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Full Length Research Paper

Experimental study of the elastic multiple scattering by a two-dimensional (2D) periodic array in the case of empty cavities

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In this work, we study the propagation of the plane waves in a solid medium (Aluminium) which contains a periodic two-dimensional (2D) array of cylindrical inclusions using a multiple scattering theory. The aim of this study is to validate the experience of certain theoretical predictions such as the frequential positions of the stop-bands and their widths like their evolution according to the geometrical characteristics of the 2D array. The cavities are excited in normal incidence. Measurements are taken in the case of the empty cavities. The acquisitions of all the transmission and reflection temporal signals are obtained by the use of transducers of contact. All the recorded signals are subjected to a signal processing (FFT, normalization).

Key words: Multiple scattering theory/periodic distributions of inclusions/stop-bands/two-dimensional (2D) array.

INTRODUCTION

The propagation of acoustic waves in the heterogeneous medium equipped with a periodic structure has been the object of an interest growing for a few decades. A great number of periodic structures were studied and various theoretical approaches were employed. The existence of original physical properties such as the presence of stop-bands, that is, frequential bands for which the waves cannot be propagated at long distance in the medium corresponding to a strong attenuation, and pass-bands of less attenuation were all highlighted. The analogy existing with the propagation of electromagnetic waves in the periodic dielectric structures stimulates today's research on the "phononic crystals" composed of elastic inclusions periodically distributed in an elastic matrix. The propagation of elastic waves in such medium is in addition the subject of studies in fields as varied as: Geophysics, medical acoustics and mechanical engineering (let us quote like applications the Non-Destructive-Testing; NDT).

The periodic medium considered in this study is a periodic two-dimensional (2D) array of cylindrical inclusions in aluminium solid.The aim of this work is to validate by the experimental study certain theoretical predictions of Robert et al. (2004) such as the frequential positions of the stop-bands, their widths and their evolution according to the geometrical characteristics of the array. By using the finite element method, Langlet (1993) could characterize the propagation of waves in such an array by analyzing the dispersion and the attenuation of the waves of Lamb being propagated in a periodically bored elastic plate. The theory developed by Robert et al. (2004) is however valid only for infinitely long inclusions with respect to the wavelengths considered. The experimental study of the propagation of Lamb waves in a bored plate was thus not followed. This study will be based on the analysis of the reflection and transmission coefficients of a periodic 2D array finite thickness.

In this study, the fundamental points of the theory are presented. This theory is an extension of the theoretical model developed by Audoly, Dumery, Mulolland and Heckl (Sigalas and Garcia, 2000; Tanaka et al., 2000) for

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the rigid or elastic obstacles arrays immersed in a fluid. This model consists to decompose the periodic 2D array into a series of periodic 1D linear array. If the scattering by each of these arrays is known, the propagation of the waves from one array to another is then deduced from the theorem of Bloch or an iterative method. The approach of these various authors is based on the work of (Mcphedran et al., 2000) on the plane waves scattering by a periodic linear 1D array cylindrical objects. Mcphedran et al. (2000) by using an exact calculation of multiple scattering, obtains an expression of the scattered field in the form of a superposition of plane waves diffracted under various angles. In the context of the scattering in an elastic medium, the formalism of Mcphedran et al. (2000) was generalized, while being based on the matrix theory of transition T, theory intensively exploited by (Mulholland and Heckl, 1994; Heckl and Mulholland, 1995) for similar multi-scatters mediums.

In the experimental study which is proposed, the various arrays are subjected to an incidental plane wave in a perpendicular plane to the axes of cylindrical inclusions. The cavities are excited in normal incidence. There will be thus never conversion between the longitudinal and transversal waves at the time of the reflection or the transmission by the 2D arrays. The study moreover will be led to the low frequencies for which the 2D arrays diffract only one plane wave in the same direction as the incidental plane wave. Consequently, in normal incidence and low frequency, the 2D arrays will have the acoustic behaviour of a stratified fluid medium (and periodic) of which it will be a question of measuring the reflected and transmitted acoustics fields.

To highlight the influence of the geometrical characteristics of the arrays on the reflected or transmitted acoustic field, two arrays are studied. Their implementation consists in practising holes in the thickness of an aluminium block, this thickness being large with respect to the wavelengths considered. Measurements are then taken in the air. The acquisition of the reflected or transmitted signal is obtained then by the use of transducers of contact to the plane interface between Aluminium and the air. The recorded signals are then subjected to a signal processing (Fast Fourier Transform (FFT), filtering, normalization).

THEORY OF THE MULTIPLE SCATTERING FOR PERIODIC 2D ARRAY: CASE OF THE NORMAL INCIDENCE

Reflection and transmission of a periodic and finite array

Figure 1 presents the geometry of the studied array. It is composed of an arbitrary number *S* of periodic 1D and finite arrays, regularly spaced of a distance *D* along axis *X*. The global array is thus finite and thickness $e = (S - 1)$ *D* according to this axis. We consider in addition that all the 1D linear arrays are identical, d is the period according to the axis *y*, $2a$ is the diameter of the cylindrical cavities.

