International Journal of the Physical Sciences Vol. 8(7), pp. 254-263, 23 February, 2013
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS12.713
ISSN 1992 - 1950 ©2013 Academic Journals

Full Length Research Paper

Transient mass transfer flow past an impulsively started infinite vertical plate with ramped plate velocity and ramped temperature

N. Ahmed* and M. Dutta

Department of Mathematics, Gauhati University, Guwahati – 781014, Assam, India.

Accepted 19 February, 2013

An exact solution to the problem of a transient free convective mass transfer flow of a Newtonian non-Grey optically thin fluid past a suddenly started infinite vertical plate embedded in a porous medium with ramped wall temperature as well as ramped plate velocity in presence of appreciable thermal radiation and first-order chemical reaction is presented. The resulting system of equations governing the flow is solved by employing Laplace Transform technique in closed form. Detailed computations of the influence of ramped velocity parameter A, radiation parameter Q, Reynolds number Re, Schmidt number Sc, porosity parameter S and chemical reaction parameter K on the variations in the fluid velocity, fluid temperature, fluid concentration, and skin friction, Nusselt number and Sherwood number at the plate are demonstrated graphically. The results show that the effect of the ramped parameter A accelerates the fluid flow substantially. Further, our investigation reveals the fact that the viscous drag at the plate gets increased due to chemical reaction in case of ramped plate temperature. Comparison of some of the results of the present work is made with previously published results under special cases, and shows a good agreement.

Key words: Ramped plate velocity, ramped plate temperature, thermal radiation, chemical reaction, optically thin, porosity.

INTRODUCTION

Natural or free convection is a physical process of heat and mass transfer involving fluids which originates when the temperature as well as species concentration change causes density variations inducing buoyancy forces to act on the fluid. Such flows exist abundantly in nature, and due to its applications in engineering and geophysical environments, these have been studied extensively in practice. The heating of rooms inside buildings using radiators is an example of application of heat transfer by free convection. Detailed areas of applications of free convection flow are found in Ghoshdastidar (2004) and Nield and Bejan (2006).

Welty et al. (2007) defines mass transfer as the transport of one constituent from a region of higher concentration to that of a lower concentration. Mass transfer is the basis for many biological and chemical processes. Biological processes include the oxygenation of blood and the transport of ions across membranes within the kidney. Chemical processes include the chemical vapour deposition of Silane (SiH_4) onto a silicon wafer, the doping of silicon wafer to form a semiconducting thin film, the aeration of wastewater, and the purification of ores and isotopes. Mass transfer also occurs in many other processes such as absorption, adsorption, drying, precipitation, membrane filtration and distillation.

Another process of heat transfer is radiation through electromagnetic waves. Radiative convective flows are encountered in several industrial and environmental processes. Some of these include heating and cooling chambers, evaporation from large open water reservoirs, solar power technology and astrophysical flows. The

*Corresponding author. E-mail: saheel_nazib@yahoo.com.

study of radiation interaction with convection for heat and mass transfer in fluids is quite significant. Various problems on steady and unsteady fluid flow past a moving plate in the presence of free convection and radiation has been studied by Cess (1966), England and Emery (1969), Raptis and Perdikis (1999), Muthucumaraswamy et al. (2001b) and Chamkha et al. (2001). These model studies have been performed on flows in a non-porous medium. Among the studies on radiative and free-convective flow past a vertical plate in porous medium, works of Raptis (1998), Sattar et al. (2000) and Hossain and Pop (2001) are significant.

In several occasions it is observed that a foreign mass reacts with the fluid and in such situations chemical reaction plays an important role in heat and mass transfer problems. In particular, the presence of foreign mass in air or water causes some kind of chemical reaction. Bird et al. (2001) states that during a chemical reaction between two species, heat is also generated. The reaction rate in chemical reactions generally depends on the concentration of the species itself. In Cussler (2009), a reaction is defined to be of first order if the rate of reaction is directly proportional to the concentration of the species. The effect of the presence of foreign mass on the free convection flow past a semi-infinite vertical plate was studied by Gebhart and Pera (1971). Several investigators have studied the effect of chemical reaction on different convective heat and mass transfer flows, among whom Anjalidevi and Kandasamy (1999), Muthucumaraswamy and Ganesan (2001a), Muthucumaraswamy and Shankar (2011) are worth mentioning.

Recently, Das et al. (2011) have studied the radiation effect on natural flow of an optically thin viscous incompressible fluid near a suddenly moving vertical plate with ramped wall temperature by adopting Cogley-Vincenti-Gilles equilibrium model introduced by Cogley et al. (1968). An optically thin fluid has no self absorption property, but it can absorb radiation emitted by the boundaries. In the investigation done by Das et al. (2011), a particular characteristic time $t_0 = \frac{\nu}{U_0^2}$ depending on the kinematic viscosity and the plate velocity has been considered.

In the present work an attempt has been made to study the effects of ramped plate velocity and chemical reaction on a natural flow of an optically thin viscous incompressible radiating non-Grey fluid past an impulsively started infinite vertical plate embedded in a porous media, with ramped wall temperature. The acceleration of the plate is sustained for a finite time interval and thereafter it moves with uniform velocity. In this proposed work, not a specific characteristic time t_0 is considered. Due to an arbitrary choice of t_0, the Reynolds number Re gets introduced into the problem.

Subsequently, the influence of the Reynolds number on the flow field in addition to other similar parameters is investigated in the preview of current study.

BASIC EQUATIONS

The equations governing the motion of an incompressible, viscous, radiating and chemically reacting fluid past a solid surface in porous medium are:

Equation of continuity:

$$\vec{\nabla} \cdot \vec{q} = 0 \quad (1)$$

Momentum equation:

$$\rho\left[\frac{\partial \vec{q}}{\partial t'} + \left(\vec{q} \cdot \vec{\nabla}\right)\vec{q}\right] = -\vec{\nabla} p + \rho \vec{g} + \mu \nabla^2 \vec{q} - \frac{\mu}{k^*}\vec{q} \quad (2)$$

Energy equation:

$$\rho C_p \left[\frac{\partial T'}{\partial t'} + \left(\vec{q} \cdot \vec{\nabla}\right) T'\right] = K_T \nabla^2 T' + \varphi - \frac{\partial q_r}{\partial n'} \quad (3)$$

Species continuity equation:

$$\frac{\partial C'}{\partial t'} + \left(\vec{q} \cdot \vec{\nabla}\right) C' = D_M \nabla^2 C' + \overline{K}\left(C'_\infty - C'\right) \quad (4)$$

All the physical quantities are defined in the list of symbols. Consider a transient, radiative and chemically reactive natural flow of an incompressible, Newtonian, non-Grey and optically thin fluid past an impulsively started vertical plate embedded in a porous medium with ramped plate velocity and temperature. For the sake of idealization of the model, the following assumptions are made:

(i) All the fluid properties are considered constants except the influence of the variation in density in the buoyancy force term.
(ii) The viscous dissipation of energy is negligible.
(iii) Permeability of the medium (k^*) is considered to be constant.
(iv)The flow is assumed to be one dimensional (parallel to the vertical plate).
(v) The radiation heat flux (q_r) in the direction of the plate velocity is considered negligible in comparison to that in the normal direction.
(vi) The chemical reaction is considered to be homogeneous and of first order.

