# Optimizing planting areas using differential evolution (DE) and linear programming (LP) 

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Accepted 25 March, 2009


#### Abstract

The application of differential evolution (DE) algorithm to a constrained optimization problem of irrigation water use is presented in this study. Irrigation water, amounting to $10 \mathrm{Mm}^{3}$ is optimized to irrigate 2500 ha of land where 16 different crops are planted on different areas of land. The crops are assumed not rain-fed but depend only on irrigation water. The DE which is an evolutionary based optimization algorithm, with codes written in MATLAB is used to solve the problem. Also a comparative study using linear programming (LP) is performed. The results are compared to study the effectiveness of the DE algorithm. The different areas of land where the crops are to be planted to maximize the total income (TI) in monetary terms (South African Rand, ZAR) are optimized. It is found that a TI of ZAR 46 060000 can be derived without any constraint violation using DE. Also ZAR 46060200 can be derived using LP. Ten strategies of DE are tested with this problem varying the population size (NP), crossover constant (CR) and weighting factor (F). It is found that strategy 1, DE/rand-1-bin, with values of NP, CR and $F$ of $160,0.95$ and 0.5 respectively, obtains the best solution most efficiently. DE is considered comparable to LP in solving this optimization problem.


Key words: Irrigation planning, linear programming, constrained optimization, differential evolution, evolutionary algorithm.

## INTRODUCTION

Rainfall in South Africa is erratic and generally, South Africa is a dry country. It has less than 500 mm rain on average annually over about two-third of its area. Most commercial farmers in South Africa therefore depend on irrigation. The major dams in the country supply irrigation water to farmers at a price. If a farmer knew the volume of water that would be available to him over a period, he would be faced with some other problems. He has to decide on which crops to plant to maximize the usage of water available and determine the appropriate cropping pattern over the period to maximize his profit. For example, a farmer may have to plant all crops on fixed hectares of land with a fixed volume of water available for irrigation over a period. So he needs to know the area of land to plant each of the crops to maximize the usage of irrigation water and hence his profit. This idea necessitate the use of mathematical models to maximize the total in-

[^0]come derived from the crops planted. Several studies (Raju and Vasan, 2004; Nagesh et al., 2006; Raju and Nagesh, 2006) have reported the application of evolutionary algorithm [Genetic algorithm (GA), simulated annealing (SA), evolutionary strategies (ES), particle swarm and ant colony optimizations] to cropping pattern as well as irrigation planning. The studies find the algorithms effective. Comparisons of the results with other optimization techniques have also been done (Raju and Nagesh, 2004; Nagesh et al., 2006). It was concluded that the results obtained are similar. Differential evolution (DE) and linear programming (LP) are applied to a case study of Bisalpur project in India (Raju and Vasan, 2004). The objective of the study was to determine cropping pattern which maximizes net benefit. Also, the study by Vasan and Raju (2007) demonstrates the applicability of differrential evolution to a case study of Mahi Bajaj Sagar Project (MBSP), India.
The study concludes that DE/rand-to-best-1-bin strategy is the best strategy giving maximum benefits taking minimum CPU time for irrigation planning. This present paper uses differential evolution (DE) and linear program-
ming (LP) optimization techniques to maximize the total income derived from planting 16 different crops on corresponding areas of land to maximize the usage of irrigation water. The optimization is not based on a specific case study but is considered adequately realistic which can be developed for farmers' specific uses.

## Linear programming (LP)

Linear programming (LP) deals with problem of optimizing limited resources like water among the competing activities (Rao, 2003). The general structure of a linear programming model is:

Maximize or minimize $\sum_{j}^{n} P_{j} x_{j}$
Subject to: $\sum_{i}^{n} a_{i j} x_{j} \leq b_{i}$ for $\mathrm{i}=1,2,3, \ldots, \mathrm{~m}$
$x_{j} \geq 0$ for all $j=1,2,3, \ldots, n$.
Subject to the equality and inequality constraints.
Where:
$x_{j}$ are unknown decision-variables.
$\mathrm{a}_{\mathrm{ij}}$ and $\mathrm{b}_{\mathrm{j}}$ are constants.
To solve Eq. (1), we have to find an unknown vector $x_{j}$ that minimizes (or maximizes) the objective function $\sum_{j}^{n} P_{j} x_{j}$ and satisfies the constraints. For this study, Linear, Interactive and Discrete Optimizer (LINDO) software which can be obtained from www.lindo.com is used.

