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Full Length Research Paper

A robust observer design for chaos synchronization of uncertain systems using LMI criterion

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In this paper, considering the effect of exoteric perturbation on system's parameters, a robust observer, based on Lyapunov theory, is designed for the output synchronization of a class of uncertain systems when the full state vector is not available. With the help of LMI criterion and then using the MATLAB LMI Toolbox, the observer gain *L* is derived, which can not only make the outputs of both master and slave systems reach H_∞ synchronization with the passage of time but also reduce the influence of the disturbance within a prescribed level. In the end, we take the perturbed Chua's circuit as an example to illustrate that the proposed method is effective and quite robust against the effect of exoteric perturbation.

Key words. Chaos synchronization; the observer method; Lyapunov theory; linear matrix inequality (LMI), PACC: 0545

INTRODUCTION

Owing to complexity and unpredictability of chaos behavior, how to control and utilize chaos has attracted much attention since the pioneering work of Oat et al. (1990). At approximately the same time, Pecora and Carroll (1990) firstly proposed chaotic synchronization phenomenon. Generally speaking, chaos synchronization refers to a process wherein two (or many) chaotic systems (either equivalent or nonequivalent) adjust a given property of their motion to a common behavior due to coupling or forcing (Li et al., 2007). Now, it has been applied to many different fields such as secure communication, information science, chemical reactions and biological system (Boccaletti et al., 2002; Bowong et al., 2006; Zhou et al., 2005; Li et al., 2004). There are many strategies for achieving synchronization of chaotic systems, which include impulsive control (Rong et al., 2007), adaptive control (Bing-Jian et al., 2007; Agiza, 2004), backstepping control (Park, 2006), sliding mode control (Tie-Gang et al., 2005) and observe-based control (Yao et al., 2006; Ting, 2007).

Though a considerable amount of research on chaos synchronization has been done in the past decade, availability of the full state vector of both master and slave systems is the basis of many approaches for achieving synchronization (Li et al., 2007; Bing-Jian et al., 2007; Agiza, 2004; Park, 2006; Tie-Gang et al., 2005; Liao et al., 2007). But in many situations, it is impossible that we can obtain all state variables of the system, so observer-based method has been naturally proposed for chaos synchronization, where the outputs of the driving system are to control the response system so that they can oscillate in a synchronized manner. By using an integral observer approach, a sufficient condition was derived for realizing chaos synchronization based on Lyapunov stability theory for a class of chaotic systems in reference (Jiang et al., 2005), where chaotic systems' parameters were taken as known constants. As we know, in practical situations, some or all of the system's parameters are uncertain and these parameters change from time to time. So for uncertain chaotic system with parameters perturbation, based on LMI technique and Lyapunov stability theory, a new sufficient criterion was established for achieving chaotic synchronization in (Chen and Zhang, 2006). One drawback, however, is that nonlinear input is introduced to eliminate the perturbation terms.

Recently, the H_∞ control method was proposed to reduce the effect of the disturbance on the available output to within a prescribed level (Anton, 1992). Based on this idea, when system's parameters were known, reference (Hou et al., 2007) investigated the output synchronization for a class of chaotic systems, where the controller was derived by means of output feedback control method. While, for some classical Rössler systems with noise perturbation, when system's parameters were taken as known constants, reference (Liao et al., 2007) studied H_∞ chaos suppression problem based on the sliding mode control method, where all state variables were used in designing controller.

In this paper, considering the effect of exoteric perturbation on system's parameters, when only the system outputs are available, the observer method is used in order to achieve robust output synchronization for a class of uncertain chaotic systems. Firstly, based on Lyapunov theory, we get a new sufficient condition that can guarantee the observer is robust against the effect of exoteric perturbation. And then, we transform this condition into linear matrix inequality (LMI) form that can be solved by using MATLAB LMI toolbox, where an appropriate the gain matrix L is derived that can not only make the system outputs achieve H_∞ synchronization but also reduce the influence of the disturbance within a prescribed level. Finally, The Chua's chaotic system is used to illustrate the effectiveness of the proposed method, and it shows that this observer we derived is assuredly robust against the effect of exoteric perturbation.

