International Journal of the Physical Sciences Vol. 6(9), pp. 2194-2204, 4 May, 2011
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS11.378
ISSN 1992 - 1950 ©2011 Academic Journals

Full Length Research Paper

Stiffness reduction index for detection of damage location: Analytical study

Moatasem Mohammed Fayyadh* and Hashim Abdul Razak

Department of Civil Engineering, University of Malaya, 50603 Kuala Lumpur, Malaysia.

Accepted 05 April, 2011

A new damage location index is developed and presented in this paper. The new index is based on the combined mode shape vector and its derivatives to overcome the problem of inaccuracy and insensitivity of existing indices to locate damage in a beam-like structure model. The proposed new index called the stiffness reduction index (SRI) is computed based on the change in eigenvectors and its curvature at each element and for individual mode shapes. In order to assess the validity of the new index, comparisons with an existing algorithm, that is, the co-ordinate model assurance criteria (COMAC) were undertaken using modal data from a finite element model. The study has adopted the first four bending modes to examine the sensitivity of both existing as well as developed damage location indices. The study has examined the new damage index for very small damage intensity, which has been presented by reducing the modulus of elasticity at certain elements along the beam length by only 1%. The results obtained from this study revealed higher and better sensitivity of the developed index to detect a low damage intensity and at different locations along the length of the beam. The study has validated the new index to detect the damage location for the case of multi damage location presented at the same time along the beam length. In addition, the index is more adept and managed to resolve the inadequacies of the COMAC index.

Key words: Damage location indices, stiffness reduction index, co-ordinate modal assurance criteria, stiffness reduction ratio.

INTRODUCTION

During the last few years, the development of damage detection using modal parameter techniques has attracted significant attention with respect to civil engineering applications. The inspection of a structure for damage is important in deciding the maintenance of that structure. Much research in the field of damage detection has been published, and a variety of methods have been developed. These methods are based mainly on the relationship between dynamic characteristics of the structure and damage parameters like crack depth and its crack location. Cracks are classified as a main cause of structural failure. Sudden failure during a high load operation could be disastrous; thus, early crack detection is important. Damage can be detected, quantified and localized using online damage assessment techniques such as measuring the vibration parameters, which indicate the global and local condition of the structure. In general, cracks will cause a reduction in the stiffness of a structure. Such changes to the structure's physical properties cause changes to its dynamic parameters, including mode shape and its derivatives. Therefore, it is possible to detect the damage location by measuring changes in the mode shape derivatives. The concept of mode shape curvature was introduced by Pandey et al. (1991), who used a finite element model of a simply supported beam with a reduction in E of 50% over one-third of the span. The authors found that modal curvature is a far more sensitive damage indicator than the MAC or coordinate-modal assurance criteria (COMAC) values. This method was further developed by Ratcliffe (1997), who proposed studying the curvature of the damaged beam without need of a priori knowledge of the undamaged structure. The method uses the Laplace operator of discrete mode shape and is proportional to the quotient.

The presence of severe damage is detectable in the

*Corresponding author. E-mail: moatasem.m.f@gmail.com.

form of a jump in the Laplacian. Stubbs et al. (1995) developed a damage index method to localize the damage. This method utilizes characteristics of mode shape curvature for a beam as the main variable in the derived damage location algorithm based on the relative differences in modal strain energy before and after damage. This method was adopted by Peterson et al. (2001) and Wahab and Roeck (1999), who used the change in mode shape curvature to detect the damage location, showing that the change in curvature has more sensitivity than the mode shapes themselves. Johan (2003) developed a new technique to detect, localize and quantify damage in reinforce concrete (RC) structures using vibration monitoring known as “direct stiffness calculation”. It was developed to assess damage in RC structures and estimate bending and torsional stiffness along the structure from experimental natural frequencies and mode shapes and their derivatives. Beams were gradually damaged and the changes of dynamic parameters were monitored from the initial state to the failure state. Direct stiffness calculation proves that modal parameters turned out as good indicators. Dutta and Talukdar (2004) carried out the Eigen analysis using the Lanczos algorithm in an adaptive h-version finite element environment to control the discretization error for an accurate evaluation of modal parameters. They found that the location of damage was better performed by considering the curvature of mode shapes. Law and Lu (2005) proposed a time domain method in which the parameters of a crack in a structural member are identified from strain or displacement measurement. The dynamic response was calculated based on modal superposition. In the inverse analysis, the optimization technique coupled with regularization of the solution is used to identify the cracks. The use of this formulation for identification was further extended to the case of multiple cracks. Computation simulations with sinusoidal and impulsive excitations on beams with single or multiple cracks show that the method is effective for identifying the parameters of the cracks accurately. Ismail and Abdul (2006) used mode shape derivatives to detect the location of damage due to a single crack as well as honeycombs in RC beams known as local stiffness indicator (LSI). LSI was proposed as a damage location indicator.

The LSI was obtained by rearranging the equation of free vibration for uniform beams, and applying the fourth order centered finite divided difference formula to regressed mode shape data. According to LSI damage index, the exact location is around the center of the detected region. Curve fitting with Chebyshev series rationales onto the mode shape also highlighted the points’ residuals around the damaged region. Choi et al. (2008) developed two existing algorithms for global nondestructive evaluation and studied localized damage in timber beams using a finite element model. These damage location algorithms were found to be ineffective in locating multiple damage scenarios and were unable to estimate the severity of damage. The modifications to the damage index algorithms, as well as the development of a hybrid algorithm, were proposed by Choi et al. (2008) to overcome the problems. Experimental modal analysis data were used to extract mode shapes for calculating the damage index in the proposed method, which utilizes change in modal strain energy between the undamaged and damaged timber beam models. The modified damage index normalizes the mode shape curvature, and the hybrid algorithm combines the modified damage index and change in flexibility algorithm, which reflect the changes of natural frequency and mode shape. Detection of damage by using limited number of natural frequencies and/or mode shape was done by Perera et al. (2008), who proposed a new damage detection method called local modal stiffness, which can be determined from the frequency response function and depends on both frequency and mode shape. The method was examined by using experimental progress on RC beams with cracking. Kim et al. (2010) offered a new proposal for a hybrid health monitoring system, using sequential vibration impedance approaches, to detect damage in pre-stress concrete bridge girders. Kopsaftopoulos and Fassois (2010) applied several vibration-based statistical time series as structural health monitoring methods on lightweight aluminum truss structures. Yan et al. (2010) developed a wavelet-based method, which not only localized multiple damage sites but also provided information on when the damage occurred. Rodríguez et al. (2010) proposed a method for detecting damage in structures without baseline state information. The study found that it is possible to identify the location and severity of damage based on singular value decomposition.

