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Full Length Research Paper

Compressed channel estimation for sparse multipath two-way relay networks

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Relay network was introduced to realize high-data rate transmission and high capacity with limited transmit power. However, channel state information (CSI) is needed due to the requirement from coherent data detection and the self-data removal at terminals in the two-way relay networks (TWRN). Traditional linear probing techniques are able to acquire the accurate CSI by using enough training sequence. However, they may lead to low bandwidth efficiency due to that the implicit assumption of a rich underlying multipath. However, in many cases, the multipath channel has a sparse structure. Unlike the previous methods, we propose a compressed channel estimation method which exploit the sparse structure in multipath TWRN and hence provide significant improvements in MSE performance when compared with conventional LS-based linear channel probing strategies in the TWRN. Simulation results confirm performance of the proposed method.

Key words: Two-way relay networks (TWRN), channel state information (CSI), compressive sensing, sparse multipath channel, compressed channel estimation.

INTRODUCTION

Relay communications have been drawing great attention in recent years. In this paper, two-way relay network (TWRN) is investigated, where two terminals, T_1 and T_2, exchange information based on the assistance of a relay R via amplify-and-forward (AF). TWRN have been intensively studied due to their capability of enhancing the transmission capacity and providing the spatial diversity for single-antenna wireless transceivers by employing the relay nodes as "virtual" antennas (Gao et al., 2008). A major difficulty is how to effectively recover the data transmitted over an unknown frequency-selective fading channel. Because of demodulation and coherence detection of each terminal, not only needs to know the channel state information (CSI) from relay to itself but also the CSI from the other terminal to relay.

Training-based linear channel estimation methods were proposed in Gao et al. (2009) for the TWRN. The authors considered optimal training sequence design and linear probing methods based on the implicit assumption of a rich underlying multipath environment. In other words, training-based methods proposed in these works are mainly composed of linear reconstruction techniques such as least square (LS), minimum mean square error (MMSE), and linear maximum signal-noise ratio (LMSNR), thus reducing the problem of channel estimation to that of designing optimal training sequences. In recent years, numerous channel measurements have shown that the multipath channels tend to exhibit cluster or sparse structures in which majority of the channel taps end up being either zero or below the noise floor (Yan et al., 2007; Gui et al., 2011).

However, traditional training-based linear methods that utilize linear reconstruction strategies at the receiver seem incapable of exploiting the potential sparse multipath channels, thereby leading to overutilization of the key communication resources, such as energy and

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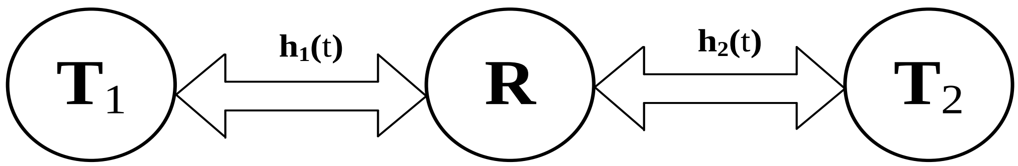

Figure 1. Two-way relay network.

bandwidth. In other words, exploiting this channel sparsity will improve bandwidth efficiency. In this paper, we first introduce a sparse channel estimation technique with CoSaMP algorithm (Needell and Tropp, 2009) for TWRN. And we present a theoretical analysis on compressed channel estimation method and then confirmed it by computer simulations.

SYSTEM MODEL

Consider a two-way relay network (TWRN) where the two terminal users, T_1 and T_2, exchange information by the assistance of an relay is shown in Figure 1. And it works under amplify-and-forward (AF) protocol. The signal exchange in TWRN is divided into two phases: transmitting period and broadcasting period. The relay and two terminals are assumed to have one antenna each. Let channel impulse responses $\mathbf{h}_i$ ($i=1,2$) be the deterministic (but unknown) K_i -sparse impulse responses of the frequency-selective fading channels between terminals and relay. Assuming that the channel vector $\mathbf{h}_i$ is comprised of L_i ($i=1,2$) paths. The multipath impulse response, $\mathbf{h}_i(\tau)$, can be expressed as (Adachi et al., 2009):

$$\mathbf{h}_i(\tau)=\sum_{l=0}^{L_i-1} h_{i,l}\delta(\tau-\tau_l)\,,\quad i=1,2\,. \tag{1}$$