Throughout this paper, $\| x \|$ represents the Euclidean norm of the vector x, while for a real matrix A, $\| A \| = \sqrt{\rho(A^T A)}$ denotes the spectral norm of A, where $\rho(M)$ is the spectral radius of the matrix M and A^T is the transpose of the matrix A. In addition, I is the identity matrix of appropriate dimensions.

System formulation

Let a class of chaotic systems be given in the following

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bf(x(t)) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

Where $x \in R^n$ is the state vector and $y \in R^m$ is the output. A, B and C are known matrices with proper dimensions respectively and (A, C) is observable. $f(x)$ is a nonlinear function satisfying Lipschitz condition, which can be written as

$$\left\| f(x) - f(\hat{x}) \right\| \le \varepsilon \left\| x - \hat{x} \right\| \tag{2}$$

Where δ is called Lipschitz constant and $\delta > 0$.
Considering the effect of exoteric perturbation on system's parameters, system (1) becomes

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A_1(t))x(t) + Bf(x(t)) \\ y(t) &= Cx(t) \end{aligned} \tag{3}$$

Where $\Delta A_1(t)$ is the perturbed matrix.

When only the system outputs are available, based on the observer method, the slave system is

$$\begin{aligned} \dot{\hat{x}}(t) &= (A + \Delta A_2(t))\hat{x}(t) + Bf(\hat{x}(t)) + L(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= C\hat{x}(t) \end{aligned} \tag{4}$$

Where $\Delta A_2(t)$ is the perturbed matrix satisfying bounded condition $\| \Delta A_2(t) \| \le \varepsilon_2$. L is the observer gain that need to be obtained .
Let us define synchronization error $e(t) = x(t) - \hat{x}(t),\ y_e(t) = y(t) - \hat{y}(t)$, so the error dynamical system is

$$\begin{aligned} \dot{e}(t) &= (A - LC)e(t) + B(f(x(t)) - f(\hat{x}(t))) + \Delta A_1(t)x(t) - \Delta A_2(t)\hat{x}(t) \\ y_e(t) &= Ce(t) \end{aligned} \tag{5}$$

For the perturbed matrix $\Delta A_1(t)$ and $\Delta A_2(t)$, we define $\omega(t) = \Delta A_1(t) - \Delta A_2(t)$ and assume $\omega(t) \in L_2$, that is $\int_0^\infty \omega(t)^T \omega(t) dt < \infty$. The H_∞ performance problem will be considered, which can formulate as follows: design the gain of matrix L such that
If $\omega(t) = 0$, the error system (5) will be asymptotically stable;
Given a positive scalar γ and under zero initial conditions, if $\omega(t) \ne 0$, the following condition will hold:

$$\int_0^\infty [y_e^T(t) y_e(t) - \gamma^2 \omega^T(t)\omega(t)]dt \le 0 \tag{6}$$

Where γ is called as the disturbance attenuation performance index.

Our aim is to determine the gain of matrix L such that the statement (a) is satisfied and the performance index (6) in statement (b) is within the upper bound, that is,

$$\sup_{\omega(t) \ne 0, \omega(t) \in L_2} \frac{\| y_e(t) \|_2}{\| \omega(t) \|_2} \le \gamma$$

MAIN RESULTS

This section is to get an appropriate observer, so that it could realize robust output synchronization of both master and slave system, that is to say, the outputs of the error system achieve H_∞ synchronization with the disturbance attenuation γ with the passage of time.

Theorem 1 Given three positive constants $\delta_1, \delta_2, \gamma$ and if there exist the gain L and positive matrices P, Q such that the following equality holds

$$P(A - LC) + (A - LC)^T P + \delta_1 PBB^T P + (\delta_2 + \gamma^{-2} d^2)P^2 + (\delta_1^{-1}\varepsilon^2 + \delta_2^{-1}\varepsilon_2^2)I + C^T C = -Q \tag{7}$$

then the outputs of the error system will satisfy the H_∞ syn chronization with the disturbance attenuation γ.

Proof. Construct the Lyapunov function in the form of

$$V(t) = e^T(t)Pe(t) \qquad (8)$$

Where $P = P^T > 0$. It is obvious that V is a positive function for all $t \geq 0$.