The 2D array is subjected by acoustic longitudinal wave in normal incidence. The goal is then to express the reflection and transmission coefficients of the array, noted *R* and *T,* in the field of the low frequencies: $f \le f_1^L$. Under these conditions we recall that there is no conversion mode during the transmission and of the reflection by the periodic 1D linear arrays and that the latter transmit plane modes purely propagates in the same direction as the incidental wave. The periodic 2D array can then be regarded as a "multi-layer" in which only a longitudinal plane wave is propagated perpendicularly with various interfaces.

From now on, the reflection and transmission coefficients of the periodic 1D linear array which are defined by Equations 4 and 6 are supposed to be known. In the continuation, to reduce the notations, we will note them *R* and *T*. The goal now is to calculate the two coefficients *R* and *T.*

Let us clarify the method for calculation of *R*. The idea consists to decompose the 2D array as shown (Figure 2) in the following way: the linear array 1 is isolated different from those that form a "sub-array", Numbered 1, of S-1 linear arrays; the array being subjected by a longitudinal plane wave. The field *L* considered is then the superposition of a wave *L* reflected by linear array 1, and a series of waves *L* transmitted by linear array 1 and reflected several times between linear array 1 and sub-array 1. The multiple reflections between the two linear arrays are described by series of Debye. The coefficient of reflection *R*-can then be written where $k = k_L$ is the number of waves in the elastic medium, and

R_1: the reflection coefficient of sub-array 1;
r_1: the reflection coefficient of the linear array 1;
t_1: the transmission coefficient of the linear array 1;
T_1: the transmission coefficient of sub-array1.

$$\begin{aligned} R &= r_1 + \left(t_1 e^{ikD}\right)\left(R_1 e^{ikD}\right)t_1 + \left(t_1 e^{ikD}\right)\left(R_1 e^{ikD}\right)\left(r_1 e^{ikD}\right)\left(R_1 e^{ikD}\right)t_1 + \\ &\left(t_1 e^{ikD}\right)\left(R_1 e^{ikD}\right)\left(r_1 e^{ikD}\right)\left(R_1 e^{ikD}\right)\left(r_1 e^{ikD}\right)\left(R_1 e^{ikD}\right)t_1 + \cdots \\ &= r_1 + t_1 e^{i2kD} R_1 t_1 \left\{1 + r_1 R_1 e^{i2kD} + \left(r_1 R_1\right)^2 e^{i4kD} + \left(r_1 R_1\right)^4 e^{i6kD} + \cdots \right\} \end{aligned} \tag{1}$$

Geometrical series (Equation 1) is convergent. It can thus be written:

$$1 + \left(r_1 R_1\right) e^{2ikd} + \left(r_1 R_1\right)^2 e^{4ikd} + \left(r_1 R_1\right)^4 e^{6ikd} + \cdots = \frac{1}{1 - \left(r_1 R_1\right) e^{2ikd}} \tag{2}$$

Finally, the reflection coefficient of the total 2D array is written:

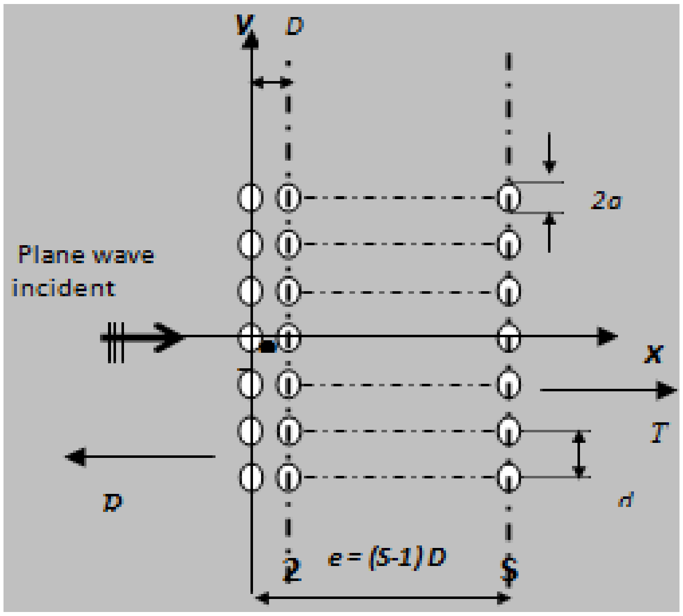

Figure 1. Geometry of the cylindrical cavities array.