From these studies, it can be seen that the damage location indices are less sensitive to being affected by the damage location; this creates disadvantages in detecting the damage location. Thus, the objective of this study is to develop a damage location index with higher sensitivity in detecting the damage location and to overcome the weaknesses of the previous indices. Here, a new damage location index, related to the changes in mode shapes curvature, and mode shape vector due to damage, is developed.

Theoretical development

In this study, a newly proposed technique that utilizes both the mode shape vector and mode shape second derivative as damage location indicators will be discussed. The indicator, called the stiffness reduction index (SRI), is the stiffness percentage ratio of the damaged case to the control beam. In addition, it will summarize another indicator found in the literature which will then be used to validate the SRI; this indicator uses the changes in mode shape through COMAC.

Developed stiffness reduction index (SRI)

This developed index is based on the free vibration equation. The control equation of free vibration is shown as follows:

$$M.\frac{d^2y}{dx^2} + K\,y = 0 \quad (1)$$

Where,

y (f(x)) is the mode shape vector;
x is the distance along the beam length from one support; and
$\frac{d^2y}{dx^2}$ is the curvature of the mode shape at each point *x*.

Let (- K/M) represent the stiffness index (SI); then:

$$SI = \left(\frac{d^2y}{dx^2}\right)/\,y = \text{Curvature} / y = C/y \quad (2)$$

Where, *C* is the curvature, or the second derivative of the mode shape vector;

$$SI_c = \frac{C_c}{y_c}$$

$$SI_d = \frac{C_d}{y_d}$$

and where "*c*" is the indicator for the control case, and "*d*" is the indicator for the damage case.

Thus, the stiffness reduction index (SRI) can be calculated as:

$$SRI = 1 - \frac{SI_d}{SI_c} = 1 - \frac{Cd*yc}{Cc*yd} \quad (3)$$

In the case of no reduction in stiffness, the SRI is equal to zero. At the point at which there is a reduction in stiffness, SI_d will be less than SI_c, and (SI_d/SI_c) will have a value less than 1. The difference between 1 and (SI_d/SI_c) will be the SRI.

Co-ordinate modal assurance criterion (COMAC)

COMAC is one of the indices primarily used to correlate analytical and experimental data. It was then used for damage location by comparing the damage and control cases.

$$COMAC\ i,c,d = \frac{|\sum_{j=1}^{m} \varnothing ijc * \varnothing ijd]^{\wedge 2}}{\sum_{j=1}^{m}(\varnothing ijc)^2 * \sum_{j=1}^{m}(\varnothing ijd)^{\wedge 2}} \quad (4)$$

Where:

Ø is the mode shape vector;
j is the mode number and *m* is total number of modes; and
i is the node number in which *c* indicate the control set data and *d* indicate the damage set data.

This study presents COMAC in terms of reduction in COMAC by meaning of (1-COMAC) in order to make it more easily understandable. The value of COMAC should be equal to 1 when there is no damage, and less than 1 for cases with damage. 1-COMAC will have zero value when there is no damage and a value between zero and 1 when there is damage.

Finite element modeling

In order to demonstrate the significance and capability of this new technique, one finite-element beamlike structure model was built to represent control and damaged cases. The span length of the beam was 3250 mm with a cross-sectional area of 150 by 250 mm. The sensitivity level of the new algorithm attempted to detect the smallest damage severity and different damage locations. It also attempted to detect damage location in case of multi damage cases. Since the dynamic parameters are related to the stiffness of the structural element (Equation 5), the damage is presented by reducing the modules of elasticity E values.

$$f \propto \sqrt{EI} \quad (5)$$

Where *f* is the natural frequency, *EI* is the flexural rigidity and, *E* is the modulus of elasticity, *I* is the second moment of inertia.

The stiffness reduction ratio (SRR), adopted as denotation for the damage level, can be calculated as:

$$SRR = (1 - E_d/E_c).100\% \quad (6)$$

Where E_d is the modulus of elasticity for the damage cases, and E_c is the modulus of elasticity for the control case.

Two levels of stiffness were adopted as the control where *E* is 200 KN/mm^2, and the stiffness reduction ratio (SRR) was 0%, and the damage case where SRR was 1% and *E* is 198000 KN/mm^2. The damage location algorithms were examined at the same damage level and two different single damage locations; the first single damage was located at the mid-span and the second single damage was located at the quarter-span. In addition, the damage location algorithms were examined when there are multi damage locations; the first multi damage location case is when there is damage located at the quarter span and at the mid span, the second multi damage locations case is when there is damage located near the support , at the quarter-span, and near the mid-span. A total of five cases for SRR of 1% and different single and multi damage locations were adopted. Table 1 shows the damage cases adopted in the present study. Figure 1 shows the beamlike structure model for the control beam and different damage cases. Utilizing a general-purpose finite-element package called DIANA TNO that is based on the displacement method; one two-dimensional finite-element model was constructed to represent the beamlike structure model. The beam model was built by using a 4-node plane stress element. Figure 2 shows a typical model for the beam constructed using software. The physical and material

Table 1. Damage cases adopted in present study.