A coordinate system (x', y', z') is now introduced with X-axis along the plate in the upward vertical direction, Y-axis normal to the plate directed into the fluid region and Z-axis along the width of the plate (Figure 1).

Initially the plate and the surrounding fluid were at rest and at the same temperature T'_∞ and concentration C'_∞. At time $t' > 0$, the plate is suddenly moved in its own plane with initial velocity U_0 and uniform acceleration $A\frac{U_0}{t_0}$, and the wall temperature raises to $T'_\infty + (T'_W - T'_\infty)\frac{t'}{t_0}$ and the concentration changes to $C'_\infty + (C'_W - C'_\infty)\frac{t'}{t_0}$ for $0 < t' \le t_0$, and a constant temperature

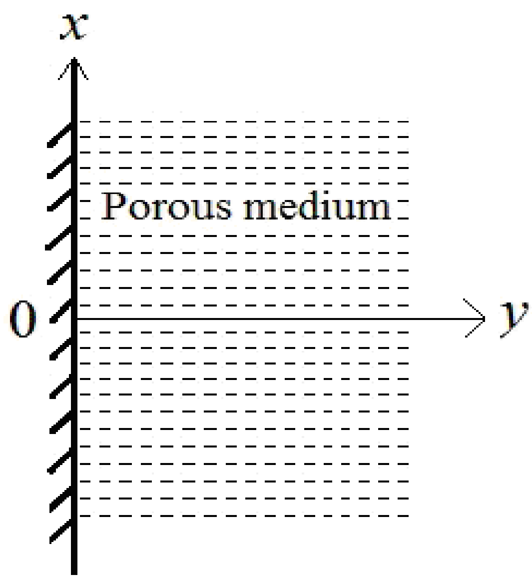

Figure 1. Geometry of the problem.

T'_W $(T'_W > T'_\infty)$, a constant concentration C'_W $(C'_W > C'_\infty)$, and a uniform plate velocity $(\mathrm{A}+1)U_0$ is maintained at $t' > t_0$. Let $\vec{q} = (u', 0, 0)$ denote the fluid velocity at the point (x', y', z', t') in the fluid. Then Equation (1) reduces to $\frac{\partial u'}{\partial x'} = 0$ which yields

$$u' = u'(y', t') \tag{5}$$

In light of the above coordinate system, Equation (2) takes the form

$$\rho \frac{\partial u'}{\partial t'} = -\frac{\partial p}{\partial x'} - \rho g + \mu \frac{\partial^2 u'}{\partial y'^2} - \frac{\mu}{k^*} u' \tag{6}$$

and $$\frac{\partial p}{\partial y'} = 0 \tag{7}$$

From Equation (7) it is inferred that p is independent of y' indicating the fact that the pressure near the plate is same as that far away from the plate in normal direction. This observation establishes the result that as one moves far away from the plate,

$$\frac{\partial p}{\partial x'} = -\rho_\infty g \tag{8}$$

Elimination of $\frac{\partial p}{\partial x'}$ from Equations (6) and (8), then yields

$$\rho \frac{\partial u'}{\partial t'} = \mu \frac{\partial^2 u'}{\partial y'^2} + (\rho_\infty - \rho) g - \frac{\mu}{k^*} u' \tag{9}$$

The equation of state according to classical Boussinesq approximation is:

$$\rho = \rho_\infty \left[1 - \beta (T' - T'_\infty) - \overline{\beta} (C' - C'_\infty) \right] \tag{10}$$

Coupling Equations (9) and (10) together and accomplishing the fact that $\frac{\rho_\infty}{\rho} \cong 1$, the following linear partial differential equation is obtained:

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta (T' - T'_\infty) + g\overline{\beta} (C' - C'_\infty) - \frac{\nu}{k^*} u' \tag{11}$$

Further, since the plate is of infinite length and in view of assumption (II), the following reduced form of the Equations (3) and (4) are obtained:

$$\rho C_p \frac{\partial T'}{\partial t'} = K_T \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} \tag{12}$$

$$\frac{\partial C'}{\partial t'} = D_M \frac{\partial^2 C'}{\partial y'^2} + \overline{K} (C'_\infty - C') \tag{13}$$

The initial and boundary conditions to be satisfied by Equations (11), (12) and (13) are:

$$u' = 0,\ T' = T'_\infty,\ C' = C'_\infty \ \text{ for } y' \geq 0 \text{ and } t' \leq 0 \tag{14}$$

$$u' = U_0 + \mathrm{A}\frac{U_0}{t_0} t',\ T' = T'_\infty + (T'_W - T'_\infty)\frac{t'}{t_0}, \quad C' = C'_\infty + (C'_W - C'_\infty)\frac{t'}{t_0} \ \text{ at } y' = 0 \text{ and } 0 < t' \leq t_0 \tag{15}$$

$$u' = (\mathrm{A}+1)U_0,\ T' = T'_W,\ C' = C'_W \ \text{ at } y' = 0 \text{ and } t' > t_0 \tag{16}$$

$$u' \to 0,\ T' \to T'_\infty,\ C' \to C'_\infty \ \text{ as } y' \to \infty \text{ for } t' > 0 \tag{17}$$

Cogley et al. (1968) emphasized that the rate of radiative heat flux in optically thin limit for a non-Grey gas near equilibrium is given by:

$$\frac{\partial q_r}{\partial y'} = 4I (T' - T'_\infty) \tag{18}$$

Where,

$$I = \int_0^\infty (K_\lambda)_W \left(\frac{\partial e_{\lambda h}}{\partial T'} \right)_W d\lambda \tag{19}$$

In view of Equation (18), Equation (12) becomes

$$\rho C_p \frac{\partial T'}{\partial t'} = K_T \frac{\partial^2 T'}{\partial y'^2} - 4I (T' - T'_\infty) \tag{20}$$

In order to normalize the flow model, the following non-dimensional variables and parameters are introduced:

$$\left.\begin{aligned}
&u=\frac{u'}{U_0},\ y=\frac{y'U_0}{\nu},\ t=\frac{t'}{t_0},\ \theta=\frac{T'-T'_\infty}{T'_W-T'_\infty},\\
&\phi=\frac{C'-C'_\infty}{C'_W-C'_\infty},\ \mathrm{Pr}=\frac{\mu C_p}{K_T},\ \mathrm{Q}=\frac{4I\nu}{\rho C_p U_0^2},\\
&\mathrm{Re}=\frac{U_0^2 t_0}{\nu},\ \mathrm{Gr}=\frac{\nu g\beta\left(T'_W-T'_\infty\right)}{U_0^3},\\
&\mathrm{Gm}=\frac{\nu g\overline{\beta}\left(C'_W-C'_\infty\right)}{U_0^3},\ \mathrm{S}=\frac{k^*U_0^2}{\nu^2},\\
&\mathrm{Sc}=\frac{\nu}{D_M},\ \mathrm{K}=\frac{\overline{K}\nu}{U_0^2}
\end{aligned}\right\}\tag{21}$$