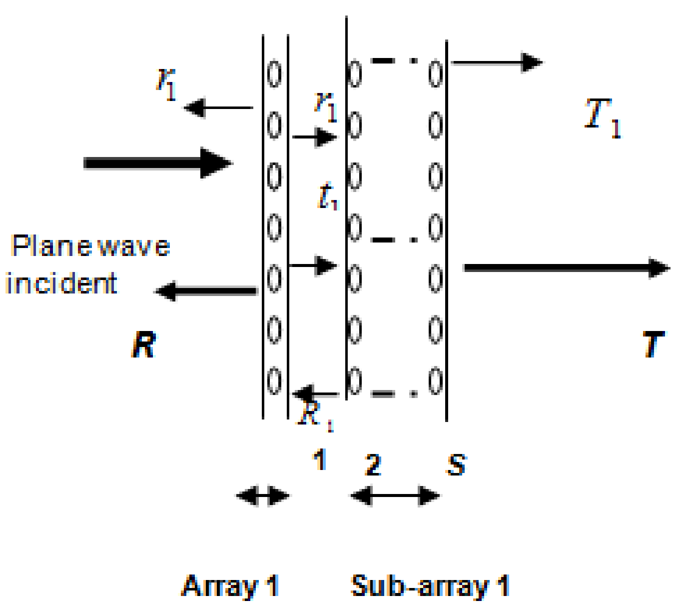

Figure 2. First decomposition to determine the global reflection coefficient.

$$R = r_1 + t_1 e^{2ikd} R_1 t_1 \frac{1}{1-(r_1 R_1)e^{2ikd}} \tag{3}$$

That is to say:

$$R = r_1 + \frac{t_1 R_1 t_1 e^{2ikD}}{1-r_1 R_1 e^{2ikD}} \tag{4}$$

We now search the global transmission coefficient T: in the same way that previously we can calculate T as follows:

$$\begin{aligned} T &= T_1 \left(t_1 e^{ikD}\right) + \left(t_1 e^{ikD}\right)\left(r_1 e^{ikD}\right)\left(R_1 e^{ikD}\right) T_1 + \\ &\left(t_1 e^{ikD}\right)\left(r_1 e^{ikD}\right)\left(R_1 e^{ikD}\right)\left(r_1 e^{ikD}\right)\left(R_1 e^{ikD}\right) T_1 + \cdots \\ &= T_1 t_1 e^{ikD}\left\{1+\left(r_1 R_1\right)e^{2ikD} + \left(r_1 R_1\right)^2 e^{4ikD} + \cdots\right\} \end{aligned} \tag{5}$$

The geometrical series (Equation 5) is convergent. It can thus be written:

$$\left\{1+\left(r_1 R_1\right)e^{2ikD} + \left(r_1 R_1\right)^2 e^{4ikD} + \cdots\right\} = \frac{1}{1-r_1 R_1 e^{2ikD}}$$

Finally, the transmission coefficient can be written in the form:

$$T = \frac{T_1 t_1 e^{ikD}}{1-r_1 R_1 e^{2ikD}} \tag{6}$$

In Equations 4 and 6, the reflection/transmission coefficients R_1 and T_1 are unknown (Figure 3). It is thus necessary to repeat this decomposition for sub-array 1. In its turn, this last can be decomposed as follows: the linear array 2 is isolated from S-2 other linear arrays which then form a sub-array 2. We then note R_2 and T_2 the reflection and the transmission coefficients of this last. This second decomposition then provides:

$$R_1 = r_2 + \frac{t_2 R_2 t_2 e^{2ikD}}{1-r_2 R_2 e^{2ikD}} \tag{7}$$

$$T_1 = \frac{T_2 t_2 e^{ikD}}{1-r_2 R_2 e^{2ikD}}. \tag{8}$$

In the same way, the reflection /transmission coefficients R_2 and T_2 are unknown. It is thus necessary to decompose in its turn sub-array 2 like the precedents. This method is thus a recurrence. With the decomposition S, Equations 7 and 8 become