Differentiating V with respect to time t

$$\begin{aligned}\dot{V}(t) &= \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t)\\ &= ((A-LC)e(t) + B(f(x(t)) - f(\hat{x}(t))) + \Delta A_1(t)x(t) - \Delta A_2(t)\hat{x}(t))^T Pe(t)\\ &\quad + e^T(t)P((A-LC)e(t) + B(f(x(t)) - f(\hat{x}(t))) + \Delta A_1(t)x(t) - \Delta A_2(t)\hat{x}(t))\\ &= e^T(t)(P(A-LC) + (A-LC)^T P)e(t) + 2e^T(t)PB(f(x(t)) - f(\hat{x}(t)))\\ &\quad + 2e^T(t)P(\Delta A_1(t)x(t) - \Delta A_2(t)\hat{x}(t))\end{aligned} \qquad (9)$$

With the help of inequality (2), given a positive constant δ_1, we derive

$$2e^T(t)PB(f(x(t)) - f(\hat{x}(t))) \leq \delta_1 e^T(t)PBB^T Pe(t) + \delta_1^{-1}\varepsilon^2 e^T(t)e(t) \qquad (10)$$

and

$$2e^T(t)P(\Delta A_1(t)x(t) - \Delta A_2(t)\hat{x}(t)) = 2e^T(t)P\Delta A_2(t)e(t) + 2e^T(t)P(\Delta A_1(t) - \Delta A_2(t))x(t) \qquad (11)$$

Due to $\| \Delta A_2(t) \| \leq \varepsilon_2$, hence, with a positive constant δ_2, we obtain

$$2e^T(t)P\Delta A_2(t)e(t) \leq \delta_2 e^T(t)P^2 e(t) + \delta_2^{-1}\varepsilon_2^2 e^T(t)e(t) \qquad (12)$$

As we know, the system is chaotic, so for any time $t \geq 0$, the state vector is bounded, i.e. $\| x(t) \| \leq d$. Combined with $\omega(t) = \Delta A_1(t) - \Delta A_2(t)$ and given a positive constant γ, the following inequality yields

$$2e^T(t)P(\Delta A_1(t) - \Delta A_2(t))x(t) \leq \gamma^{-2}d^2 e^T(t)P^2 e(t) + \gamma^2\omega^T(t)\omega(t) \qquad (13)$$

Since (7) and (10)-(13) hold, we derive

$$\dot{V}(t) \leq -e^T(t)Qe(t) - e^T(t)C^T Ce(t) + \gamma^2\omega^T(t)\omega(t) \qquad (14)$$

That is

$$\dot{V}(t) \leq -e^T(t)Qe(t) - y_e^T(t)y_e(t) + \gamma^2\omega^T(t)\omega(t)$$

Thus

$$\dot{V}(t) \leq -e^T(t)Qe(t) - y_e^T(t)y_e(t) + \gamma^2\omega^T(t)\omega(t) \qquad (15)$$

It is easy to find that if $\omega(t) = 0$

$$\dot{V}(t) < -\lambda_{\min}(Q) \| e(t) \|^2 < 0 \qquad (e \neq 0) \qquad (16)$$

therefore, based on Lyapunov stability theory, the error system (5) is asymptotically stable. But if $\omega(t) \neq 0$, integrating the two sides of (15) from 0 to ∞, we have

$$V(\infty) - V(0) + \int_0^\infty (\| y_e(t) \|^2 - \gamma^2 \| \omega(t) \|^2)dt \leq 0 \qquad (17)$$

With zero initial condition, we have $V(0) = 0$ and $V(\infty) > 0$, so the following inequality holds

$$\int_0^\infty (\| y_e(t) \|^2 - \gamma^2 \| \omega(t) \|^2)dt \leq 0 \qquad (18)$$

Thus, H_∞ performance with the disturbance attenuation γ is satisfied.

However, we can easily find that it is difficult to solve equation (7) and obtain the gain of matrix L. So we change it into linear matrix inequality by means of the following famous lemma, which can be easily solved by using the MATLAB LMI Toolbox.