All physical quantities are defined in the list of symbols as mentioned earlier. Utilizing the transformations and definitions Equation (21), the Equations (11), (13) and (20) become

$$\frac{\partial u}{\partial t}=\mathrm{Re}\left[\frac{\partial^2 u}{\partial y^2}+\mathrm{Gr}\theta+\mathrm{Gm}\phi-\frac{1}{\mathrm{S}}u\right]\tag{22}$$

$$\frac{\partial \theta}{\partial t}=\mathrm{Re}\left[\frac{1}{\mathrm{Pr}}\frac{\partial^2 u}{\partial y^2}-\mathrm{Q}\theta\right]\tag{23}$$

$$\frac{\partial \phi}{\partial t}=\mathrm{Re}\left[\frac{1}{\mathrm{Sc}}\frac{\partial^2 \phi}{\partial y^2}-\mathrm{K}\phi\right]\tag{24}$$

subject to the relevant initial and boundary conditions:

$$u=0,\ \theta=0,\ \phi=0 \text{ for } y\ge 0,\ t\le 0\tag{25}$$

$$u=\mathrm{A}t+1,\ \theta=t,\ \phi=t \text{ at } y=0,\ 0<t\le 1\tag{26}$$

$$u=\mathrm{A}+1,\ \theta=1,\ \phi=1 \text{ at } y=0,\ t>1\tag{27}$$

$$u\to 0,\ \theta\to 0,\ \phi\to 0 \text{ as } y\to\infty \text{ for } t>0\tag{28}$$

METHODS OF SOLUTION

Taking Laplace transform of the Equations (22), (23) and (24), the following equations are obtained

$$\frac{d^2\overline{u}}{dy^2}-\left(\frac{1}{\mathrm{Re}}s+\frac{1}{\mathrm{S}}\right)\overline{u}=-\mathrm{Gr}\overline{\theta}-\mathrm{Gm}\overline{\phi}\tag{29}$$

$$\frac{d^2\overline{\theta}}{dy^2}-\mathrm{Pr}\left(\frac{1}{\mathrm{Re}}s+\mathrm{Q}\right)\overline{\theta}=0\tag{30}$$

$$\frac{d^2\overline{\phi}}{dy^2}-\mathrm{Sc}\left(\frac{1}{\mathrm{Re}}s+\mathrm{K}\right)\overline{\phi}=0\tag{31}$$

Where

$$\left.\begin{aligned}
&\overline{u}(y,s)=\int_0^\infty u(y,t)e^{-st}dt\\
&\overline{\theta}(y,s)=\int_0^\infty \theta(y,t)e^{-st}dt\\
&\overline{\phi}(y,s)=\int_0^\infty \phi(y,t)e^{-st}dt
\end{aligned}\right\}\tag{32}$$

The corresponding boundary conditions for $\overline{u}$, $\overline{\theta}$ and $\overline{\phi}$ are:

$$\begin{aligned}
&\overline{u}=\mathrm{A}\frac{1}{s^2}\left(1-e^{-s}\right)+\frac{1}{s},\ \overline{\theta}=\frac{1}{s^2}\left(1-e^{-s}\right),\\
&\overline{\phi}=\frac{1}{s^2}\left(1-e^{-s}\right) \text{ at } y=0
\end{aligned}\tag{33}$$

$$\overline{u}\to 0,\ \overline{\theta}\to 0,\ \overline{\phi}\to 0 \text{ as } y\to\infty\tag{34}$$

The Equations (29), (30) and (31) are ordinary second order differential equations, the solutions of which subject to the conditions (33) and (34) are as follows:

$$\begin{aligned}
\overline{u}(y,s)=\ &\mathrm{A}\frac{1}{s^2}\left(1-e^{-s}\right)e^{-y\sqrt{P_1}}+\frac{1}{s}e^{-y\sqrt{P_1}}+\\
&\frac{P_2}{P_1-P_3^2}\left(e^{-y\sqrt{P_1}}-e^{-yP_3}\right)+\\
&\frac{P_4}{P_1-P_5^2}\left(e^{-y\sqrt{P_1}}-e^{-yP_5}\right)
\end{aligned}\tag{35}$$

$$\overline{\theta}(y,s)=\frac{1}{s^2}\left(1-e^{-s}\right)e^{-yP_3}\tag{36}$$

$$\overline{\phi}(y,s)=\frac{1}{s^2}\left(1-e^{-s}\right)e^{-yP_5}\tag{37}$$

Where

$$\left.\begin{aligned}
&P_1=\frac{1}{\mathrm{Re}}s+\frac{1}{\mathrm{S}},\ P_2=-\mathrm{Gr}\frac{1}{s^2}\left(1-e^{-s}\right),\\
&P_3=\sqrt{\mathrm{Pr}\left(\frac{1}{\mathrm{Re}}s+\mathrm{Q}\right)},\ P_4=-\mathrm{Gm}\frac{1}{s^2}\left(1-e^{-s}\right),\\
&P_5=\sqrt{\mathrm{Sc}\left(\frac{1}{\mathrm{Re}}s+\mathrm{K}\right)}
\end{aligned}\right\}\tag{38}$$

Taking inverse Laplace transform of Equations (35), (36) and (37), the representative velocity, temperature and concentration fields are obtained and are as below:

$$u=\psi_1-\overline{\psi_1}+\omega_1-\frac{\mathrm{ReGr}}{1-\mathrm{Pr}}E_4-\frac{\mathrm{ReGm}}{1-\mathrm{Sc}}E_8\tag{39}$$

$$\theta=\psi_2-\overline{\psi_2}\tag{40}$$

$$\phi=\psi_3-\overline{\psi_3}\tag{41}$$

Where

$$\psi_1=\mathrm{A}\times\psi\left(\mathrm{Re},\mathrm{S},1,y,t\right),\ \overline{\psi_1}=\mathrm{A}\times\psi\left(\mathrm{Re},\mathrm{S},1,y,t-1\right)\times\mathrm{H}\left(t-1\right),$$

$$\psi_2=\psi\left(\mathrm{Re},\frac{1}{\mathrm{Q}},\mathrm{Pr},y,t\right),\ \overline{\psi_2}=\psi\left(\mathrm{Re},\frac{1}{\mathrm{Q}},\mathrm{Pr},y,t-1\right)\times\mathrm{H}\left(t-1\right),$$

$$\psi_3=\psi\left(\mathrm{Re},\frac{1}{\mathrm{K}},\mathrm{Sc},y,t\right),\ \overline{\psi_3}=\psi\left(\mathrm{Re},\frac{1}{\mathrm{K}},\mathrm{Sc},y,t-1\right)\times\mathrm{H}\left(t-1\right),$$

$$\omega_1=\omega\left(\mathrm{Re},\mathrm{S},y,t\right),$$

$$F_1 = F\left(\mathrm{Re},\mathrm{S},a,1,y,t\right),\ F_2 = F\left(\mathrm{Re},\frac{1}{\mathrm{Q}},a,\mathrm{Pr},y,t\right),$$

$$E_1 = \frac{1}{2a^2}e^{at}\left(F_1 - F_2\right),$$

$$G_1 = G\left(\mathrm{Re},\mathrm{S},a,1,y,t\right),\ G_2 = G\left(\mathrm{Re},\frac{1}{\mathrm{Q}},a,\mathrm{Pr},y,t\right),$$

