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Full Length Research Paper

A general method for finding the exact solution of linear Volterra integral equations of the second kind

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In this paper, a general method is given for the solution of linear Volterra integral equations of the second kind, which is based on the action of the operator defined by the kernel of the integral equation on a suitable basis for the corresponding function spaces. The necessary conditions for using this method are so weak that extends its applicability. The solved examples show the strength of this method.

Key words: Volterra integral equations, Fredholm integral equations, integral operators, basis functions.

INTRODUCTION

The theory of integral equations has a close relationship with different branches of mathematics. Moreover, many problems involving ODE's and PDE's may be stated in the form of integral equations (Hochstadt, 1973). In this paper, we consider linear Volterra integral equations of the second kind in the forms:

$$f(x)=(I-\lambda K)u(x)=u(x)-\lambda\int_0^x k(x,t)u(t)dt, 0\le x\le a \quad (1)$$

where $k:[0,a]\times[0,a]\to\mathrm{R}$ is square integrable function and $f:[0,a]\to\mathrm{R}$ is a known function in $L^2[0,a]$ and the equation is supposed to be uniquely solvable.

There are many methods for finding an approximate solution of (1). Adomian Decomposition Method (Hosseini, 2009a), Direct Computation Method (Babolian and Masouri, 2007), Taylor-successive approximation method (Hosseini, 2009b), and Method of Successive Substitutions (Atkinson and Han, 2000) are some of these methods. All methods can be classified under two main categories, projection methods (Hochstadt, 1973) and collocation methods (Atkinson and Han, 2000; Atkinson, 1997; Hosseini, 2009a). Some of them use power series and some of them use Chebyshev (Babolian and Fattahzadeh, 2007), Bernstein (Bhattacharya and Mandal, 2008) or other orthogonal polynomials. Some of them use radial basis functions (Golbabai and Sifollahi, 2006) and some other means to approximate the solution function $u(x)$, that can be found in Babolian and Fattahzadeh (2007), all of them use a prescribed set of functions $\{u_j\}_{j=1}^{\infty}$ for approximating $u(x)$ as a linear combination $u(x)=\sum_{j=1}^{n}c_j u_j(x)$ and then try to find c_j 's. Here, we will present a method to find $u(x)$ such that not only u_i's are not known apriori, but also the number of them will be found at the end of the process.

THE METHOD

We consider a Volterra integral equation (VIE) in the form (1) in which $k(x,t)$ and $f(x)$ are suitable known functions, λ is a known parameter and $u(x)$ is the unknown function, to be determined. Now, if $\{u_r(x)\}_{r\in\mathrm{I}}$ be a family of suitable functions where I is an index set, then using the operator $(I-\lambda K)$ and its effect on the family $\{u_r(x)\}_{r\in\mathrm{I}}$, that is:

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$$w_r(x) = [(I - \lambda K)u_r](x) = u_r(x) - \lambda\int_0^x k(x,t)u_r(t)dt \quad (2)$$

we can construct a new family of functions $\{w_r(x)\}_{r\in 1}$, and if a linear combination of w_r's can represent the function $f(x)$, then $u(x)$ can be represented as a corresponding linear combination of u_r's. This subject is explained in the following theorem.

Theorem 1

Given any integral equation in the form (1) and any integrable function $u_1(x)$ there is another VIE with the same kernel $K(x,t)$ that $v(x) = u_1(x) - u(x)$ is its solution and

$$f_1(x) = u_1(x) - \lambda\int_0^x k(x,t)u_1(t)dt - f(x) \quad (3)$$

is its known part.

Proof. Putting $u(x) = u_1(x) - v(x)$ in (1) and doing some trivial computations we obtain:

$$v(x) = u_1(x) - f(x) - \lambda\int_0^x k(x,t)u_1(t)dt - \lambda\int_0^x k(x,t)v(t)dt \quad (4)$$

which is the desired result. As a consequence of theorem (1) we obtain a method for constructing the exact or an approximate solution of (1).