Lemma (Schur Complements [15]) for a given symmetric matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

Where $S_{11} = S_{11}^T, S_{12} = S_{21}^T$ and $S_{22} = S_{22}^T$

the condition $S < 0$ is equivalent to

$S_{22} < 0$ and $S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0$.

Accordingly, theorem 1 can be transformed into the following conclusion.

Theorem 2 Given three positive constants $\delta_1, \delta_2, \gamma$ and if suitable matrix W and positive matrix P are selected such that the following equality holds

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ \Xi_{12}^T & \Xi_{22} & 0 \\ \Xi_{13}^T & 0 & \Xi_{33} \end{bmatrix} < 0 \qquad (19)$$

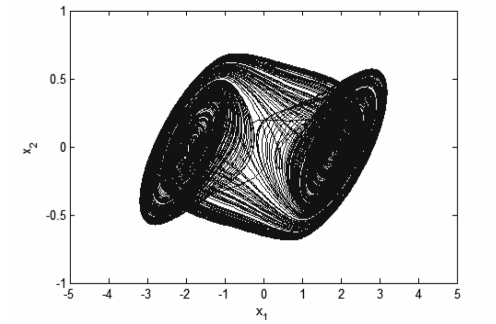
Where

$$\Xi_{11} = PA + A^T P - WC - C^T W^T + (\delta_1^{-1}\varepsilon^2 + \delta_2^{-1}\varepsilon_2^2)I + C^T C$$

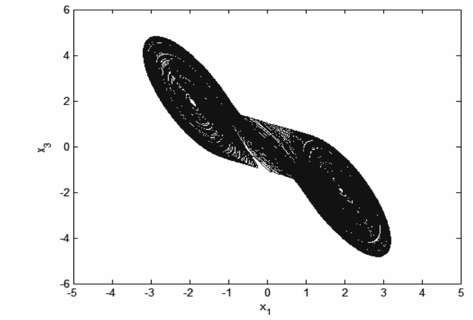
$$\Xi_{12} = PB \quad \Xi_{13} = P \quad \Xi_{22} = -\delta_1^{-1}I \quad \Xi_{33} = -l^{-1}I$$

$l = \delta_2 + \gamma^{-2}d^2$ then, with the observer gain $L = P^{-1}W$, the H_∞ synchronization problem with the disturbance attenuation γ will be satisfied.

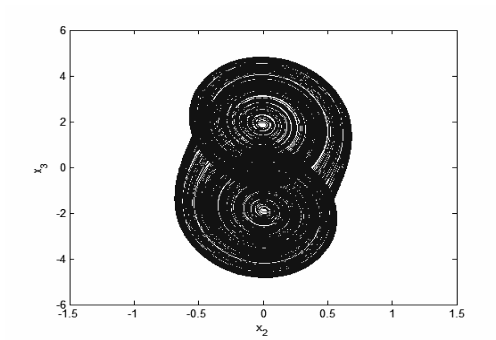
Proof. Based on Schur Complements, it is easy for us to obtain (19).



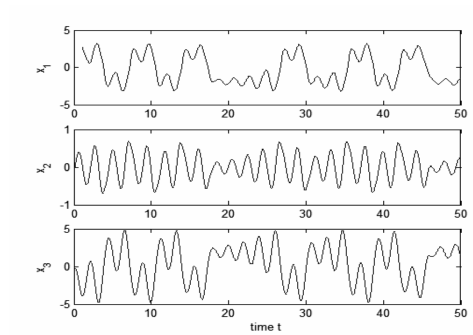
(a) Phase portrait (x_1 versus x_2)



(b) Phase portrait (x_1 versus x_3)



(c) Phase portrait (x_2 versus x_3)



(d) the time history

Figure 1. Phase portrait and the time history of Chua's system.

Numerical simulations

In this section, we will give simulation results of the Chua's chaotic system to demonstrate that the above derived criterion is effective.

The Chua's chaotic system can be written as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bf(x(t)) \\ y(t) &= Cx(t) \end{aligned} \quad (20)$$

Where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix},\ A = \begin{bmatrix} -\alpha & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta_1 & -\beta_2 \end{bmatrix},\ B = \begin{bmatrix} -\alpha & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$C = c\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}^T,\ f(x) = \begin{bmatrix} f(x_1) \\ 0 \\ 0 \end{bmatrix}.$$

Where $c \neq 0$ and $f(\cdot)$ is a piecewise linear function in the form of

$$f(x_1) = bx_1 + 0.5(a-b)(|x_1+1| - |x_1-1|)$$

with $a < b < 0$.

