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Full Length Research Paper

An evolutionary multi-objective optimization algorithm for portfolio selection problem

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Cultural algorithms (CAs) are one of the metaheuristics which can be adapted in order to work in multi-objective optimization environments. On the other hand, portfolio selection problem (PSP) is a well-know problem in literature. However, only a few articles have applied evolutionary multi-objective (EMO) algorithms to these problems and articles presenting CAs applied to the PSP have not been found. In this article, we present a bi-objective cultural algorithm (BOCA) which has been applied to the PSP, and obtaining acceptable results in comparison with other well-known EMO algorithms from the literature. The considered criteria of the problem are risk minimization and profit maximization. The different solutions obtained with the BOCA have been compared using max-delta-area metric.

Key words: Constraint programming, autonomous search, heuristic search.

INTRODUCTION

In order to solve several complex optimization problems, the evolutionary algorithms have become an efficient and effective choice for researchers principally because this kind of techniques are capable to find good solutions for most of these problems in acceptable computational times. Hence, it is reasonable to think that if these algorithms are used in this context we can reach good solutions in a multi-objective environment, in a similar way as in mono-objective optimization problems. In fact, several researches in the last two decades have developed this idea. However, the difficulties that these algorithms present (particularly genetic algorithms) in a mono-objective environment could also happen in multi-objective environments. Specifically, they easily fall into premature convergence with low evolution efficiency because the implicit information embodied in the evolution process and domain knowledge corresponding to optimization problems is not fully used (Guo et al., 2010). In order to effectively make use of implicit evolution information, Reynolds (1994) proposed Cultural algorithms (CAs) which were inspired from human culture evolution process.

CAs has a dual evolution structure which consists in both spaces: population and belief space. The population space works as any other evolutionary algorithm. However, the belief space implicit knowledge is extracted from better individuals in the population and stored in a different way. Then, they are used to guide the evolutionary process in the population space so as to induce population escaping from the local optimal solutions. It has been proved that CAs can effectively improve the evolution performance. Furthermore, the algorithms also provide a universal model for extraction and utilization of the evolution information (Guo et al., 2010).

In the last 20 years, several authors have centered their efforts in the development of several EMO algorithms in order to solve a specific group of problems

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Abbreviations: CAs, Cultural algorithms; **PSP,** portfolio selection problem; **EMO,** evolutionary multi-objective; **BOCA,** bi-objective cultural algorithm; **EVL,** extended virtual loser; **PF,** Pareto frontier; **MOEA,** multi-objective evolutionary algorithms.

which are called Multi-objective or, generalizing, Multi-Criteria. Maravall and De Lope (2007) used a genetic algorithm in order to solve the multi-objective dynamic optimization for automatic parking system, Deb et al. (2000) proposed an improvement to the well-known NSGA algorithm (and that they called NSGA-II) based on an elitist approach, and Borgulya (2008) presents an EMO algorithm applied for a specific variation of the well-studied CVRP, where he includes in the EMO algorithm an explicit collective memory method, namely the extended virtual loser (EVL). Bhattacharya and Bandyopadhyay (2010) reported a complete and extensive literature review related with EMO. However, where the CA is applied to PSP was found in literature articles, In fact, some recent published books (Coello et al., 2007, 2010; Coello and Landa, 2003) only mention as an example of CA application to solve MOPs.

In this article, we present a bi-objective cultural algorithm (BOCA) which has been applied to the portfolio selection problem (PSP) and it obtains an important improvement in comparison with other well-known EMO algorithms as P-MOEA and E-MOEA presented in Branke (2009). The criterion considered in this work included both, risk (in order to minimize it) and profit (in order to maximize it). The different solutions obtained with the CA were compared using a max-delta-are which was derived from hypervolume *S* metric proposed in Knowles and Corne (2002) and used in several works in the literature.

Evolutionary (Multi-objective) optimization

Here, we briefly introduce the main principles of MOP and particularly, MOCO problems. The following definitions were extracted from Kaliszewski (2006).

Given a set of alternatives, a feasible alternative x is called dominated if there is another feasible alternative in the set, say alternative x', such that:

i) x' is equally or more preferred than x with respect to all criteria, and
ii) x' is more preferred than x for at least one criterion

If the above holds, the alternative x' is called dominating. A pair of alternatives x and x', where x is dominated and x' in dominating, is said to be in Pareto dominance relation and is denoted by $x' \preceq x$. In a set of more than two alternatives, one alternative can be dominating and dominated at the same time.

Given a set of feasible alternatives, one which is not dominated by any other alternative of this set is called efficient. In other words, an alternative is efficient if there is no other alternative in the set:

i) equally or more preferred with respect to all criteria, and
ii) more preferred for at least one criterion

Alternatives which are not efficient are called nonefficient.

The Pareto optimal set $\left(P^*\right)$ is composed of the feasible solutions which are not dominated by any other solution. Therefore,

$$P^* = \left\{ x \in \Omega : there\ is\ no\ x' \in \Omega, x' \preceq x \right\}$$

The Efficient (or Pareto) Frontier $\left(PF^*\right)$ is the image of the Pareto optimal set in the objective space, that is;

$$PF^* = \left\{ f(x) = \left(f_1(x), \ldots\ldots, f_k(x) \right) : x \in P^* \right\}$$

As said above, many authors have worked in order to solve different MOCO problems. Particularly, EMO is an important research area for this goal. An evolutionary algorithm is a stochastic search procedure inspired by the evolution process in nature. In this process, individuals evolve and the fitter ones have a better chance of reproduction and survival. The reproduction mechanisms favor the characteristics of the stronger parents and hopefully produce better children guaranteeing the presence of those characteristics in future generations (Villegas et al., 2006). A complete review about different EMO algorithms is presented in Zitzler (1999) and Coello et al. (2007, 2010).