$$R_{S-1} = r_S + \frac{t_S R_S t_S e^{2ikD}}{1-r_S R_S e^{2ikD}} \tag{9}$$

$$T_{S-1} = \frac{T_S t_S e^{ikD}}{1-r_S R_S e^{2ikD}} \tag{10}$$

In this last decomposition, the last sub-array S does not

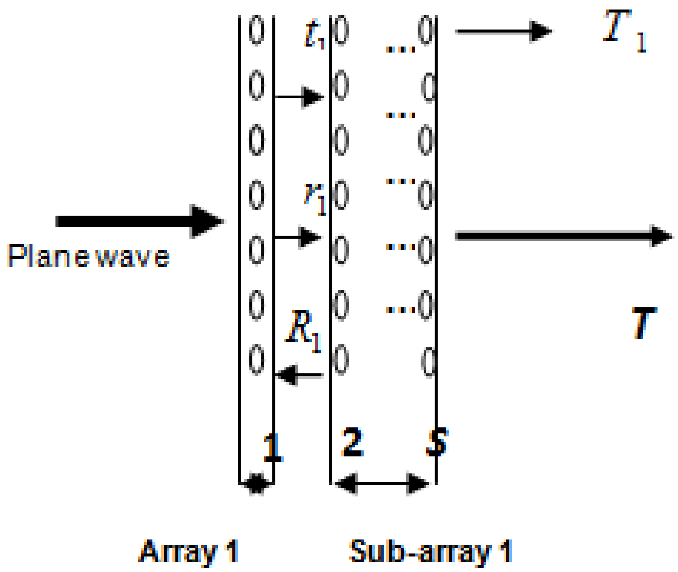

Figure 3. First decomposition to determine the global transmission coefficient.

contain any more that only one linear array whose coefficients r_S and t_S are supposed to be known:

$$R_S = r_S = r, \quad T_S = t_S = t \tag{11}$$

Consequently, the two relations in Equation 11 are relations which "close" the recurrence. To calculate *R* and *T*, we start these two last relations, and then we "descend" until the recurrence of Equations 4 and 6. This method of calculation is, numerically, very rapid when we know once and for all the reflection/transmission coefficients of all the linear arrays. In addition, the method can be generalized easily with the case of an oblique incidence (by taking in account mode conversions between the longitudinal and transversal waves) and if all the linear arrays are different from/to each other.

Characteristic equation

If the 2D array comprises a sufficiently large number of linear arrays, it can be regarded as being infinite so that the propagation of waves can be described by the theorem of Bloch. This theorem is applicable to the fields of potentials, displacements or constraints, binds the fields defined in two separate points of the array of a distance *D* applied to the fields of potentials and displacements, the theorem of Bloch is written:

$$u_i\left(x+D, y\right) = e^{i\gamma D} u_i\left(x, y\right) \tag{12}$$

$$\phi\left(x+D, y\right) = e^{i\gamma D} \phi\left(x, y\right) \tag{13}$$

$$\sigma_{ij}\left(x+D, y\right) = e^{i\gamma D} \sigma_{ij}\left(x, y\right) \tag{14}$$

Where indices *i* and *j* indicate the components according to x or y.
The complex size γ is the number of wave of Bloch defined by:

$$\gamma = \gamma^{'} + i\gamma^{''} \tag{15}$$

The real part $\gamma^{'}$ is the component according to x of the vector of a wave being propagated in the periodic array, and the imaginary part $\gamma^{''}$ is its attenuation. That is to say a linear array *S* taken randomly.
Between the arrays *S* - 1 and *S*, the scalar potential can be written;

$$\begin{aligned}\phi\left(x, y\right) &= Ae^{ikx} + Be^{-ikx} \\ &= \tilde{A} + \tilde{B}\end{aligned} \tag{16}$$

Between the arrays *S* and *S* + 1, at a distance *D*. we have

$$\begin{aligned}\phi\left(x+D, y\right) &= e^{ikD}Ce^{ikx} + e^{-ikD}De^{-ikx} \\ &= e^{ikD}\tilde{C} + e^{-ikD}\tilde{D}\end{aligned} \tag{17}$$

Projection according to *x* and *y* of the relation of Bloch on displacements lead to;

$$e^{i\gamma D}\tilde{A} - e^{i\gamma D}\tilde{B} - e^{i\xi}\tilde{C} + e^{-i\xi}\tilde{D} = 0 \tag{18}$$

With $\xi^{D} = kD$, the relation of Bloch on the constraints σ_{xx}, σ_{xy} provides directly;

$$e^{i\gamma D}\tilde{A} + e^{i\gamma D}\tilde{B} - e^{i\xi}\tilde{C} - e^{-i\xi}\tilde{D} = 0 \tag{19}$$

Remark

There are two equations to determine four unknown factors, namely: $\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}$. The two missing equations are obtained in the following way: if we refer to Figure 4, we can observe that the field *B* results from the reflection of field *A* and the transmission of the field *D*:

$$\tilde{B} = r\tilde{A} + t\tilde{D} \tag{20}$$

In a similar way it is found that;