Stiffness damage cases	Stiffness reduction ratio (SRR%)	Damage location
Control C	0	N/A
C1	1	Mid-Span EL7
C2	1	Quarter-Span EL3
C3	1	Quarter-Span+ Mid-Span EL3 and EL7

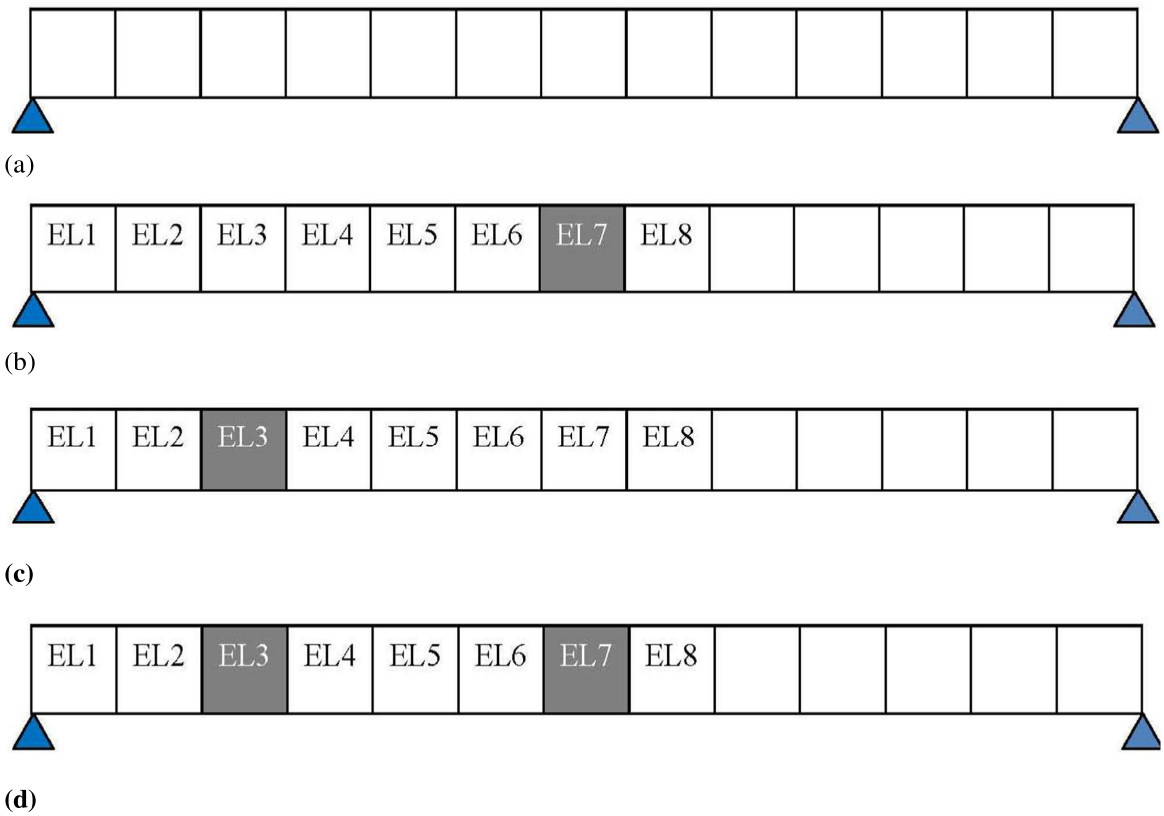

Figure 1. Beam-like structure model used in present study. (a) Control beam model, (b) Mid-span damage model, (c) quarter-span damage model and (d) multi damage model EL 3 and 7.

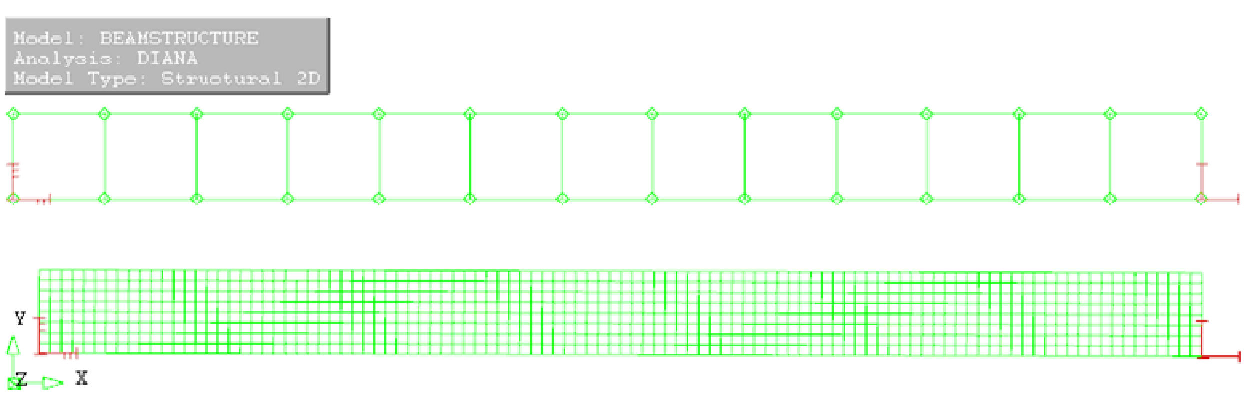

Figure 2. A typical model for the beam-like structure model.

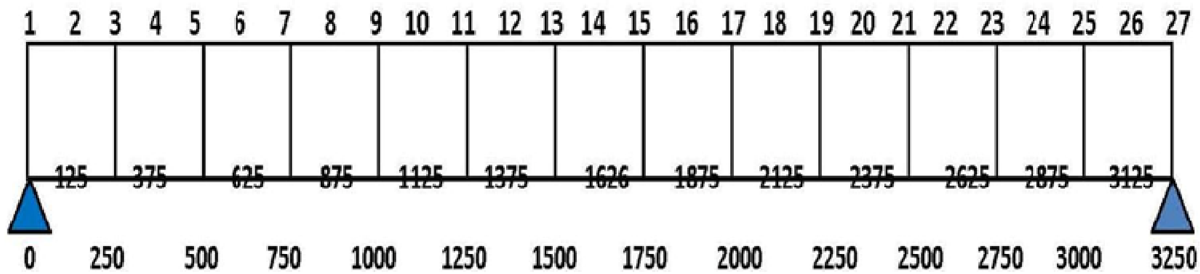

Figure 3. Distribution of 27 DOF along the beam length: top is the degree of freedom numbering and bottom is the distance in mm from the left support.

Table 2. FRI values at different damage cases.

Damage case	FRI Index Value %			
	Mode 1	Mode 2	Mode 3	Mode 4
Control C	0.00	0.00	0.00	0.00
C1	0.08	0.01	0.07	0.02
C2	0.04	0.06	0.06	0.03
C3	0.08	0.06	0.13	0.05

Table 3. MAC values at different damage cases.