Portfolio selection problem (PSP)

The PSP was presented by Markowitz (1952). He proposes an asset selection method for portfolios which considers the conduct that an investor does (or should) have. This conduct consist in searching the combination where portfolio maximize its returns for a given risk level or portfolio minimize its risk for an expected returns. From Markowitz mean-variance model, several extensions have been proposed in the literature. Crama and Schyns (2003) showed an extended list of several variations to the model and different solving approaches of the PSP. DiTollo and Rolli (2006) proposed three additional constraints to the model: cardinality, min-max constraint for assets of the portfolio and minimum size of the transaction. However, the inclusion of these constraints makes the problem intractable.

The main problem of the PSP is to solve two conflicting optimization criteria: On one hand the risk of a portfolio, represented by its variance is to be minimized, while on

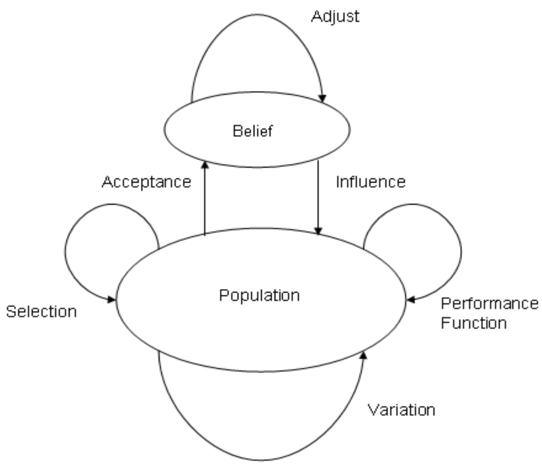

Figure 1. Spaces of a cultural algorithm.

the other hand the expected return of the portfolio is to be maximized. The model formulation for PSP is presented as follows:

$$\min \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij} \qquad 1$$

$$\max \sum_{i=1}^{N} w_i \mu_i \qquad 2$$

$$s.t. \sum_{i=1}^{N} w_i = 1, \quad w_i \geq 0, \quad i = 1, 2, \ldots, N \qquad 3$$

Where N corresponds to the number of available assets, w_i represents the investment portion in asset $i \in \{1, 2, ..., N\}$; where $w = \{w_1, ..., w_N\} \in \Re^N$ is the N-dimensional solution vector; μ_i corresponds to the expected return of asset $i \in \{1, 2, ..., N\}$; σ_{ij} corresponds to the covariance between the returns of assets $i, j \in \{1, 2, ..., N\}$, and $\sigma = (\sigma_{ij})_{i=1,.....,N; j=1,.....,N}$ denotes the corresponding $N \times N$ covariance matrix.

As said above, other approaches have been proposed in the literature, for example, Loraschi and Tettamanzi (1996), Li et al. (2001) and Lin and Liu (2006); however, in this article, we solved the well-known Markowitz mean-variance model presented in Equations 1, 2 and 3.

Bi-objective cultural algorithm (BOCA)

The experience and beliefs accepted by a community in a social system are the main motivations for the creation of CAs. They were developed by Reynolds (1994), in order to model the evolution of cultural systems based on the principles of human social evolution from the literature of social sciences, who believe that the evolution can be seen as an optimization process. The CAs are identified to guide the evolution of the population based on the knowledge. This applies to the knowledge provided to future generations, allowing them to accelerate the convergence of the algorithm to obtain good solutions (Reynolds, 1999). Besides, the domain knowledge is modeled separately from the population, because there is certain independence between both, which allow to work and to model separately each one of them, in order to enhance the overall algorithm and thus, improving the search for best solutions. Figure 1 shows this interaction.

CAs are mainly characterized by presenting two inheritance systems: at the population level and knowledge. This key feature is designed to increase the learning rates and convergence of the algorithm, and thus to do a more responsive system for a number of problems (Landa and Coello 2004). This feature identifies two significant levels of knowledge: a micro-evolutionary level (represented by the area of population) and macro-evolutionary level (represented by the space of beliefs) (Soza et al., 2007). CAs have the following components: population space (set of individuals who have independent features) (Soza et al., 2007); belief space (stored knowledge individuals have acquired in previous generations) (Landa and Coello, 2004); computer protocol, connecting the two spaces and defining the rules on the type of information to be exchanged between the spaces, by using the acceptance and influence function; and finally, knowledge sources which are described in terms of their ability to coordinate the distribution of individuals on the nature of an instance of a problem. These knowledge sources can be of the following types (Kobti, 2004): circumstantial, normative, domain, topographic, historical.

General structure of BOCA and its principal functions and operators are presented in appendix 1. In this article, we choose a continuous representation. Each individual in the population represents an alternative portfolio and is implemented by an N-length vector (with *N* a number of possible assets) where each position is in a range (0, 1). In order to treat the unfeasible individuals in the population, a normalization rule has been implemented which allows tp to keep the ratio between the weights of each asset and at the same time satisfy the constraint of the problem (Equation 3). Then, the new weight for each asset in the unfeasible individual is calculated as follows:

$$w_j = \frac{w_j}{\sum_{i=1}^{N} w_i} \qquad 4$$

The initial population is generated using a random function for each weight. Once the population has been created, the repair function is used in order to ensure the feasibility of all individuals in the population. This is a necessary evaluation criterion In order to compare the two different solutions (individuals). As previously mentioned, this criterion is difficult to define due to the multi-objective nature of the environment. Given the dominance concept explained in the introductory part of the study, many individuals are not comparable only using the objective functions of the problem. To solve this problem, we propose a grid which defines areas where they are located to different non-dominated solutions. The boundaries of this grid will be defined by the lower and upper bounds of each objective function, which will be updated every few generations. This update is not done by every generation because is very expensive in CPU time and the improvements are marginal. This grid is divided into 8 columns and 8 rows. Figure 2 shows the proposed network. Where, lb_1 is the lower bound of function z_1; lb_2

