

Full Length Research Paper

Analytical solutions for heat transfer flows of a third grade fluid in case of posttreatment of wire coating

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The steady flow of a third grade fluid which belongs to non-Newtonian fluid mechanics deserves more attention than it has been to date, because in the nonlinear systems, the literature is scarce. In this paper, we develop a formulation suitable for solution in the case of posttreatment of wire coating containing thermal effects of third grade fluids. The optimal homotopy asymptotic method (OHAM) and regular perturbation method are used to investigate the flow of the titled problem. Expressions for velocity and temperature distributions are given. It is noted that the solutions are strongly dependent on the Brinkman number, the ratio of velocity U and non-Newtonian parameter β . Finally, a physical interpretation of the results is given with the help of several graphs.

Key words: Wire coating, third grade fluid, perturbation method, optimal homotopy asymptotic method.

INTRODUCTION

Polymer extrudate is an important industrial process used for coating a wire for the purpose of high and low voltage and protection against corrosion. Wire coating have many application in the field of chemical and industrial engineering (Matallah et al., 2001; Binding et al., 1996). The basic concepts of modeling of different problems in the case wire coating for viscous fluids are given by Denn and middleman (Denn, 1980; Middleman, 1977). Many authors have studied the wire coating in the die. Akhter and Hashmi (1999, 1977) have developed the mathematical model for wire coating using power law model and investigated the effect of the change in viscosity. Siddiqui et al. (2009) studied the wire coating extrusion in a pressure-type die for the flow of a third grade fluid.

The coated wire after leaving the die is effected by the quality of the material used in coating process, the wire drawing velocity and the temperature. There are very few disclosures presenting theoretically analysis of flow in the post treatment process subsequent to the die. Kasajima and Ito, 1973; Nayfeh, 1979 have studied the post

treatment of polymer extrudate in wire coating. They used the power law model approach and derived the solutions for velocity and temperature distributions. In a wider flow and recent context, some recent attempts have been made by Mutlu et al., 1998; Mutlu et al., 1998; Matallah et al., 2002, that explain the pressure and tube-tooling wire coating and multi-mode wire coating of viscous and viscoelastic fluid flows. The exact solution of the Navier-Stokes equations is notoriously, difficult to find because the non-linearity exist in these equations due to the convection term. To handle the non-linearity, different approximate analytical and numerical methods have been widely used in fluid mechanics and engineering. The perturbation methods are the most widely applied analytic tools for non-linear problems to obtain approximate solutions of these equations. Most of the perturbation methods require the presence of the small or large parameter in that equation but every equation has not small or large parameter. Therefore, there is a strong need to develop new methods (He 2006).

Recently, Marinca and Herisanu developed a new method known as optimal homotopy asymptotic method (OHAM). The optimal homotopy asymptotic method (OHAM) combines the He's Homotopy Perturbation Method (HPM) and the method of least squares to

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optimally identify the unknown constants of the series solutions (Marinca and Herisanu, 2010). Marinca and Herisanu proposed this new homotopy technique called the optimal homotopy asymptotic method (OHAM) which is also proved to be a reliable approach to strongly nonlinear problems. In a series of papers by Marinca et al. (2008), Herisanu et al. (2008), Marinca and Herisanu (2008), Islam et al. (2010) and Javed et al. (2010) have not only applied this method successfully to obtain the solution of some important problems in engineering and fluid mechanics, but also they have shown that this method is a powerful tool than other perturbation tools for non linear problems.

To the authors' knowledge, no previous investigation has been reported to develop the governing equations for steady incompressible flow containing thermal effects of a third grade fluid in case of posttreatment of wire coating. In this work, it is intended to construct the equations for an incompressible flow of a third grade fluid containing thermal effects. The non-linear differential equations are made dimensionless and solved for velocity and temperature distributions by means of perturbation and optimal homotopy asymptotic method.

BASIC GOVERNING EQUATIONS

The basic equations governing the flow of an incompressible third grade fluid with thermal effects are:

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\rho \frac{D \mathbf{u}}{Dt} = \nabla \cdot \mathbf{T} + \rho \mathbf{f}_i, \tag{2}$$

$$\rho c_p \frac{D \Theta}{Dt} = k \nabla^2 \Theta + \mathbf{S} \cdot \mathbf{L} \tag{3}$$

where \mathbf{u} is the velocity vector, ρ is the constant density, \mathbf{f} is the body force, \mathbf{T} is the Cauchy stress tensor, D/Dt denote the material derivative, Θ is the fluid temperature, k is the thermal conductivity, c_p is the specific heat and \mathbf{L} is the gradient of velocity vector \mathbf{u} . The Cauchy stress tensor \mathbf{T} is defined as:

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S}, \tag{4}$$

where p is the dynamic pressure, \mathbf{I} course is the identity tensor, and \mathbf{S} is the shear stress tensor. For third grade fluid, the extra stress tensor \mathbf{S} is defined as:

$$\mathbf{S} = \mu \mathbf{A}_1 + \alpha_1 \mathbf{A}_2 + \alpha_2 \mathbf{A}_1 + \beta_1 \mathbf{A}_3 + \beta_2 (\mathbf{A}_1 \mathbf{A}_2 + \mathbf{A}_2 \mathbf{A}_1) + \beta_3 (tr \mathbf{A}_2) \mathbf{A}_1, \tag{5}$$

in which μ is the coefficient of viscosity of the fluid,

$\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3$ are the material constants and $\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3$ are the line kinematic tensors defined by:

$$\mathbf{A}_1 = \mathbf{L}^T + \mathbf{L}, \tag{6}$$

$$\mathbf{A}_n = \mathbf{A}_{n-1} \mathbf{L}^T + \mathbf{L} \mathbf{A}_{n-1} + \frac{D \mathbf{A}_{n-1}}{Dt}, \quad n = 2, 3 \tag{7}$$

where the superscript T denotes the transpose of the matrix.