Corollary 1

Suppose $\{u_j(x)\}_{j=1}^{\infty}$ is a given sequence of integrable functions and $u(x)$ is the solution of (1). Then

$$u(x) = \sum_{j=1}^{n}(-1)^{j-1}u_j(x) \quad (5)$$

if and only if

$$f(x) = \sum_{j=1}^{n}(-1)^{j-1}[u_j(x) - \lambda\int_0^x k(x,t)u_j(t)dt] \quad (6)$$

Proof. Given $\{u_j(x)\}_{j=1}^{\infty}$ we can use theorem (1) to construct a sequence of functions $\{v_j(x)\}_{j=1}^{\infty}$ such that $v_1(x) = u_1(x) - u(x)$ and $v_j(x) = u_j(x) - v_{j-1}(x)$, $2 \le j \le n$. Then we have

$$u(x) = u_1(x) - v_1(x) = u_1(x) - (u_2(x) - v_2(x)) = \ldots = \sum_{j=1}^{n}(-1)^{j-1}u_j(x) + (-1)^n v_n(x) \quad (7)$$

and

$$v_n(x) = \sum_{j=0}^{n-1}(-1)^j[u_{n-j}(x) - \lambda\int_0^x k(x,t)u_{n-j}(t)dt] + (-1)^n f(x) + \lambda\int_0^x k(x,t)v_n(t)dt. \quad (8)$$

Now, it is trivial that (8) has solution $v_n(x) = 0$ if and only if (6) is satisfied, and then (5) is also true.

Theorem 2

Let $u(x)$ be the unique solution of the equation:

$$f(x) = (I - \lambda K)u(x) \quad (9)$$

where K is a compact $L^2([0,a]\times[0,a])$ kernel and λ is not an eigenvalue of K also $\{u_i(x)\}$ is an $L^2[0,a]$ sequence of functions such that $\sum_{i=1}^{\infty}u_i(x)$ is a pointwise convergent series to an $L^2[0,a]$ function. Then $u(x) = \sum_{i=1}^{\infty}u_i(x)$ for every x in $[0,a]$ if and only if $f(x) = (I - \lambda K)\sum_{i=1}^{\infty}u_i(x)$.

Proof. Let x be an arbitrary point in $[0,a]$. Then,

$$f(x) - \sum_{i=1}^{\infty}(I - \lambda K)u_i(x) = (I - \lambda K)u(x) - (I - \lambda K)\sum_{i=1}^{\infty}u_i(x) = (I - \lambda K)[u(x) - \sum_{i=1}^{\infty}u_i(x)]. \quad (10)$$

Now, if $\sum_{i=1}^{\infty} u_i(x) = u(x)$, then from (10) we have:

$$f(x) - \sum_{i=1}^{\infty} (I - \lambda K) u_i(x) = 0.$$

Conversely if

$$f(x) = \sum_{i=1}^{\infty} (I - \lambda K) u_i(x)$$

then from (10), we obtain $(I - \lambda K)(u(x) - \sum_{i=1}^{\infty} u_i(x)) = 0$. Moreover, since K is compact and the integral Equation (9) is unisolvable, then by the Fredholm Alternative Theorem, $u(x) = \sum_{i=1}^{\infty} u_i(x)$ is the unique solution of (9). Corollary (1) gives an efficient method for constructing the exact solution and approximate solution of (1). For this purpose, we first choose a suitable complete sequence of functions $\{u_j(x)\}_{j=1}^{\infty}$, which is chosen such that the integrals $\int_0^x k(x,t) u_j(t) dt$ can be computed easily and then use the following algorithm.