$$\tilde{C} = r\tilde{D} + t\tilde{A} \tag{21}$$

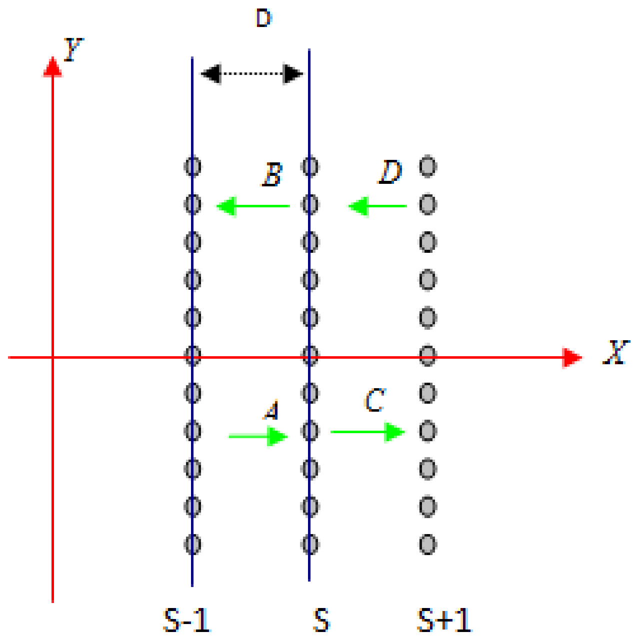

Figure 4. 2d array composed of an infinite number of identical arrays and periodically spaced; the amplitudes of the waves are presented.

There are now four equations for four unknown factors, the problem is thus well posed. There is finally the system of equation according to;

$$e^{i\gamma D}\left[\tilde{A}+\tilde{B}\right]=\tilde{C}e^{ikD}+\tilde{D}e^{-ikD} \quad \text{(I)}$$

$$e^{i\gamma D}\left[\tilde{A}-\tilde{B}\right]=\tilde{C}e^{ikD}-\tilde{D}e^{-ikD} \quad \text{(II)} \qquad (22)$$

$$\tilde{B}=r\tilde{A}+t\tilde{D}$$

$$\tilde{C}=r\tilde{D}+t\tilde{A}$$

That we can still write in the matrix form;

$$\begin{vmatrix} e^{i\gamma D} & e^{i\gamma D} & -e^{ikD} & -e^{-ikD} \\ e^{i\gamma D} & -e^{i\gamma D} & -e^{ikD} & e^{-ikD} \\ r & -1 & 0 & t \\ t & 0 & -1 & r \end{vmatrix} \begin{vmatrix} \tilde{A} \\ \tilde{B} \\ \tilde{C} \\ \tilde{D} \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \end{vmatrix} \qquad (23)$$

The determinant associated with the system is thus;

$$D=\begin{vmatrix} e^{i\gamma D} & e^{i\gamma D} & -e^{ikD} & -e^{-ikD} \\ e^{i\gamma D} & -e^{i\gamma D} & -e^{ikD} & e^{-ikD} \\ r & -1 & 0 & t \\ t & 0 & -1 & r \end{vmatrix}=0 \qquad (24)$$

The system of Equations 22 has a solution different to zero provided that the determinant associated with the system is null, we obtain the characteristic equation which gives γ.

We can develop calculations further. If we eliminate $\tilde{B},\tilde{C}$ in (I) and (II) of Equation 22, it becomes:

$$\left[te^{ikD}-(1+r)e^{i\gamma D}\right]\tilde{A}+\left[re^{ikD}+e^{-ikD}-te^{i\gamma D}\right]\tilde{D}=0 \qquad (25)$$

$$\left[te^{ikD}-(1-r)e^{i\gamma D}\right]\tilde{A}+\left[re^{ikD}-e^{-ikD}+te^{i\gamma D}\right]\tilde{D}=0 \qquad (26)$$

The determinant associated with this new system is thus;

$$\begin{vmatrix} te^{ikD}-(1+r)e^{i\gamma D} & re^{ikD}+e^{-ikD}-te^{i\gamma D} \\ te^{ikD}-(1-r)e^{i\gamma D} & re^{ikD}-e^{-ikD}+te^{i\gamma D} \end{vmatrix}=0 \qquad (27)$$

And by simplification, we have:

$$\begin{vmatrix} -re^{i\gamma D} & e^{-ikD}-te^{i\gamma D} \\ te^{ikD}-e^{i\gamma D} & re^{ikD} \end{vmatrix}=0 \qquad (28)$$

The characteristic equation D = 0 can be reduced to:

$$ch(\gamma D)=\frac{1}{2t}\left\{\left((t)^2-(r)^2+1\right)\cos\xi^D+i\left((t)^2-(r)^2-1\right)\sin\xi^D\right\} \qquad (29)$$

Because of the absence of mode conversions, this equation is identical to that established by Hecklin as in the case of submerged tubes array in water.

EXPERIMENTAL DEVICE

Description of the material

Figure 5 shows the experimental device. The measurements are performed in the case of empty cavities and the two types of arrays 2D are excited at normal incidence. The array consists of cylindrical cavities dug in an aluminum block to form an elastic matrix. Firstly, the excitation and reception of temporal signals reflected and transmitted is obtained by the use of contact transducer with a center frequency of 0.25 MHz. The bandwidth of this transducer is the order of twice the center frequency. The transmitter transducer is excited by a pulsed electric signal delivered by a pulse generator. The latter includes amplification and filtering for processing the signal received by the receiver transducer. This pulse generator comprises a swingable door on two positions. In the first position, it is used as a single transmitter and receiver transducer at a time. In the second position, it uses two transducers of the same center frequency as a transmitter and the other as a receiver. The received signal is displayed on an oscilloscope type Lecroy 9430. Average of 200 scans is then performed in order to filter the signal. The averaged signal is sampled and thereafter stored in a computer. All signals are recorded and then subjected to signal processing FFT.