When parameters are taken as $\beta = 10, \beta_1 = 15, \beta_2 = 0.0385, a = -1.28, b = -0.69$, the system (20) exhibits chaotic, this can be seen in Figure1. Figure 1a, b and c show us the phase portrait, while Figure 1d gives us the time history of Chua's system.

Due to the existence of exoteric perturbation, system (20) becomes

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A_1(t))x(t) + Bf(x(t)) \\ y(t) &= Cx(t) \end{aligned} \quad (21)$$

and we select the perturbed matrix simply in the following:

$$\Delta A_1(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta\beta_{21}(t) \end{bmatrix}$$

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A_1(t))x(t) + Bf(x(t)) \\ y(t) &= Cx(t) \end{aligned} \quad (21)$$

and we select the perturbed matrix simply in the following:

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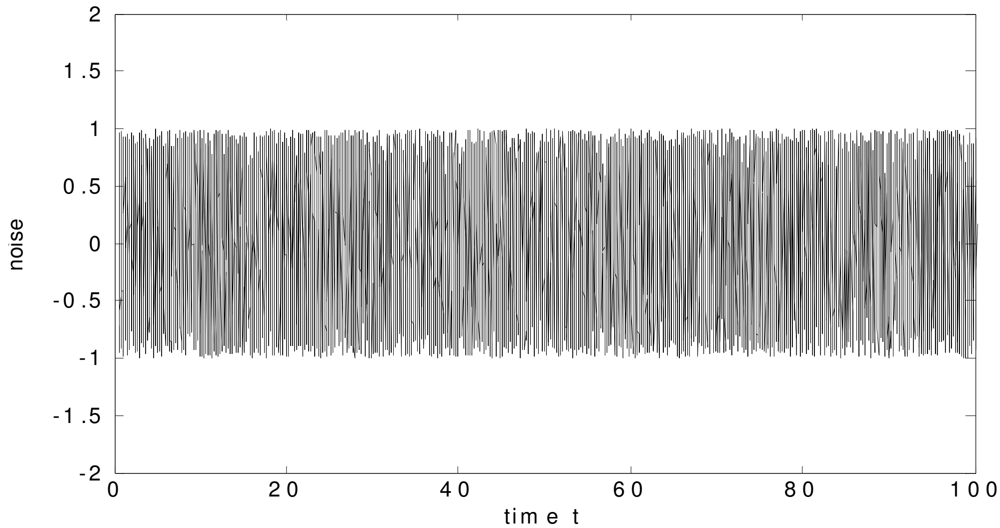

Figure 2. Uniformly distributed random noise in the range [-1.0,1.0].