## Differential evolution (DE)

The operation and working principles of differential evolution (DE) are widely documented in the literature. However, brief concepts are reported in this paper. DE is an improved version of genetic algorithm (GA), a type of evolutionary algorithm (EA) for faster optimization (Price and Storn, 1997). The principal difference between GA and DE is that GA relies on uniform crossover, a mechanism of probabilistic and useful exchange of information among solutions to locate better solutions; DE uses a non uniform crossover in that the parameter values of the child vector are inherited in unequal proportions from the parent vectors. This can take child vector parameters from one parent more often than it does from others. By using components of existing population members to construct trial vectors, recombination efficiently shuffles information about successful combinations, enabling the search for an optimum to focus on the most promising area of solution space (Onwubolu and Babu, 2004).
Unlike traditional simple GA that uses binary coding for representing problem parameters (though some GA also
uses real number), DE is a simple yet powerful population based, direct search algorithm for globally optimizing functions with real value parameters (Babu and Jehan, 2003). DE uses real coding of floating point numbers. The advantages of DE are its simple structure, ease of use, speed and robustness. The crucial idea behind DE is a scheme for generating trial parameter vectors. Basically, DE adds the weighted difference between two population vectors to a third vector. The key parameters of control in DE are: NP - the population size, CR - the crossover constant and F - the weight applied to random differential scaling factor.
The simple adaptive scheme used by DE ensures that the mutation increments are automatically scaled to the correct magnitude. The advantage of DE over GA is also the manipulation of floating point numbers with arithmetic operators instead of "bit flipping" approach of traditional GA. Hence DE has the following advantages over GA; ease of use, efficient memory utilization, lower computational complexity that is scales better on large problems, lower computational effort, that is faster convergence and finally greater freedom in designing a mutation distribution (Price et al., 2005).

Price and Storn (1997) gave the working principles of DE with single strategy. Later on, they suggested 10 different strategies namely, DE/rand-1-bin, DE/best-1-bin, DE/best-2-bin, DE/rand-2-bin, DE/randtobest-1-bin, DE /rand-1-exp, DE/best-1-exp, DE/best-2-exp, DE/rand-2exp, $D E /$ rand to best-1-exp. DE/x-y-z indicates $D E$ for differential evolution, x is a string which denotes the vector to be perturbed, $y$ denotes the number of different vectors taken for perturbation of x and z is the crossover method(exp: exponential; bin: binomial). A strategy that works out to be best for a given problem may not work well when applied to a different problem. The formulations of different strategies are given in Table 1. DE has been successfully applied to solve a wide range of optimisation problems such as optimisation of non-linear functions (Angira and Babu, 2003), optimal design of shell and tube heat exchangers (Babu and Munawar, 2007), optimization of process synthesis and design problems (Angira and Babu, 2006), optimization of non-linear chemical processes (Babu and Angira, 2006) and Estimation of heat transfer parameters in a trickle-bed reactor (Babu and Sastry, 1999).

## The pseudo code for DE used in this study

The pseudo code of DE for solving the constrained optimization problem of irrigation water use in this study is given below:

1) Initialize the values of $\mathrm{D}=16, \mathrm{NP}=(5-10)^{*} \mathrm{D}, \mathrm{CR}=0.5$ to $1.0, \mathrm{~F}=0.5$ to 1.0 , and maxgen (maximum generation) $=6000$
2) Initialize all the constants

For $\mathrm{i}=1$ to D
Enter price(i), yield(i), Duration (i), CWR(i), Dlower (i) and