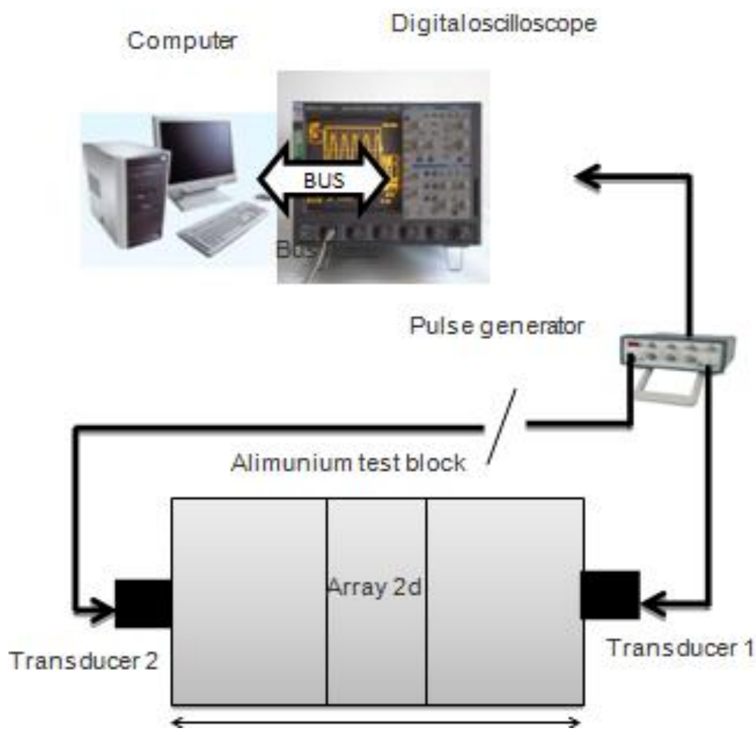

Figure 5. Experimental device.

Table 1. Extraction of all the normalization transmission and reflection temporal signals and with diffusers.

Stop band	Numerical (MHz)	Experimental (MHz)
S.B	0.25 - 0.52	0.26 - 0.47

THE COMPARISON OF RESULTS BETWEEN NUMERICAL AND EXPERIMENTAL STUDY: CASE OF THE EMPTY CAVITIES AND NORMAL INCIDENCE

Results obtained on the first 2D array with *S* = 6 linear arrays and finite

i) The ray of the cylindrical cavities: α = 2 mm,
ii) The period of the cavities according to the axis$y$: d = 5 mm,
iii) The period of the 1D arrays according to the axis$x$: D = 6 mm,
iv) The thickness of the 2D array: $e = (S - 1) D$ = 30 mm,
v) The cut-off frequency is given by: f_c = 0.620 MHz,
vi) Curve in red color represent reflection coefficient in energy,
vii) Curve in blue color represent transmission coefficient in energy.

In the experimental study, we extracted all the normalization transmission and reflection temporal signals and with diffusers (Table 1). Then, they underwent several processing like the elimination of the undesirable parts (echoes), retiming, the FFT in order to obtain the normalized function of form, so that we can compare it with the result obtained numerically, by using several transducers.

According to the curves (Figures 6 and 7), we see well that the stop-band obtained theoretically and the stop-band obtained in experiments are superimposed, which means that this theoretical result was checked in experiments.

Results obtained on the second 2D array with *S* = 5 1D arrays and limited

i) The ray of the cylindrical cavities: α = 2 mm,
ii) The period of the cavities according to the axis: d = 6 mm,
iii) The period of the 1D arrays according to the axis: D = 20 mm,
iv) The thickness of the 2D array: $e = (S - 1)$ D = 80 mm,
v) The cut-off frequency is given by: f_c = 0.520 MHz,
vi. Curve in blue color represent reflection coefficient in energy,
vii. Curve in red color represent transmission coefficient in energy.