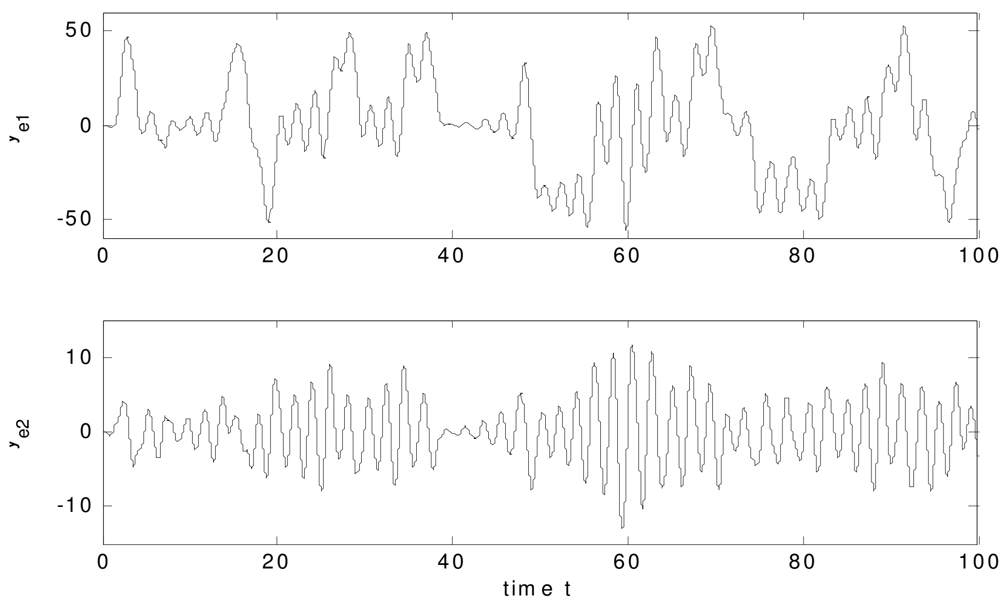

Figure 3. The time history of the output error with disturbance when any control input is not applied

Now, based on the observer method, equation (21) is regarded as the master system and the slave system is

$$\begin{aligned} \dot{\hat{x}}(t) &= (A+\Delta A_2(t))\hat{x}(t) + Bf(\hat{x}(t)) + L(y(t)-\hat{y}(t)) \\ \hat{y}(t) &= C\hat{x}(t) \end{aligned} \tag{22}$$

$\Delta A_2(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta\beta_{22}(t) \end{bmatrix}$ and the gain matrix $L = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}$.

Where the perturbed matrix

Thus, the error system can be derived as

$$\begin{aligned} \dot{e}(t) &= (A-LC)e(t) + B(f(x(t)) - f(\hat{x}(t))) + \Delta A_1(t)x(t) - \Delta A_2(t)\hat{x}(t) \\ y_e(t) &= Ce(t) \end{aligned} \tag{23}$$

Firstly, some constants are given in the following:

$c = 10, \gamma = 0.15, \delta_1 = 0.3, \delta_2 = 0.1$ and $\Delta\beta_{21} = 0.3\cos(6t)$. Simultaneously, we adopt $\Delta\beta_{21}$ as to be uniformly distributed random noise in the range [-1.0, 1.0], which can be seen in Figure 2. According to calculation, we get $\|\Delta A_2(t)\| \le 0.3$ and $d = 10, \varepsilon = 1.0$. The initial conditions

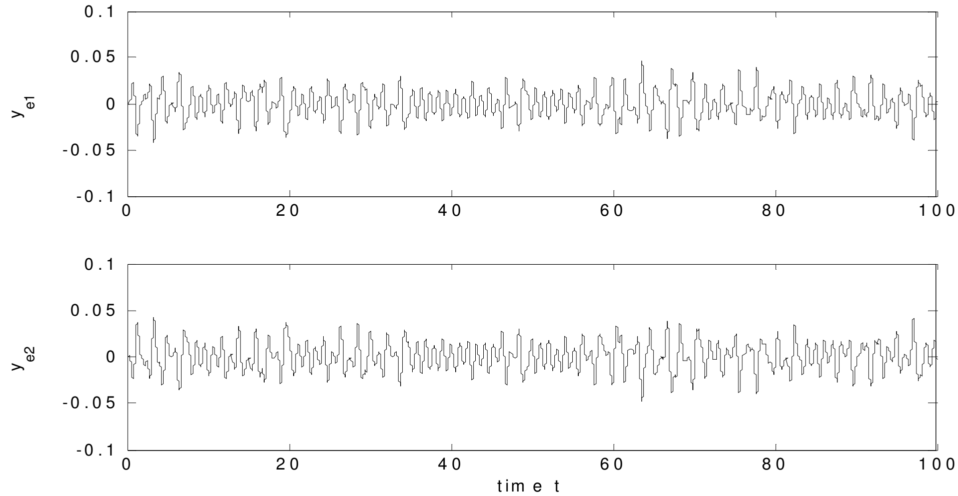

Figure 4. The time history of the output error with disturbance via the observer method.

