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Full Length Research Paper

Fuzzy resolvability modulo fuzzy ideals

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In this paper, the notion of fuzzy resolvability modulo ideal, with respect to the fuzzy local function in a fuzzy topological space, was investigated. A trial was made to explain the notions of resolvability of complementary fuzzy dense set, fuzzy local closed set and fuzzy I- condense set.

Key words: Fuzzy topology, fuzzy sets, fuzzy closure operator, fuzzy local function, fuzzy ideals, fuzzy dense set.

INTRODUCTION

The notion of fuzzy sets was introduced by Zadeh (1965), and since then many eminent researchers used this notion in their respective fields of study. Chang (1968) introduced this notion in topology and put forward the idea of fuzzy topological space. Various aspects of general topology have been investigated in fuzzy topology. Study based upon the ideals has been under discussion for many years. Kurtowski (1966) and Vaidyanathaswami (1945) and several others contributed a lot to lay the foundation of this notion. Fuzzy Ideal in the context of fuzzy sets and fuzzy mapping was carried out by Sarkar (1997) and many others, to extend the work in fuzzy ideal topological space. The definition of fuzzy local function was also carried out in this reference. Our main aim in this paper is to introduce and study fuzzy resolvability based on the fuzzy local functions and fuzzy ideals. These terms used in accomplishing this goal laid foundation to $*-\text{fuzzy}$ topological space; this is the generalization of fuzzy topological space.

PRELIMINARIES

Throughout this paper, we used mean (X, τ) as fuzzy topological space similar to the sense of Chang (1968). *A* fuzzy point is represented as x_λ with the support of $x \in X$ and the value $0 < \lambda \leq 1$. The constant fuzzy set takes the value of 0 and 1, and closure of fuzzy set *A* is represented as cl(*A*). Let *X* be a set, by a fuzzy set on *X,* we mean a function μ: *X* → (0, 1). In fact, μ(*x*) represents the degree of membership of *x*. For example, every characteristic function of a set is a fuzzy set. The characteristic functions of subsets of a set *X* are referred to as the crisp fuzzy sets in *X*.

A fuzzy set *A* in (X, τ) is called q-neighborhood if there exists a fuzzy open set μ such that $x_\lambda q \mu \subseteq A$. We will denote the set of all neighborhood of x_λ in (X, τ) by $N(x_\lambda)$.

Definition 1

A fuzzy topology is a family τ of fuzzy sets in *X* which satisfies the following conditions (Chang, 1968):

1. $0_x, 1_x \in \tau$
2. If A, B $\in$ τ, then A $\vee$ B $\in$ τ,
3. $A_\alpha \in \tau$ for each $\alpha \in \Delta$, then $\sup_{\alpha \in \Delta} A_\alpha \in \tau$. τ is called a fuzzy topology on *X* and the pair (*X*, τ) is a fuzzy topological space. Every element of τ is called a τ-open fuzzy set. A fuzzy set is τ-closed if its complement is τ -open.

Definition 2

A non empty collection of fuzzy sets ϑ of a set *X* is called a fuzzy ideal on *X* if and only if (Jankovic and Hamlett, 1990; Sarkar, 1997):

1. $\mu \in \vartheta$ and $\lambda \subseteq \mu \Rightarrow \lambda \in \vartheta$. [heredity]

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2. For each $\lambda, \mu \in \vartheta \Rightarrow \lambda \vee \mu \in \vartheta$. [finite additivity]

Definition 3

Let (X, τ , ϑ) be fuzzy ideal topological space with ϑ as the fuzzy ideal on X (Sarkar, 1997). Let A be any fuzzy set of X. Then, the fuzzy local function $A^*(\vartheta,\tau)$ of A is the union of all fuzzy points x_λ, such that if $\mu \in N(x_\lambda)$ and $I \in \vartheta$, then there is at least one $y \in X$ for which $\mu(y) + A(y) - 1 > I(y)$. In Fadil and Cemil (2011), this has been described as, the fuzzy local function of $A \le X$ with respect to τ and ϑ which is denoted by A^* or $A^*(\vartheta)$ for which $A^*(\vartheta,\tau)$ will cause no ambiguity, is defined as $\vee\{x \in X : A \wedge U \notin I \text{ for every } U \in \tau\}$ for every $U \in \tau$ and $I \in \vartheta$.

MAIN RESULTS

Definition 4

A subset A of fuzzy ideal topological space (X, τ , ϑ) is called fuzzy I-dense set if $A^*(x) = \bar{1}$.

Definition 5

A fuzzy ideal topological space (X, τ, ϑ) is called fuzzy I-resolvable, if there exists fuzzy I-dense subsets $A,B \in \mathrm{I}^X$ such that, max($A(x),B(x)$) = $\bar{1}$ and min($A(x),B(x)$) = $\bar{0}$.