$$E_2 = \frac{1}{2a}\left(G_2 - G_1\right),$$

$$E_3 = E_1 + E_2,$$

$$\overline{F_1} = F\left(\mathrm{Re},\mathrm{S},a,1,y,t-1\right),\ \overline{F_2} = F\left(\mathrm{Re},\frac{1}{\mathrm{Q}},a,\mathrm{Pr},y,t-1\right),$$

$$\overline{E_1} = \frac{1}{2a^2}e^{a(t-1)}\left(\overline{F_1} - \overline{F_2}\right)\times \mathrm{H}\left(t-1\right),$$

$$\overline{G_1} = G\left(\mathrm{Re},\mathrm{S},a,1,y,t-1\right),\ \overline{G_2} = G\left(\mathrm{Re},\frac{1}{\mathrm{Q}},a,\mathrm{Pr},y,t-1\right),$$

$$\overline{E_2} = \frac{1}{2a}\left(\overline{G_2} - \overline{G_1}\right)\times \mathrm{H}\left(t-1\right),\ \overline{E_3} = \overline{E_1} + \overline{E_2},\ E_4 = E_3 - \overline{E_3}$$

$$F_3 = F\left(\mathrm{Re},\mathrm{S},b,1,y,t\right),\ F_4 = F\left(\mathrm{Re},\frac{1}{\mathrm{K}},b,\mathrm{Sc},y,t\right),$$

$$E_5 = \frac{1}{2b^2}e^{bt}\left(F_3 - F_4\right),$$

$$G_3 = G\left(\mathrm{Re},\mathrm{S},b,1,y,t\right),\ G_4 = G\left(\mathrm{Re},\frac{1}{\mathrm{K}},b,\mathrm{Sc},y,t\right),$$

$$E_6 = \frac{1}{2b}\left(G_4 - G_3\right),$$

$$E_7 = E_5 + E_6,$$

$$\overline{F_3} = F\left(\mathrm{Re},\mathrm{S},b,1,y,t-1\right),\ \overline{F_4} = F\left(\mathrm{Re},\frac{1}{\mathrm{K}},b,\mathrm{Sc},y,t-1\right),$$

$$\overline{E_5} = \frac{1}{2b^2}e^{b(t-1)}\left(\overline{F_3} - \overline{F_4}\right)\times \mathrm{H}\left(t-1\right),$$

$$\overline{G_3} = G\left(\mathrm{Re},\mathrm{S},b,1,y,t-1\right),\ \overline{G_4} = G\left(\mathrm{Re},\frac{1}{\mathrm{K}},b,\mathrm{Sc},y,t-1\right),$$

$$\overline{E_6} = \frac{1}{2b}\left(\overline{G_4} - \overline{G_3}\right)\times \mathrm{H}\left(t-1\right),\ \overline{E_7} = \overline{E_5} + \overline{E_6},\ E_8 = E_7 - \overline{E_7},$$

$$a = \frac{\mathrm{Re}}{1-\mathrm{Pr}}\left[\mathrm{Pr}\,\mathrm{Q} - \frac{1}{\mathrm{S}}\right],$$

$$b = \frac{\mathrm{Re}}{1-\mathrm{Sc}}\left[\mathrm{K}\mathrm{Sc} - \frac{1}{\mathrm{S}}\right]$$

The real valued functions ψ, ω, F and G are defined as follows:

$$\psi\left(\xi,\eta,\alpha,y,t\right) = \frac{1}{2}\left[\left(t + \frac{y}{2\xi}\sqrt{\alpha\eta}\right)e^{y\sqrt{\frac{\alpha}{\eta}}}erfc\left(\frac{y}{2}\sqrt{\frac{\alpha}{\xi t}} + \sqrt{\frac{\xi t}{\eta}}\right) + \left(t - \frac{y}{2\xi}\sqrt{\alpha\eta}\right)e^{-y\sqrt{\frac{\alpha}{\eta}}}erfc\left(\frac{y}{2}\sqrt{\frac{\alpha}{\xi t}} - \sqrt{\frac{\xi t}{\eta}}\right)\right]$$

$$\omega\left(\xi,\eta,y,t\right) = \frac{1}{2}\left[e^{\frac{y}{\sqrt{\eta}}}erfc\left(\frac{y}{2\sqrt{\xi t}} + \sqrt{\frac{\xi t}{\eta}}\right) + e^{\frac{-y}{\sqrt{\eta}}}erfc\left(\frac{y}{2\sqrt{\xi t}} - \sqrt{\frac{\xi t}{\eta}}\right)\right]$$

$$F\left(\xi,\eta,\alpha,\delta,y,t\right) = e^{y\sqrt{\frac{\alpha\delta}{\xi}+\frac{\delta}{\eta}}}erfc\left(\frac{y}{2}\sqrt{\frac{\delta}{\xi t}} + \sqrt{\left(\alpha + \frac{\xi}{\eta}\right)t}\right) + e^{-y\sqrt{\frac{\alpha\delta}{\xi}+\frac{\delta}{\eta}}}erfc\left(\frac{y}{2}\sqrt{\frac{\delta}{\xi t}} - \sqrt{\left(\alpha + \frac{\xi}{\eta}\right)t}\right)$$

$$G\left(\xi,\eta,\alpha,\delta,y,t\right) = \left(t + \frac{y}{2\xi}\sqrt{\delta\eta} + \frac{1}{\alpha}\right)e^{y\sqrt{\frac{\delta}{\eta}}}erfc\left(\frac{y}{2}\sqrt{\frac{\delta}{\xi t}} + \sqrt{\frac{\xi t}{\eta}}\right) + \left(t - \frac{y}{2\xi}\sqrt{\delta\eta} + \frac{1}{\alpha}\right)e^{-y\sqrt{\frac{\delta}{\eta}}}erfc\left(\frac{y}{2}\sqrt{\frac{\delta}{\xi t}} - \sqrt{\frac{\xi t}{\eta}}\right)$$

Where $\xi,\ \eta,\ \alpha,\ \delta,\ y,\ t$ are real variables.

LOCAL CO-EFFICIENT OF SKIN FRICTION

The viscous drag at the plate per unit area in the direction of the plate velocity is given by the Newton's law of viscosity in the form:

$$\tau' = \mu\frac{\partial u'}{\partial y'}\bigg]_{y'=0} = \mu\frac{U_0^2}{\nu}\frac{\partial u}{\partial y}\bigg]_{y=0} \tag{42}$$

The co-efficient of skin friction at the plate is given by

$$\tau = \frac{\tau'}{\frac{\mu U_0^2}{\nu}} = \frac{\partial u}{\partial y}\bigg]_{y=0} \tag{43}$$

LOCAL CO-EFFICIENT OF RATE OF HEAT TRANSFER

The heat flux q^* from the plate to the fluid is given by Fourier's law of conduction in the form

$$q^* = -K_T\frac{\partial T'}{\partial y'}\bigg]_{y'=0} = -K_T\left(T'_W - T'_\infty\right)\frac{U_0}{\nu}\frac{\partial\theta}{\partial y}\bigg]_{y=0} \tag{44}$$