Damage case	MAC value			
	Mode 1	Mode 2	Mode 3	Mode 4
Control C	0	0	0	0
C1	99.999960%	99.999978%	99.999923%	99.999943%
C2	99.999984%	99.999935%	99.999775%	99.999869%
C3	99.999962%	99.999934%	99.999839%	99.999830%

and material properties of the beam were Poisson's ratio of 0, mass density of 7850 kg/m^3 and Young's modulus of 200,000 MPa for the control case and 198,000 Mpa for the damage case. The self-weight was computed by taking gravitational acceleration as 9.81 m/s^2 in the $-y$ direction. Initially, Eigen analyses were performed so that modal parameters for the control beam model could be approximated. Next, the damage was created on the beam model by changing the value of the modules of elasticity first at the mid-span element (Figure 1b), secondly at the quarter-span element (Figure 1c), then at quarter-span and mid-span elements (Figure 1d). At each damaged case, Eigen value analysis was again performed to obtain the modal parameters relevant to the damage case induced.

The modal parameters were carried out with the natural frequency as the global characteristic and the mode shape as the local characteristic. The beam was divided into 27 degrees of freedom (DOF), starting from the left support where the first DOF is at 0 distance from the support and the second DOF is at 125 mm from the support. The next degree of freedom is located at 125 mm from the previous one up to the 27th degree of freedom which is located at the right support (Figure 3). The change in the natural frequency was used to indicate the damage level with each damage case adopted in the present study. The frequency reduction index (FRI) was adopted to present the damage level based on the change in the natural frequency as in the following:

$$\mathrm{FRI} = \left(1 - \frac{f_{i,d}}{f_{i,c}}\right) \cdot 100\% \quad (7)$$

Where $f_{i,c}$ and $f_{i,d}$ are the natural frequency at ith mode for control and damaged beam, respectively.

The FRI values for different damage cases are illustrated in Table 2. The results prove that SRR of 1% corresponds to a very small damage level whereas at worse case, when there is multi-damage at different elements, the maximum FRI value was 0.13% which is too small. Modal assurance criterion (MAC), which is a correlation between experimental mode shapes and curve-fitted mode shapes, is used in this study to monitor the change in the mode shape for different damage cases. The correlation for the ith element is given by the following formula:

$$\mathrm{MAC} = \frac{\left|\sum_{i=1}^{n} \varphi_{i,c} \cdot \varphi_{i,d}\right|^2}{\left(\sum_{i=1}^{n} \varphi_{i,c} \cdot \varphi_{i,c}\right)\left(\sum_{i=1}^{n} \varphi_{i,d} \cdot \varphi_{i,d}\right)} \quad (4)$$

Where $\varphi_{i,c}$ and $\varphi_{i,d}$ are the mode shapes at ith mode for control and damaged cases, respectively.

The MAC values for different damage cases are illustrated in Table 3. The results show an excessively small change in the MAC value which can be neglected in some cases. The maximum change in

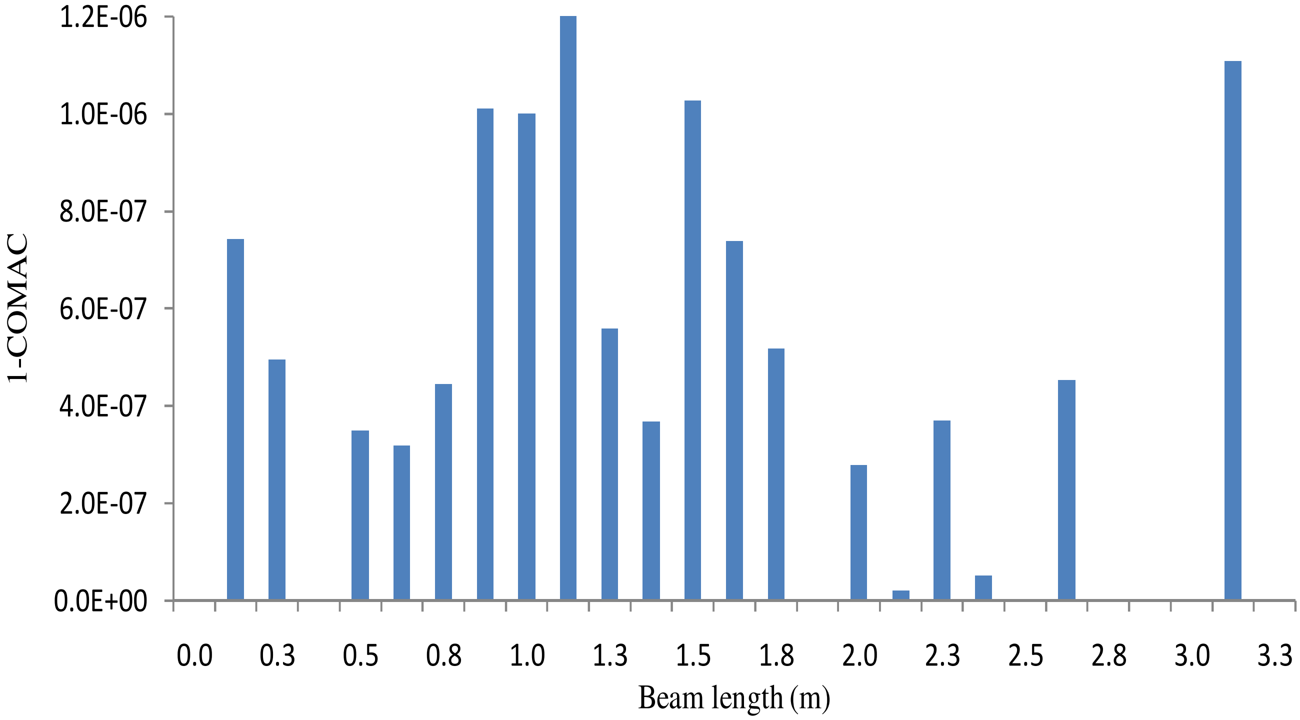

Figure 4. COMAC index for damaged located at mid-span.

the MAC value for the fourth mode when there is multi-damage located at different elements was 0.999983, the difference being located at the fifth digit after the decimal point. This proves the very small damage level for SRR of 1% and it also indicates the very small change in the mode shapes which will be used in the form of its derivatives to localize the damage. Any damage location index which can detect the location of such a damage level can be considered a good sensitive index. This allows the conclusion that the SRR of 1% is a quite good enough damage level to examine the existing algorithms as well as the modified algorithms.