According to the curves (Figures 8 and 9), we see well

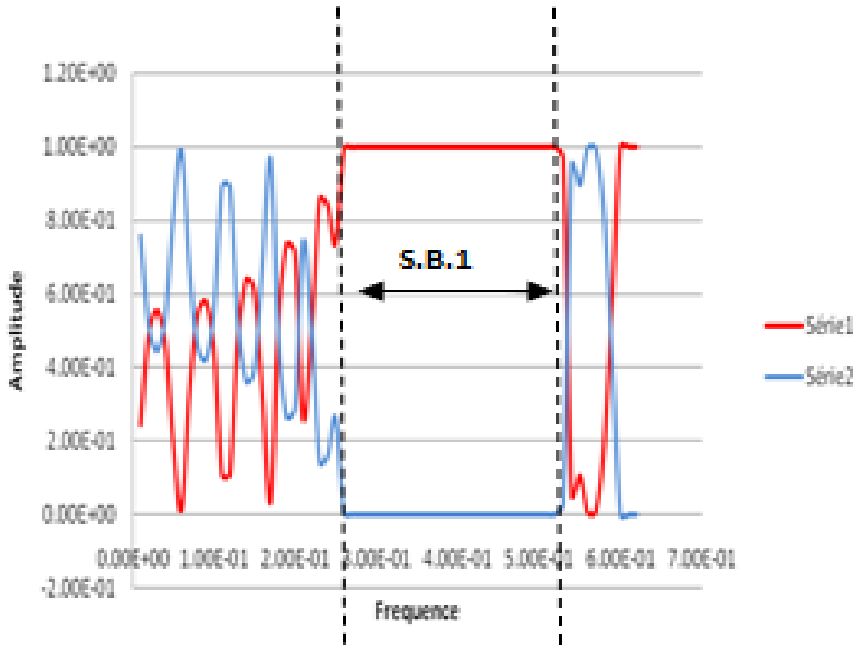

Figure 6. Frequential evolution of the reflection and transmission coefficients in energy $|R^{LL}|^2$ and $|T^{LL}|^2$ empty cavities of the array in normal incidence with *d/a=2.5, D/a=3*,the cut-off frequency f_c = 0.620 MHz.

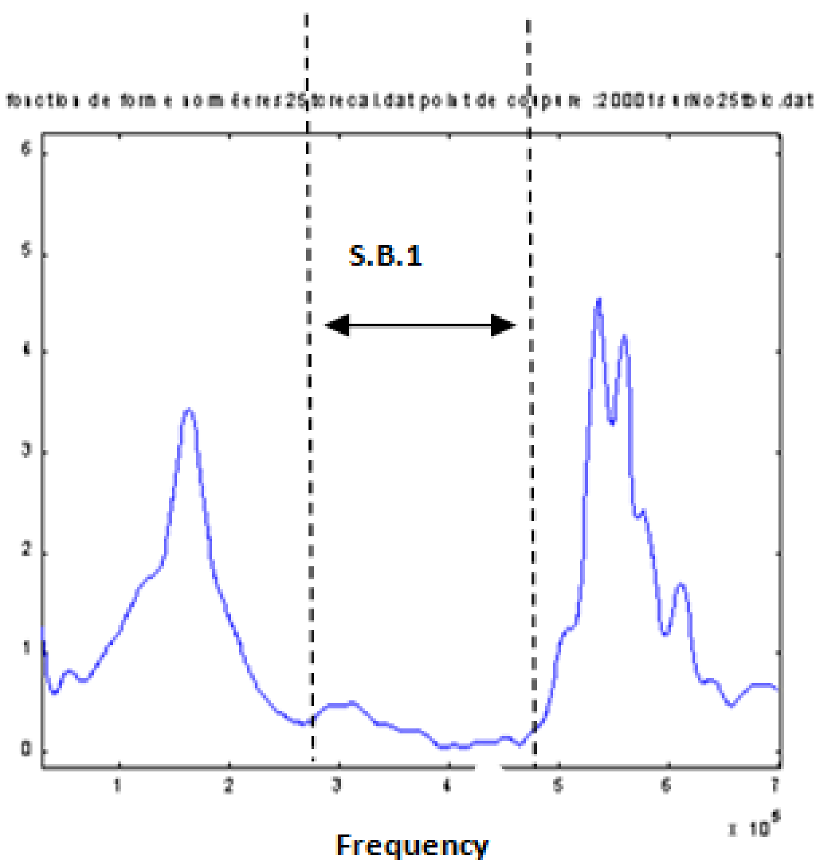

Figure 7. Function form normalized.

