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Differential transform method for solving non-linear systems of partial differential equations

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The differential transform method (DTM) has been applied to solve many functional equations so far. In this paper, we propose this method (DTM), for solving nonlinear Jaulent-Miodek and the Hirota-Satsuma equation. Numerical solutions obtained by differential transform method are compared with the exact solutions and that obtained by Adomian decomposition methods. The results for some values of the variables are shown in tables and the solutions are presented as plots as well, showing the ability of the method.

Key words: Differential transform method, Jaulent-Miodek (JM) equation, the Hirota-Satsuma equation, numerical.

INTRODUCTION

The differential transform method has been successfully used by Zhou (1986) to solve linear and nonlinear initial value problems in electric circuit analysis. In recent years, differential transform method (DTM) has been used to solve one-dimensional planar Bratu problem, differential-difference equation, delay differential equations, differential algebraic equation, integro-differential systems (Rostam et al., 2011; Kanth and Aruna, 2008; Zhou, 1986; Raslan and Zain, 2013; Raslan et al., 2012; Arikoglu and Ozko, 2008; Adbel-Halim, 2008; Wazwaz, 2000). We reformulate DTM to solve nonlinear Jaulent-Miodek and the Hirota-Satsuma equation and compare our results with the exact solutions.

The structure of this paper is organized as follows: firstly, we begin with some basic definitions and the use of the proposed method. Later, we apply the differential transformation method to solve Jaulent -Miodek(JM) equation and the Hirota-Satsuma equation in order to show its ability and efficiency.

DESCRIPTION OF THE METHODS

The differential transform method (DTM)

The differential transformation of the kth-order derivative of function $u(x)$ is defined as follows:

$$U(k) = \frac{1}{k!}\left[\frac{d^k u(x)}{dx^k}\right]_{x=x_0} \quad (1)$$

Where $u(x)$ is the original function, $U(k)$ is the transformed function and $\frac{d^k}{dx^k}$ is the kth derivative with respect to x .The differential inverse transform of $U(k)$ is defined as:

$$u(x) = \sum_{k=0}^{\infty} U(k)\,(x - x_0)^k, \quad (2)$$

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Combining Equations 1 and 2 we obtain

$$u(x)=\sum_{k=0}^{\infty}\frac{1}{k!}\left[\frac{d^k u(x)}{dx^k}\right](x-x_0)^k \quad \text{at } x=x_0 \qquad (3)$$

The following theorems that can be deduced from Equations 1 and 2 are given (Rostam et al., 2011; Kanth and Aruna, 2008; Raslan and Zain, 2013; Raslan et al., 2012).

Theorem 1: If the original function is $u(x)=w(x)\pm v(x)$, then the transformed function is $U_k(x)=W_k(x)\pm V_k(x)$.

Theorem 2: If the original function is $u(x)=a\,v(x)$. then the transformed function is $U_k(x)=aV_k(x)$.

Theorem 3: If the original function is $u(x)=\frac{\partial^m w(x)}{\partial x^m}$. then the transformed function is $U_k(x)=\frac{(k+m)!}{k!}W_k(x)$.

Theorem 4: If the original function is $u(x)=\frac{\partial}{\partial x}w(x)$, then the transformed function is. $U_k(x)=\frac{\partial}{\partial x}W_k(x)$

Theorem 5: If the original function is $u(x)=x^n$, then the transformed function is

$$U(k)=\delta(k-n)=\begin{cases}1 & k=n\\ 0 & k\neq n\end{cases}.$$

Theorem 6: If the original function is $u(k)=w(x,t)v(x,t)$, then the transformed function is

$$U_k(x)=\sum_{r=0}^{k}W_r(x)V_{k-r}(x)$$

APPLICATIONS

In this section, we propose the extended DTM, for solving Jaulent-Miodek(JM) equation and the Hirota-Satsuma equation. Numerical solutions obtained by the DTM are compared with the exact solutions, and compared with that obtained by Adomian decomposition methods (ADM) methods (Jafar and Mostfa, 2011; Raslan, 2004). The results for some values of the variables are shown in tables and the solutions are presented as plots as well, showing the ability of the method.