ALGORITHM

To solve integral equations by this method, in step one we choose a suitable family of functions $\{v_r\}_{r \in \mathrm{I}}$ where I is an index set, as basis functions. In step two, we find the family of functions $\{w_r(x)\}_{r \in \mathrm{I}}$, such that $w_r(x) = (I - \lambda K) v_r(x)$. In step three, we choose a set of functions $v_{i_1}, v_{i_2}, \cdots, v_{i_n}$ such that $w_{i_1}, w_{i_2}, \cdots, w_{i_n}$ can expend $f(x)$. In step four, we rename the set $\{v_{i_1}, \cdots, v_{i_n}\}$ as $\{u_1, u_2, \cdots, u_n\}$ and $\{w_{i_1}, w_{i_2}, \cdots, w_{i_n}\}$ as $\{w_1, w_2, \cdots, w_n\}$. In step five, if there exists coefficients $c_1, c_2, \cdots, c_n$ such that

$$f(x) = \sum_{i=1}^{n} (-1)^i c_i w_i(x) \tag{11}$$

then we obtain that

$$u(x) = \sum_{i=1}^{n} (-1)^i c_i u_i(x) \tag{12}$$

is the unique solution of integral equation. If the family $\{u_r\}_{r \in \mathrm{I}}$ is not suitable then we must choose a new family of functions and go to step two. We state more details subsequently.

EXAMPLES

Here, we demonstrate the strength of our method by using it to solve some examples.

Example 1: Consider the equation

$$u(x) = x + \int_0^x (x - t) u(t) dt \tag{13}$$

Solution: The family of functions $\{v_r(x) = e^{rx}, r \in \mathrm{R}\}$ seems to be suitable, and moreover we have

$$[(I - \lambda K) v_r](x) = v_r(x) - \int_0^x (x-t) v_r(t) dt = e^{rx} - \int_0^x (x-t) e^{rt} dt =$$
$$(1 - \frac{1}{r^2}) e^{rx} + \frac{x}{r} + \frac{1}{r^2}, r \in \mathrm{R} \tag{14}$$

Now, we must choose those r's which are necessary for generating $f(x) = x$. So r's must be such that the coefficients of e^{rx}'s become zero, and this gives $r = \pm 1$. So, according to step four of our algorithm for $r = \pm 1$ and (13) we respectively have

$u_1(x) = v_1(x) = e^x, w_1(x) = (I - \lambda K) u_1(x) = (I - \lambda K) e^x = x + 1$ and
$u_2(x) = v_{-1}(x) = e^{-x}, w_2(x) (I - \lambda K) u_2(x) = (I - \lambda K) e^{-x} = -x + 1.$

So according to Equation(10), we must choose c_1 and c_2 such that:

$$f(x) = c_1 (I - \lambda K) u_1(x) - c_2 (I - \lambda K) u_2(x).$$

That is

$$x = c_1 (x + 1) - c_2 (-x + 1)$$

which gives $c_1 = c_2 = \frac{1}{2}$ and so by (11)

$$u(x) = c_1 u_1(x) - c_2 u_2(x) = \frac{1}{2} e^x - \frac{1}{2} e^{-x} = sinhx$$

is the desired solution of (12).

Example 2: Consider the equation

$$u(x)-\int_0^x y^{x-y}u(y)dy=0 \tag{15}$$

Solution: Using the functions of the form $u_{\alpha,\beta}(x)=x^{\alpha x+\beta},\alpha,\beta\in\mathrm{R}$, we have:

$$[(\lambda K)u_{\alpha,\beta}](x)=\int_0^x y^x y^{(\alpha-1)y+\beta}dy=\int_0^x y^{x+(\alpha-1)y+\beta}dy.$$

To have some simple primitive functions, we put $\alpha=1$ and then we have,

$$[(\lambda K)u_{\alpha,\beta}](x)=[(\lambda K)u_{1,\beta}](x)=\int_0^x y^{x+\beta}dy=\frac{x^{x+\beta+1}}{x+\beta+1}$$

and then we obtain,

$$[(I-\lambda K)u_{1,\beta}](x)=x^{x+\beta}-\frac{x}{x+\beta+1}x^{x+\beta}=\frac{\beta+1}{x+\beta+1}x^{x+\beta}$$

So according to (11), we must choose c such that

$$0=f(x)-c(\frac{\beta+1}{x+\beta+1})x^{x+\beta}=0-c(\frac{\beta+1}{x+\beta+1})x^{x+\beta}$$

which gives $\beta=-1$ and so $u(x)=cx^{x-1},c\in\mathrm{R}$ is the exact solution of (14).

