 0. \tag{9}$$

In this paper, the focus is going to be on obtaining the localized stationary solution to (8) in the form [10]

$$q(x,t) = \phi(x)e^{-i\lambda t}, \tag{10}$$

Where; λ is a constant and the function ϕ depends on the variable x alone. Thus, from (9) and (10), $\phi(x)$ satisfies the time independent inhomogeneous nonlinear equation that is given by

$$\lambda\phi + a\phi'' + 2b\phi\ln\phi = 0. \tag{11}$$

Equation (11) has a single Lie point symmetry, namely $X = \partial/\partial x$. This symmetry will be used to integrate equation (11) once. It can be easily seen that the two invariants are

$$u = \phi \tag{12}$$

and

$$v = \phi' \tag{13}$$

Treating u as the independent variable and v as the dependent variable, (11). can be rewritten as

$$\frac{dv}{du} = \frac{-\lambda u - 2b\ln u}{av}. \tag{14}$$

Integrating (14) yields

$$v^2 = \frac{b-\lambda}{a}u^2 - \frac{2b}{a}u^2 \ln u + \frac{c_1}{a}, \tag{15}$$

Where; c_1 is an arbitrary constant of integration. Now, rewriting (15) in terms of the variable ϕ gives

$$\left(\frac{d\phi}{dx}\right)^2 = \frac{b-\lambda}{a}\phi^2 - \frac{2b}{a}\phi^2 \ln \phi + \frac{c_1}{a} \tag{16}$$

that leads to the quadrature;

$$x + c_2 = \int \frac{\sqrt{a}\, d\phi}{\sqrt{(b-\lambda)\phi^2 - 2b\phi^2 \ln \phi + c_1}}, \tag{17}$$

Where; c_2 is an arbitrary constant of integration.

For $c_1 = 0$, (17) leads to

$$x + c_2 = -\frac{\sqrt{a}}{b}\sqrt{-2b \ln \phi - b - \lambda}\ , \tag{18}$$

which is the stationary solution to (9). Thus, from (18), it is possible to observe that it is necessary to have

$$2b \ln \phi + b + \phi < 0 \tag{19}$$

for the solution to exist. Thus, finally, the stationary solution to (9) is given by (10) where

$$\phi(x) = \exp\left\{-\frac{a(b+\lambda) + b^2(x+c_2)^2}{2ab}\right\}. \tag{20}$$

Conclusion

In this paper, the stationary solution is obtained for NLSE with log-law nonlinearity. Here, the Lie symmetry approach is used to carry out the integration of the NLSE. The resulting solution is in quadratures

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