RESULTS AND DISCUSSION

Here the results obtained from the analytical modeling. The first part of the section shows the 1-COMAC results of damage location for different damage locations. The results were presented to validate the newly developed index. The second part shows the results of the newly developed damage location index for the same damage cases, presented for 1-COMAC, in order to show the higher sensitivity of the developed index. 1-COMAC was calculated based on the total number of bending mode shapes of six modes. The damage location using the 1-COMAC index for damage with SRR of 1% and located at mid-span and quarter-span respectively are shown in Figures 4 and 5. The results show less sensitivity of 1-COMAC in detecting the damage location. 1-COMAC detects damage at the zone between 875 and 1750 mm from the left support for the case when the actual damage is located at the zone between 1500 and 1750 mm from the left support, and it also detects damage near both supports, which is incorrect. 1-COMAC detected damage along the zone between 750 and 1250 mm, at 1625 mm and at 2000 mm from the left support for the case when the actual damage is located at the zone between 500 and 750 mm, also detecting damage near the supports which is again incorrect. In order to investigate 1-COMAC sensitivity for detecting multi damage locations, Figure 6 show the damage location using the 1-COMAC index for multi-damage with SRR of 1% and located at quarter-span as well as mid-span. The result shows that 1-COMAC is less sensitive to detect cases of multi damage locations. 1-COMAC detects damage located at the supports which does not match the actual damage locations. Next are presented the damage location results using the newly developed SRI. As mentioned earlier, the results are presented for the same damage intensity and location as the 1-COMAC results in order to compare the sensitivity for each damage intensity and location along the beam length. Figure 7 shows the results of the first four bending mode shapes when SRR is 1% and the damage located at the mid-span on the zone between 1500 and 1750 mm from the left support. SRI will be calculated for each mode separately to show the influence of the damage on each mode. The results show the very good sensitivity of SRI index to localize the damage location, although when the damage intensive is very low, the SRR is only 1%.

The results show that all the first four bending modes

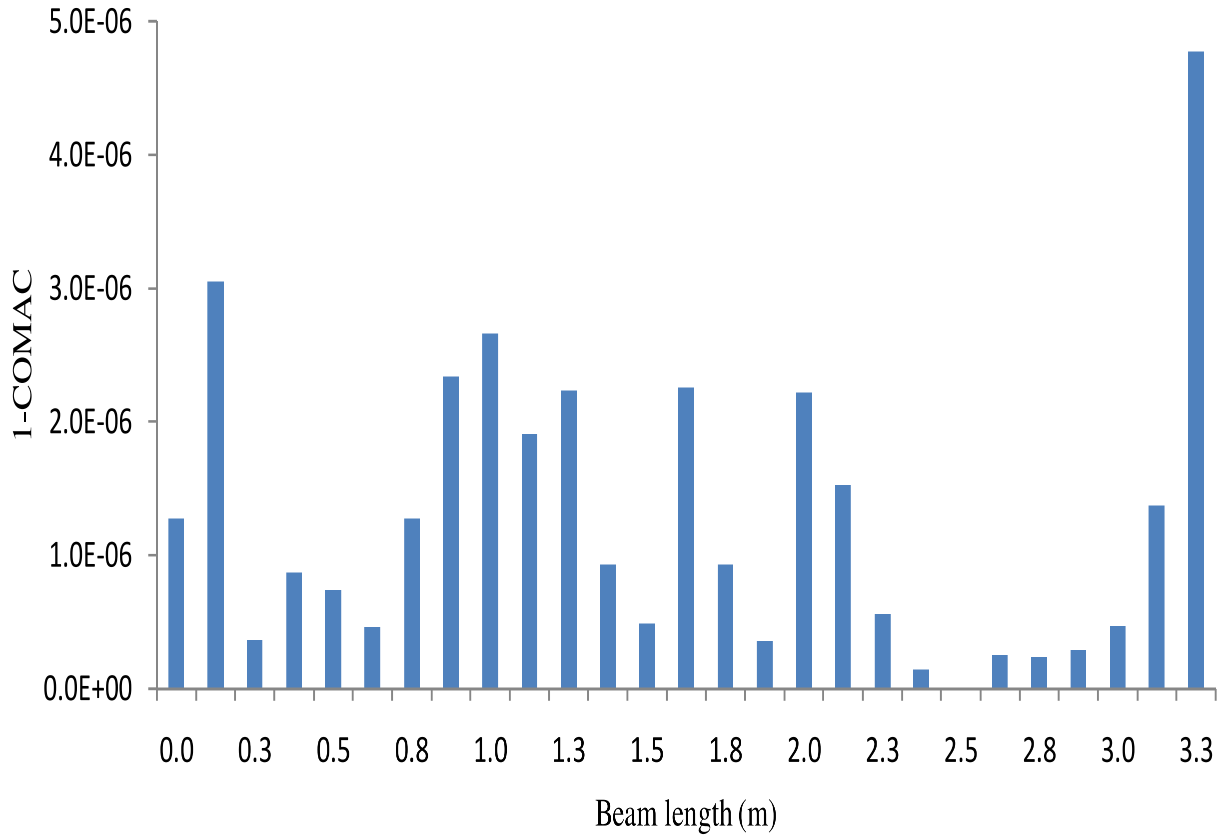

Figure 5. COMAC index for damaged located at quarter-span.