Example 3: Consider the singular homogeneous integral equation

$$\varphi(x)=\lambda\int_0^\infty e^{-|x-y|}\varphi(y)dy. \tag{16}$$

Solution: If we choose the family of functions $\{u_r(x)=\{e^{rx}:r\in\mathrm{R}\}$ then we have

$$\lambda\int_0^\infty k(x,y)u_r(y)dy=\lambda\int_0^\infty e^{-|x-y|}e^{ry}dy=$$

$$\lambda\int_0^x e^{-(x-y)}e^{ry}dy+\lambda\int_x^\infty e^{+(x-y)}e^{ry}dy= \tag{17}$$

$$(\frac{\lambda}{r+1}-\frac{\lambda}{r-1})e^{rx}-\frac{\lambda}{r+1}e^{-x},r\neq-1,r<1.$$

Moreover for $r=-1$ or $r\geq1$ the improper integral diverges, and then:

$$(I-\lambda K)u_r(x)=e^{rx}-\frac{-2\lambda}{r^2-1}e^{rx}+\frac{\lambda}{r+1}e^{-x}=(\frac{r^2+2\lambda-1}{r^2-1})e^{rx}+\frac{\lambda}{r+1}e^{-x}.$$

So according to (10), we must choose those r's which are necessary for generating $f(x)=0$, that is r's must be such that the coefficients of e^{rx}'s become zero, and this gives $r^2+2\lambda-1=0$ and then $r=\pm\sqrt{1-2\lambda}$. Moreover, for $r_1=\sqrt{1-2\lambda}$ and $r_2=-\sqrt{1-2\lambda}$, we respectively have:

$$u_1^*=e^{r_1x}=e^{x\sqrt{1-2\lambda}},u_2^*=e^{r_2x}=e^{-x\sqrt{1-2\lambda}}.$$

Now

$$0=f(x)=c_1(I-\lambda K)u_1(x)-c_2(I-\lambda K)u_2(x)=c_1(I-\lambda K)e^{x\sqrt{1-2\lambda}}$$

$$-c_2(I-\lambda K)e^{-x\sqrt{1-2\lambda}}=-c_1(\frac{\lambda}{1+\sqrt{1-2\lambda}})e^{-x}+c_2(\frac{\lambda}{1-\sqrt{1-2\lambda}}e^{-x}).$$

If we put $c_1=\beta(1+\sqrt{1-2\lambda})$ and $c_2=\beta(1-\sqrt{1-2\lambda})$ then according to Equation (11), we see that:

$$u(x)=c_1u_1^*(x)-c_2u_2^*(x)=\beta[(\sqrt{1-2\lambda})e^{x\sqrt{1-2\lambda}}+(\sqrt{1-2\lambda}-1)e^{-x\sqrt{1-2\lambda}}]$$

is the exact solution of (16).

Example 4: Consider the integral equation (Bhattacharya and Mandal, 2008):

$$u(x)=1-x-\frac{3}{2}x^2+\frac{x^3}{2}+\int_0^x\frac{1+x}{1+t}u(t)dt \tag{18}$$

Solution: If we choose the sequence of functions $\{u_n(x)=(1+x)^n,n\in\mathrm{N}\cup\{0\}\}$ as the basis functions then we have:

$$\int_0^x\frac{1+x}{1+t}u_n(t)dt=\int_0^x\frac{1+x}{1+t}(1+t)^n dt=\frac{(1+x)^{n+1}}{n}-\frac{1+x}{n},n\geq2$$ and for $n=0,1$ we respectively obtain:

$$\int_0^x\frac{1+x}{1+t}dt=(1+x)ln(1+x),\int_0^x\frac{1+x}{1+t}(1+t)dt=x(1+x).$$