Due to the $\mathbf{h}_i$, $i=1,2$ are sparse multipath channels, it should be noted that $K_i << L_i$,, that is to say, on these kind of channels which were supported by few significant channel taps, while most of channel taps are approximate to zero or under the noise floor. A example of sparse multipath channel is shown in Figure 2. They are assumed to be zero-mean circularly symmetric complex Gaussian random variables with variance $\sigma^2_{\mathbf{h}_i}$. The average transmission powers of T_1, T_2 and relay are P_1, P_2 and P_r, respectively. For the time being, we assume perfect synchronization among three terminals.

At the first time slot, two terminals T_1 and T_2 send out data symbols $\mathbf{d}_i \in C^N$ ($i=1,2$), the power constraints of the transmission is $E\{\|\mathbf{d}_i\|_2^2\}=P_i$, $i=1,2$, where $E[\cdot]$ is the ensemble average operation. The received signal $\mathbf{y}_r \in C^{N+L-1}$ at the relay R can be expressed:

$$\mathbf{y}_r=\mathbf{H}_1\mathbf{d}_1+\mathbf{H}_2\mathbf{d}_2+\mathbf{n}_r, \tag{2}$$

where $\mathbf{H}_1$ and $\mathbf{H}_2$ are two circulant matrices with first columns of $[\mathbf{h}_1^T,\mathbf{0}_{1\times(N-L)}]^T$ and $[\mathbf{h}_2^T,\mathbf{0}_{1\times(N-L)}]^T$, respectively; $\mathbf{n}_r \in C^N$ is a complex Gaussian noise with zero mean and covariance matrix $E\{\mathbf{n}_r\mathbf{n}_r^H\}=\sigma_r^2\mathbf{I}_N$.

To minimize the computational burden at R, which broadcasts the scaled version of this superposition signal, that is, R works under the amplify-and-forward (AF) protocol. The received signal in Equation (2) is then amplified by a relay factor:

$$\alpha=\sqrt{\frac{P_r}{\sigma^2_{\mathbf{h}_1}P_1+\sigma^2_{\mathbf{h}_2}P_2+\sigma_n^2}}, \tag{3}$$

and then is broadcasted. Without loss of generality, we will only consider the channel estimation problem at T_1, while the discussion for T_2 can be made correspondingly. The received signal at T_1 can be written as:

$$\begin{aligned}\mathbf{y}_1&=\alpha\mathbf{H}_1\mathbf{y}_r+\mathbf{n}_1\\&=\alpha\mathbf{H}_1\mathbf{H}_1\mathbf{d}_1+\alpha\mathbf{H}_1\mathbf{H}_2\mathbf{d}_2+\mathbf{n}_2,\end{aligned} \tag{4}$$

where $\mathbf{n}_2=\alpha\mathbf{H}_1\mathbf{n}_r+\mathbf{n}_1$, $\mathbf{n}_1 \in C^N$ is a complex Gaussian noise with zero mean and covariance matrix $E\{\mathbf{n}_2\mathbf{n}_2^H\}=\sigma_n^2(\alpha^2|\mathbf{H}_1|^2+\mathbf{I}_N)$. According to the System model (4), the maximum likelihood data detection (MLD) is done at the terminal T_1 as:

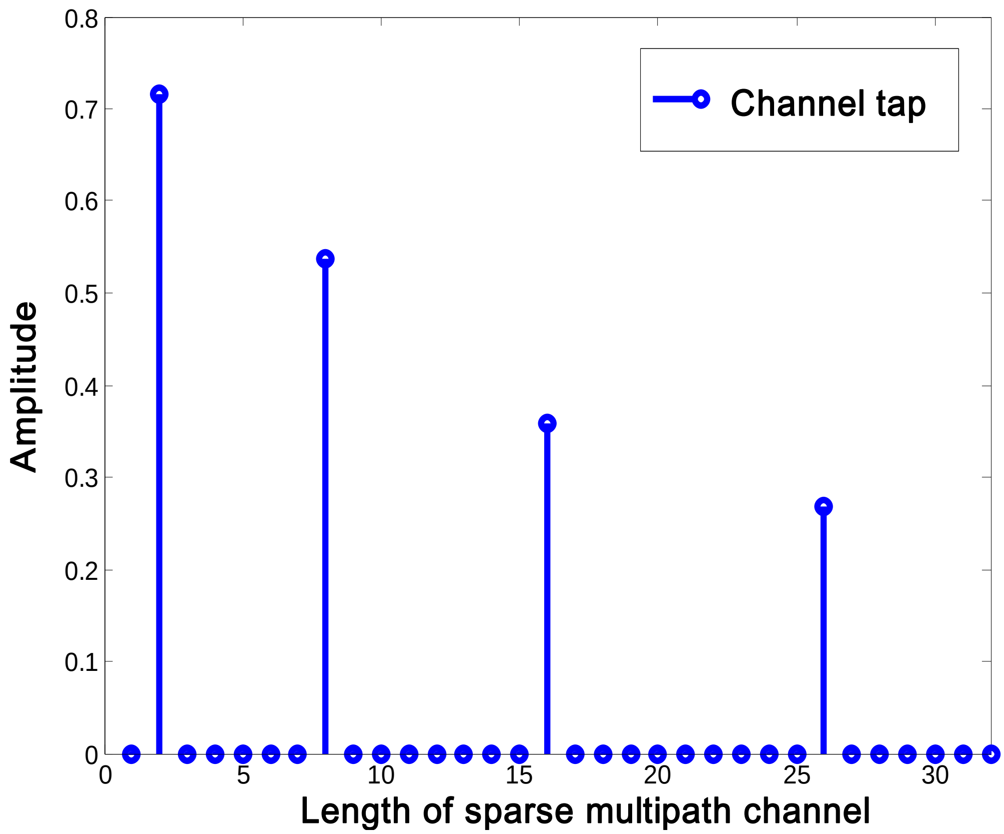

Figure 2. An example of sparse multipath channel.

$$\begin{aligned}\tilde{\mathbf{d}}_2 &= \arg\max_{\mathbf{d}_2} \mathrm{P}(\mathbf{y}|\mathbf{d}_1,\mathbf{d}_2)\\ &= \arg\max_{\mathbf{d}_2} \frac{1}{\pi\sigma_n^2(\alpha^2|\mathbf{H}_1|^2+\mathbf{I}_N)}\\ &\quad\times \exp\left\{-\frac{|\mathbf{y}-\mathbf{H}_1\mathbf{H}_1\mathbf{d}_1-\mathbf{H}_1\mathbf{H}_2\mathbf{d}_2|^2}{\sigma_n^2(\alpha^2|\mathbf{H}_1|^2+\mathbf{I}_N)}\right\}\\ &= \arg\min_{\mathbf{d}_2}|\mathbf{y}-\mathbf{H}_1\mathbf{H}_1\mathbf{d}_1-\mathbf{H}_1\mathbf{H}_2\mathbf{d}_2|,\end{aligned} \tag{5}$$

Observing Equation (5) we find that the exact knowledge of $\mathbf{H}_1$ and $\mathbf{H}_2$ are not required for ML data detection. Instead, the self- and the cross-products of the circulant channel matrices, that is, $\mathbf{H}_1\mathbf{H}_1$ and $\mathbf{H}_1\mathbf{H}_2$, are those only needed. Hence, these channel matrices can be directly estimated at T_1. Comparing to the channel estimation at R, which the estimators $\tilde{\mathbf{H}}_1$ and $\tilde{\mathbf{H}}_2$ feed back the estimates to T_1, there exist some advantages (Gao et al., 2009), that is, estimation error and spectrum efficiency. However, channel estimation According to circulant matrix theory, $\mathbf{H}_i$, $i=1,2$ can be decomposed as:

$$\mathbf{H}_i = \mathbf{W}^H \Xi_i \mathbf{W},\ i=1,2, \tag{6}$$

where $\Xi_i = \mathrm{Diag}[H_{i,0},...,H_{i,k},...,H_{i,N-1}] \in \mathrm{C}^{N\times N}$ is diagonal matrix about frequency-domain channel coefficients, and $H_{i,k} = \sum_{l=0}^{L-1} h_i(l) e^{-j2\pi kl/K}, k=0,...,N-1$, and $\mathbf{W}$ is the unitary discrete Fourier transform matrix with:

$$\mathbf{W}^{mn} = \frac{1}{\sqrt{N}} e^{-j2\pi kl/N},\ m,n=0,...,N-1. \tag{7}$$