FORMULATION OF THE PROBLEM

In wire coating, the quality of the material and wire drawing velocity are important within the die, after leaving the die temperature and the shape of the transverse sectioning is also very important. Considering the flow of the polymer extrudate given in Figure 1 denoted by the solid line; to analyze the flow behavior of a polymer used in wire coating, it is convenient to divide the flow transversely into many short sections as shown in broken lines in Figure 1; with the assumption that each section has almost the same shape, we analyze only one section because each section can be assumed to be approximately of the shape shown in Figure 2 and readily analyzable. Considering the wire of radius R_1 and temperature

Θ_1 is dragged in the z direction through an incompressible third grade polymer (II) with a velocity V_1 and the gas (III) surrounding the polymer (II) is at temperature Θ_2 and flowing with a velocity V_2 ; considering the cylindrical coordinates (r, θ, z) , such that r is perpendicular to the direction of flow; boundary conditions are:

$$w = V_1, \quad \Theta = \Theta_1 \quad \text{at } r = R_1, \quad w = V_2, \quad \Theta = \Theta_2 \quad \text{at } r = R_2, \tag{8}$$

Assuming that the flow is steady, laminar, unidirectional and axisymmetric:

We seek the velocity field of the form:

$$\mathbf{u} = [0, 0, w(r)], \quad \mathbf{S} = S(r), \quad \Theta = \Theta(r) \tag{9}$$

The following are some more assumptions, which are made during the formulation of the problem:

1. The fluid flow is incompressible.
2. Polymer II holds the third order fluid model for shear rate.

In Figure 2, the wire I, the polymer II and gas III are in contact with each other and consider no slippage occurs along the contacting surfaces of the wire, polymer, and the gas.

Using the velocity field, the continuity Equation (1) is satisfied identically, and the non zero components of Equation (5) become:

$$S_{rr} = (2\alpha_1 + \alpha_2) \left(\frac{dw}{dr} \right)^2 \tag{10}$$

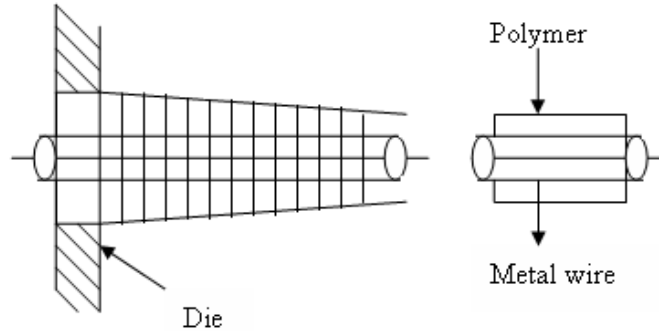


Figure 1. Schematic profile of polymer extrudate in wire coating.

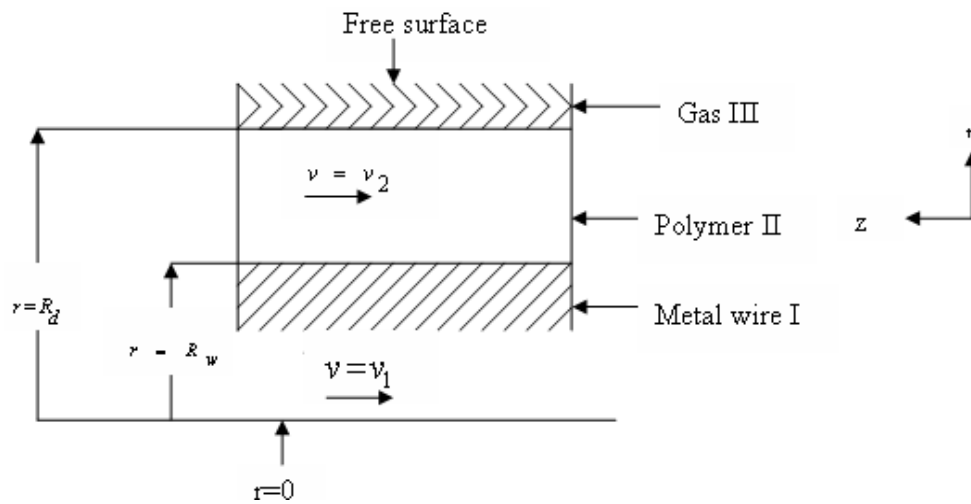


Figure 2. Drag flow in wire coating.

$$S_{zz} = \alpha_2 \left(\frac{dw}{dr} \right)^2 \quad (11)$$

$$S_{rz} = \mu \frac{dw}{dr} + 2(\beta_2 + \beta_3) \left(\frac{dw}{dr} \right)^3 \quad (12)$$

Substituting the velocity field and equation (10) to (12) in the equation of balance of momentum (2) in the absence of body force takes the form:

$$-\frac{\partial p}{\partial r} = (2\alpha_1 + \alpha_2) \frac{1}{r} \frac{d}{dr} \left\{ r \left(\frac{dw}{dr} \right)^2 \right\} \quad (13)$$

$$\frac{\partial p}{\partial \theta} = 0 \quad (14)$$

$$\frac{\partial p}{\partial z} = \frac{\mu}{r} \frac{d}{dr} \left(r \frac{dw}{dr} \right) + 2(\beta_2 + \beta_3) \frac{d}{dr} \left(r \left(\frac{dw}{dr} \right)^3 \right) \quad (15)$$

Equation (15) represents the flow due to pressure gradient. After leaving the die, there is only drag flow. Hence, we consider:

$$2(\beta_2 + \beta_3) \frac{d}{dr} \left(r \left(\frac{dw}{dr} \right)^3 \right) + \frac{\mu}{r} \frac{d}{dr} \left(r \frac{dw}{dr} \right) = 0 \quad (16)$$

and the energy Equation (3) becomes:

$$k \left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} \right) \Theta + \mu \left(\frac{dw}{dr} \right)^2 + 2(\beta_2 + \beta_3) \left(\frac{dw}{dr} \right)^4 = 0. \quad (17)$$

Introduce the dimensionless parameters:

$$r^* = \frac{r}{R_1}, \quad w^* = \frac{w}{V_1}, \quad \Theta^* = \frac{\Theta - \Theta_1}{\Theta_2 - \Theta_1}, \quad (18)$$

$$\beta = \beta_2 + \beta_3, \quad \frac{R_2}{R_1} = \delta > 1, \quad \frac{V_2}{V_1} = U < 1$$

The system of Equation (16) to (17) and (8) after dropping the “*” take the following form:

$$r \frac{d^2 w}{dr^2} + \frac{dw}{dr} + 2\beta \left(3r \frac{d^2 w}{dr^2} \left(\frac{dw}{dr} \right)^2 + \left(\frac{dw}{dr} \right)^3 \right) = 0, \quad (19)$$

$$w(1) = 1, \text{ and } w(\delta) = U, \quad (20)$$

$$\frac{d^2 \Theta}{dr^2} + \frac{1}{r} \frac{d\Theta}{dr} + Br \left(\frac{dw}{dr} \right)^2 + 2Br\beta \left(\frac{dw}{dr} \right)^4 = 0 \quad (21)$$

$$\Theta(1) = 0, \text{ and } \Theta(\delta) = 1. \quad (22)$$

where Br is the Brinkman number and

$$Br = \frac{\mu V_1^2}{k(\Theta_2 - \Theta_1)}, \beta^* = \frac{\beta}{\mu \left(\frac{R_1^2}{V_1^2} \right)}, \quad (23)$$

The traditional perturbation method is used to solve Equation (19) and (21) with the corresponding boundary conditions (21) and (22).