Consider the following Jaulent-Miodek equation (Jafar and Mostfa, 2011)

$$\frac{\partial u}{\partial t}+\frac{\partial^3 u}{\partial x^3}+\frac{3}{2}v\frac{\partial^3 u}{\partial x^3}+\frac{9}{2}\frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial x^2}-6u\frac{\partial u}{\partial x}-6uv\frac{\partial v}{\partial x}-\frac{3}{2}\frac{\partial u}{\partial x}v^2=0,$$
$$\frac{\partial v}{\partial t}+\frac{\partial^3 v}{\partial x^3}-6v\frac{\partial u}{\partial x}-6u\frac{\partial v}{\partial x}-\frac{15}{2}\frac{\partial v}{\partial x}v^2=0 \qquad (4)$$

with an initial condition

$$u(x,0)=\frac{1}{4}\alpha-\frac{1}{4}\beta^2-\frac{1}{2}\beta\sqrt{\alpha}\,\mathrm{sech}(\sqrt{\alpha}x)-\frac{3}{4}\alpha\,\mathrm{sech}^2(\sqrt{\alpha}x), \qquad (5)$$
$$v(x,0)=\beta+\sqrt{\alpha}\,\mathrm{sech}(\sqrt{\alpha}x).$$

with the exact solution

$$u(x,t)=\frac{1}{4}(\alpha-\beta^2)-\frac{1}{2}\beta\sqrt{\alpha}\,\mathrm{sech}(\sqrt{\alpha}(x+\gamma t))-\frac{3}{4}\alpha\,\mathrm{sech}^2(\sqrt{\alpha}(x+\gamma t)),$$
$$v(x,t)=\beta+\sqrt{\alpha}\,\mathrm{sech}(\sqrt{\alpha}(x+\gamma t)).$$

Then, by using the basic properties of the reduced differential transformation, we can find the transformed form of Equation 4 as

$$(k+1)U_k=-\frac{\partial^3}{\partial x^3}U_k(x)-\frac{3}{2}\sum_{r=0}^{k}(V_{k-r}\frac{\partial^3 V_k}{\partial x^3}))-\frac{9}{2}\sum_{r=0}^{k}(\frac{\partial V_{k-r}}{\partial x}\frac{\partial^2 V_r}{\partial x^2})$$
$$+6\sum_{r=0}^{k}(U_{k-r}\frac{\partial U_k}{\partial x})+6\sum_{s=0}^{k}\sum_{r=0}^{s}(U_{k-r}V_{k-s}\frac{\partial V_k}{\partial x})+\frac{3}{2}\sum_{s=0}^{k}\sum_{r=0}^{s}(V_{k-r}V_{k-s}\frac{\partial U_k}{\partial x})$$

$$(k+1)V_k=-\frac{\partial^3 V_k}{\partial x^3}+\sum_{r=0}^{k}(V_{k-r}\frac{\partial U_k}{\partial x})+\sum_{r=0}^{k}(U_{k-r}\frac{\partial V_k}{\partial x})$$
$$+\frac{15}{2}\sum_{s=0}^{k}\sum_{r=0}^{s}(V_{k-r}V_{k-s}\frac{\partial V_k}{\partial x}) \qquad (6)$$

Using the initial condition (5), we have

$$U_0=\frac{1}{4}\alpha-\frac{1}{4}\beta^2-\frac{1}{2}\beta\sqrt{\alpha}\,\mathrm{sech}(\sqrt{\alpha}\,x)-\frac{3}{4}\alpha\,\mathrm{sech}^2(\sqrt{\alpha}\,x), \qquad (7)$$
$$V_0=\beta+\sqrt{\alpha}\,\mathrm{sech}(\sqrt{\alpha}\,x).$$

Now, substituting Equation 7 into Equation 6, we obtain the following $U_k(x)$ values successively

$$U_1=\frac{1}{4}\Big(-3\alpha^2\beta\,\mathrm{sech}[x\sqrt{\alpha}]\tanh[x\sqrt{\alpha}]+6\alpha\beta^3\mathrm{sech}[x\sqrt{\alpha}]\tanh[x\sqrt{\alpha}]+3\alpha^{5/2}\mathrm{sech}[x\sqrt{\alpha}]^2\tanh[x\sqrt{\alpha}]$$
$$+18\alpha^{3/2}\beta^2\mathrm{sech}[x\sqrt{\alpha}]^2\tanh[x\sqrt{\alpha}]+4\alpha^2\beta\,\mathrm{sech}[x\sqrt{\alpha}]^3\tanh[x\sqrt{\alpha}]$$
$$+4\alpha^2\beta\,\mathrm{sech}[x\sqrt{\alpha}]\tanh[x\sqrt{\alpha}]^3\Big)$$