Therefore, Equation (4) can be rewritten as:

$$\mathbf{y}_1 = \mathbf{W}^H \alpha \Xi_1 \Xi_1 \mathbf{W}\mathbf{d}_1 + \mathbf{W}^H \alpha \Xi_1 \Xi_2 \mathbf{W}\mathbf{d}_2 + \mathbf{n}_2. \tag{8}$$

Based on the discrete fourier transform (DFT), with a condition of $(2L-1) \le N$, we observe that $\mathbf{W}^H \alpha \Xi_1 \Xi_i \mathbf{W}$, $i=1,2$ are the decomposition of a circulant matrix which are constructed from a convolution impulse response vector $\mathbf{h} = \alpha(\mathbf{h}_1 * \mathbf{h}_1)$ and $\mathbf{g} = \alpha(\mathbf{h}_1 * \mathbf{h}_2)$, respectively. The channel length of $\mathbf{h}$ or $\mathbf{g}$ is $(2L-1)$. If the (8) is pre-multiplied by $\mathbf{W}$, then the received signal at terminal user T_1 is given by:

$$\mathbf{y} = \mathrm{Diag}(\mathbf{W}\mathbf{d}_1)\mathbf{F}\mathbf{h} + \mathrm{Diag}(\mathbf{W}\mathbf{d}_2)\mathbf{F}\mathbf{g} + \mathbf{n}, \qquad (9)$$

where $\mathbf{F}$ is the first $(2L-1)$ columns of $\sqrt{N}\mathbf{W}$ and $\mathbf{n} = \alpha\mathbf{\Xi}_1\mathbf{W}\mathbf{n}_r + \mathbf{W}\mathbf{n}_1$ denote complex Gaussian random noise vector with zero mean and covariance matrix of $\mathrm{E}\{\mathbf{n}\mathbf{n}^H\} = \sigma_n^2(\alpha^2|\mathbf{\Xi}_1|^2 + \mathbf{I}_N)$. To be the convenient for compressive channel sensing, we rewritten (8) as simply system model:

$$\mathbf{y} = \mathbf{D}_1\mathbf{h} + \mathbf{D}_2\mathbf{g} + \mathbf{n}, \qquad (10)$$

where $\mathbf{D}_i = \mathrm{Diag}(\mathbf{W}\mathbf{d}_i)\mathbf{F},\ i = 1,2$ denote transformed data symbols. After discussing the system model, we consider the signal self-interference removal and detection at the terminal T_1.

COMPRESSED CHANNEL ESTIMATION

From Equation (6), it can be understood that the exact knowledge of channel vectors $\mathbf{h}_1$ and $\mathbf{h}_2$ are not required for self-information removal and coherent detection. Only composite channel estimators $\hat{\mathbf{b}}$ and $\hat{\mathbf{c}}$ are necessary. Suppose that $\mathbf{x}_1$ and $\mathbf{x}_2$ are training sequences which are independent of data $\mathbf{d}_i$ ($i = 1,2$). Hence, training-based sparse channel model and received signal can be given by:

$$\mathbf{y}_{tr} = \alpha\mathbf{x}_1 * \mathbf{b} + \alpha\mathbf{x}_2 * \mathbf{c} + \mathbf{n}, \qquad (11)$$

According to the random matrix theorem (Gray, 2006), circulant matrices have convolutional structure inherent to linear systems identification problems. Thus, system model (7) can be rewritten using linear matrix-vector product form as:

$$\mathbf{y}_{tr} = \alpha\mathbf{X}_1\mathbf{b} + \mathbf{X}_2\mathbf{c} + \mathbf{n} = \alpha\mathbf{X}\boldsymbol{\theta} + \mathbf{n}, \qquad (12)$$

where $\mathbf{X} = \mathbf{X}_1, \mathbf{X}_2 \in \mathrm{C}^{\tilde{N}\times(4L-2)}$ and $\boldsymbol{\theta} = [\mathbf{b}^T, \mathbf{c}^T]^T \in \mathrm{C}^{4L-2}$. Define $\tilde{N} = N + 2L - 2$ where $\mathbf{X}_{i=1,2}$ are $\tilde{N}\times(2L-1)$ partial circulant channel matrix whose first row and first column can written as $[x_i(0), \mathbf{0}_{2L-2}^T]$ and $[\mathbf{x}_i, \mathbf{0}_{2L-2}^T]^T$, respectively.