Table 1. Formulation of the ten different strategies of differential evolution

| Strategy | Description | Formulation |
| :---: | :---: | :---: |
| 1 | DE/rand/1/bin | $v(g, i, j)=x\left(g, r_{3}, j\right)+\mathrm{F} *\left[x\left(g, r_{1}, j\right)-x\left(g, r_{2}, j\right)\right]$ |
| 2 | DE/best/1/bin | $v(g, i, j)=x(g, \text { best }, \mathbf{j})+\mathrm{F} *\left[x\left(g, r_{1}, \mathrm{j}\right)-x\left(g, r_{2}, \mathrm{j}\right)\right]$ |
| 3 | DE/best/2/bin | $v(g, i, j)=x(g, \text { best }, j)+\mathrm{F} *\left[x\left(g, r_{1}, j\right)+x\left(g, r_{2}, j\right)-x\left(g, r_{3}, j\right)-x\left(g, r_{4}, j\right)\right]$ |
| 4 | DE/rand/2/bin | $v(g, i, j)=x\left(g, r_{5}, j\right)+\mathrm{F} *\left[x\left(g, r_{1}, j\right)+x\left(g, r_{2}, j\right)-x\left(g, r_{3}, j\right)-x\left(g, r_{4}, j\right)\right]$ |
| 5 | DE/rand-to-best /1/bin | $v(g, i, j)=x(g, i, j)+\mathrm{F} *[\mathrm{x}(\mathrm{g}, \mathrm{best}, \mathrm{j})-\mathrm{x}(\mathrm{g}, \mathrm{i}, \mathrm{j})]+\mathrm{F} *\left\lfloor\mathrm{x}\left(\mathrm{g}, \mathrm{r}_{1}, j\right)-\mathrm{x}\left(\mathrm{g}, \mathrm{r}_{2}, \mathrm{j}\right)\right\rfloor$ |
| 6 | DE/rand/1/exp | $v(g, i, j)=x\left(g, r_{3}, j\right)+\mathrm{F} *\left[x\left(g, r_{1}, j\right)-x\left(g, r_{2}, j\right)\right]$ |
| 7 | DE/best/1/exp | $v(g, i, j)=\mathcal{X}(\boldsymbol{q}$, best, $\mathbf{j})+\mathrm{F} *\left[x\left(g, r_{1}, \mathrm{j}\right)-x\left(g, r_{2}, \mathrm{j}\right)\right]$ |
| 8 | DE/best/2/exp | $v(g, i, j)=x(g$, best,$j)+\mathrm{F} *\left[x\left(g, r_{1}, j\right)+x\left(g, r_{2}, j\right)-x\left(g, r_{3}, j\right)-x\left(g, r_{4}, j\right)\right]$ |
| 9 | DE/rand/2/exp | $v(g, i, j)=x\left(g, r_{5}, j\right)+\mathrm{F} *\left[x\left(g, r_{1}, j\right)+x\left(g, r_{2}, j\right)-x\left(g, r_{3}, j\right)-x\left(g, r_{4}, j\right)\right]$ |
| 10 | DE/rand-to-best /1/exp | $v(g, i, j)=x(g, i, j)+\mathrm{F} *[\mathrm{x}(\mathrm{g}, \mathrm{best}, \mathrm{j})-\mathrm{x}(\mathrm{g}, \mathrm{i}, \mathrm{j})]+\mathrm{F} *\left\lfloor\mathrm{x}\left(\mathrm{g}, \mathrm{r}_{1}, j\right)-\mathrm{x}\left(\mathrm{g}, \mathrm{r}_{2}, \mathrm{j}\right)\right\rfloor$ |

Dupper (i) (lower and upper bound constraints of area respectively). End for
3) Initialize all the vectors of the population randomly

For $\mathrm{i}=1$ to NP
For $\mathrm{j}=1$ to D
$\mathrm{x}(0, \mathrm{i}, \mathrm{j})=\mathrm{D}$ lower $(\mathrm{j})+$ random number
$(0,1)^{*}$ [D upper ( j )-D lower ( j$\left.)\right]$

## Next j

Next i
4) Select the strategy and crossover method to use
5) Perform mutation, crossover, selection and evaluation of the objective function for maxgen number of generations.
Initialize gen = 1

While (gen<maxgen)
for $\mathrm{i}=1$ to NP
Evaluate the objective function, $\mathrm{f}(\mathrm{xgen}, \mathrm{i})$, check for constraint violation if violated,
$f(x g e n, i)=8^{*} 109$. End for
Evaluate the objective function, $f\left(\mathrm{x}_{\text {gen, }}\right)$; check for constraint violation if violated,
$f\left(\mathrm{x}_{\mathrm{gen}, \mathrm{i}}\right)=8^{\star 1} 10^{9}$. End for
6) Find out the minimum objective function

For $i=1$ to NP
Perform mutation and crossover for the strategy and crossover method selected for example, strategy 1 ( $\mathrm{DE} / \mathrm{rand} / 1 / \mathrm{bin}$ ) is used for this pseudo code for each vector x (gen, j,i) (target vector), select 3 distinct vectors $x\left(\right.$ gen $\left., r_{1}, j\right), x\left(g e n, r_{2}, j\right)$ and $x\left(g e n, r_{3}, j\right)$ randomly from the

Table 2. Crop areas for DE/rand-1-bin and linear programming for the 16 crops.