both master and slave system are adopted as: $x_1(0)=0.8, x_2(0)=0.1, x_3(0)=0.1$ and $\hat{x}_1(0)=0.8, \hat{x}_2(0)=0.1, \hat{x}_3(0)=0.1$. So the initial values of the error system are $e_1(0)=0, e_2(0)=0$ and $e_3(0)=0$. Figure 3 illustrates the time history of the output error $y_e(t)$ with disturbance when any control input is not applied. It is found that, when the system's parameters are perturbed, the output error $y_e(t)$ with any control input cannot converge to zero as time t tends to infinity though the initial values of the master system are the same to the ones of the slave system. In order to realize the output synchronization, the observer method is adopted here. By use of the MATLAB LMI toolbox, from (19), we derive that the gain of matrix is $L=\begin{bmatrix}35.0897 & 126.8107 & 478.9436\end{bmatrix}^T$. Figure 4 depicts the time history of the output error vector $y_e(t)$ when the observer method is applied, we see that the outputs of the error system $y_{e_1}(t), y_{e_2}(t)$ all fluctuate in the range of [-0.05,0.05], that is to say, the fluctuations of the output error vector is very small compared with $\gamma=0.15$. Therefore, the appropriate gain L assuredly reduces the effect of the exoteric perturbation within a prescribed level $\gamma=0.15$. At this time , we conclude that robust output synchronization achieve via the proposed method in our paper.

Secondly, we select constants $c=20, \gamma=0.25, \delta_1=0.3, \delta_2=0.1, \Delta\beta_{22}=0.3\cos(6t)$ and $\Delta\beta_{21}$ is taken as Gaussian noise with mean 0 and variance 1 that shows in Figure 5. The initial conditions of both master and slave system are adopted as: $x_1(0)=0.8, x_2(0)=0.1, x_3(0)=0.1$ and $\hat{x}_1(0)=0.8,\ \hat{x}_2(0)=0.1, \hat{x}_3(0)=0.1$. Therefore, we get that the initial values of the error system are $e_1(0)=0, e_2(0)=0$ and $e_3(0)=0$. It is in Figure 6 that depicts the time history of the output error vector $y_e(t)$ with disturbance when any control input is not applied. The same conclusion as the above is drawn that, the initial values of the master system are the same to the ones of the slave system, but the output error $y_e(t)$ with any control input cannot converge to zero with the passage of time t. To observe the H_∞ performance with disturbance attenuation, the gain of matrix is obtained $L=\begin{bmatrix}26.2390 & 93.8341 & 354.5015\end{bmatrix}^T$ from (19) with the aid of the MATLAB LMI toolbox. Figure 7 shows the time history of the output error vector $y_e(t)$ of both master and slave system with disturbance via the observer method. It is found that the outputs of the error system $y_{e_1}(t), y_{e_2}(t)$ all change in the range [-0.1, 0.1] with zero initial conditions, that is, compared with $\gamma=0.25$; the fluctuations of the output error vector are very small. So robust output synchronization achieves with the disturbance attenuation $\gamma=0.25$ with the passage of time, which is consistent with our theory analysis.

Conclusions

Due to the existence of exoteric perturbation, a robust observer, based on Lyapunov theory, is designed for the output synchronization of a class of uncertain systems when the full state vector is not available. A new sufficient condition is derived that can not only make the outputs of both master and slave systems reach H_∞ synchronization

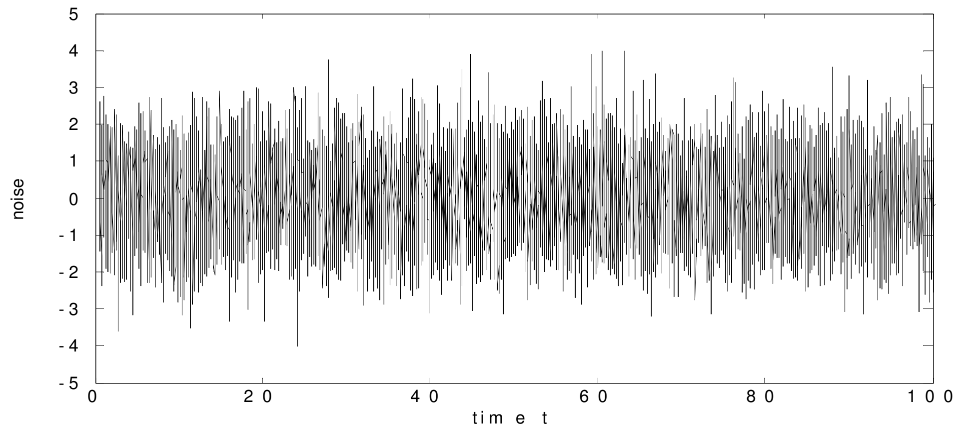

Figure 5. Gaussian noise with mean 0 and variance 1.