PERTURBATION SOLUTION

The approximate solution to Equation (19) subject to the boundary conditions (20) can be obtained by selecting $\beta = \mathcal{E}$, as perturbation parameter. The velocity field is chosen:

$$w(r, \mathcal{E}) = w_0(r) + \mathcal{E} w_1(r) + \mathcal{E}^2 w_2(r) + \dots \quad (24)$$

Substituting the aforementioned consideration in Equation (19) and (20), and comparing the coefficient of $\mathcal{E}^0, \mathcal{E}^1, \mathcal{E}^2$ we obtain the zeroth, first and second order problem in the following form:

Zeroth-order problem:

$$r \frac{d^2 w_0}{dr^2} + \frac{dw_0}{dr} = 0 \quad (25)$$

with the boundary conditions:

$$w_0(1) = 1, w_0(\delta) = U. \quad (26)$$

First-order problem:

$$r \frac{d^2 w_1}{dr^2} + \frac{dw_1}{dr} + 6r \frac{d^2 w_0}{dr^2} \left(\frac{dw_0}{dr} \right)^2 + 2 \left(\frac{dw_0}{dr} \right)^3 = 0 \quad (27)$$

with the boundary conditions:

$$w_1(1) = 0, w_1(\delta) = 0. \quad (28)$$

Second-order problem:

$$r \frac{d^2 w_2}{dr^2} + \frac{dw_2}{dr} + 6r \frac{d^2 w_1}{dr^2} \left(\frac{dw_0}{dr} \right)^2 + 6 \frac{dw_1}{dr} \left(\frac{dw_0}{dr} \right)^2 + 12r \frac{dw_0}{dr} \frac{dw_1}{dr} \frac{d^2 w_0}{dr^2} = 0 \quad (29)$$

with the boundary conditions:

$$w_2(1) = 0, w_2(\delta) = 0. \quad (30)$$

Now we solve this sequence of problems and generate the series solution.

Zeroth-order problem solution:

$$w_0(r) = 1 + \frac{\ln r}{\ln \delta} (U - 1), \quad (31)$$

which is the Newtonian solution.

First-order problem solution:

$$w_1(r) = \left(\frac{U-1}{\ln \delta} \right)^3 \left[\frac{\ln r}{\ln \delta} \left(1 - \frac{1}{\delta^2} \right) - \left(1 - \frac{1}{r^2} \right) \right]. \quad (32)$$

Second-order problem solution:

$$w_2(r) = 3 \left(\frac{U-1}{\ln \delta} \right)^5 \frac{\ln r}{\ln \delta} \left[\left(1 - \frac{1}{r^4} \right) - \left(1 - \frac{1}{\delta^4} \right) - \frac{1}{\ln \delta} \left(1 - \frac{1}{\delta^2} \right) \left(\frac{1}{\delta^2} - \frac{1}{r^2} \right) \right] \quad (33)$$

Next, we find the approximate solution for temperature profile, for which we write:

$$\Theta(r, \mathcal{E}) = \Theta_0(r) + \mathcal{E} \Theta_1(r) + \mathcal{E}^2 \Theta_2(r) + \dots \quad (34)$$

Submitted Equation (24), (34) and $\beta = \mathcal{E}$ in Equation (21) and (22) and collecting the same power of \mathcal{E} , yields different order problems. Zeroth-order problem with boundary conditions:

$$\frac{d^2 \Theta_0}{dr^2} + \frac{1}{r} \frac{d\Theta_0}{dr} + Br \left(\frac{dw_0}{dr} \right)^2 = 0 \quad (35)$$

$$\Theta_0(\delta) = 1, \quad \Theta_0(1) = 0 \quad (36)$$

with the solution:

$$\Theta_0(r) = \frac{\ln r}{2(\ln \delta)^2} \left(2 \ln \delta - Br (U - 1)^2 \ln \left(\frac{r}{\delta} \right) \right) \quad (37)$$

First-order problem with boundary conditions:

$$\frac{d^2 \Theta_1}{dr^2} + \frac{1}{r} \frac{d\Theta_1}{dr} + 2Br \left(\frac{dw_0}{dr} \right)^4 + 2Br \frac{dw_0}{dr} \frac{dw_1}{dr} = 0 \quad (38)$$

$$\Theta_1(\delta) = 0, \quad \Theta_1(1) = 0 \quad (39)$$

with the solution:

$$\Theta_1(r) = \frac{1}{2(\ln \delta)^5} Br (U-1)^4 \left[\left(1 - \frac{1}{r^2} \right) \ln \delta + \left(1 - \frac{1}{\delta^2} \right) (2(\ln r)^2 - \ln r - 2 \ln r \ln \delta) \right] \quad (40)$$

Second-order problem with boundary conditions:

$$\frac{d^2\Theta_2}{dr^2} + \frac{1}{r} \frac{d\Theta_2}{dr} + 8Br \left(\frac{dw_0}{dr} \right)^3 \frac{dw_1}{dr} + 2Br \frac{dw_0}{dr} \frac{dw_2}{dr} + Br \left(\frac{dw_1}{dr} \right)^2 = 0 \quad (41)$$

$$\Theta_2(\delta) = 0, \quad \Theta_2(1) = 0 \quad (42)$$

with the solution:

$$\Theta_2(r) = \frac{Br}{4} \left(\frac{U-1}{\ln \delta} \right)^6 \left(8 \frac{\ln r}{(\ln \delta)^2} \left(1 - \frac{1}{\delta^2} \right)^2 - 14 \left(\frac{\ln r}{\ln \delta} \right)^2 \left(1 - \frac{1}{\delta^2} \right)^2 + 12 \frac{(\ln r)^2}{\ln \delta} \left(1 - \frac{1}{\delta^4} \right) - 17 \frac{\ln r}{\delta^2 \ln \delta} \left(1 - \frac{1}{\delta^2} \right) + 11 \frac{\ln r}{\ln \delta} \left(1 - \frac{1}{\delta^2} \right) - 8 \frac{1}{\ln \delta} \left(1 - \frac{1}{r^2} \right) \left(1 - \frac{1}{\delta^2} \right) - 12 \ln r \left(1 - \frac{1}{\delta^4} \right) + 3 \left(1 - \frac{1}{r^4} \right) \right) \quad (43)$$