| S/N | Crops | Crop areas (ha) |  |
| :---: | :--- | :---: | :---: |
|  |  | DE strategy 1, <br> DE/rand/1/bin | LP |
| 1 | Tobacco | 400.00 | 400.000 |
| 2 | Maize | 111.00 | 111.000 |
| 3 | Sorghum | 199.99 | 200.000 |
| 4 | Wheat | 166.99 | 167.000 |
| 5 | Groundnut | 222.00 | 222.000 |
| 6 | Soy beans | 257.07 | 286.000 |
| 7 | Sunflower | 67.94 | 24.074 |
| 8 | Green | 125.00 | 125.000 |
|  | beans | 500.00 | 500.000 |
| 9 | Dry beans | 333.00 | 333.000 |
| 10 | Pea | 10.00 | 24.926 |
| 11 | Dry peas | 29.00 | 29.000 |
| 12 | Potato | 12.00 | 12.000 |
| 13 | Cabbage | 22.00 | 22.000 |
| 14 | Onion | 15.00 | 15.000 |
| 15 | Tomato | 29.00 | 29.000 |
| 16 | Water | 2500.00 | 2500.00 |
|  | melon | Total | 10000000 |
|  | area(ha) | 10000000 |  |
|  | Total Vol. | (m ${ }^{3}$ ) |  |

current population other than vector $X_{i}$
$u($ gen, $\mathrm{i}, \mathrm{j})=x\left(\right.$ gen, $\left.\mathrm{r}_{1}, \mathrm{j}\right)+\mathrm{F}^{*}\left(\mathrm{x}\left(\right.\right.$ gen, $\left.\mathrm{r}_{2}, \mathrm{j}\right)-\mathrm{x}\left(\right.$ gen, $\left.\left.\mathrm{r}_{3}, \mathrm{j}\right)\right)$
7) The $u(g e n, i, j)$ vector generated must satisfy the boundary constraints, perform crossover for each target vector using any crossover method, binary crossover method is used here;
$\mathrm{p}=$ random number
jrand $\left.=\operatorname{int}[\operatorname{rand}(0,1)]^{*} \mathrm{D}\right)+1$
for $n=1$ to $D$
if $p<C R$ or $n=j r a n d$,
Then
$x(t, i, j)=u(g e n, i, j)$
else
$x(t, i, j)=x($ gen, $\mathrm{i}, \mathrm{j})$
Calculate $f\left(x_{t, j}\right)$
The generated vectors should satisfy the boundary constraints. Perform selection, compare the trial objective function value with $f\left(x_{i}\right)$
If $f\left(\mathrm{X}_{\mathrm{t}, \mathrm{i}}\right)<\mathrm{f}\left(\mathrm{X}_{\mathrm{i}}\right)$
then
$x($ gen $+1, i, j)=x(t, i, j)$ else
$x($ gen $+1, i, j)=x($ gen, $, j, j)$
end
end
end
gen $=$ gen +1
Print out results
end

## METHODOLOGY

In this study, we assume we have 2500 hectares area of land that needs to be irrigated. On the area of land of 2500 hectares, 16 different crops namely, tobacco, maize, sorghum, wheat, ground-nut, soybeans, sunflower, green beans, dry beans, green pea, dry pea, potato, cabbage, onion, tomato and watermelon will be planted. Each of the crops must be planted in at least 10 hectares of land and at most in known maximum irrigated area of land in hectares for each crop. The minimum planting areas ensure the availability of all the crops in the market while the maximum planting areas make sure that they will not have storage or selling problem if the yield exceeds the storage facilities available or the demand is less than the supply which will make the selling price fall. The study is to find out the corresponding areas of land in hectares where each of the 16 crops should be planted to maximize the total income on the land using the available irrigation water maximally. The problem is solved using the 10 strategies of DE and the results obtained are compared with that obtained by LP.
The objective function of the problem is formulated to maximize the total income in monetary value of South African Rand (ZAR) derived from planting all the 16 crops on the known total area of land (2500 ha) for a planting season using $10 \mathrm{Mm}^{3}$ of water for the whole period. It can be expressed as:

Maximize $T I=\sum_{i=1}^{N} T I_{i} A_{i}$
Where:
$\mathrm{N}=$ number of crops
$T l_{i}=$ total income of $\mathrm{i}^{\text {th }}$ crop in rand in one season(R/ha)
$A_{i}=\quad$ area where it ${ }^{\text {th }}$ crop is grown in ha (unknown)
$\mathrm{TI}=$ total income on the whole area (ZAR)
To compute the total income (ZAR/ha) from each crop, the selling price (ZAR/ton) of crop(i) from Table 2 (Agriculture, 2007) is multiplied by yield (ton/ha) also from the same reference.
$\mathrm{Tl}_{\mathrm{i}}(\mathrm{ZAR} / \mathrm{ha})=$ Price $_{\mathrm{i}}(\mathrm{R} / \text { ton })^{*}{ }^{\text {Yield }}{ }_{\mathrm{i}}($ ton/ha)
The constraints of the problem are as follows: The irrigation release for the whole season is taken to be $\mathrm{V}=10 \mathrm{Mm}^{3}$. The total area where the crops are to be grown is also taken to be $\mathrm{A}<=2500 \mathrm{ha}$.
$\mathrm{V}<=10000000 \mathrm{~m}^{3}$
The volume of water used is equal to the sum of crop water requirement for each crop multiplied by the area where the crop is grown.
$V \geq \sum_{i=1}^{N}\left[C W R{ }_{i} * A_{i}\right]$
Where:
$\mathrm{CWR}_{\mathrm{i}}=$ average gross crop water requirement for crop i over the period

$$
\begin{align*}
& \mathrm{A}_{\mathrm{i}}=\text { area of land where crop } \mathrm{i} \text { is grown in } \mathrm{m}^{2} \\
& \sum_{i=1}^{N} A_{i} \leq A \leq 25000000 \tag{14}
\end{align*}
$$



Figure 2. Total incomes for different strategies of DE.

The total area, A is equal to the sum of all the areas of land where each of the crops is grown.

To make sure that all the crops are grown in at least $100000 \mathrm{~m}^{2}$ of land, each area, $A_{i}$ must be equal or greater than $100000 \mathrm{~m}^{2}$ and less than or equal to the maximum areas for each crop.
$100000<=A_{i}>=A_{\text {imax }}(i=1,2, \ldots 16)$
Where, $A_{\text {imax }}$ is the maximum area where each crop should be grown

## RESULTS AND DISCUSSION

By using the linear programming method, the maximum total income (TI) of ZAR 46,060,200 is obtained using equation (11). The differential evolution algorithm used to solve the equation (11) is written in MATLAB. The objecttive is to maximize the total income in monetary value (ZAR) derived from planting the 16 crops on 2,500 ha of land and irrigating with $10 \mathrm{Mm}^{3}$ of water. The problem is solved by converting a constrained problem to an unconstrained problem using the method of penalty function value (Deb, 1995). If the constraints are violated, the objective value turns to a very high value. In this study, a high value of $8^{\star} 10^{9}$ is used. This ensures that any solu-tion that violates the constraints will not be selected. The 10 DE strategies are tested using the program to deter-mine the best strategy for the problem in terms of the highest objective function value, lowest number of func-tion evaluations and iterations before convergence. The control parameters; population size (NP), crossover constant (CR) and weighting factor $(\mathrm{F})$ are varied to determine the best combinations that will give the best result in term of
highest objective function values, lowest number of function evaluations and lowest number of iterations. The complete results are presented in Tables 2 to 9 and Figures 2 and 3.
From the results in Table 2, DE Strategy 1, DE/rand-1bin and LP are comparable in optimizing the planting areas. The crop areas for the 16 different crops are given. The total areas for the two methods, DE and LP are the same. It is found that both techniques give the same areas for almost all the crops except soybeans, sunflower and dry peas. Both give the same total volume of water ( $10 \mathrm{Mm}^{3}$ ) to irrigate the crops.
In Table 3 and Figure 2, the results of the 10 different strategies of DE are given. It is found that for the combination of $N P=160, C R=0.95$ and $F=0.5$, Strategies 1 ( $\mathrm{DE} /$ rand-1-bin) and 4 (DE/rand-2-bin) give the maximum total income of ZAR 46060000 without any constraint violation. The total area used is 2,500 ha and the total volume of water is $10 \mathrm{Mm}^{3}$ as desired in the problem. The best strategy for this problem is strategy 1 ( $\mathrm{DE} /$ rand $/ 1 / \mathrm{bin}$ ) which gives the solution to the problem in 1994 iterations. All other strategies give lower values of total income with total area greater than or less than 2,500 ha and total volume of water less than $10 \mathrm{Mm}^{3}$. Population size (NP), crossover constant (CR) and weighting factor ( F ) were varied on Strategy 1, DE/rand-1-bin to determine their optimal combination. The results obtained for different combinations presented in Tables 4 to 9 and Figure 3. F is varied from 0.5 to 0.9 , CR is varied from 0.3 to 1.0 while NP, from 100 to 200 at the step of 10 . It is found that DE performs well for the combination of NP, CR and F of 160, 0.95 and 0.5 re-

