Full Length Research Paper

Stresses in a spherical shell by using Lebesgue measure concept

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The problem of elastic-plastic transition stresses in a spherical shell under internal pressure has been solved by using Lebesgue measure temperature with the concept of generalized strain measure. It has been seen that the results are similar to those given by Hulsurkar (1963).

Key words: Creep, compressibility, spherical shell, pressure, Lebesgue measure, transitional stress.

INTRODUCTION

Classical measures of deformation are inadequate to deal with such transition and hence different constitutive equations are used for each state. The constitutive equations are further simplified by using assuming theory, which does not only bypass these transitions but also makes design engineers estimate them; and then plastic flow or creep or fatigue sets in. This, combined with experiment data, leads to over-designing of the structures. Thus, there arises the need to construct a generalized measure to avoid adhoc assumption and semi-empirical laws; and also to have better agreement with experimental results

It is known that if, in a very small interval, the number of fluctuations is very large and the ordinary measure based on the Riemanian integral concept fails, measures like that of Lebesgue have to be used. In a similar way, the generalized measures given by weighted integral representations (Seth, 1972, 1966) give very satisfactory results in problems such as that of plasticity and creep. Seth (1972, 1966) has defined the generalized principal strain measure ε_{ii} by taking the Lebsgue integral of the weighted function $(1 - 2e_{ii})^{\frac{n}{2}-1}$ as:

$$\varepsilon_{ii} = \int_0^{\overset{A}{e_{ii}}} \left[1 - 2 \overset{A}{e_{ii}} \right]^{\frac{n}{2}-1} d \overset{A}{e_{ii}} = \frac{1}{n} \left[1 - \left(1 - 2 \overset{A}{e_{ii}} \right)^{\frac{n}{2}} \right],$$

$(i = 1,2,3)$ (1)

where n is the measure and $\overset{A}{e_{ii}}$ is the principal Almansi strain component. For the uniaxial case it is given by Seth (1972) as:

$$e = \left[\frac{1}{n} \left\{ 1 - \left(\frac{l_0}{l} \right)^n \right\} \right]^m$$ (2)

Where m is the irreversibility index and l_0 and l are the initial and strained lengths of the rod, respectively. In Cartesian framework we can readily write down the generalized measure in terms of any other measures. If n = 0, 1, 2, -1, -2, it gives the Hencky, Swainger, Almansi, Cauchy and Green measure respectively, where in all cases m = 1. Seth (1972, 1966) said that the generalized measure gives the well-known creep strain law; creep strain laws used in current literature such as Norton, Kachonov, Odqvist and Andrade laws etc. can be derived from the generalized measure.

The most important contribution made by the generalized measures is that they make use of empirical laws and jump conditions unnecessary. If such laws exist, they come out of the analytical treatment as a particular case. Thus, an important function of non-linear measure is to explain transition without assuming condition to match the two solutions at transitions. In this research paper we discuss the problems "elastic - plastic transition stresses in a spherical shell under internal pressure by using concept of Lebesgue measure and it is seen that the results are similar to those obtained by Seth's strain measure.

LEBESGUE STRAIN MEASURE

Taking the power of a weighted function as a principal measure, we take the derivative of the function of a weighted function as the principal measure and define the Lebesgue strain measure ε_{ii} in terms of the principal Almansi strain components $\overset{A}{e}_{ii}$ as:

$$\varepsilon_{ii} = \frac{1}{2}\int_0^e F'\left[1 - 2\overset{A}{e}_{ii}\right] d\overset{A}{e}_{ii} = \frac{1}{2}\left[F(1) - F\left(1 - 2\overset{A}{e}_{ii}\right)\right];$$

(3)

Such that $F(0) = 0, F(\infty) = \infty, F'(0) = 0$.

Problem in spherical shell under internal pressure has been analyzed by using strain measure in Section 2. It is shown that the results are obtained by using the concept of generalized strain measure as given by Seth's.

Governing equations

Consider a spherical shell of constant thickness under uniform internal pressure. Let a be the inner radius and b be the external radius of spherical shell as shown in Figure 1. The symmetry of structure and the loading about the centre of spherical shell, the displacement components in spherical co-ordinates (r, θ, ϕ) can be taken as (Seth, 1972, 1963):

$$u = r(1 - \beta) ; v = 0; w = dz,$$

(4)

where β is function of $r = \sqrt{x^2 + y^2}$ only and d is a constant.