SOLUTION BY OHAM

Here we apply the optimal homotopy asymptotic method (OHAM) (11) to (17) to Equation (19). Using:

$$L[\phi(r, p)] = r \frac{\partial^2 \phi(r, p)}{\partial r^2} + \frac{\partial \phi(r, p)}{\partial r}, \quad g(r) = 0$$

and

$$N[\phi(r, p)] = 2\beta \left(3r \frac{\partial^2 \phi(r, p)}{\partial r^2} \left(\frac{\partial \phi(r, p)}{\partial r} \right)^2 + \left(\frac{\partial \phi(r, p)}{\partial r} \right)^3 \right). \quad (44)$$

where L and N are linear and nonlinear operators, respectively and $p \in [0, 1]$ is an embedding parameter. We construct a homotopy $\phi(r, p) : R \times [0, 1] \rightarrow R$ which satisfies:

$$(1-p)[L(\phi(r, p)) + g(r)] = H(p)[L(\phi(r, p)) + g(r) + N(\phi(r, p))], \quad (45)$$

with boundary conditions:

$$\phi(1, p) = 1, \quad \phi(\delta, p) = U. \quad (46)$$

Here $H(p)$ is a nonzero auxiliary function for $p \neq 0$, $H(0) = 0$ and $\phi(r, p)$ is an unknown function. Obviously, when $p = 0$ and $p = 1$, $\phi(r, 0) = w_0(r)$ and $\phi(r, 1) = w(r)$ respectively. Thus, as p varies from 0 to 1, the solution $\phi(r, p)$ approaches from $w_0(r)$ to $w(r)$, where $w_0(r)$ is obtained from equation (45) for $p = 0$. The auxiliary function $H(p)$ depends either upon some constants (11) to (17) or upon some functions depending on a physical parameter (Herisanu and Marinca, 2010). It was shown in the paper (Herisanu and Marinca, 2010) that a more complex function $H(p)$ leads to more accurate results.

We choose the auxiliary function $H(p)$ in the form:

$$H(p) = pC_1 + p^2C_2 + \dots \quad (47)$$

where C_1, C_2, \dots are constants to tackled the solution easily.

To get an approximate solution, we expand $\phi(r, p, C_i)$ in Taylor's series about p in the following manner:

$$\phi(r, p, C_i) = w_0(r) + \sum_{k=1}^{\infty} w_k(r, C_1, C_2, \dots, C_k) p^k. \quad (48)$$

Substituting Equations (47) and (48) into equation (45) and equating the coefficient of like powers of p , we obtain the following linear equations:

Zeroth-order problem:

$$p^0 : r \frac{d^2 w_0}{dr^2} + \frac{dw_0}{dr} = 0 \quad (49)$$

subject to the boundary conditions:

$$w_0(1) = 1, \quad w_0(\delta) = U. \quad (50)$$

First-order problem

$$p^1 : r \frac{d^2 w_1}{dr^2} + \frac{dw_1}{dr} - \frac{dw_0}{dr} - C_1 \frac{dw_0}{dr} - 2\beta C_1 \left(\frac{dw_0}{dr} \right)^3 - r \frac{d^2 w_0}{dr^2} - rC_1 \frac{d^2 w_0}{dr^2} - 6r\beta C_1 \left(\frac{dw_0}{dr} \right)^2 \frac{d^2 w_0}{dr^2} = 0$$

subject to boundary conditions:

$$w_1(1) = 0, \quad w_1(\delta) = 0. \quad (52)$$

Second-order problem:

$$p^2 : r \frac{d^2 w_2}{dr^2} + \frac{dw_2}{dr} - C_2 \frac{dw_0}{dr} - 2\beta C_2 \left(\frac{dw_0}{dr} \right)^3 - \frac{dw_1}{dr} - C_1 \frac{dw_1}{dr} - rC_2 \frac{d^2 w_0}{dr^2} - 6\beta C_1 \left(\frac{dw_0}{dr} \right)^2 \frac{dw_1}{dr} - 6r\beta C_2 \left(\frac{dw_0}{dr} \right)^2 \frac{d^2 w_0}{dr^2} - 12r\beta C_1 \frac{dw_0}{dr} \frac{dw_1}{dr} \frac{d^2 w_0}{dr^2} - r \frac{d^2 w_1}{dr^2} - rC_1 \frac{d^2 w_1}{dr^2} - 6r\beta C_1 \left(\frac{dw_0}{dr} \right)^2 \frac{d^2 w_1}{dr^2} = 0$$

subject to boundary conditions:

$$w_2(1) = 0, \quad w_2(\delta) = 0. \quad (54)$$

The convergence of the series (48) depends upon the auxiliary constants C_1, C_2, \dots . If it is convergent at $p = 1$:

$$\phi(r, C_1, C_2, \dots, C_m) = w_0(r) + \sum_{i=1}^m w_i(r, C_1, C_2, \dots, C_i). \quad (55)$$

Substitution of Equation (55) into Equation (45), yields the following expression for residual:

$$R(r, C_1, C_2, \dots, C_m) = L(\phi(r, C_1, C_2, \dots, C_m)) + g(r) + N(\phi(r, C_1, C_2, \dots, C_m)). \quad (56)$$