$$V_1=\frac{1}{2}(-3\alpha^2\mathrm{Sech}[x\sqrt{\alpha}]\mathrm{Tanh}[x\sqrt{\alpha}]-6\alpha\beta^2\mathrm{Sech}[x\sqrt{\alpha}]\mathrm{Tanh}[x\sqrt{\alpha}]+2\alpha^2\mathrm{Sech}[x\sqrt{\alpha}]^3\mathrm{Tanh}[x\sqrt{\alpha}]$$
$$+2\alpha^2\mathrm{Sech}[x\sqrt{\alpha}]\mathrm{Tanh}[x\sqrt{\alpha}]^3) \qquad (8)$$

Substituting Equation 6 into Equation 3, we obtain the following $U_2(x)$ and $V_2(x)$ values successively. Finally the differential inverse transform of $U_k(x)$ and $V_k(x)$ gives:

$$u_n(x,t)=\sum_{k=0}^{\infty}U_k\,t^k \quad , \; v_n(x,t)=\sum_{k=0}^{\infty}V_k\,t^k \qquad (9)$$

For numerical study, terms approximations have been considered, and the results are presented in Tables 1 to 2 and Figures 1 to 2.

Table 1. The numerical results un(x,t) and vn(x,t) in comparison with the analytical solution u(x,t)and v(x,t), when $\alpha = \beta = 0.01$ [6]for solution of Equation 1.

x	T	Uexact.	Uapprox.	Abs. error	Vexact.	Vapprox.	Abs. error.
0.1	0.1	-0.00552422	-0.00552422	-7.31186×10^{-15}	0.109995	0.109995	4.09811×10^{-14}
0.1	0.15	-0.00552421	-0.00552421	-1.64166×10^{-14}	0.109995	0.109995	9.20652×10^{-14}
0.15	0.1	-0.00552324	-0.00552324	-1.74314×10^{-14}	0.109989	0.109989	1.03195×10^{-13}
0.2	0.3	-0.00552185	-0.00552185	-2.85525×10^{-13}	0.10998	0.10998	1.73363×10^{-12}
0.35	0.25	-0.00551544	-0.00551544	-6.2866×10^{-13}	0.109938	0.109938	3.92535×10^{-12}
0.45	0.45	-0.00550916	-0.00550916	-3.38924×10^{-12}	0.109898	0.109898	2.13474×10^{-11}
0.45	0.7	-0.00550907	-0.00550907	-8.18441×10^{-12}	0.109897	0.109897	5.15872×10^{-11}
0.8	0.8	-0.00547509	-0.00547509	-3.39431×10-11	0.109677	0.109677	2.16775×10^{-10}
0.85	0.95	-0.00546861	-0.00546861	-5.39471×10^{-11}	0.109636	0.109636	3.45017×10^{-10}
0.9	0.9	-0.0054619	-0.0054619	-5.42248×10^{-11}	0.109592	0.109592	3.47153×10^{-10}
1	1	-0.00544719	-0.00544719	-8.23443×10^{-11}	0.109497	0.109497	5.28343×10^{-10}