Definition 6

An ideal ϑ on fuzzy topological space(X, τ) is said to be fuzzy I-codense, if for $\mathrm{T} \in \tau$ and $I \in \vartheta$, we can write that min (T(x), I(x)) = $\bar{0}$

Theorem 1

A fuzzy ideal ϑ on fuzzy topological space (X, τ) is said to be fuzzy I-codense, if and only if $\bar{1} = \bar{1}^*$.

Proof

Sufficiency, let $\bar{1} = \bar{1}^*$, then we prove that ϑ is fuzzy I-codense. Let $\bar{0} \ne \mu \in \tau$, since $\mu(x) \le \bar{1}(x) \ \ \forall\, x \in X$, then for $x \in \bar{1}^*$.

$$\Rightarrow \mu(x) + \bar{1}(x) - 1 > \vartheta(x)$$

$$\Rightarrow \mu(x) > \vartheta(x) \Rightarrow \mu \notin \vartheta$$
$$\Rightarrow \min(\mu(x), \vartheta(x)) = \bar{0}$$

where $\mu \in \tau$ means that ϑ is fuzzy I-codense

Theorem 2

For fuzzy ideal topological space (X, τ, ϑ), the following statements are equivalent:

(a) (X, τ) is fuzzy I-resolvable.
(b) (X, τ) is fuzzy I-dense.

Proof

(a)$\Rightarrow$(b), for X to be fuzzy resolvable and A and B are fuzzy resolution of X, then note that:

$$cl^*(A) = \max(A(x), A^*(x))$$

$$\Rightarrow cl^*(A) = \max(A(x), \bar{1}) = \bar{1}$$

$\Rightarrow$ (X, τ) is fuzzy I-dense set.

(b)$\Rightarrow$(a), assuming that max(A(x), B(x)) = 1 and min(A(x), B(x)) = 0, where A and B are the fuzzy I-dense sets. Let $x \notin A^*$, then $x \in U$ whereas U is a fuzzy open set, then U(x) +A(x)-1$\le \vartheta$(x). Let $V(x)$ = U(x)+A(x)-1$\ne 0$, moreover U(x) > A(x) otherwise B fails to be a fuzzy I-Idense set. Clearly for W(x) = U(x)/V(x) $\Rightarrow$ min (W(x), A(x)) = 0 which contradicts the initial assumption.

Corollary 1

If X is fuzzy resolvable, then there exists disjoint fuzzy dense subsets A and B each *-fuzzy-dense in itself, that is, X has any fuzzy resolution of sets which are *-fuzzy-dense in itself.

Definition 7

A subset S of a fuzzy ideal topological space (X, τ , ϑ) is called fuzzy I-locally closed if S(x) = min(U(x), F(x)), where U is fuzzy open and $F(x)$ = F^*(x) (Jankovic and Hamlett, 1990).

Theorem 3

Let (X, τ, ϑ) be fuzzy ideal topological space and $A \in I^X$, then the following are equivalent:

1. *A* is locally fuzzy I-closed.
2. $A(x) = \min(U(x), A^*(x))$ for some fuzzy open set *U*.
3. $A(x) < A^*(x)$, then A^*-A is fuzzy closed set.
4. $A(x) < A^*(x)$, then $A- A^*$ is fuzzy open.
5. $A(x) < A^*(x)$ and $A(x) < \text{int}(A(x), 1- A^*(x))$.

Proof

(1)$\Rightarrow$(2), suppose *A* is locally I-fuzzy closed subset of *X* and $A(x) = \min(U(x), V(x))$, where $V^*(x) = V(x)$. Then $A(x) < V(x)$ which implies that $A^*(x) < V(x)$ (Sarkar, 1997; Theorem 4).

$\Rightarrow A(x) = \min(U(x), V(x)) = \min(U(x), V^*(x)) < (\min(U(x), V(x)))^* = A^*(x)$.

And so, $A(x) = \min(A(x), A^*(x)) = \min(\min(U(x), V(x)), A^*(x)) = \min(U(x), \min(V(x), A^*(x))) = \min(U(x), A^*(x))$.

(2)$\Rightarrow$(3), suppose $A(x) = \min(U(x), A^*(x))$ for some fuzzy open set *U*. Clearly, $A(x) < A^*(x)$ and furthermore,

$(A^* - A)(x) = \min(A^*(x), 1-A(x)) = \min(A^*(x), 1-\min(U(x), A^*(x))) = \min(A^*(x), 1-U(x))$

$\Rightarrow$Since A^* is fuzzy closed set, therefore A^*-A *will* also be a fuzzy closed set as required.

(3)$\Rightarrow$(4), consider that, $\Rightarrow \min(A^*(x), 1-A(x))$ is fuzzy closed. The compliment of the aforementioned, $\Rightarrow \min(A(x), 1-A^*(x))$ is fuzzy open. This implies that $A- A^*$ is fuzzy open.