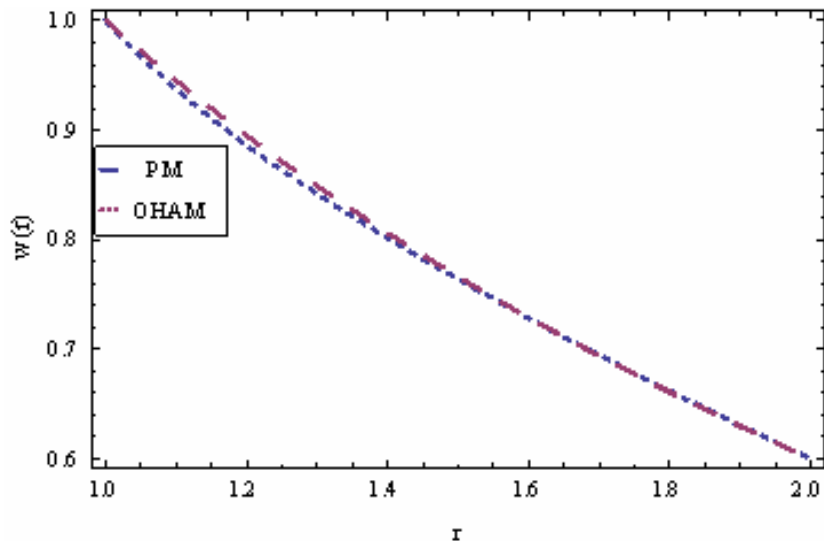


Figure 3. Comparison of dimensionless velocity profiles using PM and OHAM, when $U = 0.6$, $\beta = 0.01$, $C_1 = -0.001357286$, $C_2 = -0.0027125721$.

To find the optimal values $C_i, i = 1, 2, 3, \dots$, one can apply different methods like method of least squares, Galerkin's, Ritz, and collocation method. Here we used method of least squares as:

$$J(C_1, C_2, \dots, C_m) = \int_a^b R^2(y, C_1, C_2, \dots, C_m) dy \quad (57)$$

$$\frac{\partial J}{\partial C_1} = \frac{\partial J}{\partial C_2} = \dots = \frac{\partial J}{\partial C_m} = 0 \quad (58)$$

where a and b are properly chosen numbers to locate the desired $C_i (i = 1, 2, \dots, m)$. With these constants known, the approximate solution (of order m) is well-determined.

In view of all these, the corresponding solutions of Equations (49), (51) and (53) together with the boundary conditions (50), (52) and (54) are given as follows:

$$w_0(r) = 1 + \Psi_{10} \ln r \quad (59)$$

$$w_1(r) = \frac{\Psi_{11}}{r^2} + \Psi_{12} + \Psi_{13} \ln r \quad (60)$$

$$w_2(r) = \frac{\Psi_{14}}{r^4} + \frac{\Psi_{15}}{r^2} + \Psi_{16} + \Psi_{17} \ln r \quad (61)$$

where $\Psi_{10}, \Psi_{11}, \Psi_{12}, \Psi_{13}, \Psi_{14}, \Psi_{15}, \Psi_{16}$ and Ψ_{17} are constants which holding the auxiliary constants C_1 and C_2 are given in appendix.

Now since the second order approximation is:

$$w(r) = w_0(r) + w_1(r) + w_2(r) \quad (62)$$

the second order approximate solution to velocity distribution is given by:

$$w(r) = \frac{\Psi_{14}}{r^4} + \frac{1}{r^2}(\Psi_{11} + \Psi_{15}) + (\Psi_{12} + \Psi_{16} + 1) + (\Psi_{13} + \Psi_{17}) \ln r \quad (63)$$

Finally, we solve the energy Equation (21) with respect to the boundary conditions (22). For this we substitute equation (63) in Equation (21) and integrate twice, we obtain the temperature distribution function as follows:

$$\Theta(r) = \frac{\tau_0}{r^{18}} + \frac{\tau_{10}}{r^{16}} + \frac{\tau_{11}}{r^{14}} + \frac{\tau_{12}}{r^{12}} + \frac{\tau_{13}}{r^{10}} + \frac{\tau_{14}}{r^8} + \frac{\tau_{15}}{r^6} + \frac{\tau_{16}}{r^4} + \frac{\tau_{17}}{r^2} + \tau_{18} + \tau_{19} \ln r \quad (64)$$

where $\tau_0, \tau_{10}, \tau_{11}, \tau_{12}, \tau_{13}, \tau_{14}, \tau_{15}, \tau_{16}$ and Ψ_{17} are constants which involve the auxiliary constants C_1 and C_2 are given in appendix.

RESULTS AND DISCUSSION

In this paper, approximate analytical solutions for the fluid velocity and temperature distribution have been found for the heat transfer flow of a third grade fluid for the posttreatment of wire coating. The governing non-linear ordinary differential equations are solved using the traditional perturbation method as well as the recently introduced optimal homotopy asymptotic method and the results are compared. The graphs for the functions $w(r)$ and $\Theta(r)$ are plotted against r in Figures 3 to 8. It is found in Figures 3 and 6 that PM and OHAM are in

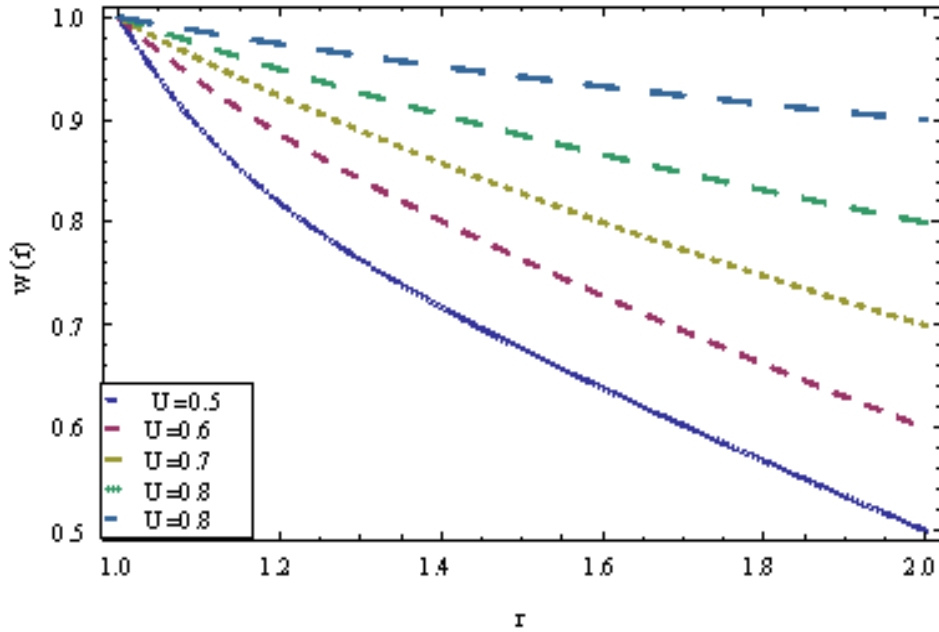


Figure 4. Velocity profile for different values of the velocities ratio U for $\beta = 0.01$.