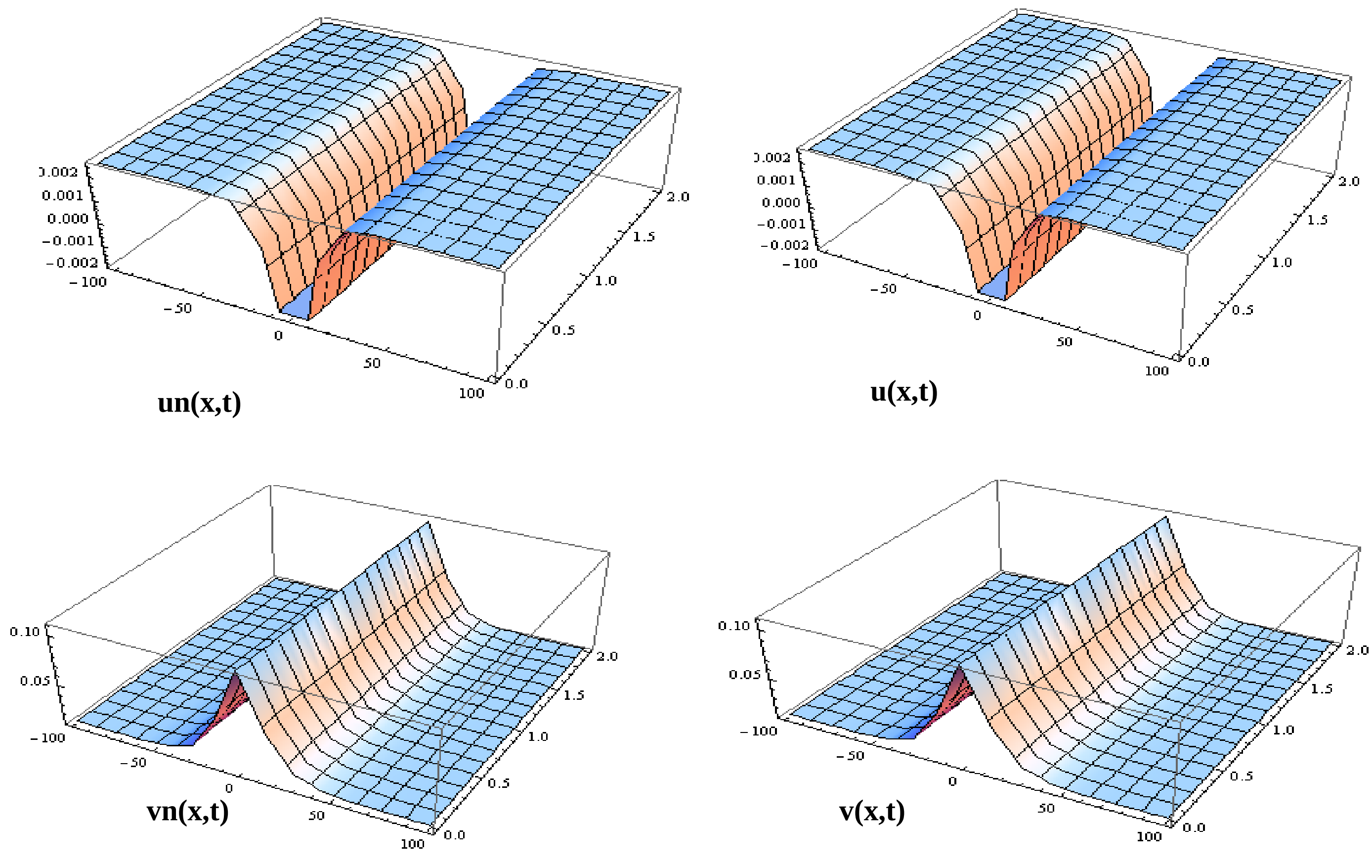

Figure 1. The numerical results for un(x,t) and vn(x,t) in comparison with the analytical solution u(x,t) and v(x,t).

Consider the Hirota-Satsuma equation (Raslan, 2004)

$$\frac{\partial u}{\partial t} = \frac{1}{2}.\frac{\partial^3 u}{\partial x^3} - 3u\frac{\partial u}{\partial x} + 3v\frac{\partial w}{\partial x} + 3w\frac{\partial v}{\partial x},$$

$$\frac{\partial v}{\partial t} = -\frac{\partial^3 v}{\partial x^3} + 3u\frac{\partial v}{\partial x} \tag{10}$$

$$\frac{\partial w}{\partial t} = -\frac{\partial^3 w}{\partial x^3} + 3u\frac{\partial w}{\partial x}$$

Table 2. Comparison between the approximation solutions (DTM) and approximation solutions (ADM) when $\alpha = \beta = \eta = 1, \xi$=0.1 and t =1 (Raslan, 2004).

X	Approximation solutions (DTM),N=4			Approximation solutions (ADM),N=4		
	Uapprox.	Vapprox.	Wapprox.	Uapprox.	Vapprox.	Wapprox.
-50	0.346645	0.0657245	1.99978	0.346645	0.0657245	1.99978
-40	0.346506	0.0656679	1.99836	0.346505	0.0656678	1.99836
-30	0.345492	0.0652524	1.98798	0.345487	0.0652518	1.98796
-20	0.338608	0.0623122	1.91447	0.338603	0.0623089	1.91437
-10	0.310292	0.0462485	1.51288	0.310816	0.0462566	1.51308
0	0.307185	0.0261333	1.01000	0.307183	0.0261307	1.00993
10	0.348145	0.0513519	1.64047	0.348596	0.0513653	1.64080
20	0.348941	0.0634004	1.94168	0.348992	0.0634044	1.94178
30	0.347039	0.0654094	1.9919	0.347044	0.0654099	1.9919 2
40	0.346718	0.0656893	1.9989	0.346719	0.0656894	1.99890
50	0.346674	0.0657274	1.99978	0.346674	0.0657274	1.99985