For compressed channel estimation in the AF-TWRN, we employ totally separated training period and data transmission period. Hence, we will only consider training-based channel estimation problem in this part. According to Equation 12, the detail of the sparse channel estimation can be implemented by CoSaMP (Needell and Tropp, 2009) as follows:

(i) Input: $\mathbf{X}_1$, $\mathbf{X}_2$, S and $\mathbf{y}$, that are training sequences transmitted from T_1 and T_2, the number of dominant channel taps of $\boldsymbol{\theta}$, observation vector $\mathbf{y}$ at T_1. The superimposed training sequence $\mathbf{X} = \mathbf{X}_1, \mathbf{X}_2$ is combined with $\mathbf{X}_1$ and $\mathbf{X}_2$ which satisfy RIP with overwhelming probability.
(ii) Output: Channel estimator $\hat{\boldsymbol{\theta}}$ which includes $\hat{\mathbf{b}}$ and $\hat{\mathbf{c}}$.
Initialize the number of nonzero taps: $\Omega^0 = \varnothing$.
Initialize the residual: $\mathbf{r}^0 = \mathbf{y}$.

While stopping criterion $\{i : i \geq 4S \mid i = 100\}$ is not satisfied, Do
Channel identification: $Q_i = \mathbf{X}^*\mathbf{r}^i$, select $\Omega_{P_i}^i = \mathrm{supp}(Q_i, 2S)$.
Update the channel dominant taps: $\Omega^i = \Omega^{i-1} \cup \Omega_{P_i}^i$.
Channel estimation with LS: $\hat{\boldsymbol{\theta}}^i = \mathbf{X}_{\Omega^i}^{\dagger}\mathbf{y}$.
Prune non-dominant channel coefficients: $\Omega_D^i = \mathrm{supp}(\hat{\boldsymbol{\theta}}^i, S)$ and $\hat{\boldsymbol{\theta}}_{(\Omega_D^i)^c}^i = 0$.
Update the estimation residual: $\mathbf{y}_r^i = \mathbf{y} - \mathbf{X}_{\Omega_D^i}(\hat{\boldsymbol{\theta}}^i)_{\Omega_D^i}$
End while

(iii) Remark: The channel estimates $\hat{\boldsymbol{\theta}}$ by the proposed estimation method, and then separate the $\hat{\boldsymbol{\theta}}$ into $\hat{\mathbf{b}}$ and $\hat{\mathbf{c}}$. $\Omega_{Q_i}^i = \mathrm{supp}(Q_i, 2S)$ means to that it select the maximum $2S$ dominant channel coefficients in Q_i. Ω^c denotes the complementary set of Ω.

SIMULATION RESULTS

In this study, the mean square error (MSE) performance of the proposed method with CoSaMP algorithm (Needell and Tropp, 2009) is evaluated by simulations.

For the purpose of comparison, the MSE performance of other existing algorithms such as LS channel estimator and lower bound (the positions of dominant taps are perfect known) will also be evaluated as for reference. The simulation parameters are given in Table 1. We always fix $P_1 = P_2 = P_r = P$ and SNR is defined as P/σ_n^2. At first, MSE performances of channel estimators are plotted as a function of signal-to-noise (SNR) in Figures 3 and 4, with the training sequence length $M = 40, 60, 80$, as a parameter. The proposed method has a small gap to lower bound but has better than LS-based linear channel estimation. By increasing the more training sequence the better MSE performance can be obtained. Secondly, we have evaluated the complexity ratio between proposed

Table 1. Simulation parameters.