Thus, we have

$$(I-\lambda K)u_n(x)=(1+x)^n-\frac{(1+x)^{n+1}}{n}+\frac{1+x}{n},n\geq 2$$

and

$$(I-\lambda K)u_0(x)=1-(1+x)ln(1+x),$$

$$(I-\lambda K)u_1(x)=(1+x)-x(1+x)=1-x^2.$$

Now, since $f(x)$ is a polynomial of degree 3, so $1\leq n\leq 2$ and

$$1-x-\frac{3}{2}x^2+\frac{x^3}{2}=f(x)=c_1(I-\lambda K)u_1(x)-c_2(I-\lambda K)u_2(x)=$$

$$c_1(1-x^2)-c_2[(1+x)^2-\frac{(1+x)^3}{2}+\frac{1+x}{2}]=$$

$$c_1-c_2-c_2x+(\frac{1}{2}c_2-c_1)x^2+\frac{1}{2}c_2x^3$$

and then we have $c_1=2$ and $c_2=1$. So $u(x)=2(1+x)-(1+x)^2=1-x^2$ is the exact solution of Equation (18). In examples (5) and (6), we will show that (11) and (12) can be extended as series.

Example 5: In this example we will solve integral equation

$$u(x)=x-\int_0^x(x-t)u(t)dt \tag{19}$$

in example (1) with an other family of functions.

Solution: By choosing the sequences of functions $\{v_n(x)=x^n,n\in \mathrm{N}\cup\{0\}\}$, we have

$$(I-\lambda K)v_n(x)=(I-\lambda K)x^n=x^n-\frac{x^{n+2}}{(n+2)(n+1)} \tag{20}$$

According to (11), we find constants $c_1,c_2,\cdots$ and functions $u_1(x)$, $u_1(x),\cdots$ recursively, such that

$$x=f(x)=c_1(I-\lambda K)u_1(x)-c_2(I-\lambda K)u_2(x)+\cdots$$

We put $u_1(x)=v_1(x)=x$ then by (20) we have,

$$x=f(x)=\ c_1(I-\lambda K)x-c_2(I-\lambda K)u_2(x)+\cdots=$$

$$c_1(x-\frac{x^3}{3\cdot 2})-c_2(I-\lambda K)u_2(x)+\cdots, \tag{21}$$

$$u(x)=c_1u_1(x)-c_2u_2(x)+\cdots=c_1x-c_2u_2(x)+\cdots$$

So $c_1=1$ and then

$$x=f(x)=\ (x-\frac{x^3}{3\cdot 2})-c_2(I-\lambda K)u_2(x)+\cdots,$$

$$u(x)=x-c_2u_2(x)+c_3u_3(x)-\cdots \tag{22}$$

Now we put $u_2(x)=v_3(x)=x^3$ and then

$$x=f(x)=\ (x-\frac{x^3}{3\cdot 2})-c_2(x^3-\frac{x^5}{5\cdot 4})+\cdots, \tag{23}$$

$$u(x)=x-c_2x^3+c_3u_3(x)-\cdots$$

So $c_2=-\frac{1}{3\cdot 2}$ and then

$$x=f(x)=(x-\frac{x^3}{3\cdot 2})+\frac{1}{3\cdot 2}+c_3(I-\lambda K)u_3(x)-\cdots,$$

$$u(x)=x+\frac{x^3}{3!}+c_3u_3(x)-\cdots$$

Finally, we find that $u_n(x)=x^{2n-1}$ and $c_n=(-1)^{n+1}\frac{1}{(2n-1)!}$ and so

$$u(x)=\sum_{n=1}^{\infty}(-1)^{n+1}(-1)^{n+1}\frac{x^{(2n-1)}}{(2n-1)!}=\sum_{n=1}^{\infty}\frac{x^{(2n-1)}}{(2n-1)!}=sinh(x)$$

is the exact solution of (19).