The co-efficient of the rate of heat transfer from the plate to the fluid in terms of Nusselt number is given by

$$\mathrm{Nu} = \frac{q^*\nu}{K_T\left(T'_W - T'_\infty\right)U_0} = -\frac{\partial\theta}{\partial y}\bigg]_{y=0} \tag{45}$$

LOCAL CO-EFFICIENT OF MASS TRANSFER

The mass flux from the plate to the fluid is governed by Fick's law in the following form:

$$\mathrm{M_W} = -D_M \left.\frac{\partial C'}{\partial y'}\right]_{y'=0} = -D_M \left(C'_W - C'_\infty\right)\frac{U_0}{\nu}\left.\frac{\partial \phi}{\partial y}\right]_{y=0} \quad (46)$$

The rate of mass transfer from the plate to the fluid in terms of the Sherwood number Sh is given by

$$\mathrm{Sh} = \frac{\mathrm{M_W}\nu}{D_M\left(C'_W - C'_\infty\right)U_0} = -\left.\frac{\partial \phi}{\partial y}\right]_{y=0} \quad (47)$$

Detailed computations of skin friction, Nusselt number and Sherwood number are obtained but not presented here for the sake of brevity.

RESULTS AND DISCUSSION

In order to get clear insight of the physical problem, numerical calculations from the analytical solutions for the representative velocity field, temperature field, concentration field, the co-efficient of skin friction, the rate of heat transfer at the plate in terms of Nusselt number and the rate of mass transfer near the plate in terms of Sherwood number have been carried out by assigning some arbitrarily chosen specific values to the physical parameters like ramped velocity parameter A, Reynolds number Re, Schmidt number Sc, the radiation parameter Q, the porosity parameter S, and the chemical reaction parameter K. The numerical values computed from the analytical solutions of the problem have been visualized in Figures 2 to 13. Comparison has been made in Figure 14 with Figure 15 (viz. Figure 4 of Das et al., 2011). Figure 14 clearly shows how the problem taken by Das et al. (2011) is a special case of the current problem for the parameters A = 0, Gm = 0, Re = 1, Pr = 0.71, Sc = 0.6, Q = 2, S = 0.04, K = 0.001 and *t* = 0.1. Both Figures 14 and 15 establish the conformity of the two problems (viz. the current problem and the work of Das et al. (2011)) in the special case when the effect of A, Gm and K (that is, ramped velocity parameter, solutal Grash of number and chemical reaction) are absent. These figures uniquely indicate that an increase in thermal Grash of number causes an overall increase in fluid velocity. Further the velocity profiles for both the problems are almost identical, thereby showing an excellent agreement between the results obtained by Das et al. (2011) and the present authors.

The velocity profiles under the influence of A and K are exhibited in Figures 2 and 3. Figure 2 shows that an increase in ramped velocity parameter A causes the fluid velocity to increase steadily indicating the fact that the fluid motion is accelerated under the influence of parameter A. This figure further establishes the fact that the fluid velocity first increases in a thin layer adjacent to the plate and thereafter it decreases asymptotically as we move away from the plate. It is found that an increase in Reynolds number leads the velocity to increase gradually. We recall that an increase in Reynolds number means a decrease in internal friction. That is, the fluid velocity increases for small viscosity and this phenomenon is in excellent agreement with the physical fact that fluid become freer to move when the internal friction gets decreased. It is observed that our chemical reaction decelerates the flow as is evident from Figure 3.

The temperature profiles have been depicted in Figures 4 and 5. From Figure 4, we notice that an increase in Reynolds number raises the fluid temperature considerably. The thermal radiation has an inhibiting effect on the temperature distribution (Figure 5) which is supported by the phenomena that radiation causes a faster dissipation of heat and consequently lowers the temperature.

Figures 6 and 7, corresponds to concentration profiles versus normal coordinate *y* under the effect of the parameters Sc and K. It is inferred from Figures 6 and 7 that an increase in Sc or K leads the concentration level of the fluid to drop steadily. We recall that an increase in Schmidt number Sc means a fall in mass diffusivity. It interprets that an increase in mass diffusivity causes the species concentration of the fluid to rise upwards. This observation is in agreement with the outcome of Figure 6. Moreover, chemical reaction results in gradual decrease in the concentration level of the fluid as validated by Figure 7.

The variation in skin friction versus Reynolds number under the influence of Q, S and K are presented in Figures 8 to 10. All these three figures uniquely indicate that an increase in Q or S or K causes a corresponding increase in skin friction. It indicates that the viscous drag at the plate gets increased under the influence of radiation, porosity of the medium and chemical reaction. Further, we observe that the effect of the above parameters on the skin friction are more pronounced for small Reynolds number and for higher Reynolds number these parameters ceases to affect τ . This phenomenon is well supported by the fact that for large Reynolds number (that is, for small viscosity) the internal friction decreases significantly. The trend of behaviour of skin friction under Q, S and K is identical in case of ramped plate temperature and isothermal plate temperature.

Figure 11 shows how the rate of heat transfer from the plate to fluid in terms of Nusselt number Nu is influenced by Q and Re. An interesting observation is made in this figure that for ramped wall temperature, the Nusselt number decreases at a faster rate for small Reynolds number and it decreases slowly and steadily for higher Reynolds number that is, for small viscosity; on the other hand in case of uniform temperature (isothermal plate) Nu becomes steady after an initial increase for large viscosity.

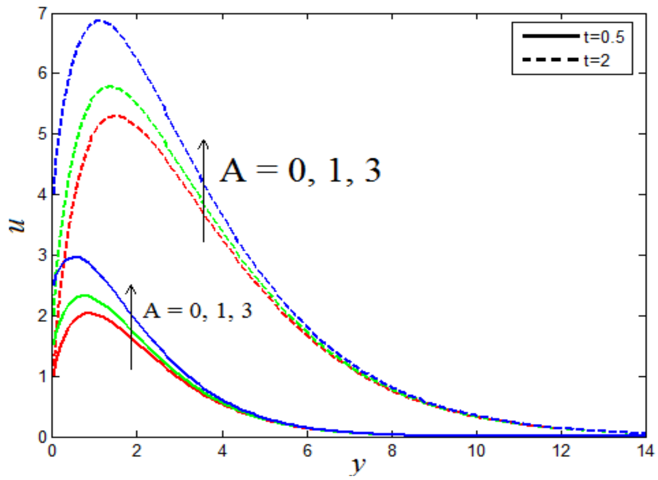

Figure 2. Velocity versus y for Gr = 10, Gm = 5, Re = 10, Pr = 0.71, Sc = 0.60, Q = 5, S = 4, K = 0.4.

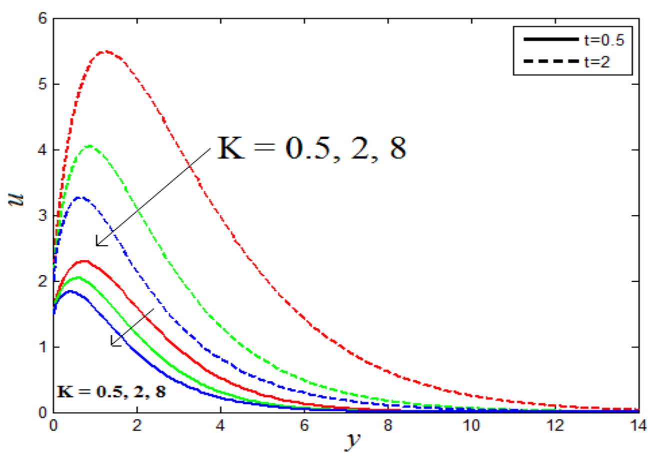

Figure 3. Velocity versus y for A = 1, Gr = 10, Gm = 5, Re = 10, Pr = 0.71, Sc = 0.60, Q = 5, S = 4.