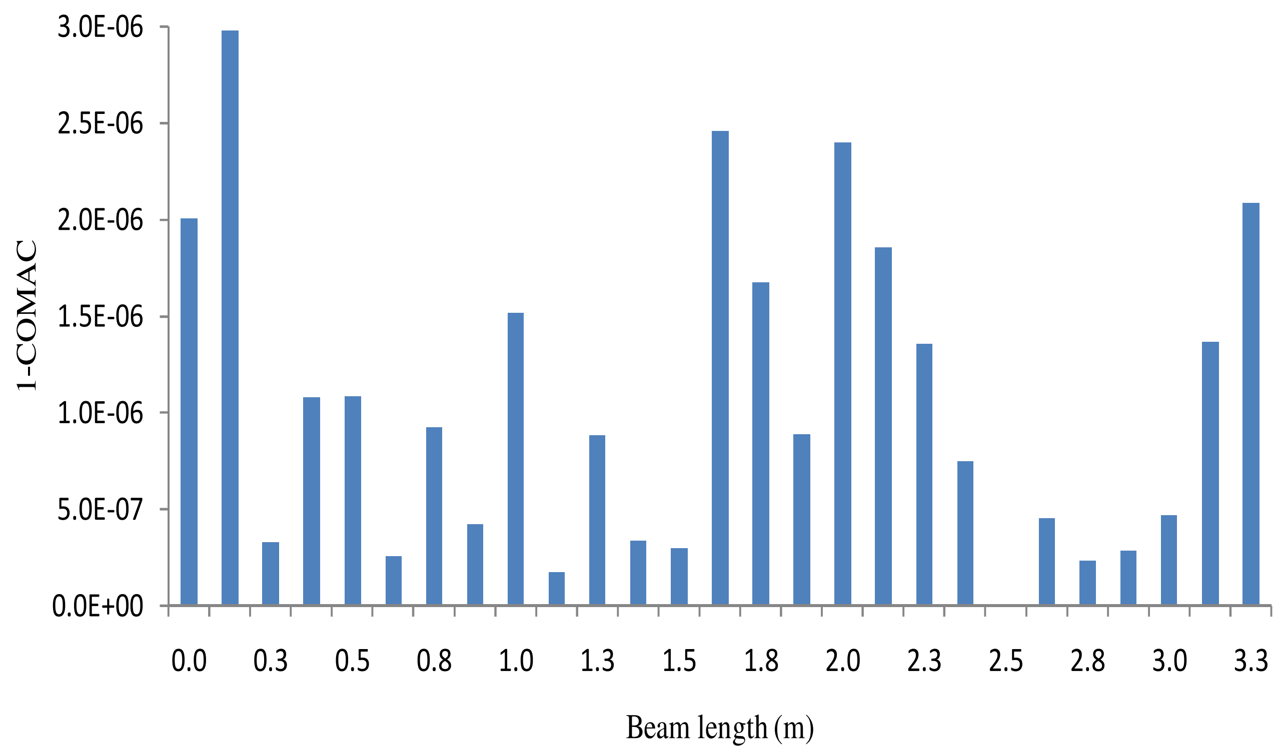

Figure 6. COMAC index for multi-damaged located at quarter and mid span.

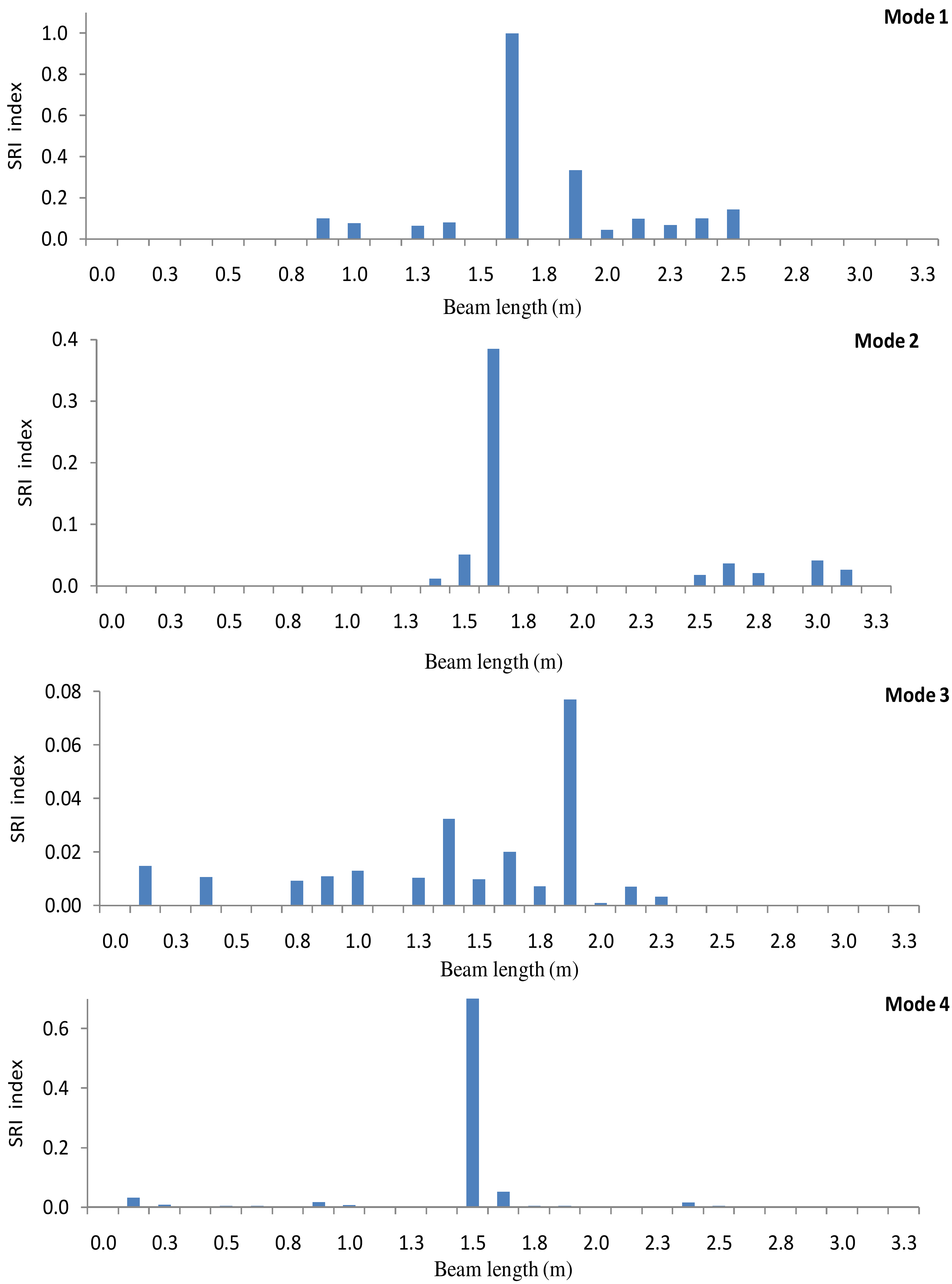

Figure 7. SRI values at damage with SRR of 1% and located at the mid-span.