The strain components for infinitesimal deformation are given by [2]:

$$e_{rr} = \frac{1}{2}\left[1 - \left(r\beta' + \beta\right)^2\right]; e_{\theta\theta} = e_{\phi\phi} = \frac{1}{2}\left[1 - \beta^2\right]; \quad (5)$$

$$e_{r\theta} = e_{\theta\phi} = e_{\phi r} = 0$$

where $\beta' = d\beta / dr$.

The generalized components of strain from equation (3) are:

$$e_{rr} = \frac{1}{2}\left[F(1) - F\left\{\left(r\beta' + \beta\right)^2\right\}\right];$$

$$e_{\theta\theta} = e_{\phi\phi} = \frac{1}{2}\left[F(1) - F\left(\beta^2\right)\right]$$
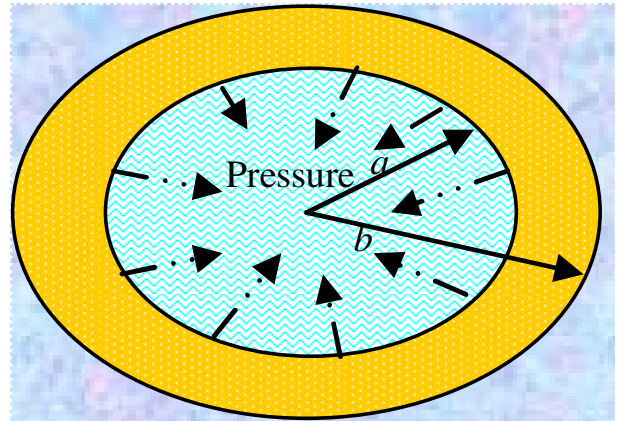

Figure 1. Spherical shell under internal.

$$e_{r\theta} = e_{\theta\phi} = e_{\phi r} = 0 \tag{6}$$

The stress –strain relations for isotropic media is given by (Sokolnikoff, 1954) [3]:

$$T_{ij} = \lambda\delta_{ij}I_1 + 2\mu e_{ij}, (i, j = 1, 2, 3) \tag{7}$$

Where T_{ij} are the stress components, λ and μ are Lame's constants, $I_1 = e_{kk}$ is the first strain invariant, δ_{ij} is the Kronecker's delta. Non –vanishing components of stress are obtained by substituting equation (6) in equation (7) are:

$$\backslash T_{rr} = \lambda I_1 + \mu\left[F(1) - F\left\{\left(r\beta' + \beta\right)^2\right\}\right];$$

$$T_{\theta\theta} = T_{\phi\phi} = \lambda I_1 + \mu\left[F(1) - F\left(\beta^2\right)\right];$$

$$e_{r\theta} = e_{\theta\phi} = e_{\phi r} = 0 \tag{8}$$

where

$$I_1 = \frac{1}{2}\left[3F(1) - F\left\{\left(r\beta' + \beta\right)^2\right\} - F\left(\beta^2\right)\right]$$

The equations of equilibrium are all satisfied except:

$$\frac{\partial T_{rr}}{\partial r} + \frac{2\left(T_{rr} - T_{\theta\theta}\right)}{r} = 0 \tag{9}$$

Substituting the stresses from equation (8) into equation (9), one gets a non -linear differential equation in

$$\beta^2 p\frac{dp}{d\beta} = \frac{-2(1-c)\beta^2 pF'\left(\beta^2\right) + c\left[F\left(\beta^2\right) - F\left\{\left(r\beta' + \beta\right)^2\right\}\right]}{\beta(p+1)F'\left\{\left(r\beta' + \beta\right)^2\right\}} - \beta p(p+1)$$

(10)

where c is the compressibility factor of the material in term of Lame's constant, and   is given by $c = 2\mu / \lambda + 2\mu$

Equation (10) becomes:

$$\frac{d\beta}{dp} = \frac{\beta^3 p(p+1)^2 F'\{(r\beta'+\beta)^2\}}{\left[\begin{array}{c} -2(1-c)\beta^2 pF'(\beta^2) + c\left[F(\beta^2) - F\{(r\beta'+\beta)^2\}\right] \\ -\beta^2 p(p+1)^2 F'\{(r\beta'+\beta)^2\} \end{array}\right]}$$

(11)

Transition points of β in equation (10) are $P \to 0, \pm\infty, -1$.