Table 3. Results obtained for the ten strategies of differential evolution with population size, $\mathrm{NP}=160$, crossover constant, $\mathrm{Cr}=0.95$ and weighting factor, $\mathrm{F}=0.5$.

| Stra No | Strategy | $\begin{array}{c}\text { Total } \\ \text { Income (*10 }\end{array}$ |
| :---: | :--- | :---: | :---: | :---: | :---: |
|  | ZAR) |  | \(\left.\begin{array}{c}No of <br>

iteration\end{array} $$
\begin{array}{c}\text { Total } \\
\text { Area (ha) }\end{array}
$$ \quad $$
\begin{array}{c}\text { Total } \\
\text { Volume (Mm }{ }^{3} \text { ) }\end{array}
$$\right]\)

Table 4. Variation of crossover constant, Cr for population size, $\mathrm{NP}=160$ and weighting factor, $\mathrm{F}=0.5$ for strategy DE/rand-1-bin.

| CR | Total <br> Area(ha) | Total <br> Volume $\left(\mathbf{M m}^{\mathbf{3}}\right)$ | Total <br> Income $\left({ }^{*} \mathbf{1 0}^{6} \mathbf{Z A R}\right)$ | N0 of <br> Iterations |
| :---: | :---: | :---: | :---: | :---: |
| 0.30 | 2499.99 | 10.00 | 46.059 | 1988 |
| 0.40 | 2500.00 | 10.00 | 46.060 | 1990 |
| 0.50 | 2500.00 | 10.00 | 46.060 | 1998 |
| 0.60 | 2500.00 | 10.00 | 46.060 | 1996 |
| 0.70 | 2500.00 | 10.00 | 46.060 | 1999 |
| 0.80 | 2500.00 | 10.00 | 46.060 | 1982 |
| 0.85 | 2500.00 | 10.00 | 46.060 | 1997 |
| 0.90 | 2500.00 | 10.00 | 46.060 | 1994 |
| 0.95 | 2500.00 | 10.00 | 46.060 | 1980 |
| 1.00 | 2498.20 | 10.00 | 46.044 | 1999 |

Table 5. Variation of crossover constant, Cr for population size, $\mathrm{NP}=160$ and weighting factor, $\mathrm{F}=0.6$ for strategy DE/rand/1/bin

| CR | Total <br> Area (ha) | Total <br> Volume $\left(\mathbf{M m}^{\mathbf{3}}\right)$ | Total <br> Income $\left.\mathbf{*}^{*} \mathbf{0}^{6} \mathbf{Z A R}\right)$ | N0 of <br> Iterations |
| :---: | :---: | :---: | :---: | :---: |
| 0.30 | 2499.80 | 10.00 | 46.056 | 1911 |
| 0.40 | 2499.80 | 10.00 | 46.056 | 1889 |
| 0.50 | 2500.00 | 10.00 | 46.056 | 1966 |
| 0.60 | 2499.90 | 10.00 | 46.055 | 1972 |
| 0.70 | 2499.90 | 10.00 | 46.055 | 1966 |
| 0.80 | 2499.80 | 10.00 | 46.053 | 1992 |
| 0.85 | 2499.80 | 10.00 | 46.052 | 1992 |
| 0.95 | 2499.90 | 10.00 | 46.056 | 1995 |
| 1.00 | 2499.90 | 10.00 | 46.055 | 1994 |

spectively without any constraints violation. This is in agreement with the practical advice on DE (Price and Sworn, 2007). It is advised to choose NP to be about 10
times the number of parameters, F to be a little lower or higher than 0.8 and that higher values of CR like $\mathrm{CR}=1$ give faster convergence as desired.


Figure 4. Crop areas for 10 strategies of differential evolution

Table 6. Variation of crossover constant, Cr for population size, NP $=160$ and weighting factor, $\mathrm{F}=0.7$ for strategy DE/rand/1/bin

| CR | Total <br> Area <br> (ha) | Total <br> Volume <br> $\left(\mathbf{M m}^{\mathbf{3}}\right)$ | Total <br> Income <br> $\left(* 10^{6}\right.$ ZAR) | N0 of <br> Iterations |
| :---: | :---: | :---: | :---: | :---: |
| 0.30 | 2499.80 | 10.00 | 46.051 | 1922 |
| 0.40 | 2499.00 | 10.00 | 46.045 | 1971 |
| 0.50 | 2499.40 | 9.99 | 46.039 | 1999 |
| 0.60 | 2499.70 | 10.00 | 46.029 | 1949 |
| 0.70 | 2497.90 | 10.00 | 46.014 | 1988 |
| 0.80 | 2499.70 | 9.99 | 45.999 | 1882 |
| 0.85 | 2498.00 | 10.00 | 45.948 | 1968 |
| 0.95 | 2495.60 | 9.99 | 45.924 | 1935 |
| 1.00 | 2493.00 | 9.99 | 45.924 | 1953 |