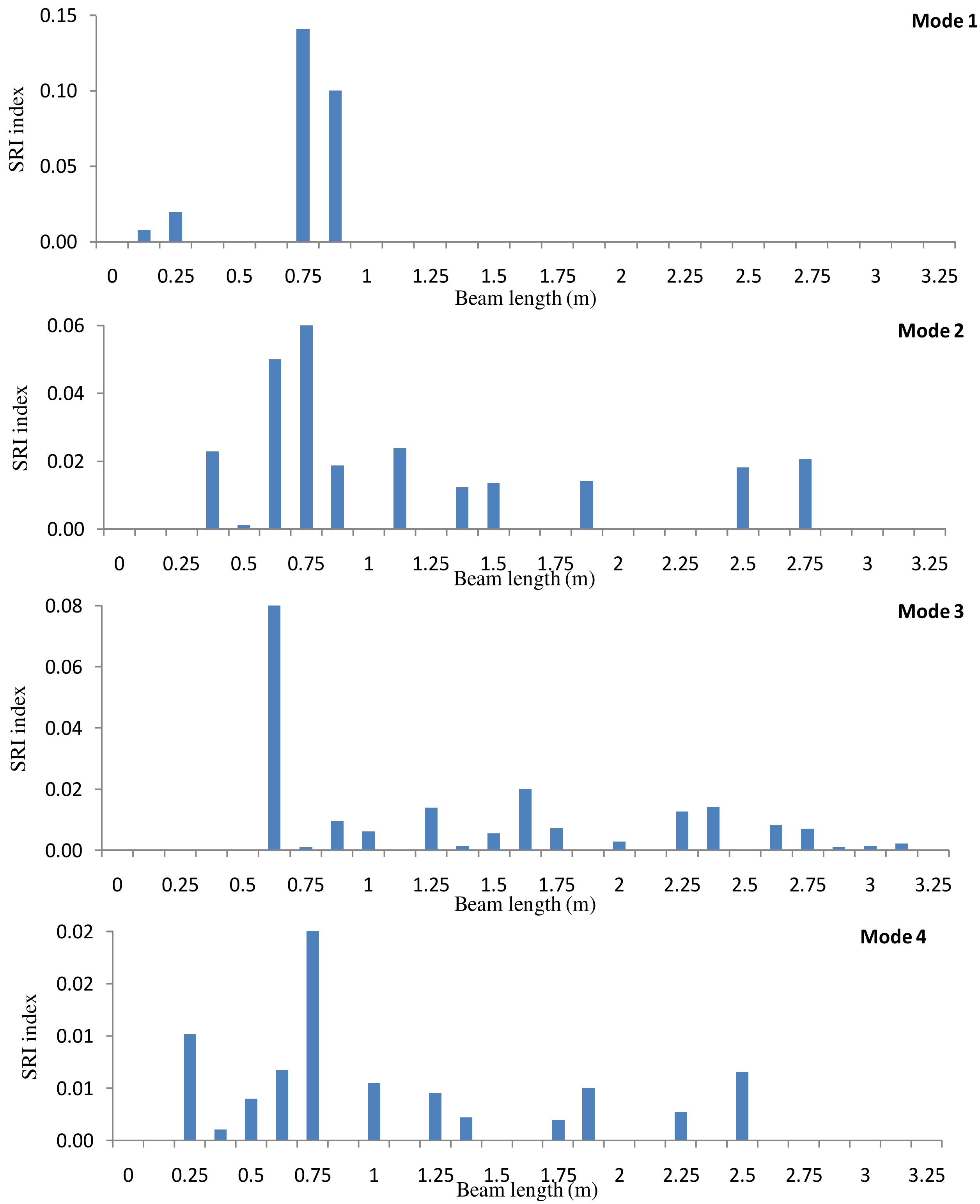

Figure 8. SRI values at damage with SRR of 1% and located at the quarter-span.

have a good ability to localize the damage. Figure 8 shows the SRI results for the first four bending modes with damage with SRR of 1% and located at the quarter span. The results have validated the ability of SRI index to detect the damage when it is located at the quarter span. All the first four bending modes have a good ability