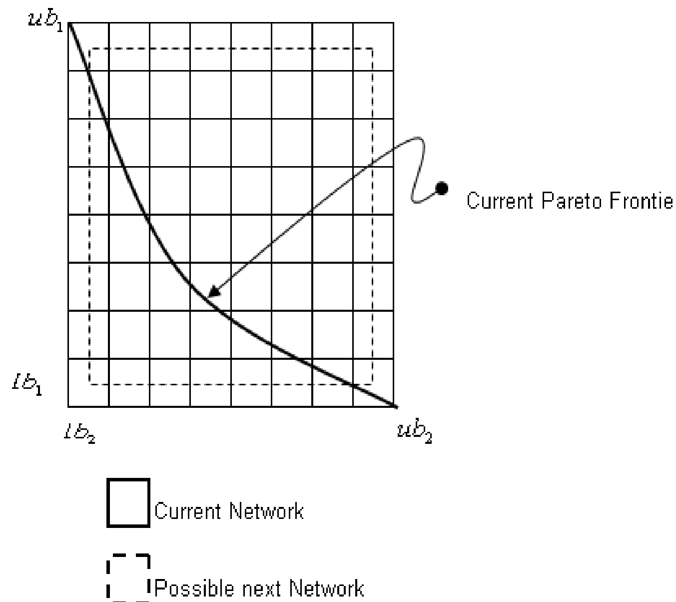

Figure 2. Classification network.

Table 1. Parameters of BOCA algorithm.

Parameter	Description
N	# of variable decision.
K	# assets to include in the portfolio
SizaPop	Size population
G	Number of generation
pMut	Mutation Probability
Gnorm	Number of generation before update the normative knowledge (boundaries of the grid)

is the lower bound of function z_2. Finally, ub_1 and ub_2 are the upper bounds of z_1 and z_2 function, respectively.

The most distinctive feature of CAs is the integration of knowledge, which through an influence function affects future generations. In this paper, the influence function that was used is based on normative knowledge. This normative knowledge keeps the upper and lower bounds raised for the maximization and minimization function respectively in past population. The initialization of the beliefs space is done after the first generation, because the candidates for this space are the non-dominated individuals. Van Veldhuizen (1999) reported that for any population, there is always an individual that is not dominated by the rest of the population.

The influence of belief space occurs when generating the next population. At that time the genetic information of the new generation should be influenced not only by the individuals of the current generation (such as genetic algorithms) but also by individuals who reside in the belief space. The tournament selection proposed in Landa (2002) was used, which apply the following rules:

i) If $x' \preceq x$, x' is selected

ii) If are not comparable

iii) If $x \, and \, x'$ are located into the grid boundaries we select to the individual that is located in the less populated cell in the grid.

iv) If $x \, and \, x'$ are located into the grid boundaries we select to the individual that is located in the less populated cell in the grid.

v) If x is within the limits of the grid and x' is outside these limits, we select x'

The algorithm finishes when the total generation number is raised. Table 1 shows the parameters of our BOCA algorithm.

NON DOMINATED SETS (NDSs): METRICS

Metrics for comparing different NDSs

One of most important problems in MOCO is how to compare two NDSs. Numerous quality assessment metrics have been developed by researchers to compare the performance of different MOEA. These metrics show different properties and address various aspects of solution set quality (Farhang-Mehr and Azarm, 2003).

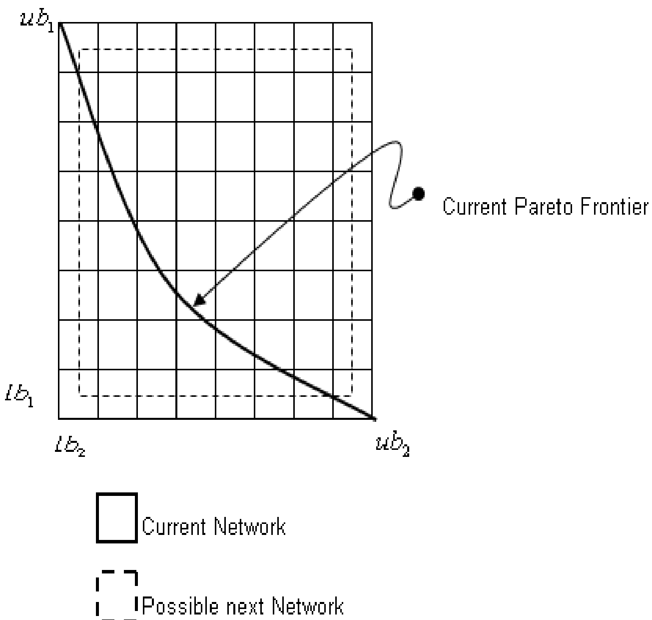

Figure 2. Classification network.

Farhang-Mehr and Azarm (2003) describes several excellence relations. These relations establish strict partial orders in the set of all NDSs with respect to different aspects of quality. Previously, Hansen and Jaszkiewicz (1998) and Zitzler (1999) consider several outperformance relations to address the closeness of NDSs to the Pareto frontier (PF). Besides the above, is also necessary to measure in a quantitative way the approximation to PF. In this way, we need to identify desirable aspects of NDSs. Zitzler et al. (2000) define those desirable aspects:

1. The distance of the resulting non-dominated set to the Pareto-optimal frontier should be minimized.
2. A good (in most cases uniform) distribution of the solutions found is desirable. The assessment of this criterion might be based on a certain distance metric.
3. The extension of the obtained non-dominated frontier should be maximized, that is, for each objective a wide range of values should be covered by the non-dominated solutions.