Table 7. Variation of crossover constant, Cr for population size, $N P=160$ and weighting factor, $\mathrm{F}=0.8$ for strategy DE/rand/1/bin

| CR | Total <br> Area(ha) $)$ | Total <br> Volume <br> $\left.\mathbf{( M m}^{\mathbf{3}}\right)$ | Total <br> Income <br> $\left({ }^{*} \mathbf{1 0}^{6} \mathbf{Z A R}\right)$ | N0 of <br> Iterations |
| :---: | :---: | :---: | :---: | :---: |
| 0.30 | 2499.10 | 10.00 | 46.044 | 1970 |
| 0.40 | 2497.90 | 9.99 | 46.023 | 1996 |
| 0.50 | 2500.00 | 10.00 | 45.998 | 1941 |
| 0.60 | 2499.10 | 9.99 | 45.977 | 1982 |
| 0.70 | 2493.90 | 9.99 | 45.907 | 1985 |
| 0.80 | 2490.70 | 9.99 | 45.818 | 1915 |
| 0.85 | 2485.40 | 10.00 | 45.785 | 1921 |
| 0.95 | 2487.50 | 9.94 | 45.638 | 1820 |
| 1.00 | 2454.10 | 9.96 | 45.452 | 1998 |

In Figure 4 and Table 10, the results of crop areas for 10 different strategies of differential evolution are pre sented. The total areas for different strategies are also gi-

Table 8. Variation of crossover constant, Cr for population size, NP = 160 and weighting factor, $\mathrm{F}=0.9$ for strategy DE/rand/1/bin

| CR | Total <br> Area <br> (ha) | Total <br> Volume <br> $\left(\mathbf{M m}^{3}\right)$ | Total <br> Income <br> $\left({ }^{*} \mathbf{1 0}^{6}\right.$ ZAR) | N0 of <br> Iterations |
| :---: | :---: | :---: | :---: | :---: |
| 0.30 | 2498.00 | 10.00 | 46.024 | 1991 |
| 0.40 | 2497.00 | 9.99 | 46.003 | 1951 |
| 0.50 | 2496.60 | 10.00 | 45.938 | 1813 |
| 0.60 | 2497.10 | 10.00 | 45.864 | 1991 |
| 0.70 | 2489.60 | 9.95 | 45.725 | 1752 |
| 0.80 | 2491.80 | 9.92 | 45.628 | 1684 |
| 0.85 | 2482.20 | 9.98 | 45.585 | 1998 |
| 0.95 | 2415.10 | 9.79 | 45.203 | 1704 |
| 1.00 | 2440.50 | 9.94 | 44.973 | 1892 |

Table 9. Variation of population size, NP for crossover constant, $\mathrm{Cr}=0.95$ and weighting factor, $\mathrm{F}=0.5$ for strategy DE/rand/1/bin

| NP | Total <br> Area <br> (ha) | Total <br> Volume <br> $\left(\mathbf{M m}^{\mathbf{3}}\right)$ | Total <br> Income <br> $\left(* 10^{6}\right.$ ZAR) | N0 of <br> Iterations |
| :---: | :---: | :---: | :---: | :---: |
| 110 | 2500.00 | 10.00 | 46.060 | 1996 |
| 120 | 2500.00 | 10.00 | 46.060 | 2000 |
| 130 | 2500.00 | 10.00 | 46.060 | 2000 |
| 140 | 2500.00 | 10.00 | 46.060 | 1990 |
| 150 | 2500.00 | 10.00 | 46.060 | 1994 |
| 160 | 2500.00 | 10.00 | 46.060 | 1980 |
| 170 | 2500.00 | 10.00 | 46.060 | 1985 |
| 180 | 2500.00 | 10.00 | 46.060 | 1989 |
| 190 | 2500.00 | 10.00 | 46.060 | 1999 |
| 200 | 2500.00 | 10.00 | 46.060 | 2000 |

ven. It is found from Figure 4 that dry bean is planted on the highest area of land followed by tobacco and pea for all the strategies. Crops like potato, cabbage, onion, to-