channel estimation	***Linear estimation method***	**LS**
	Compressed channel estimation	**CoSaMP**
Channel fading	Frequency-selective block fading	
Channel length ($\mathbf{h}_1$, $\mathbf{h}_2$)	$L_i = 16 \quad i = 1,2$	
Channel length ($\mathbf{b}$, $\mathbf{c}$)	$(2L_i - 1) = 31$	
Taps coefficients	Random Gaussian independent variables	
SNR (dB)	0~30	
Training signal	Random Toeplitz structure	
Length of training signal	*M* = 40,60,80	
Monte Carlo	*M* = 1000 trails	

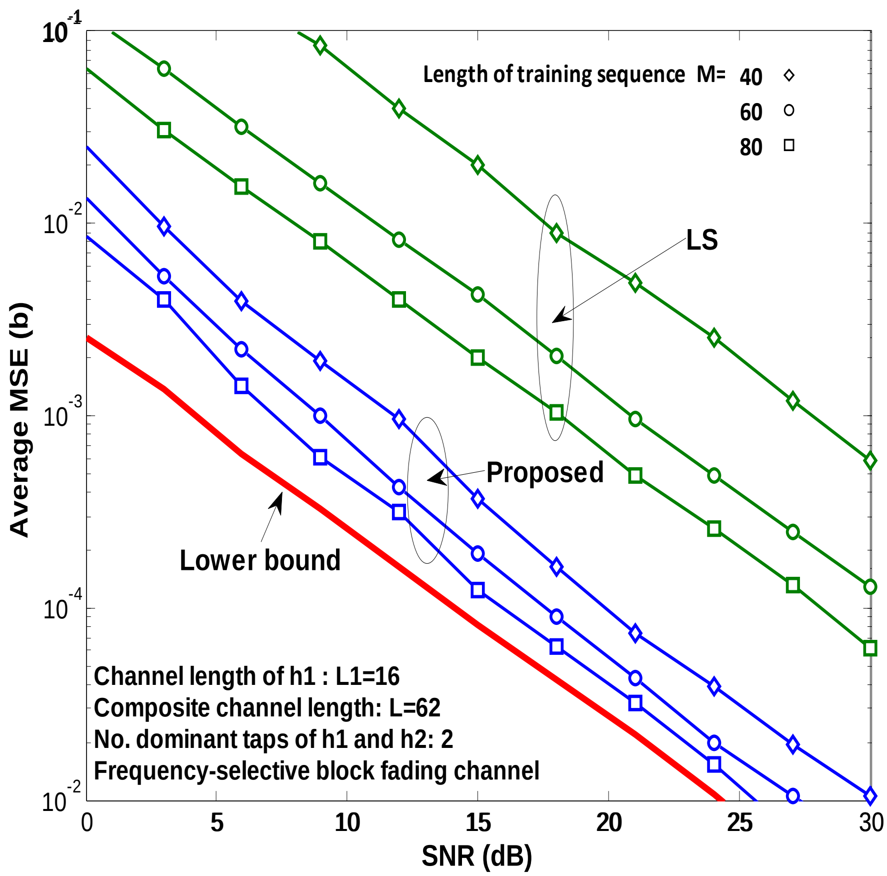

Figure 3. MSE performance of (b) versus SNR.

method and LS-base channel estimation. It can be seen that the computational complexity (CC) of proposed method is about 3.5 times higher than the LS-based channel estimation as shown in Figure 5. Therefore, the proposed method can achieve better channel estimation than LS based method at the cost of increased computational complexity.

CONCLUSIONS

In this paper, we proposed a compressed channel estimation method to address the problem in this paper. Both the theoretic analysis and computer simulations have confirmed performance of the proposed method. Furthermore, the computational complexity of our