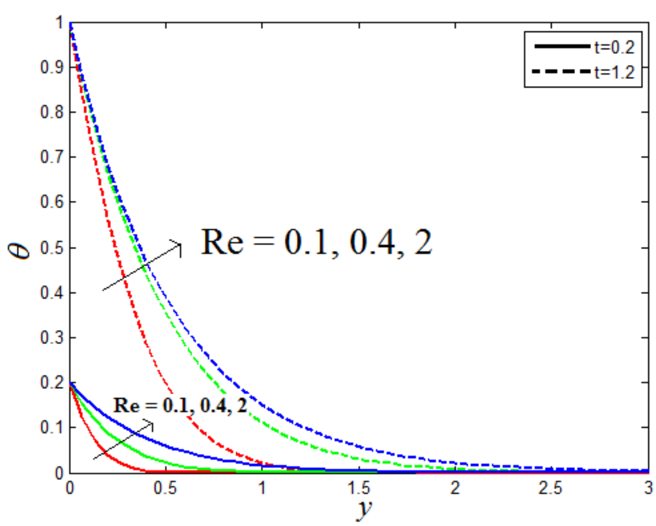

Figure 4. Temperature versus y for Pr = 0.71, Q = 5.

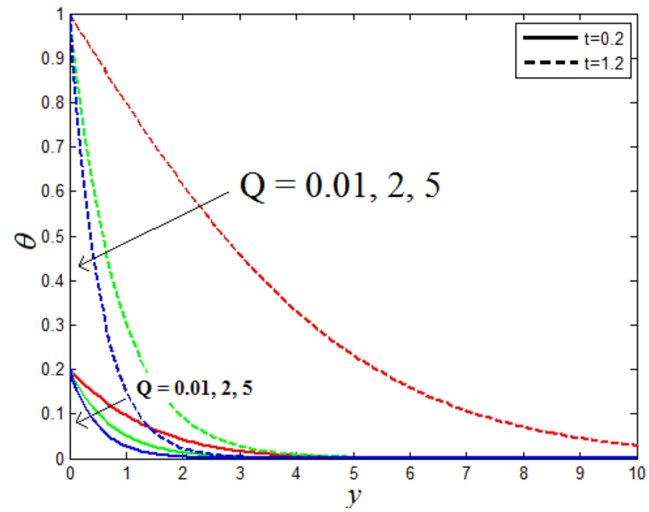

Figure 5. Temperature versus y for Re = 10, Pr = 0.71.

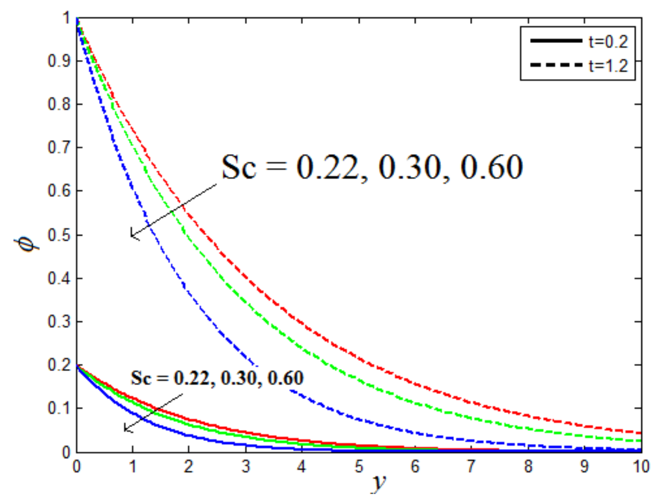

Figure 6. Concentration versus y for Re =10, K = 0.4.

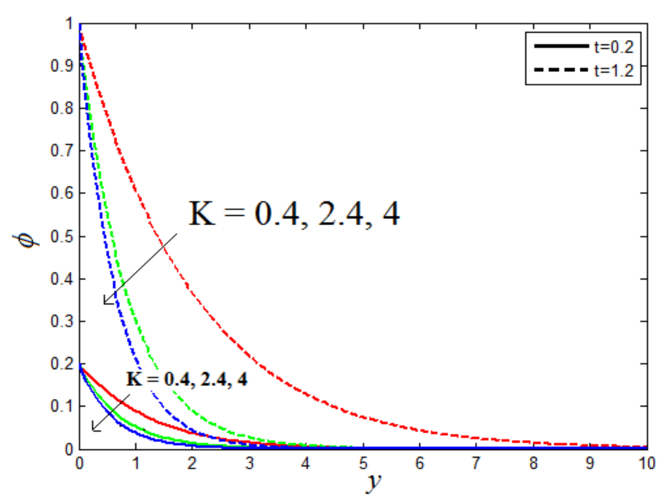

Figure 7. Concentration versus y for Re = 10, Sc = 0.60.

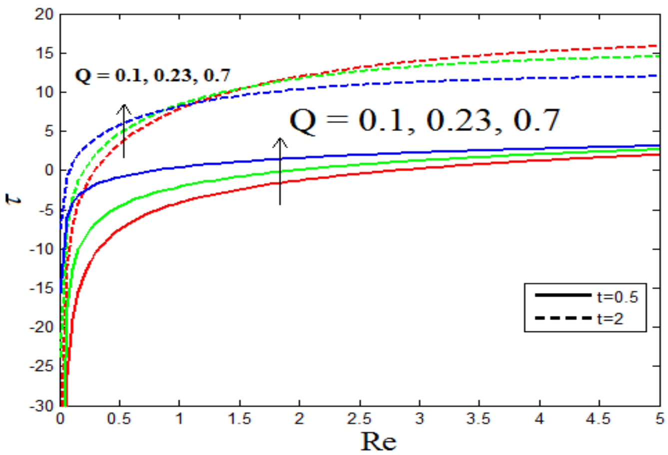

Figure 8. Skin friction versus Re for A = 1, Gr = 10, Gm = 5, Pr = 0.71, Sc = 0.60, S = 4, K = 0.4.

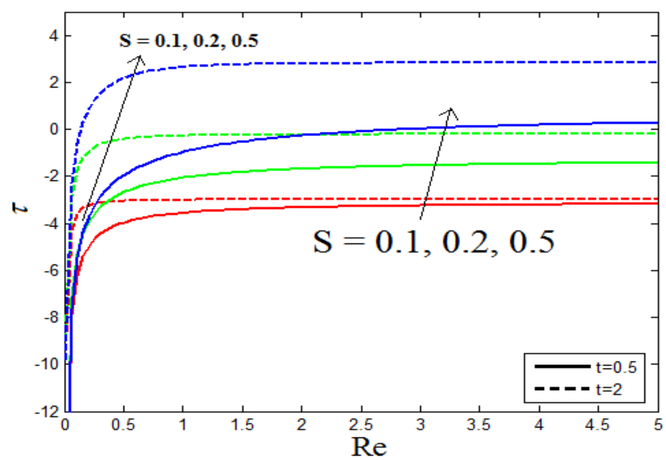

Figure 9. Skin friction versus Re for A = 1, Gr = 10, Gm = 5, Pr = 0.71, Sc = 0.60, Q = 5, K = 0.4.