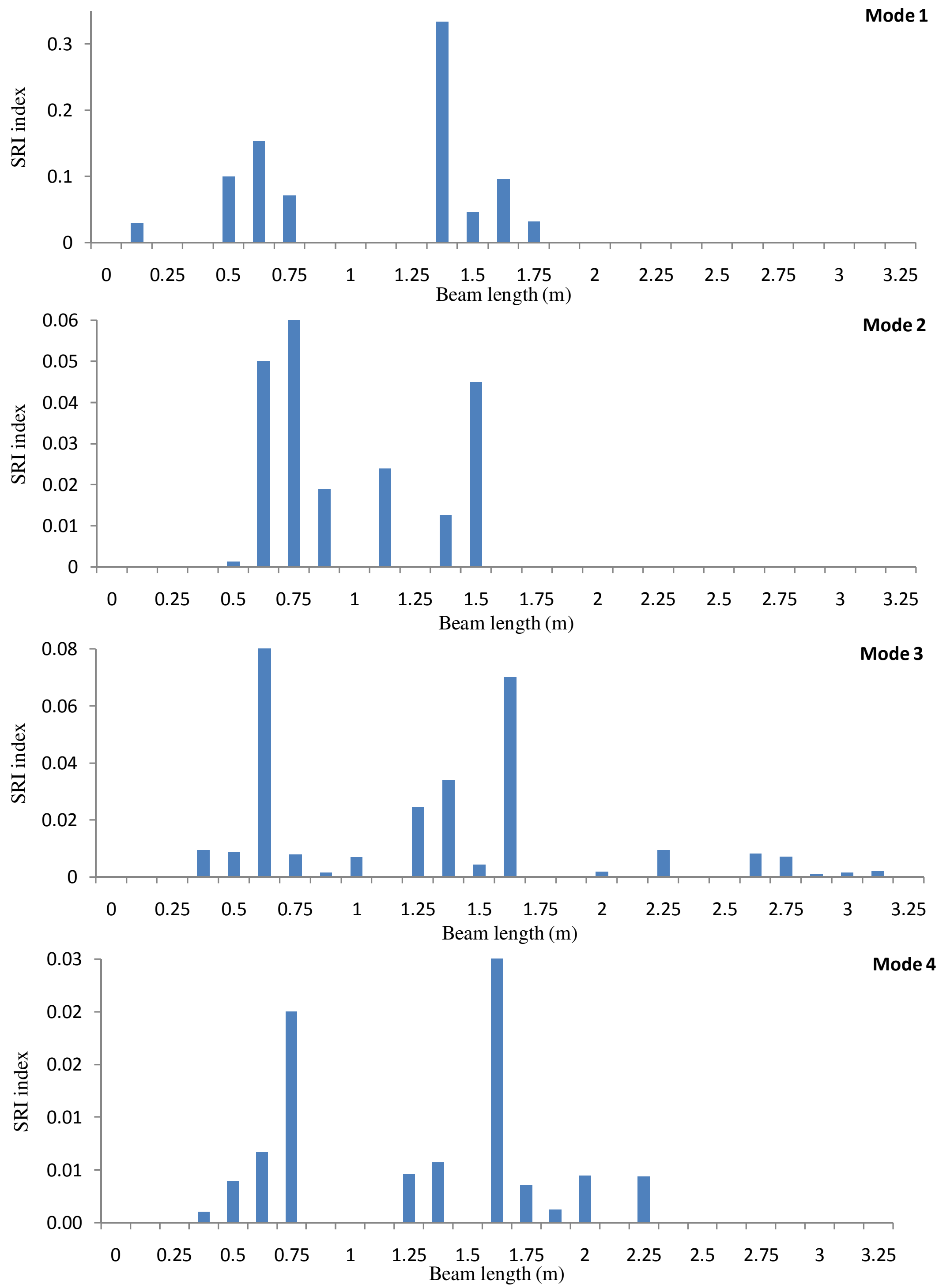

Figure 9. SRI values at multi damage with SRR of 1%, located at quarter-span and mid-span.

to localize the damage location. Mode one was the most sensitive mode, while the other three modes show some anomalous values along the beam length at the zones where there is no damage. After validating the ability of the new index for localizing a single damage located at different elements along the beam length, it will validate for the cases when there are multi damages located at different elements along the beam length. Figure 9 show SRI results for the first four bending modes when SRR is 1% and multi damage located at quarter-span and mid-span at the same time. The results have validated SRI index as a good sensitive index, although in the cases of multi damage locations. All the adopted modes have shown a good ability to detect the two damage locations. Since one of the damage is located at the quarter-span and then another is located at the mid-span, SRI has detected both locations and has detected damage in the zone between both actual damages as well. Comparing SRI results to 1-COMAC, it's clear that the new developed algorithm has higher sensitivity for the adopted damage cases. The study has validated SRI as an acceptable damage location algorithm since it can detect the damage no matter what its location and intensity, and even in the case of multi damage locations.

Conclusions

The main aim of the present study is to develop a damage location algorithm which has higher sensitivity to localize the damage regardless of its location and intensity. Consequently, the SRI index has been developed as a new damage location algorithm. SRI index has been examined to validate its sensitivity to localize the damage location for the cases of very small intensive where the stiffness reduction ratio adopted (SRR) was only 1%. SRI has been proven to be a desirably sensitive damage location algorithm, as it could detect a very small damage intensive when the damage was at the mid-span, at the quarter-span, and even when there were multi damage locations. SRI has a higher sensitivity comparing to 1-COMAC for all the investigated damage cases. The study adopted the first four bending modes, all of which show acceptable ability of the SRI algorithm to detect the damage location.

REFERENCES

Choi FC, Li J, Samali B, Crews K (2008). Application of the Modified Index Method to Timber Beams. J. Eng. Struct., 30: 1124–1145.

Dutta A, Talukdar S (2004). Damage Detection in Bridges Using Accurate Modal Parameters. J. Finite Elem. Anal. Des., 40: 287–304.

Ismail Z, Abdul Razak H (2006). Determination of Damage Location In R.C. Beams Using Mode Shape Derivatives. J. Eng. Struct., 28: 1566-1573.

Johan M (2003). Damage Assessment of Civil Engineering Structures By Vibration Monitoring. Katholieke University Leuven, Faculteit toegepaste Wetenschappen, Arenbergkastell, B-3001 Heverlee (Belgium), ISBN 90-5682-390-6.

Kim JT, Park JH, Hong DS, Park WS (2010). Hybrid health monitoring of prestressed concrete girder bridges by sequential vibration-impedance approaches. Eng. Struct.,. 32(1): 115-128.

Kopsaftopoulos FP, Fassois SD (2010). Vibration based health monitoring for a lightweight truss structure: Experimental assessment of several statistical time series methods. Mech. Syst. Signal Process., 24: 1977-1997.

Law SS, Lu ZR (2005). Crack Identification in Beam from Dynamic Response. J. Sound Vib., 285: 967-987.

Pandey AK, Biswas M, Samman MM (1991). Damage Detection from Change in Curvature Mode Shapes. J. Sound Vib., 145(2): 321-332.

Perera R, Huerta C, Orquin JM (2008). Identification of damage in RC beams using index based on local modal stiffness. Constr. Build. Mater., 22: 1659–1667.

Peterson ST, Mclean DI, Symans MD, Pollock DJ, Coder WF, Emerson RN, Fridley KJ (2001). Application of Dynamic System Identification to Timber Beam. ASCE J. Struct. Eng., 127(4): 418-25.

Ratcliffe CP (1997). Damage Detection Using A Modified Laplacian Operator On Mode Shape Data. J. Sound Vib., 204(3): 505–517.

Rodríguez R, Escobar JA, Gómez R (2010). Damage detection in instrumented structures without baseline modal parameters. Eng. Struct.,. 32(6): 1715-1722.

Stubbs N, Kim J (1995). Field Verification of Non Destructive Damage Location and Severity Estimation Algorithm. Processing of the 13th international Modal Analysis Conference, pp. 210-218.

Wahab MMA, Roeck GD (1999). Damage Detection in Bridges Using Modal curvature: Application To A Real Damage Scenario. J. Sound Vib., 226: 217-235.

Yan G, Duan Z, Ou J, De Stefano A (2010). Structural damage detection using residual forces based on wavelet transform. Mech. Syst. Signal Process., 24(1): 224-239.