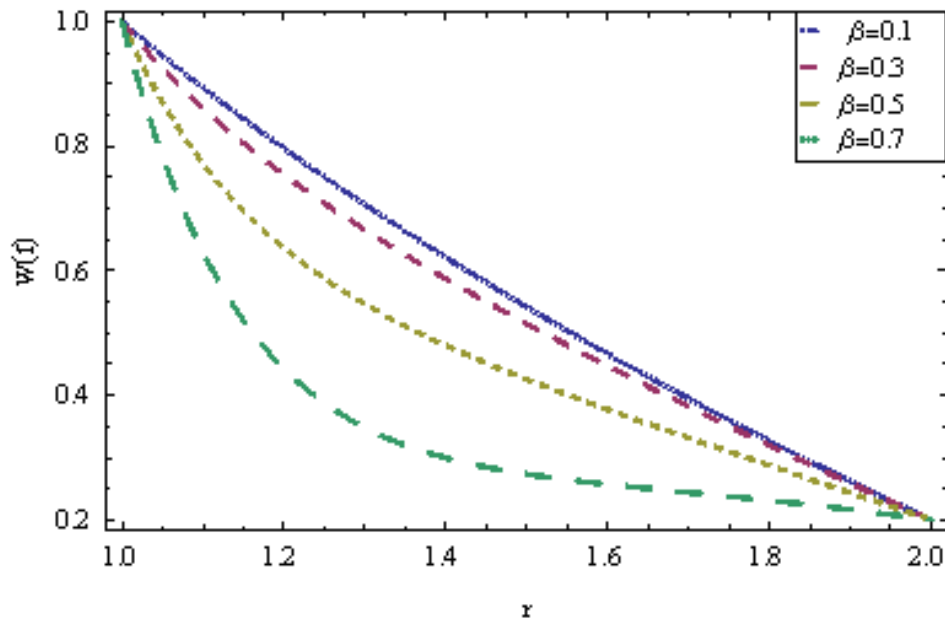


Figure 5. Velocity profile for different values of the dimensionless parameter β fixing $U = 0.2$.

good agreement for small values of β ($\beta \rightarrow 0$). But since PM can not be applicabale for large values of β and for highly non-linear problems while; OHAM works also for large values of the non-Newtonian parameter β

and non-highly non-linear problems (Islam et al., 2010; Javed et al., 2010). As the polymer applied to wire coating is considered here to be the third grade fluid, so the non-Newtonian parameter β can not approaches to zero physically, therefore OHAM can better capture the

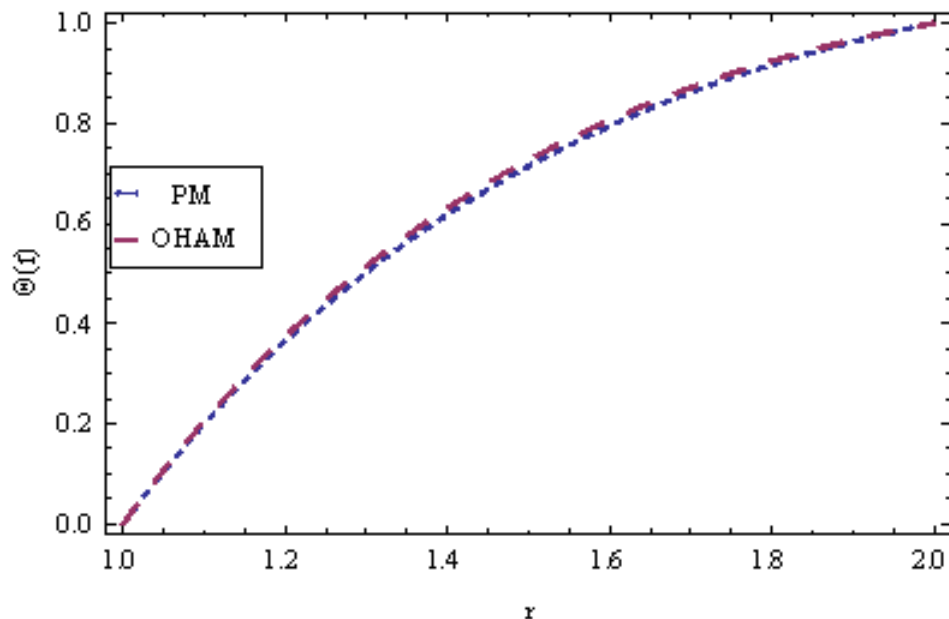


Figure 6. Comparison of dimensionless temperature distribution using PM and OHAM, when $U = 0.7$, $\beta = 0.01$, $Br = 10$, $C_1 = -0.001473286$, $C_2 = -0.0002569261$.

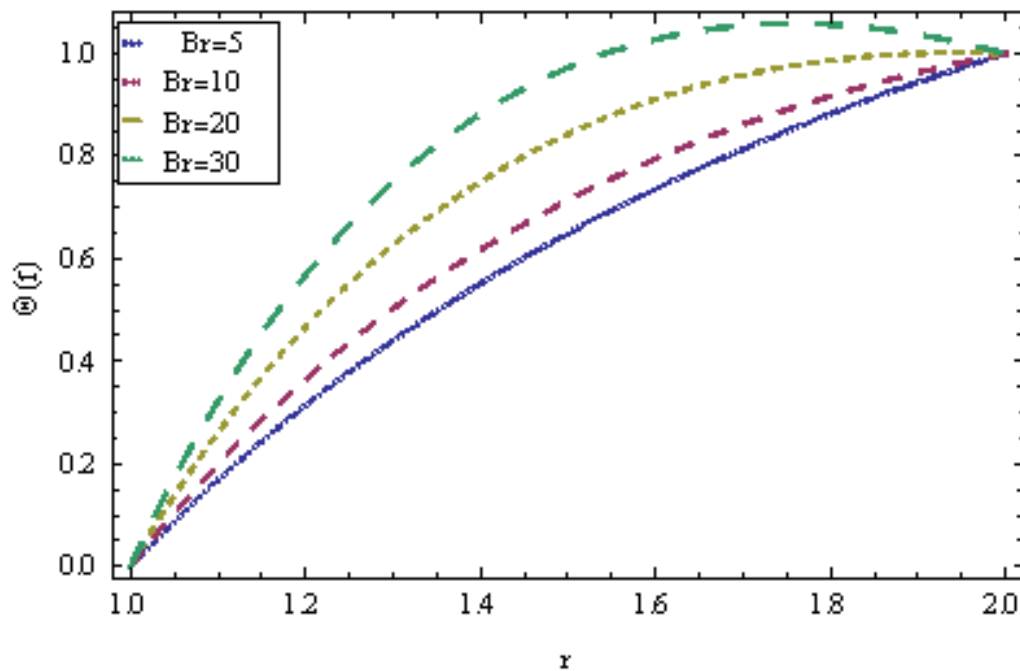


Figure 7. Temperature distribution for different values of Brinkman number, taking $U = 0.7$.