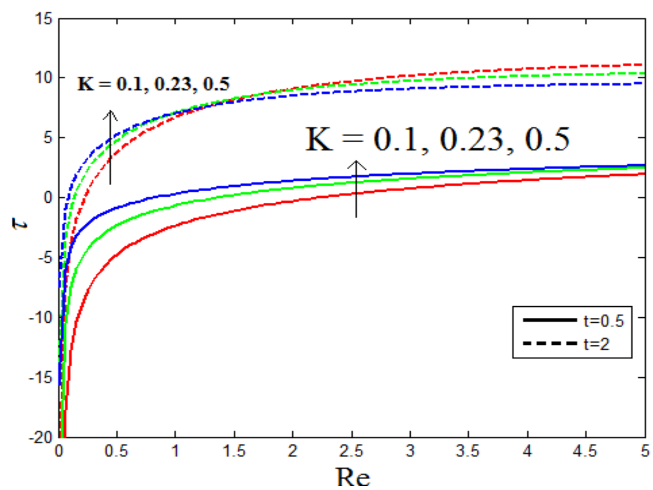

Figure 10. Skin friction versus Re for A = 1, Gr = 10, Gm = 5, Pr = 0.71, Sc = 0.60, Q = 2, S = 4.

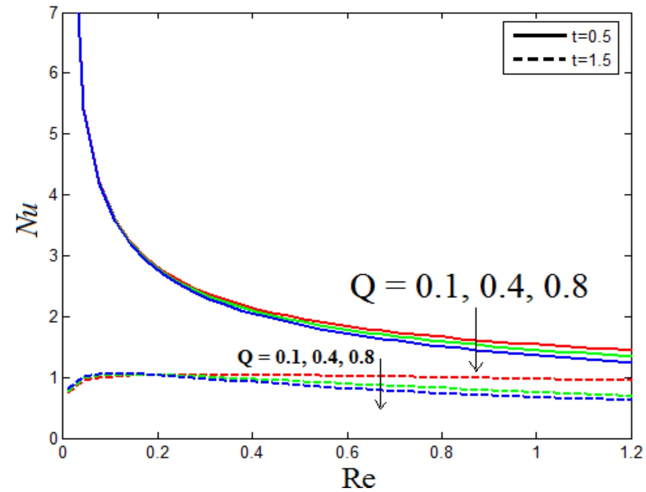

Figure 11. Nusselt number versus Re for Pr = 0.71.

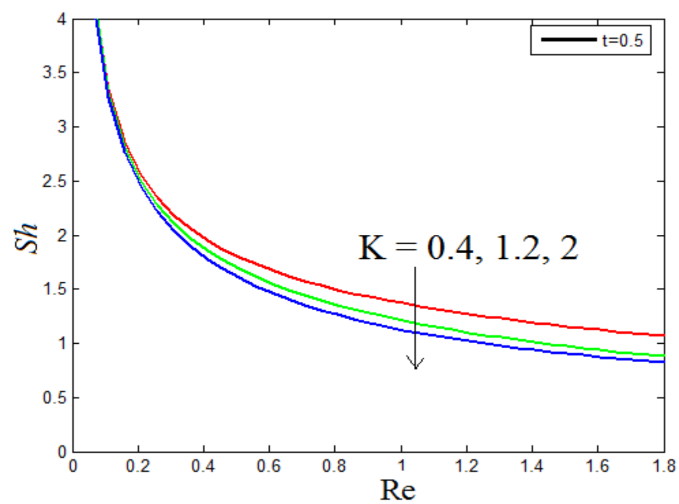

Figure 12. Sherwood number versus Re for Sc = 0.60, t = 0.5.

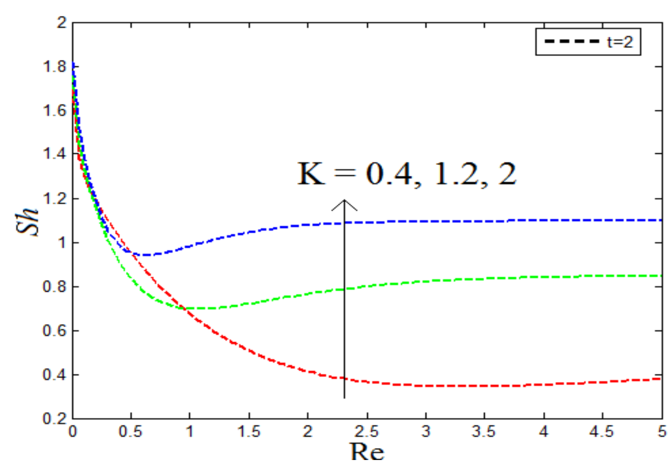

Figure 13. Sherwood number versus Re for Sc = 0.60, t = 2.

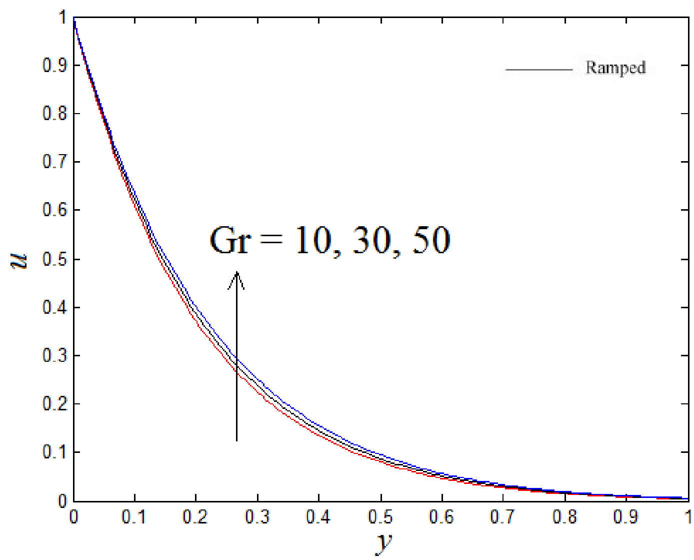

Figure 14. Velocity versus y for A = 0, Gm = 5, Re = 1, Pr = 0.71, Sc = 0.6, Q = 2, S = 0.04, K = 0.001, t = 0.1.