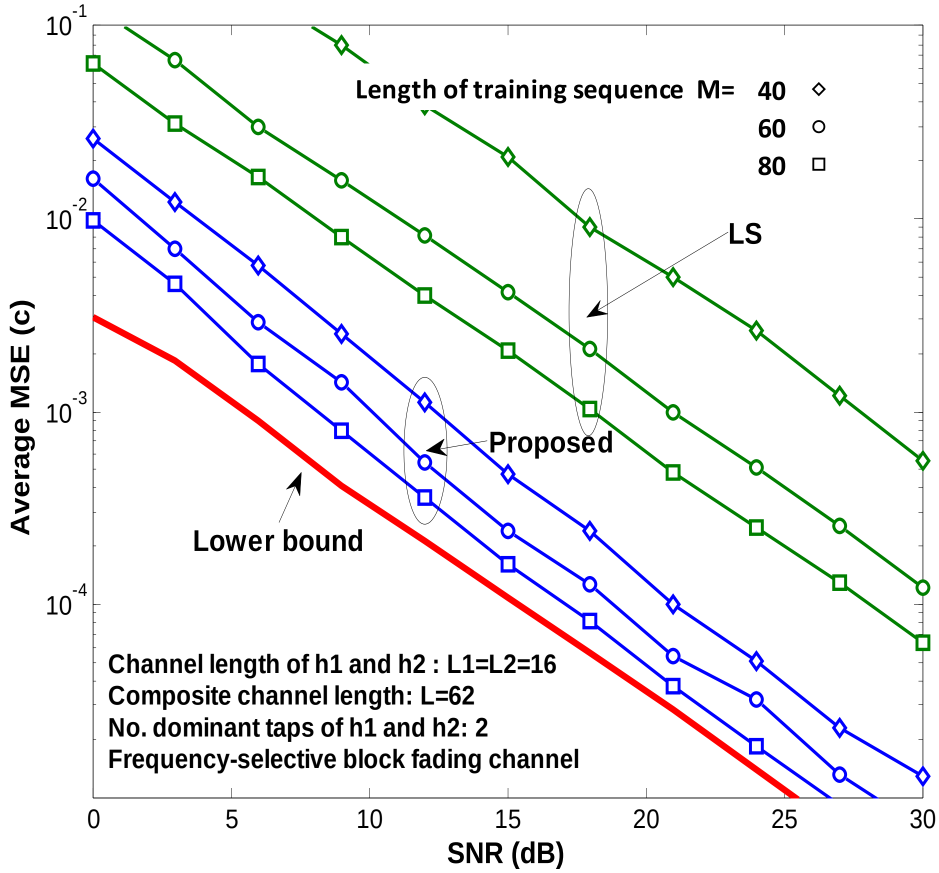

Figure 4. MSE performance of (c) versus SNR.

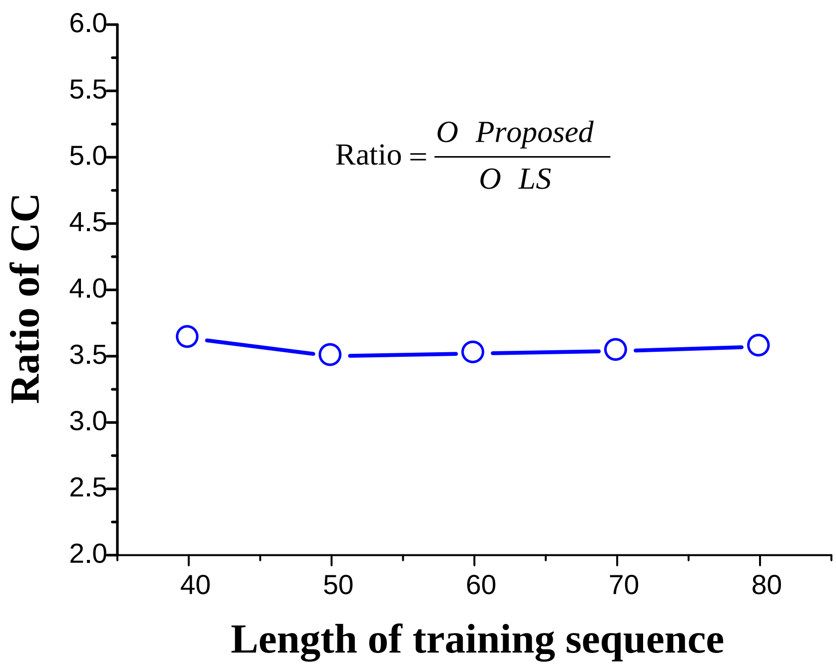

Figure 5. Computational complexity comparion (CCC).

proposed algorithm has a low complexity that of the LS-based channel estimation. According to the above studies, we conclude that the proposed method combined the robust to noise and efficient to complexity.

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