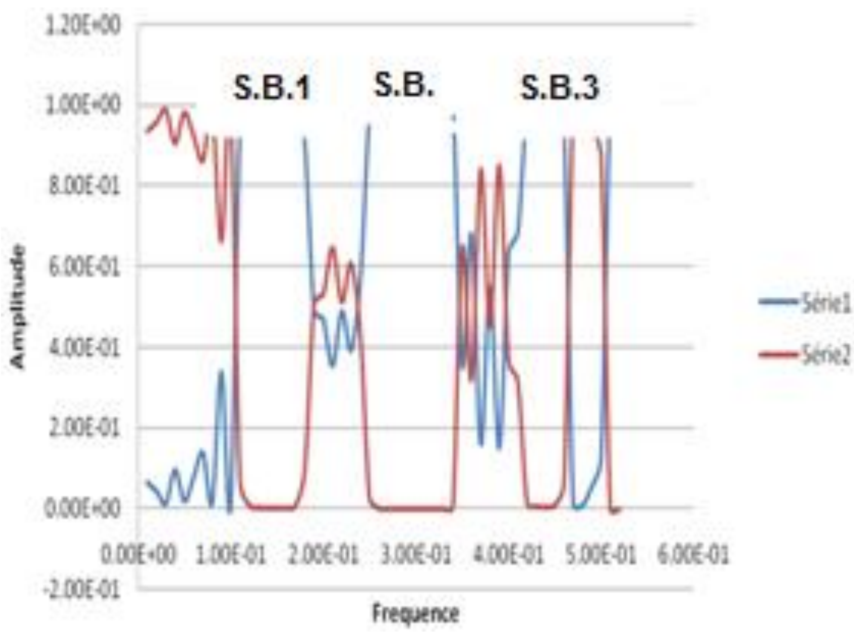

Figure 8. Frequential evolution of the reflection and transmission coefficients in energy $|R^{LL}|^2$ and $|T^{LL}|^2$ the empty cavities of an array in normal incidence with d/a=3, D/a = 10, the cut-off frequency f_c = 0.520 MHz.

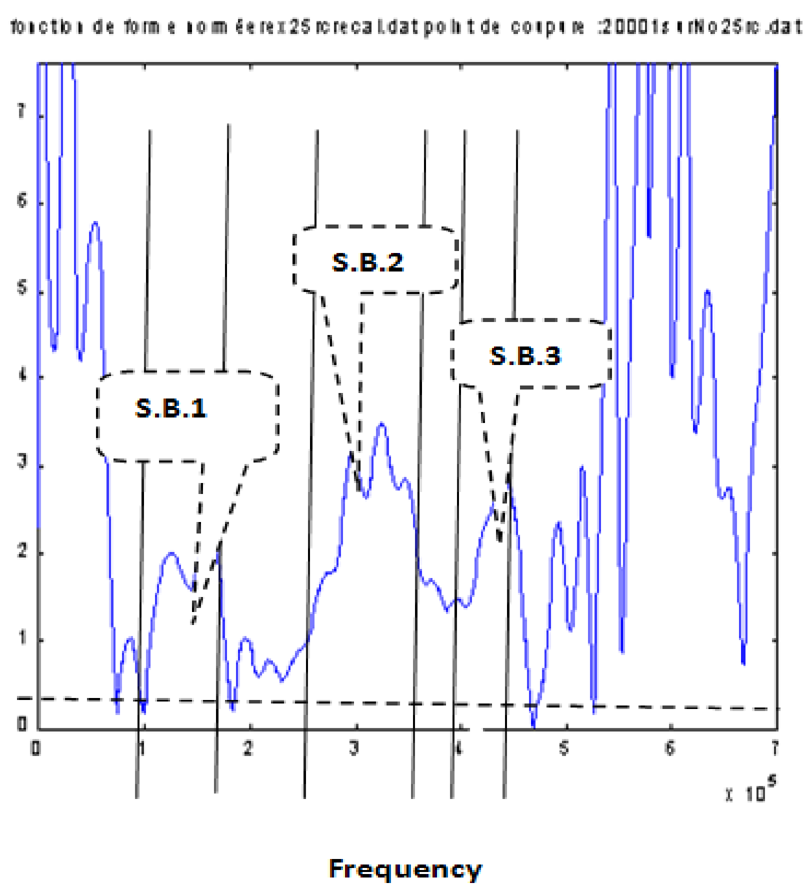

Figure 9. The function form normalized.

Table 2. The three stop-bands obtained theoretically and the three stop-bands obtained in experiments.

Stop-band	Numerical (MHz)	Experimental (MHz)
S.B.1	0.12 - 0.18	0.11-0.17
S.B.2	0.25 - 0.35	0.25-0.38
S.B.3	0.42 - 0.45	0.41-0.46

that the three stop-bands obtained theoretically and the three stop-bands obtained in experiments are superimposed (Table 2).

CONCLUSIONS AND PERSPECTIVES

A method of calculation of the scattering by a periodic 2D array in an elastic medium was presented. This method rest on the decomposition of the 2D array in a limited number of periodic 1D arrays. The scattering by each 1D array is determined by an exact calculation of multiple scattering which was adapted here to the scattering in an elastic medium. The scattering plan after plan is calculated using the theorem of Bloch.

The numerical and experimental study verified the existence of two interesting properties which are: Stop-bands and pass-bands. The appearance of one or more stop-bands is due to the choice of the geometry of the studied 2D array in the case of empty cavities and normal incidence.

The theoretical study developed here can be the later development object and can be applied to an array with variable, a periodic or random porosity, according to the direction of propagation of plane waves.

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