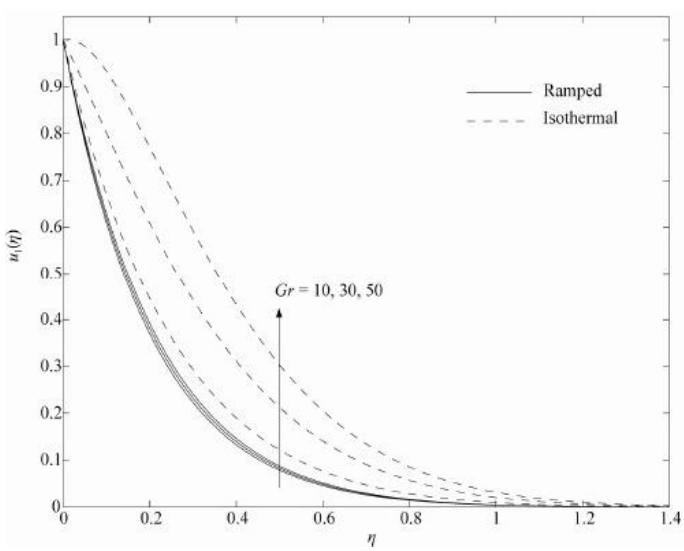

Figure 15. Figure 4 of the work of Das et al. (2011).

Figures 12 to 13 exhibit the change of behavior of the rate of mass transfer from the plate to the fluid in terms of Sherwood number Sh for $t<1$ (ramped temperature) and $t>1$ (isothermal temperature) respectively. Under chemical reaction, the mass transfer rate falls down in case of ramped plate temperature; but a reverse trend of behaviour is observed in case of isothermal plate temperature. For small Reynolds number, this rate is very sharp in either case. However, in case of uniform temperature of the plate, for small value of Reynolds number, Sh sharply decreases and as Reynolds number increases an opposing behavior is marked.

Conclusions

The fluid flow is accelerated with a corresponding increase in ramped velocity parameter A. Chemical reaction has a retarding effect on the fluid flow. The fluid temperature decreases with increasing thermal radiation and it increases as viscosity drops. The species concentration in the fluid decreases with increasing Schmidt number. The viscous drag at the plate gets increased under the influence of radiation, porosity of the medium and chemical reaction. In case of velocity, temperature and concentration profiles, as well as the skin friction, the trend of behavior is similar for both isothermal $(t>1)$ and ramped $(t<1)$ plate temperature. For ramped wall temperature, the Nusselt number decreases at a faster rate for small Re and it decreases slowly and steadily for higher Re; however, in case of uniform temperature Nu becomes steady after an initial increase for large viscosity. Under chemical reaction, the mass transfer behavior is grossly different for ramped and isothermal plate temperature.

NOMENCLATURE

A, Ramped velocity parameter (dimensionless); C_p, specific heat at constant pressure; C'_W, Reference concentration; C'_∞, concentration far away from plate; C', fluid concentration; D_M, mass diffusivity; $e_{\lambda h}$, Plank function; $\vec{g}$, gravitational acceleration vector; g, acceleration due to gravity; Gr, thermal Grashof number; Gm, solutal Grashof number; k^*, permeability of porous medium; K_T, thermal conductivity; $\overline{K}$, rate of first order homogeneous chemical reaction; $(K_\lambda)_W$, absorption co-efficient; K, chemical reaction parameter; p, pressure; Pr, Prandtl number; $\vec{q}$, fluid velocity vector; q_r, radiative heat flux; Q, radiation parameter; Re, Reynolds number; S, porosity parameter; Sc, Schmidt number; t', time; T', fluid temperature; T'_W, reference temperature; T'_∞, temperature far away from the plate; t, non-dimensional time; t_0, characteristic time; U_0, plate velocity; u', x-component of $\vec{q}$; u, non-dimensional fluid velocity; (x', y', z'), cartesian coordinates; y, non-dimensional y-coordinate; $\bar{\mathrm{f}}(y,s)$, Laplace transform of function $\mathrm{f}(y,t)$; ρ, fluid density; ρ_∞, fluid density far away from plate; μ, co-efficient of viscosity; φ, viscous dissipation of energy per unit volume; $\delta n'$, an element of the outward normal; β, co-efficient of volume expansion for heat transfer; $\overline{\beta}$, co-efficient of volume expansion for mass transfer; ν, kinematic viscosity; θ, non-dimensional temperature; ϕ, non-dimensional concentration; λ, wavelength; $(\text{Subscript})_w$ refers to the values of the physical quantities at the plate; $(\text{Subscript})_\infty$, refers to the values of the physical quantities far away from the plate.

REFERENCES

Anjalidevi SP, Kandasamy R (1999). Effects of chemical reaction, heat and mass transfer on laminar flow along a semi-infinite horizontal plate. Heat Mass Transf. 35(6):465-467.

Bird RB, Stewart WE, Lightfoot EN (2001). Transport phenomena. John Wiley and Sons, USA. P. 551.

Cess RD (1966). The interaction of thermal radiation with free convection heat transfer. Int. J. Heat Mass Transf. 9(11):1269-1277.

Chamkha AJ, Takhar HS, Soundalgekar VM (2001). Radiation effects on free convection flow past a semi-infinite vertical plate with mass transfer. Chem. Eng. J. 84(3):335-342.

Cogley AC, Gilles SE, Vincenti WG, Ishimoto S (1968). Differential approximation for radiative heat transfer in a non grey gas near equilibrium. Am. Inst. Aeron. Astron. 6(3):551-553.

Cussler EL (2009). Diffusion: Mass transfer in fluid systems. Cambridge University Press, UK. pp. 460-461.

Das S, Jana M, Jana RN (2011). Radiation effect on natural convection near a vertical plate embedded in a porous medium with ramped wall temperature. Open J. Fl. Dyn. 1(1):1-11.

England WG, Emery AF (1969). Thermal radiation effects on the laminar free convection boundary layer of an absorbing gas. J. Heat Transf. 91(1):37-44.

Gebhart B, Pera L (1971). The nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion. Int. J. Heat Mass Transf. 14(12):2025-2050.

Ghoshdastidar PS (2004). Heat transfer. Oxford University Press, Oxford. P. 225.

Hossain MA, Pop I (2001). Radiation effects on free convection over a vertical flat plate embedded in a porous medium with high porosity. Int. J. Therm. Sci. 40:289-295.

Muthucumaraswamy R, Ganesan P (2001a). First order chemical reaction on flow past an impulsively started vertical plate with uniform heat and mass flux. Acta Mech. 147:45-57.

Muthucumaraswamy R, Ganesan P, Soundalgekar VM (2001b). Heat and mass transfer effects on flow past an impulsively started vertical plate. Acta. Mech. 146:1-8.

Muthucumaraswamy R, Shankar MR (2011). First order chemical reaction and thermal radiation effects on unsteady flow past an accelerated isothermal infinite vertical plate. Ind. J. Sci. Technol. 4(5):573-577.

Nield DA, Bejan A (2006). Convection in porous media. Springer, USA. pp. 94-97.

Raptis A (1998). Radiation and free convection flow through a porous medium. Int. Comm. Heat Mass Transf. 25(2):289-295.

Raptis A, Perdikis C (1999). Radiation and free convection flow past a moving plate. Int. J. Appl. Mech. Eng. 4(4):817-821.

Sattar MA, Rahman MM, Alam MM (2000). Free convection flow and heat transfer through a porous vertical flat plate immersed in a porous medium with variable suction. J. Energy Heat Mass Transf. 21(2):17-21.

Welty JR, Wicks CE, Wilson RE, Rorrer GL (2007). Fundamentals of momentum, heat and mass transfer. John Wiley and Sons, USA. P. 398.