Table 10. Crop areas for ten strategies of differential evolution

|  | Crop Area(ha) |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| strategies | DE/rand /1/bin | $\begin{aligned} & \text { DE/best/ } \\ & \text { 1/bin } \end{aligned}$ | DE/bes t/2/bin | DE/ <br> rand/ <br> 2/bin | ```DE/rand- to- best/1/ bin``` | $\begin{gathered} \text { DE/ran } \\ \text { d/1// } \\ \exp \end{gathered}$ | DE/bes <br> t/1/exp | $\begin{aligned} & \text { DE/bes } \\ & \text { t/2/exp } \end{aligned}$ | $\begin{aligned} & \hline \text { DE/rand } \\ & \text { /2/exp } \end{aligned}$ | $\begin{gathered} \text { DE/rand } \\ \text {-to- } \\ \text { best/1/ } \\ \exp \\ \hline \end{gathered}$ |
| tobacco | 400.00 | 400.00 | 399.90 | 400.00 | 399.97 | 391.31 | 399.96 | 399.57 | 392.86 | 399.59 |
| maize | 111.00 | 111.00 | 102.63 | 111.00 | 40.66 | 110.80 | 110.03 | 86.96 | 73.59 | 104.82 |
| sorghum | 200.00 | 200.00 | 184.29 | 199.99 | 106.36 | 116.85 | 80.80 | 10.00 | 151.82 | 10.00 |
| wheat | 167.00 | 167.00 | 166.99 | 167.00 | 134.55 | 151.33 | 159.69 | 159.47 | 164.72 | 158.00 |
| groundnut | 222.00 | 222.00 | 221.77 | 222.00 | 218.66 | 220.03 | 221.23 | 209.96 | 216.28 | 221.51 |
| soy beans | 257.07 | 257.07 | 70.28 | 257.07 | 210.97 | 246.67 | 217.25 | 259.84 | 190.88 | 215.94 |
| sunflower | 67.93 | 67.93 | 231.04 | 67.94 | 263.85 | 146.89 | 206.05 | 257.02 | 195.74 | 254.53 |
| green beans | 125.00 | 125.00 | 124.99 | 125.00 | 123.11 | 124.38 | 124.88 | 124.98 | 124.95 | 124.99 |
| dry beans | 500.00 | 500.00 | 499.97 | 500.00 | 497.07 | 499.43 | 491.31 | 481.46 | 499.76 | 499.82 |
| pea | 333.00 | 333.00 | 333.00 | 333.00 | 332.61 | 332.96 | 332.97 | 331.92 | 332.94 | 332.96 |
| dry peas | 10.00 | 10.00 | 12.52 | 10.00 | 10.47 | 26.73 | 17.52 | 10.12 | 10.00 | 15.65 |
| potato | 29.00 | 29.00 | 29.00 | 29.00 | 28.76 | 29.00 | 17.90 | 29.00 | 29.00 | 28.99 |
| cabbage | 12.00 | 15.00 | 11.90 | 12.00 | 10.91 | 12.00 | 11.29 | 11.98 | 12.00 | 10.57 |
| onion | 22.00 | 22.00 | 21.99 | 22.00 | 18.77 | 22.00 | 20.97 | 18.97 | 22.00 | 16.65 |
| tomato | 15.00 | 15.00 | 15.00 | 15.00 | 14.66 | 15.00 | 12.44 | 14.10 | 15.00 | 14.97 |
| water melon | 29.00 | 29.00 | 20.94 | 29.00 | 29.00 | 29.00 | 28.95 | 28.21 | 29.00 | 29.00 |
| total <br> area(ha) | 2500.0 | 2503.0 | 2446.2 | 2500.0 | 2440.4 | 2474.4 | 2453.2 | 2433.6 | 2460.5 | 2438.0 |

mato and green beans have almost the same areas for all the strategies.

## Conclusions

The application of differential evolution (DE) and linear programming (LP) to maximize total income for 2500 ha planting area where 16 crops are planted and constrained by water availability has been demonstrated. Both LP and DE obtain an income of ZAR 46.06 bn. The convergence speed of $D E$ is effective and efficient. The effect of different combination of population size (NP), crossover constant (CR) and weighting factor ( F ) on the 10 different strategies of DE is studied. It is observed that Strategy 1, DE/rand-1-bin performs best for the problem with combination of NP, CR and F of 160, 0.95 and 0.5 respectively. The results obtained indicate that $D E$ is capable of obtaining the global optimum of optimization problems like LP. Unlike LP, DE has the advantage of not being limited to linear problems like the LP and can therefore be used to a wider variety of applications especially those that are not easy to linearised and methods that call for combined simulation - optimization. Possible areas of application include optimizing the operation of reservoir systems and water distribution system design and rehabilitation.

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