demand solution for this physical problem. Figure 4 shows the variation of velocity field $w(r)$. It is observed that the fluid velocity increases as the velocity ratio U increases and vice versa. Figure 5 is displayed for the

variation of the fluid velocity. It is observed that the velocity decreases as the value of the dimensionless parameter β increases by keeping U and Br fixed. Figure 7 indicates that the temperature distribution

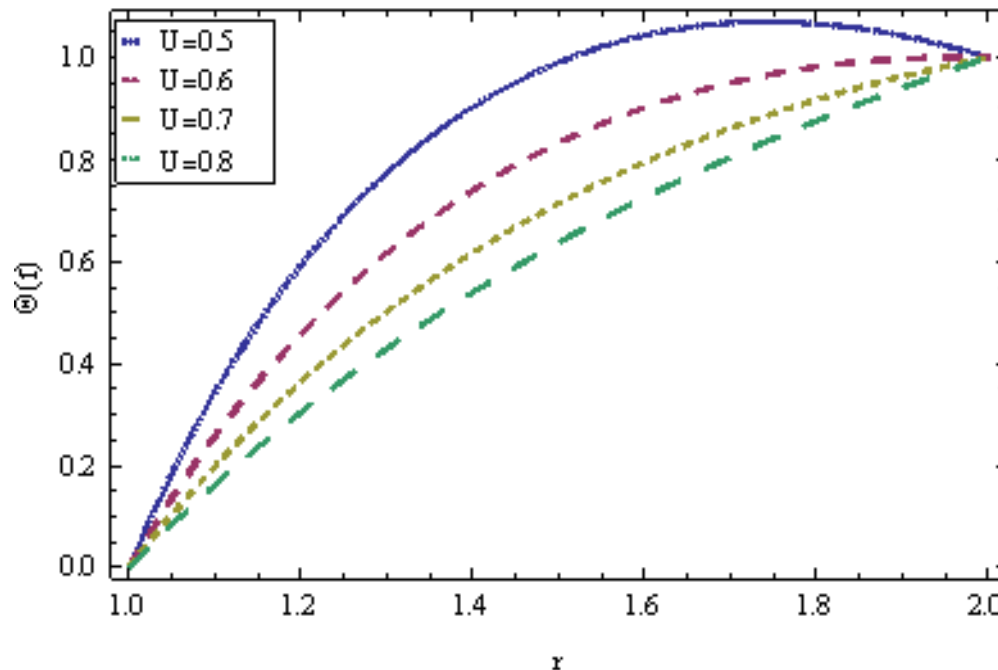


Figure 8. Temperature distribution for different values of the velocities ratio U , taking Brinkman $Br = 10$.

increases with an increase in the Brinkman number Br , while Figure 8 admits that temperature distribution decreases as the velocity ratio increases.

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APPENDIX

$$\Psi_{10} = \frac{1}{\ln \delta}(U - 1), \Psi_{11} = -\Psi_{10}^3 \beta C_1, \Psi_{12} = \Psi_{10}^3 \beta C_1, \Psi_{13} = \frac{1}{\delta^2 \ln \delta}(1 - C_1 \delta^2) \Psi_{10}^3 \beta$$

$$\Psi_{14} = 3\Psi_{10}^2 \Psi_{11} \beta C_1, \Psi_{15} = (1 + C_1) \Psi_{11} - 3\Psi_{10}^2 \Psi_{13} \beta C_1 - \Psi_{10}^3 \beta C_2$$

$$\Psi_{16} = 3\Psi_{10}^2 \Psi_{13} \beta C_1 - (1 + C_1) \Psi_{11} - 3\Psi_{10}^2 \Psi_{11} \beta C_1 + \Psi_{10}^3 \beta C_2$$

$$\Psi_{17} = \frac{1}{\delta^2 \ln \delta} \left((\delta^2 - 1)(\Psi_{11}(1 + C_1) - 3\Psi_{10}^2 \Psi_{13} \beta C_1 - \Psi_{10}^3 \beta C_2) - \frac{3}{\delta^2} \Psi_{10}^2 \Psi_{11} \beta C_1 + 3\Psi_{11} \Psi_{10}^2 \delta^2 \beta C_1 \right)$$

$$\tau_0 = -1.56\Psi_{14}^4 (\Psi_{10} + \Psi_{13} + \Psi_{17}) \beta \lambda, \tau_{10} = -4\Psi_{14}^3 (\Psi_{11} + \Psi_{15}) \beta \lambda$$

$$\tau_{11} = -3.918\Psi_{14}^2 (\Psi_{11} + \Psi_{15})^2 \beta \lambda + 0.261\Psi_{14}^3 (\Psi_{10} + \Psi_{13} + \Psi_{17}) \beta \lambda$$

$$\tau_{12} = -1.77\Psi_{14} (\Psi_{11} + \Psi_{15})^3 \beta \lambda + 5.33\Psi_{14}^2 (\Psi_{11} + \Psi_{15}) (\Psi_{10} + \Psi_{13} + \Psi_{17}) \beta \lambda$$

$$\tau_{13} = -0.32(\Psi_{11} + \Psi_{15})^4 \beta \lambda + 0.368\Psi_{14} (\Psi_{11} + \Psi_{15})^2 (\Psi_{10} + \Psi_{13} + \Psi_{17}) \beta \lambda \\ - 1.92\Psi_{14}^2 (\Psi_{10} + \Psi_{13} + \Psi_{17})^2 \beta \lambda$$