The main problem of these is that the three criteria should be combined in different ways to establish the performance of an EMO algorithm in a quantitative way. However, such combinations are simply linear combinations of weights.

In this article, we choose the max-delta-area which derives from the *S* metric. Zitzler (1999) shows the formal definition for the *S* metric can be found. This metric calculates the hyper-volume of the multi-dimensional region (Knowles and Corne, 2002) and allows the integration of aspects that are individually measured by other metrics. In this article, the total area is bounded by lb_1, lb_2, ub_1 and ub_2 previously defined. It is easy to note that, given the mentioned area, the ideal point (maximum coverage at minimum cost) will have a value for the metric equivalent to $S = 1$ (equivalent to 100% of the area). An advantage of the S metric is that each MOEA can be assessed independently of the other MOEAs. However, the S values of two sets A, B cannot be used to derive whether either set entirely dominates the other. Fortunately, this disadvantage does not affect the metric used in this article.

Table 2. Adjust Instances from OR-Library.

Instance	Source	Assets
port1	HongKong	31
port2	Germany	85
port3	UK	89
port4	USA	98
port5	Japan	225

EXPERIMENTAL RESULTS

The results obtained and the main characteristics of our BOCA implementation are discussed. We presents the adjust parameter phase and its results. Following, we show the results for the definitive instances extracted from OR-Library (Beasley, 1990) and the comparison with the results obtained by Branke (2009). Our implementation has been written in ANSI C. Tests have been performed on a 1.86 GHz Intel Core2 Duo with 2GB RAM running Windows XP.

Parameter adjust

The instances used in this article in order to adjust were extracted from the OR-Library (Beasley, 1990). Table 2 shows the main characteristics of these instances. Some results related with this adjust phase are showed in Figure 3. Figure 3a illustrates the improvement on the average return when the size of population is incremented. This effect can be observed for three tested instances. Figure 3b shows how the increment in population size influences the mean risk. If we consider only both port2 and port5 instances we can said that this increment decreases the medium risk obtained. However, for instance port1 this effect is different, because the increment in population size increases the medium risk obtained. This situation may be explained by the size of the problem or the K used value.

Figure 3c depicts the mean size of NDS. It is clear that the increment on the population size allows increment the size of NDS. However, this situation does not ensure that the diversity or covering of this NDS will be better. Finally, Figure 3d shows the execution time necessary for each population size. As occurs in other evolutionary algorithms, the execution time increases when the population size does it too.

Figure 4a illustrates the improvement on the average return when the *g* value increases. This effect can be observed for three tested instances, however is important to note that this increment is marginal after g=100. Figure 4b shows how the increment in *g* value influences the