Example 6: Consider the equation

$$u(x)=2\sqrt{x}-\int_0^x \frac{1}{\sqrt{x-t}}u(t)dt \tag{24}$$

Solution: By choosing the sequences

$$\{w_n(x)=x^n, n\in \mathrm{N}\cup\{0\}\}$$

and

$\{v_m(x)=x^{\frac{m}{2}}, m=1,3,\cdots\}$ we have

$$\int_0^x \frac{t^n}{\sqrt{x-t}}dt=\frac{\Gamma(\frac{1}{2})\Gamma(n+1)}{\Gamma(n+\frac{3}{2})}x^{n+\frac{1}{2}},\quad n\in \mathrm{N}\cup\{0\} \tag{25}$$

$$\int_0^x \frac{t^{\frac{m}{2}}}{\sqrt{x-t}}dt=\frac{m}{2^m}\pi x^{\frac{m+1}{2}}, m=1,3 \tag{26}$$

where Γ is the Gamma function. Consequently we have:

$$(I-\lambda K)w_n(x)=x^n+\int_0^x \frac{t^n}{\sqrt{x-t}}dt=x^n+\frac{\Gamma(\frac{1}{2})\Gamma(n+1)}{\Gamma(n+\frac{3}{2})}x^{n+\frac{1}{2}},\quad n\in \mathrm{N}\cup\{0\} \tag{27}$$

$$x^{\frac{m}{2}}+\int_0^x \frac{t^{\frac{m}{2}}}{\sqrt{x-t}}dt=x^{\frac{m}{2}}+\frac{m}{2^m}\pi x^{\frac{m+1}{2}},\quad m=1,3$$

According to (10), we must find functions $u_1(x)$, $u_2(x)$, $\cdots$ and constants c_1, c_2, $\cdots$ such that

$$2\sqrt{x}=f(x)=c_1(I-\lambda K)u_1(x)-c_2(I-\lambda K)u_2(x)+\cdots$$

For $u_1(x)=v_1(x)=\sqrt{x}$ and Equation (27) we have

$$2\sqrt{x}=f(x)=c_1(\sqrt{x}+\frac{\pi}{2}x)-c_2(I-\lambda K)u_2(x)+\cdots$$

So, we put $c_1=2$ and then

$$2\sqrt{x}=f(x)=2(\sqrt{x}+\frac{\pi}{2}x)-c_2(I-\lambda K)u_2(x)+\cdots$$

and

$$u(x)=c_1u_1(x)-c_2u_2(x)+\cdots=2\sqrt{x}-c_2u_2(x)+\cdots$$

Now we put $u_2(x)=w_1(x)=x$ and then by Equation (27),

$$2\sqrt{x}=f(x)=2(\sqrt{x}+\frac{\pi}{2}x)-c_2(x+\frac{4}{3}x^{\frac{3}{2}})+c_3(I-\lambda K)u_3(x)-\cdots$$

So $c_2=\pi$ and then

$$2\sqrt{x}=f(x)=2(\sqrt{x}+\frac{\pi}{2}x)-\pi(x+\frac{4}{3}x^{\frac{3}{2}})+c_3(I-\lambda K)u_3(x)-\cdots$$

and

$$u(x)=2\sqrt{x}-x+c_3u_3(x)-\cdots.$$

By iterations, we will fined $c_3=\frac{4\pi}{3}$ and $c_4=\frac{\pi^2}{2}$, $\cdots$ and so

$$2\sqrt{x}=f(x)=2(\sqrt{x}+\frac{\pi}{2}x)-\pi(x+\frac{4}{3}x^{\frac{3}{2}})+\frac{4\pi}{3}(x^{\frac{3}{2}}+\frac{3\pi}{8}x^2)+\cdots$$

and

$$u(x)=2\sqrt{x}-\pi x+\frac{4\pi}{3}x^{\frac{3}{2}}-\frac{\pi^2}{2}x^2+\cdots$$

that is the solution of (24) as series.

CONCLUSION

In this paper, we have given a new attitude to solve integral equations, which can be applied in various situations. The major difference between our method and the other methods for solving linear Volterra integral equations is in manner for trying to represent the known function $f(x)$. We predict that Fredholm integral equations and linear integro- differential equations can also be solved by this method.

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