$$\tau_{14} = -0.25(\Psi_{11} + \Psi_{15})^2 \lambda + (\Psi_{11} + \Psi_{15})^3 (\Psi_{10} + \Psi_{13} + \Psi_{17}) \beta \lambda \\ - 3\Psi_{14} (\Psi_{11} + \Psi_{15}) (\Psi_{10} + \Psi_{13} + \Psi_{17})^2 \beta \lambda$$

$$\tau_{15} = -0.454\Psi_{14} (\Psi_{11} + \Psi_{15}) \lambda - 1.33(\Psi_{11} + \Psi_{15})^2 (\Psi_{10} + \Psi_{13} + \Psi_{17})^2 \beta \lambda \\ + 0.88\Psi_{14} (\Psi_{10} + \Psi_{13} + \Psi_{17})^3 \beta \lambda$$

$$\tau_{16} = -0.25(\Psi_{11} + \Psi_{15})^2 \lambda + 0.5\Psi_{14} (\Psi_{10} + \Psi_{13} + \Psi_{17}) \beta \lambda \\ + (\Psi_{11} + \Psi_{15}) (\Psi_{10} + \Psi_{13} + \Psi_{17})^3 \beta \lambda$$

$$\tau_{17} = (\Psi_{11} + \Psi_{15}) (\Psi_{10} + \Psi_{13} + \Psi_{17}) \lambda - 0.5\Psi_{14} (\Psi_{10} + \Psi_{13} + \Psi_{17})^4 \beta \lambda$$

$$\tau_{18} = 0.25\Psi_{14}^2 \lambda + 0.44\Psi_{14} (\Psi_{11} + \Psi_{15}) \lambda + 0.25(\Psi_{11} + \Psi_{15})^2 \lambda - 0.5\Psi_{14} (\Psi_{10} + \Psi_{13} + \Psi_{17}) \lambda \\ - (\Psi_{11} + \Psi_{15}) (\Psi_{10} + \Psi_{13} + \Psi_{17}) \lambda + 1.58\Psi_{14}^4 \beta \lambda + 4\Psi_{14}^3 (\Psi_{11} + \Psi_{15}) \beta \lambda \\ + 3.918\Psi_{14}^2 (\Psi_{11} + \Psi_{15})^2 \beta \lambda + 1.77\Psi_{14} (\Psi_{11} + \Psi_{15})^3 \beta \lambda + 0.32(\Psi_{11} + \Psi_{15})^4 \beta \lambda \\ - 2.61\Psi_{14}^3 (\Psi_{10} + \Psi_{13} + \Psi_{17}) \beta \lambda - 5.33\Psi_{14}^2 (\Psi_{11} + \Psi_{15}) (\Psi_{10} + \Psi_{13} + \Psi_{17}) \beta \lambda \\ - 3.84\Psi_{14} (\Psi_{11} + \Psi_{15})^2 (\Psi_{10} + \Psi_{13} + \Psi_{17}) \beta \lambda - (\Psi_{11} + \Psi_{15})^3 (\Psi_{10} + \Psi_{13} + \Psi_{17}) \beta \lambda \\ + 1.92\Psi_{14}^2 (\Psi_{10} + \Psi_{13} + \Psi_{17})^2 \beta \lambda + 3\Psi_{14} (\Psi_{11} + \Psi_{15}) (\Psi_{10} + \Psi_{13} + \Psi_{17})^2 \beta \lambda \\ + 1.33(\Psi_{11} + \Psi_{15})^2 (\Psi_{10} + \Psi_{13} + \Psi_{17})^2 \beta \lambda - 0.88\Psi_{14} (\Psi_{10} + \Psi_{13} + \Psi_{17})^2 \beta \lambda \\ - (\Psi_{11} + \Psi_{15}) (\Psi_{10} + \Psi_{13} + \Psi_{17})^3 \beta \lambda + 0.5(\Psi_{10} + \Psi_{13} + \Psi_{17})^4 \beta \lambda$$

$$\begin{aligned}
\tau_{19} = & \frac{1}{\ln \delta} \left(1 + \left(1 - \frac{1}{\delta^2} \right) JK \lambda - 0.5 \left(1 - \frac{1}{\delta^2} \right) K^4 \beta \lambda + \left(1 - \frac{1}{\delta^4} \right) JK^3 \beta \lambda + 0.5 \left(1 - \frac{1}{\delta^4} \right) \Psi_{14} K \lambda \right. \\
& - 0.25 \left(1 - \frac{1}{\delta^4} \right) J^2 \lambda + 0.88 \left(1 - \frac{1}{\delta^6} \right) \Psi_{14} K^3 \beta \lambda - 1.33 \left(1 - \frac{1}{\delta^6} \right) J^2 K^2 \beta \lambda - 0.44 \left(1 - \frac{1}{\delta^6} \right) \Psi_{14} J \lambda \\
& - 3 \left(1 - \frac{1}{\delta^8} \right) \Psi_{14} JK^2 \beta \lambda + \left(1 - \frac{1}{\delta^8} \right) J^3 K \beta \lambda - 0.25 \left(1 - \frac{1}{\delta^8} \right) \Psi_{14}^2 \lambda - 1.92 \left(1 - \frac{1}{\delta^{10}} \right) \Psi_{14}^2 K^2 \beta \lambda \\
& + 3.84 \left(1 - \frac{1}{\delta^{10}} \right) \Psi_{14} J^2 K \beta \lambda - 0.32 \left(1 - \frac{1}{\delta^{10}} \right) \Psi_{14} J^4 \beta \lambda + 5.33 \left(1 - \frac{1}{\delta^{12}} \right) \Psi_{14}^2 JK \beta \lambda \\
& - 1.77 \left(1 - \frac{1}{\delta^{12}} \right) \Psi_{14} J^3 \beta \lambda + 2.61 \left(1 - \frac{1}{\delta^{14}} \right) \Psi_{14}^3 K \beta \lambda - 3.91 \left(1 - \frac{1}{\delta^{14}} \right) \Psi_{14}^2 J^2 \beta \lambda \\
& \left. - 4 \left(1 - \frac{1}{\delta^{16}} \right) \Psi_{14}^3 J \beta \lambda \right)
\end{aligned}$$

where $J = \Psi_{11} + \Psi_{15}$ and $K = \Psi_{10} + \Psi_{13} + \Psi_{17}$.