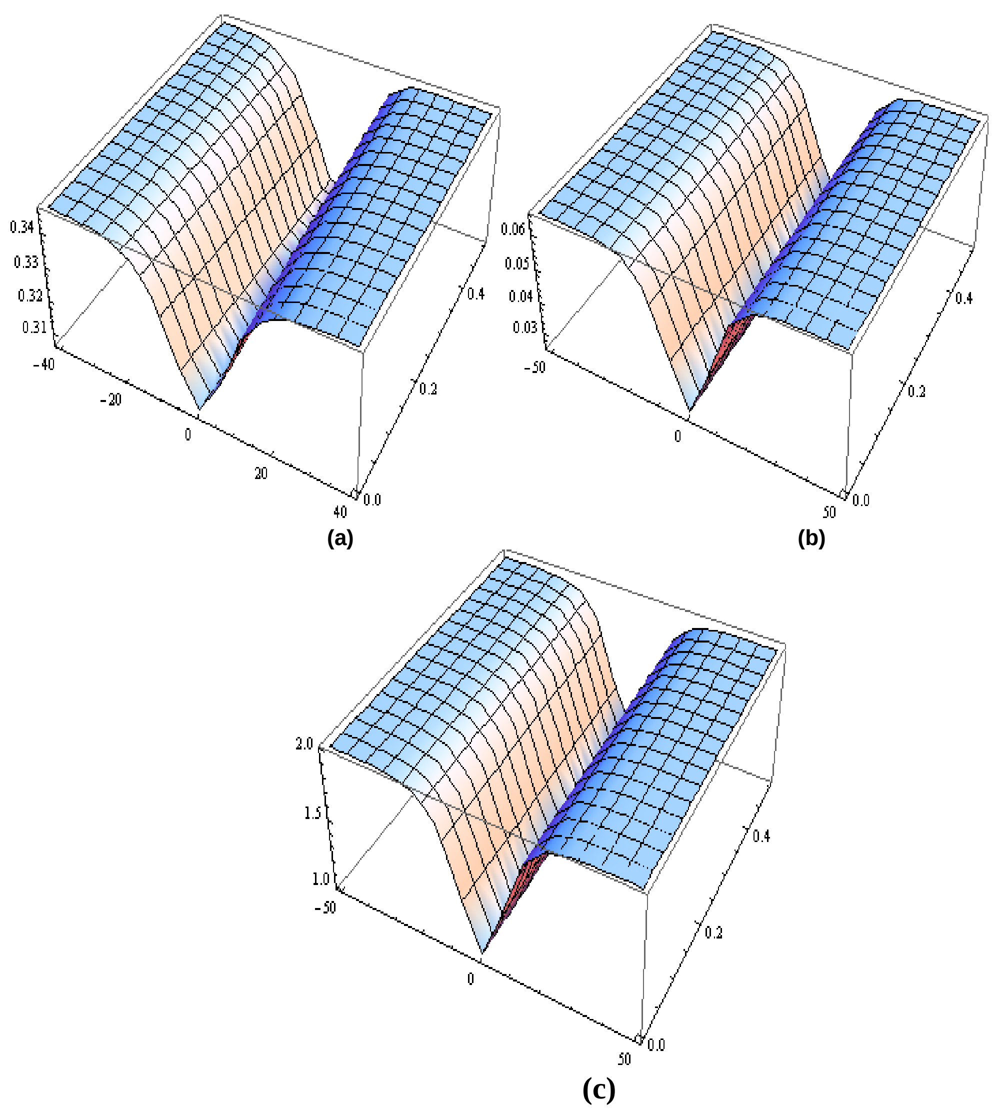

Figure 2. The approximation solutions of un(x,t), vn(x,t) and wn(x

with an initial condition

$$
\begin{aligned}
u(x,0) &= \frac{1}{3}(\beta - 8\xi^2) + 4\xi^2 \tanh^2(\xi x), \\
v(x,0) &= \frac{-4(3\alpha\xi^4 - 2\beta\eta\xi^2 + 4\eta\xi^4)}{3\eta^2} + \frac{4\xi^2}{\eta}\tanh^2(\xi x), \\
w(x,0) &= \alpha + \eta \tanh^2(\xi x),
\end{aligned} \tag{11}
$$