(4) $\Rightarrow$(5), this result is quite obvious.

(5) $\Rightarrow$(1), $1- A^*(x) = \text{int}(1- A^*(x)) < \text{int}(\min(A(x), 1- A^*(x))$

Since, $\min(A(x), 1- A^*(x))$ is fuzzy open, we can write that, $A(x) = \min(A^*(x), \min(A(x), 1- A^*(x))$; which proves the required result that A is fuzzy I-locally closed.

Theorem 4

Consider fuzzy topological space (X, τ, I) and A be fuzzy I-locally closed set, then the following conditions are equivalent:

1. I is completely fuzzy I-codense.
2. Every fuzzy dense set is fuzzy I-dense.

Proof

(1)$\Rightarrow$(2), let D be fuzzy dense subset in (X, τ) and $\mu \in N(x_\lambda)$. Clearly, $A(x) = \min(D(x), C(x)) \neq 0_x$ is a fuzzy subset in I^X. By (1), $A \notin$ I then $x_\lambda \in D$. *D* is thus fuzzy I-dense.

(2)$\Rightarrow$(1): let $A \in I^X$ and *A* is non empty open fuzzy subset and $A \in$ I. And $A(x) = \min(U(x), D(x))$ where *U* is fuzzy open set and *D* is fuzzy dense set. Now for $x_\lambda \in A$ as *U* is fuzzy open set containing *x* and $\min(U(x), D(x)) \in$ I and $x_\lambda \in D^*$, and hence $D^* \neq X$. So, D is fuzzy-dense set which fails to be a fuzzy I-dense. By contradicting $\min(\tau(x), I)$.

Theorem 5

Let (X, τ, ϑ) be an fuzzy ideal topological space and *A* be a fuzzy I-dense subset of *X*. Then *A* is open if and only if *A* is fuzzy I-locally closed.

Proof

Suppose *A* is fuzzy I-dense and fuzzy open. Then, $A = \min(A(x), 1) = \min(A(x), A^*(x))$ and so *A* is fuzzy I-locally closed. Conversely, if *A* is fuzzy I-locally closed and fuzzy I-dense, then $A(x) = \min(G(x), A(x))$, where *G* is fuzzy open which implies $A(x) = \min(G(x), 1) = G(x)$ and so *A* is fuzzy open.

Definition 8

For fuzzy subset *A* of a fuzzy ideal topological space (X, τ, ϑ) is classified as (Fadil and Cemil, 2011): fuzzy *-perfect if $A^* = A$; fuzzy $\tau^*-\text{closed}$ if $A^* \leq A$ and fuzzy regular I-closed if $A = (\text{int}\, A)^*$, respectively.

Theorem 6

In a fuzzy ideal topological space (X, τ, ϑ), the following statements holds (Fadil and Cemil, 2011):

1. Every *-perfect is fuzzy I-locally closed.
2. Fuzzy regular I-closed is fuzzy *-perfect.
3. Every fuzzy *-perfect is fuzzy $\tau^*-\text{closed}$.

MAXIMAL FUZZY I-RESOLVABILITY

Definition 9

A fuzzy ideal topological space (X, τ, ϑ) is called maximal fuzzy I-resolvable if (X, τ, ϑ) is fuzzy I-resolvable and (X, σ, ϑ) is not fuzzy I-resolvable space and σ is strictly finer than τ. A fuzzy subset S of a fuzzy ideal topological space (X, τ, ϑ) within an ideal ϑ:

$I_s = \{I \in \vartheta : I(x) \leq \vartheta(x)\} = \{\min (I_{(x)}, S_{(x)}) : I \in \vartheta\}$ on S.

By $IR(X)$, we denote the collection of all I-fuzzy resolvable subspaces of a given fuzzy topological space (X, τ, ϑ).

Theorem 7

For a fuzzy topological space (X, τ, ϑ), the following conditions are equivalent:

1. (X, τ, ϑ) is maximally fuzzy I-resolvable.
2. τ / ϕ = IR(X).

Proof

(1) $\Longrightarrow$ (2), open non empty fuzzy subspaces of fuzzy I-resolvable spaces are fuzzy I-resolvable. Let S $\in$ IR(X), from the definition of $\tau(s)$ is a fuzzy I-resolvable topology on X finer than τ. By (1) S $\in$ $\tau(s) = \tau$.

(2)$\Longrightarrow$(1), since $X \in \tau$, then by (2), X is fuzzy I-resolvable. Assume that X is not maximally fuzzy I-resolvable and let σ be fuzzy I-resolvable topology strictly finer than τ. Let $U \in \sigma / \phi$. Clearly, U is fuzzy I-resolvable in (X,σ,I) and hence in (X, τ ,I). By (2), $U \in \tau$. By contradiction, (1) is proved.

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