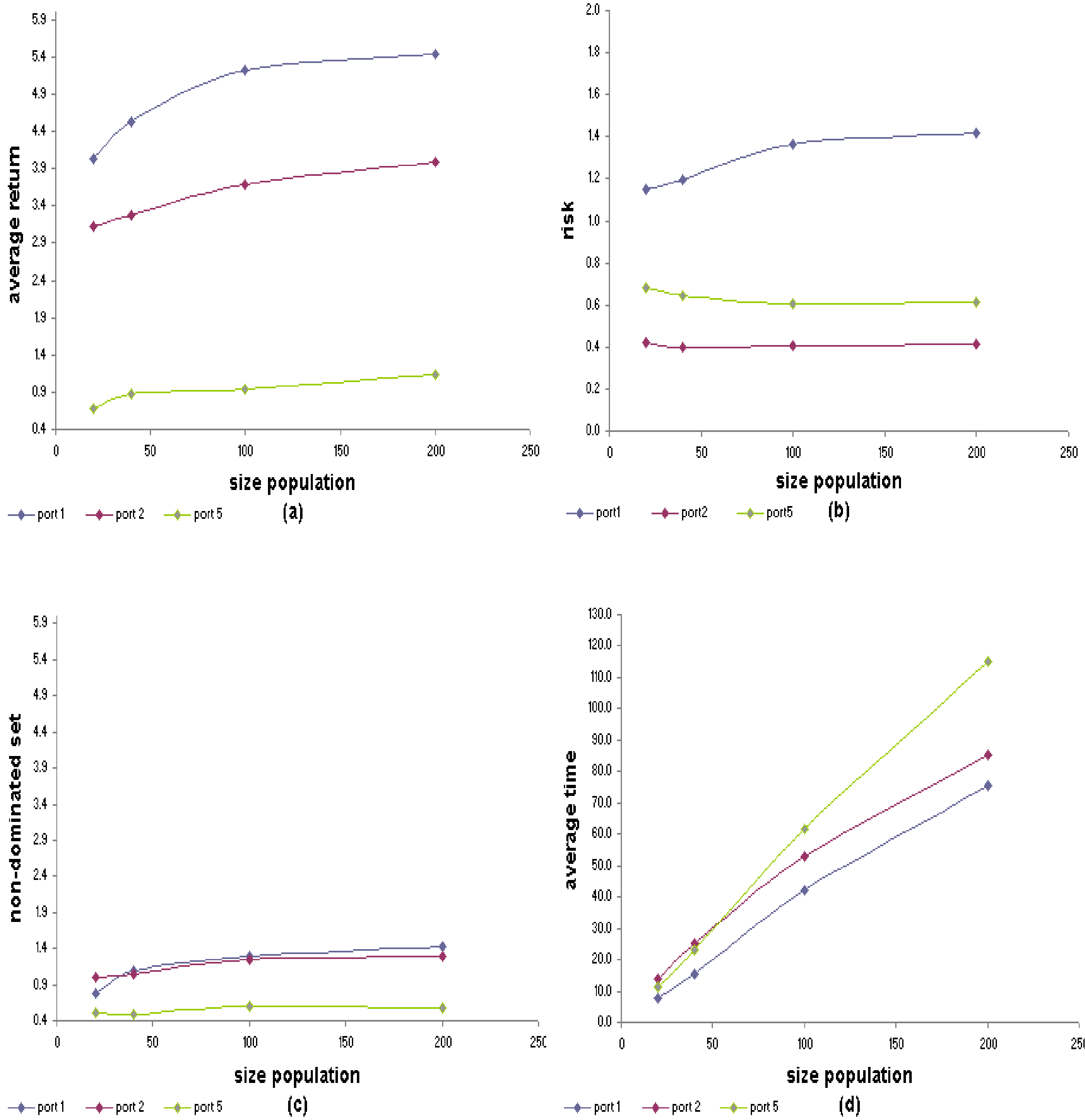

Figure 3. Adjust of population size (s*izePop)* parameter.

mean risk. If we consider only both port2 and port5 instances we can say that this increment decreases the medium risk obtained. However, for instance port1, this effect is different because the increment in g value increases the medium risk obtained. The above is similar to the effect in the Figure 3b.

Figure 4c depicts the mean size of NDS. It is clear that the increment on the population size allows increment the size of NDS. As we said above, this situation does not ensure that the diversity or covering of this NDS will be better. Finally Figure 4 (d) shows the execution time necessary for each value of g. As occurs in other evolutionary algorithms, the execution time increases when the number of generation does it too. Naturally, the

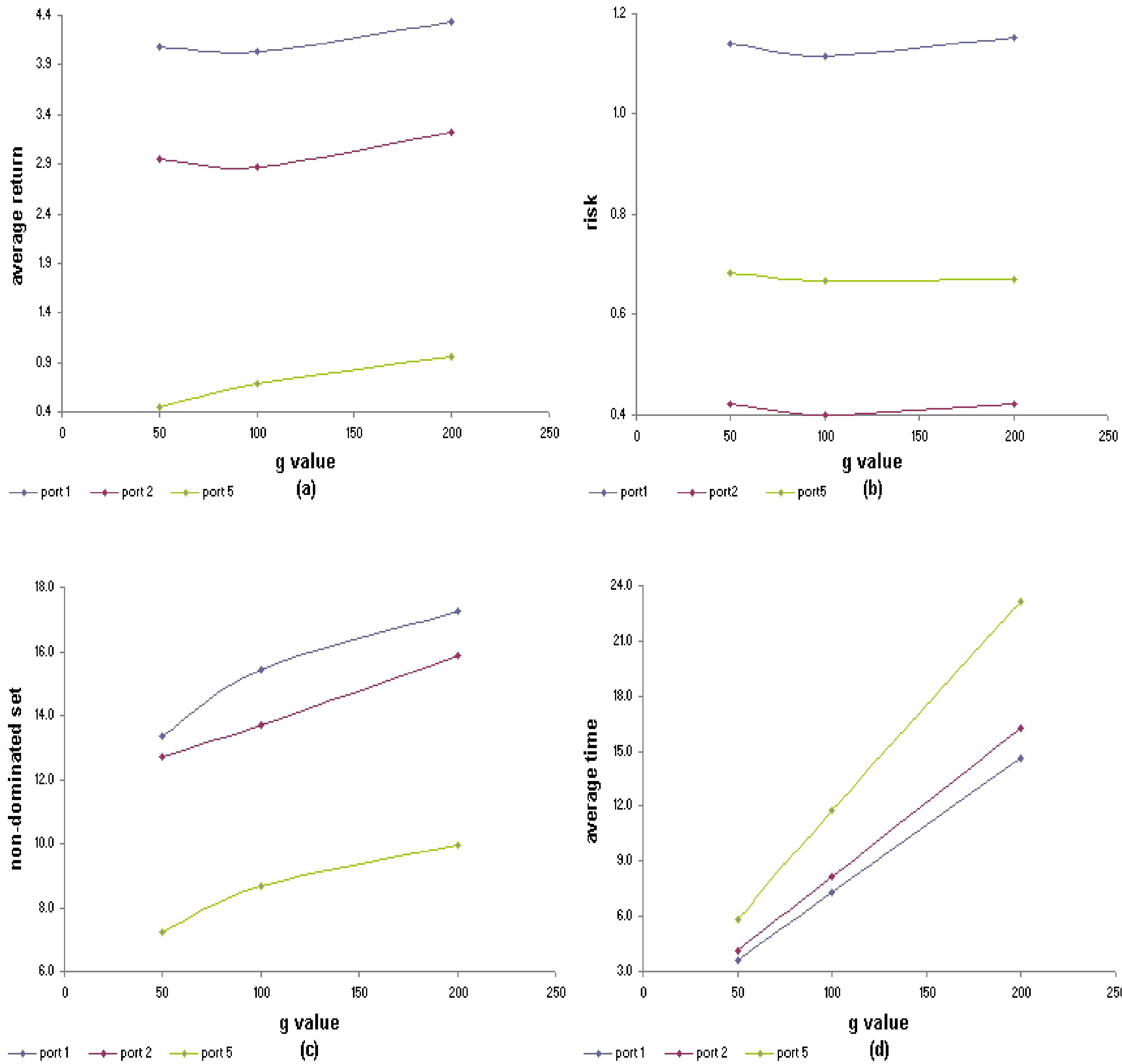

Figure 4. Adjust number of generation (*g)* parameter.

behavior of BOCA algorithm is too similar in term of growth in both size population and number of generation. This behavior generates a tradeoff between the increases in both size population and g value and execution time. For this reason, in order to minimize the execution time of our BOCA, the size population used for experiments was fixed in 40, and g value was fixed in 100 generations.