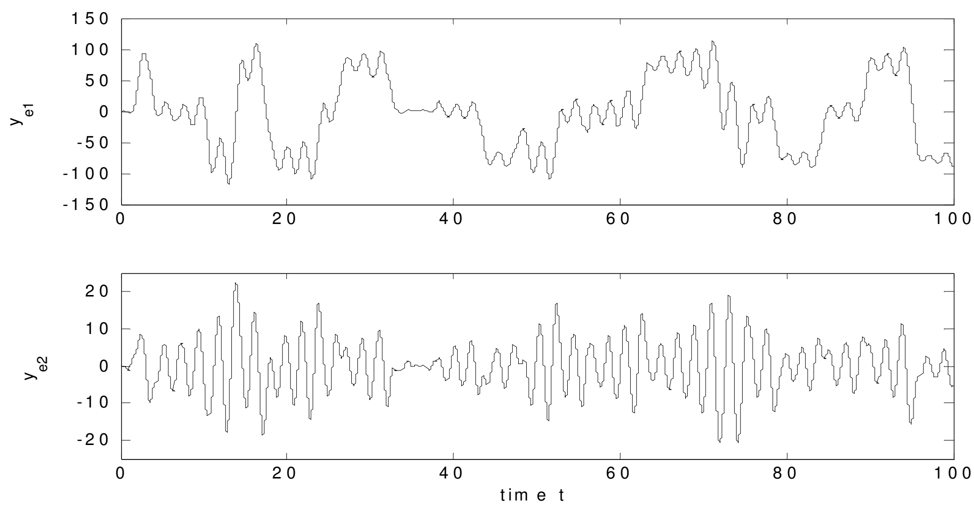

Figure 6. The time history of the output error with disturbance when any control input is not applied.

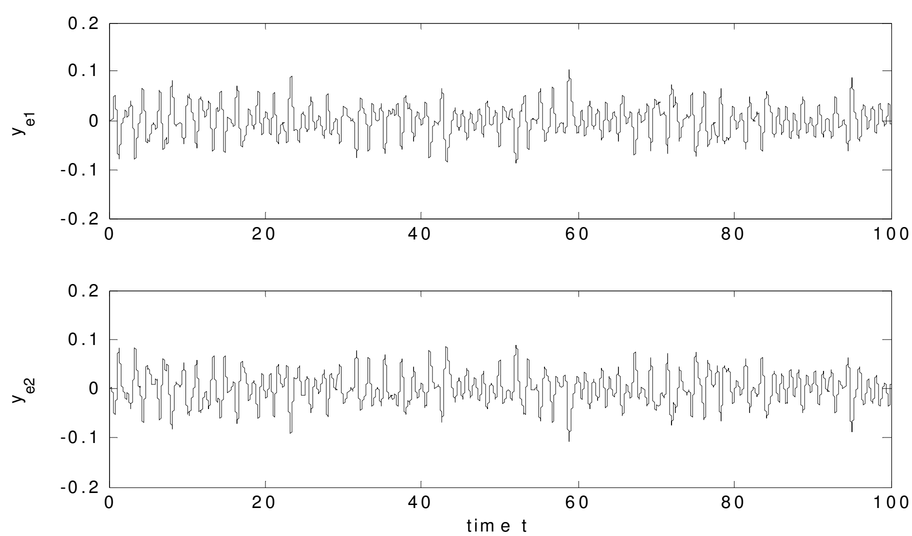

Figure 7. The time history of the output error with disturbance via the observer method

with the passage of time but also reduce the influence of the disturbance within a prescribed level. Finally, we take the perturbed Chua's circuit as an example to illustrate the effectiveness of the proposed method. It is found that our controller only depends on the outputs of the system, so it is relatively easy to implement in practical applications.

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