The boundary conditions are: $T_{rr} = -p$ at $r = a$ and $T_{rr} = 0$ at $r = b$ (12)

Solution through the principal stress-difference

For finding the creep stresses, the transition function through principal stress difference (Seth, 1963, 1970; Pankaj, 2009c; Pankaj, and Sonia, 2008a; Pankaj and Sonia, 2008b; Pankaj and Gupta, 2007a; Pankaj and Gupta, 2007b; Pankaj and Gupta, 2008; Pankaj and Sharma, 2009; Pankaj, 2009a, 2009b) at the transition point $P \to -1$ leads to the creep state .The transition function 'R' is defined as:

$$R = T_{rr} - T_{\theta\theta} = \mu\left[F(\beta^2) - F\{(r\beta'+\beta)\}\right]$$ (13)

Taking the logarithmic differentiation of equation (13) with respect to r and using equation (10), one gets:

$$\frac{d(\log R)}{d(\log r)} = \frac{-2c(6-4c)\beta^2 pF'(\beta^2)}{F(\beta^2) - F\{\beta^2(\{P+1\}^2)\}}$$ (14)

Taking the asymptotic value of equation (14) as $P \to -1$, one gets:

$$T_{rr} - T_{\theta\theta} = R = A_0 r^{-2c}\left[F(\beta^2)\right]^{3-2c}$$ (15)

where A_0 is a constant of integration. Putting the boundary condition (12) in equation (15), we get the transitional creep stresses as:

$$T_{rr} = -2A_0 \int r^{-2c-1}\left[F(\beta^2)\right]^{3-2c} dr + A_1$$

$$T_{\theta\theta} = T_{\phi\phi} = T_{rr} - A_0 r^{-2c}\left[F(\beta^2)\right]^{3-2c}$$ (17)

Where $A_0 = \dfrac{-p}{2\displaystyle\int_a^b r^{-2c-1}\left[F(\beta^2)\right]^{3-2c} dr}$

CASE STUDY

As a case study, function (1) equal to Seth's generalized measure, that is

$$\frac{1}{2}\left[F(1) - F\left(1 - 2\underset{ii}{e}^{A}\right)\right] = \frac{1}{n^m}\left[1 - \left(1 - 2\underset{ii}{e}^{A}\right)^{n/2}\right]^m$$

which gives

$$F\left(1 - 2\underset{ii}{e}^{A}\right) = F(1) - \frac{2}{n^m}\left[1 - \left(1 - 2\underset{ii}{e}^{A}\right)^{n/2}\right]^m$$

$$F(\beta^2) = F(1) - \frac{2}{n^m}\left[1 - \beta^n\right]^m$$ (18)

If we put $F(1) = 2/n^m$, equation (18) becomes:

$$F(\beta^2) = \frac{2}{n^m}\left(1 - \left[1 - \beta^n\right]^m\right)$$ (19)

Putting equation (19) in equation (15), one gets:

$$T_{rr} - T_{\theta\theta} = A_1 r^{-2c}\left[\left(1 - \left[1 - \beta^n\right]^m\right)\right]^{3-2c}$$ (20)

where $A_1 = A_0\left(\dfrac{2}{n^m}\right)^{3-2c}$ constant of integration Equation (20) are same as obtained by Seth (1972).

Using equation (12) in equation (17), one gets the transitional creep stress for steady state creep as:

$$T_{rr} = \frac{-p\left[\left(\dfrac{b}{r}\right)^{3n-2c(n-1)} - 1\right]}{\left[\left(\dfrac{b}{a}\right)^{3n-2c(n-1)} - 1\right]}$$

$$T_{\theta\theta} = T_{\phi\phi} = \frac{p\left[\dfrac{1}{2}\{n(3-2c) - 2(1-c)\}(b/r)^{3n-2c(n-1)} + 1\right]}{\left[(b/a)^{3n-2c(n-1)} - 1\right]}$$

(21)

which are the results obtained by Hulsurkar (1963). It may be noted that in deriving equations (12) and (21), none of the assumptions adopted current literature such as (1) creep strain laws of Norton, (2) the incompressibility condition and (3) the yield condition of Tresca have been used.

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