In order to adjust the mutation probability (*pMut* parameter) we consider only two instances: port1 and port2. The values for *pMut* that were used in the test are: 0.2; 0.5; 0.8. 1s 5a and b showed that the influence of the *pMut* value in each objective function separately is marginal. However, Figure 5c illustrates that when the *pMut* value increases, the size of NDS does it too. This situation can be explained because the mutation functions improve the exploration level of the algorithm. Finally, Figure 5d shows that the increases in the *pMut* value are not relevant w.r.t. the execution time. By above,

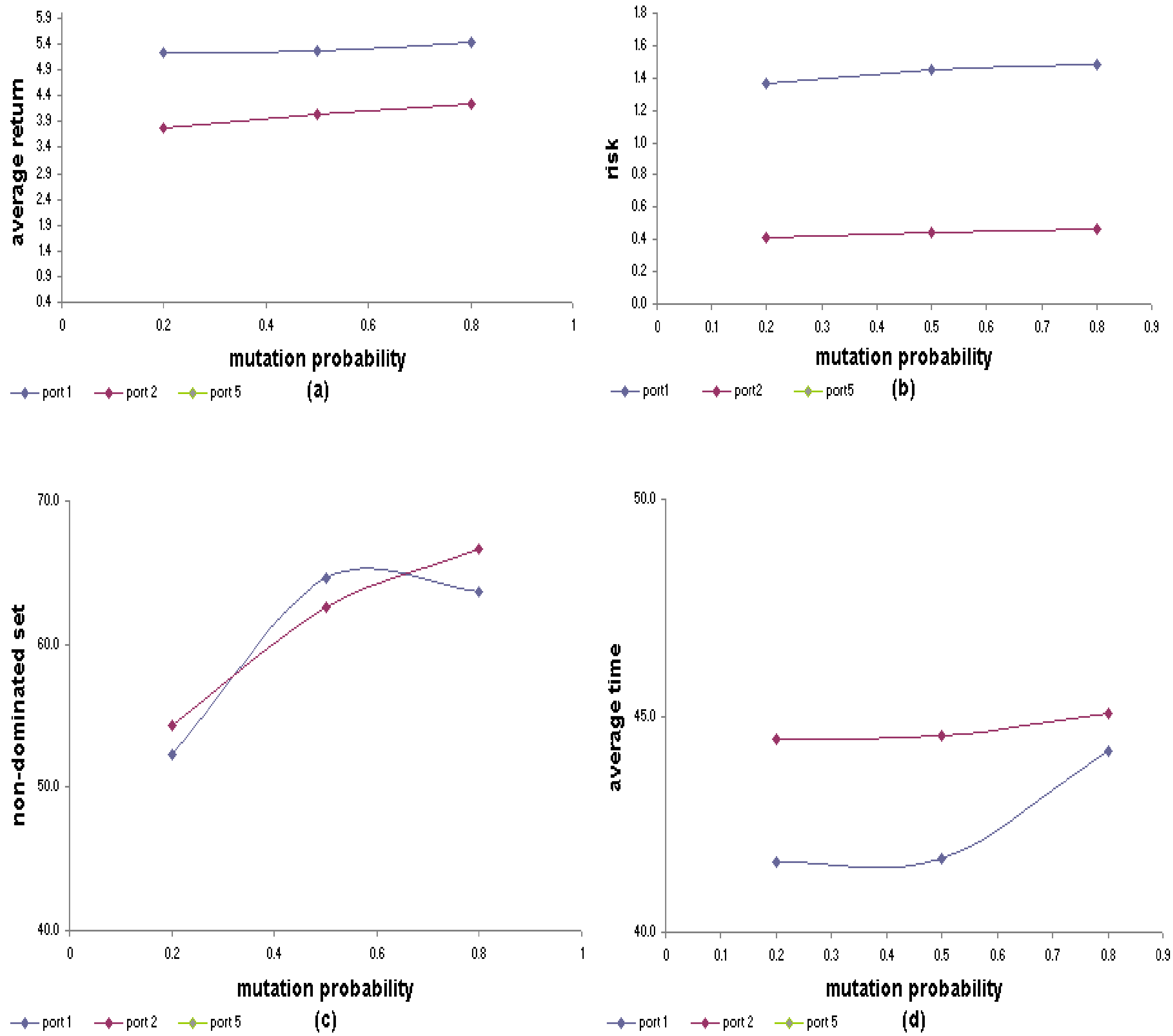

Figure 5. Adjust mutation probability (*pMut)* parameter.

Table 3. Used *K* values for each instance.

Instance	Total possible assets	10%	20%	50%
Port1	31	4	8	16
Port2	85	9	22	43
Port5	225	23	57	113

we have fixed the *pMut* value in 0.6.

In order to adjust the *K* parameter which represent the maximum assets that can be included in a specific portfolio, we evaluate our BOCA algorithm using three instances of the literature: port1, port2 and port5. The tested values for *K* parameter were: 0.1; 0.25; 0.5, respectively.

These values correspond to a portion of the total possible assets. This portion will correspond to the *K* value. Table 3 shows the used values for each instance.

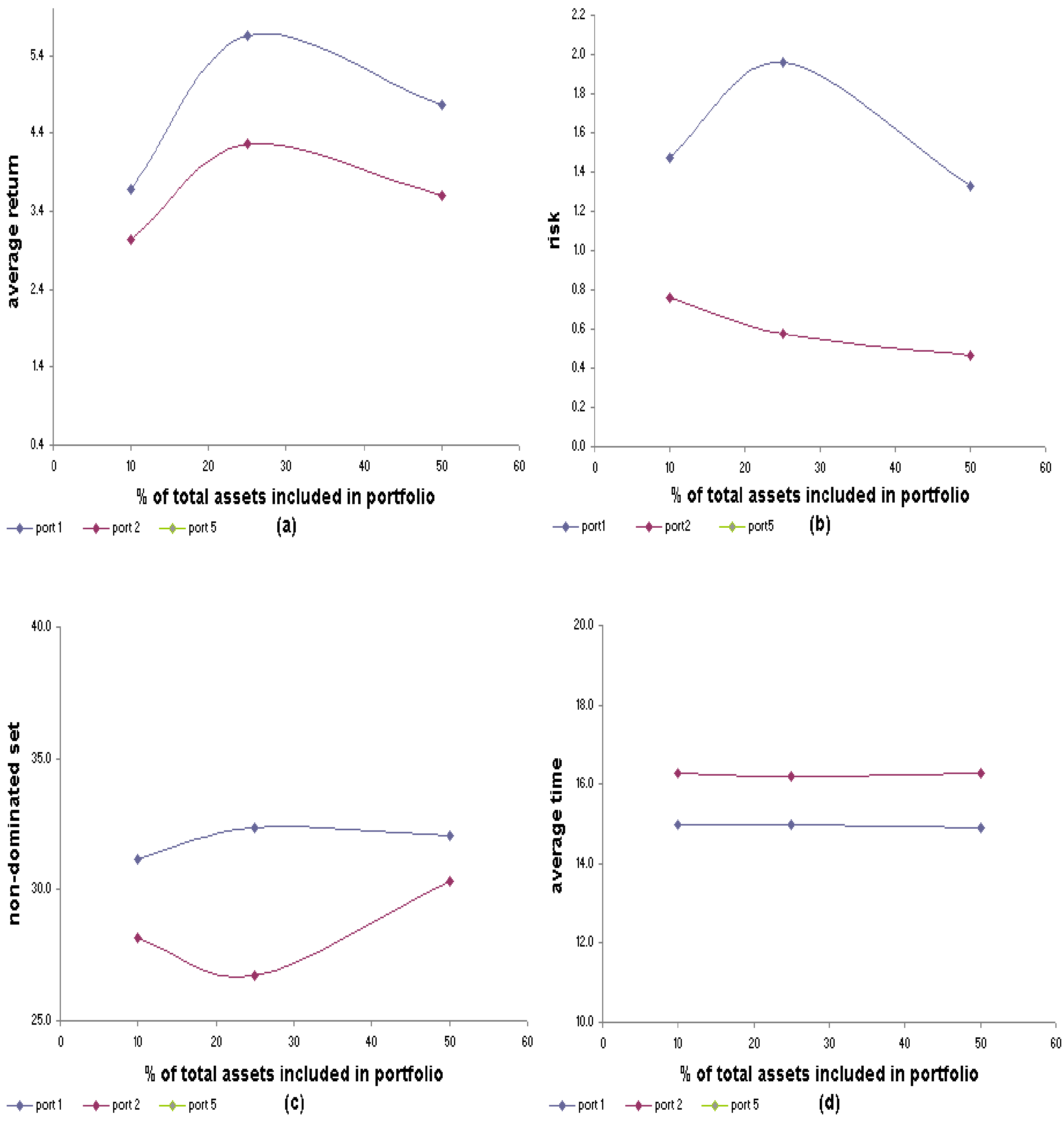

Figure 6. Adjust % of total assets that can be included in portfolio (*K* parameter).

As we can see in Figure 6 (a) and (b), K value influences the behavior of BOCA algorithm in both objective functions separately. In fact, better solutions for them have been obtained for K=0.5. However, Figure 6 (c) and (d) illustrates that this influence is marginal w.r.t. size of NDS and execution time".

DISCUSSION

The adjust parameters used for final experiments are: N= 40; g = 100; pMut = 60%. For parameter *K* we use two values (4 - 8) depending on the characteristics of the instance to evaluate. The obtained results were

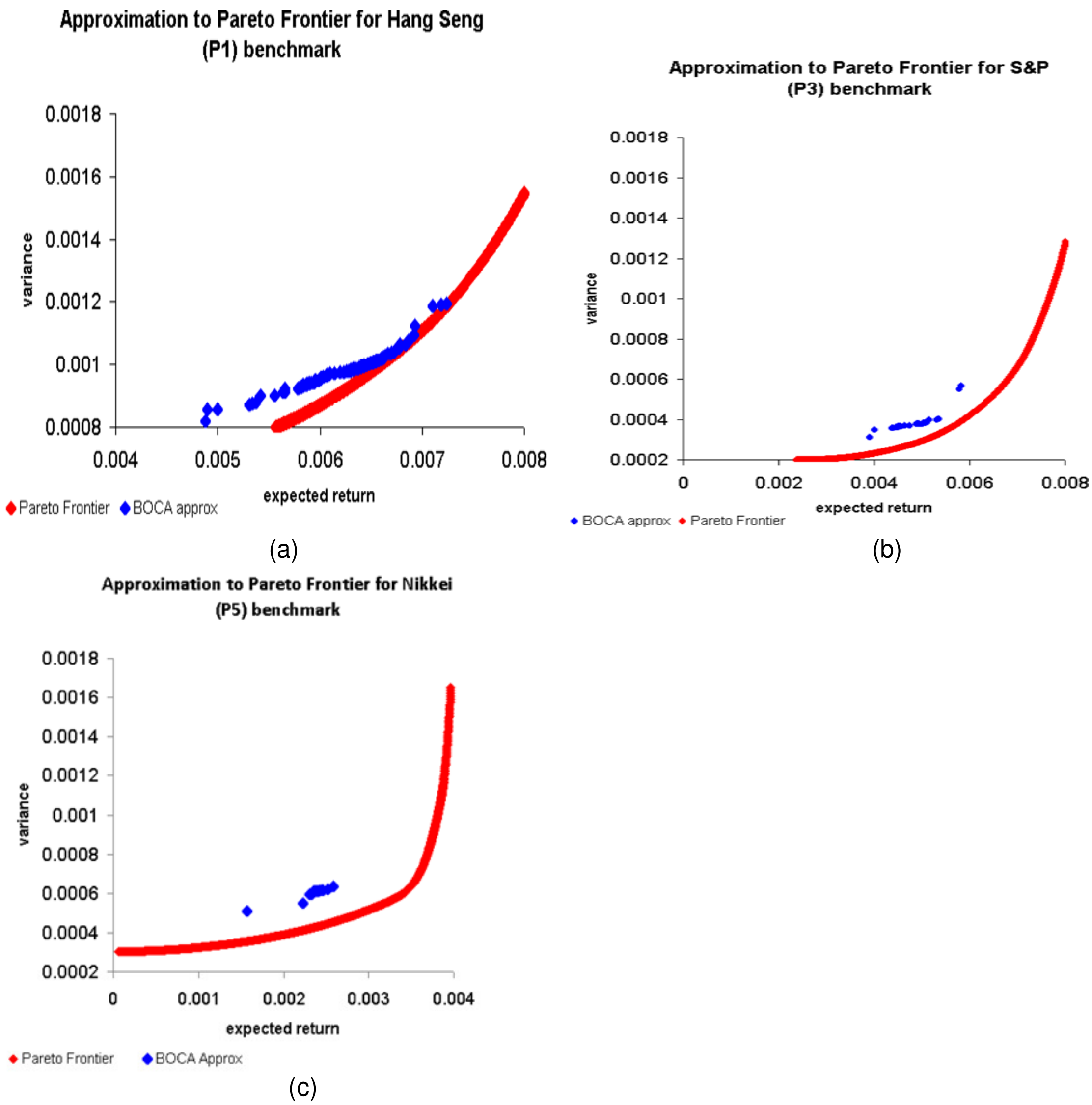

Figure 7. Approximation to Pareto frontier for P1.

compared with that obtained in Branke (2009) which uses two envelope-based multi-objective evolutionary algorithms (MOEA) called P-MOEA and E-MOEA. In this article, the used instances were: Hang Seng (31 assets, K=4), S and P (98 assets, K=4) and Nikkei (225 assets, K=8). Figures 7a, b and c show the performance of our BOCA algorithm and are related with the Pareto frontier (which is given in the OR-Library and was obtained using an exact algorithm without including any constraint to the model) for all benchmark instances.

BOCA algorithm provides a good solution for P1 instance obtaining a closest approximation to the Pareto frontier. In the case of the P3 and P5 instances, we obtained an acceptable result considering the size of them and that the Pareto frontier is obtained without constraint for the model. Thus, the constraint affects mainly the extension of the approximation and not its proximity to the front.

In order to compare our results w.r.t. [Branke, 2009], Table 4 shows the difference using the max-delta-area which is derived from the hyper-volume metric explained above. BOCA algorithm obtains acceptable solution for all instances, even improves the performance of P-MOEA for the P5 instance.

We can say that BOCA algorithm showed that it is capable of achieving good results for bi-objective problems (particularly for PSP). However, additional diversification strategies must be implemented in order to achieve greater dispersion of NDS.

Conclusion

In this work, we have confirmed that the CAs are competitive w.r.t. other evolutionary techniques. This

Table 4. Comparison between P-MOEA, E-MOEA and BOCA.

Algorithms	P1	P3	P5
P-MOEA	1.1613 ± 0.0159	2.7787 ± 0.0521	9.3292 ± 0.2287
E-MOEA	0.2275 ± 0	0.8048 ± 0.00003	0.0561 ± 0.0052
BOCA	2.4514 ± 0.0126	10.998 ± 0.0034	4.5547 ± 0.0087

better performance of CAs has been widely reported in mono-objective problems; however, the research in a multi-objective context has not achieved that maturity level yet.

The main goal of this article was to study the behavior and performance of an implementation of a CA in a multi-objective environment. In this sense, our implementation has achieved promising results. Thus, confirming that the CAs are real alternative, and a line of development that has not been sufficiently developed.

We implement a BOCA which is compared with two well known envelope-based evolutionary algorithms (P-MOEA and E-MOEA), obtaining acceptable results.

As future work, we envision two lines of work that are relevant and presented thus:

i) Regarding the type of knowledge used. In this investigation a normative knowledge has been inserted in the space of belief. This opens a line of work for experimenting with other kinds of knowledge to further improve the performance of the algorithm.

ii) Regarding the metrics. It seems interesting to evaluate the performance of cultural algorithm using other metrics to measure the level of consistency and quality of the approximations obtained.

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Appendix 1.

General Structure of BOCA

```
Begin
t = 0
initialize Population P(t)
initialize belief Space B(t)
evaluate individual fitness P(t)
while (not finish condition)do
t=t+1
parents = select parents from P(t-1) and Influence from B(t)
child = crossover (parents, Influence(B(t))
Evaluate (child)
Evaluate (child)
P(t) = child
Update (B(t))
Accept ( (P(t))
end while
while (not finish condition)do
i=i+1
Baux(i) = Mutate(B(t))
Evaluate (Baux(t))
Update (B(t))
Accept (Baux(t))
end while
end
```