Then, by using the basic properties of the reduced differential transformation, we can find the transformed form of Equation 10 as

$$
(k+1)U_k = \frac{1}{2}\frac{\partial^3}{\partial x^3}U_k(x) - 3\sum_{r=0}^{k}(U_{k-r}\frac{\partial U_r}{\partial x})) + 3\sum_{r=0}^{k}(V_{k-r}\frac{\partial W_k}{\partial x}) + 3\sum_{r=0}^{k}(W_{k-r}\frac{\partial V_k}{\partial x}), \tag{12}
$$

$$
(k+1)V_k = -\frac{\partial^3 V_k}{\partial x^3} + 3\sum_{r=0}^{k}(U_{k-r}\frac{\partial V_k}{\partial x}),
$$

$$
(k+1)W_k = -\frac{\partial^3 W_k}{\partial x^3} + 3\sum_{r=0}^{k}(U_{k-r}\frac{\partial W_k}{\partial x})
$$

Using the initial condition (11), we have

$$
\begin{aligned}
U_0 &= \frac{1}{3}(\beta - 8\xi^2) + 4\xi^2 \tanh^2(\xi x), \\
V_0 &= \frac{-4(3\alpha\xi^4 - 2\beta\eta\xi^2 + 4\eta\xi^4)}{3\eta^2} + \frac{4\xi^2}{\eta}\tanh^2(\xi x) \\
W_0 &= \alpha + \eta \tanh^2(\xi x),
\end{aligned} \tag{13}
$$

Now, substituting Equation 13 into Equation 12, we obtain the following $U_k(x)$ values successively

$$
U_1 = -\frac{1}{\eta}8(-3\alpha\xi^3 \mathrm{Sech}[x\xi]^2 \mathrm{Tanh}[x\xi] - \beta\eta\xi^3 \mathrm{Sech}[x\xi]^2 \mathrm{Tanh}[x\xi] + 3\alpha\xi^5 \mathrm{Sech}[x\xi]^2 \; 4\eta\xi^5 \mathrm{Sech}[x\xi]^2 \mathrm{Tanh}[x\xi] + 4\eta\xi^5 \mathrm{Sech}[x\xi]^4 \mathrm{Tanh}[x\xi] - 6\eta\xi^3 \mathrm{Sech}[x\xi]^2 \mathrm{Tanh}[x\xi]^3 + 10\eta\xi^5 \mathrm{Sech}[x\xi]^2 \mathrm{Tanh}[x\xi]^3)
$$

$$
V_1 = \frac{8(\beta\xi^3 \mathrm{Sech}[x\xi]^2 \mathrm{Tanh}[x\xi] - 8\xi^5 \mathrm{Sech}[x\xi]^2 \mathrm{Tanh}[x\xi] + 8\xi^5 \mathrm{Sech}[x\xi]^4 \mathrm{Tanh}[x\xi] + 8\xi^5 \mathrm{Sech}[x\xi]^2 \mathrm{Tanh}[x\xi]^3)}{\eta} \tag{14}
$$

$$
W_1 = 2(\beta\eta\xi \mathrm{Sech}[x\xi]^2 \mathrm{Tanh}[x\xi] - 8\eta\xi^3 \mathrm{Sech}[x\xi]^2 \mathrm{Tanh}[x\xi] + 8\eta\xi^3 \mathrm{Sech}[x\xi]^4 \mathrm{Tanh}[x\xi] + 8\eta\xi^3 \mathrm{Sech}[x\xi]^2 \mathrm{Tanh}[x\xi]^3)
$$

Substituting Equation 14 into Equation 12, we obtain $U_2(x)$ and $V_2(x)$ values. Finally the differential inverse transform of $U_k(x)$ and $V_k(x)$ gives:

$$
un(x,t) = \sum_{k=0}^{\infty} U_k \; t^k \;\;,\;\; vn(x,t) = \sum_{k=0}^{\infty} V_k \; t^k \;\;,\;\; wn(x,t) = \sum_{k=0}^{\infty} W_k \; t^k \tag{15}
$$

Conclusion

The DTM has been successfully applied for solving the nonlinear Jaulent-Miodek and the Hirota-Satsuma equation. The solutions obtained by DTM are compared with the exact solution. The results show that DTM method is a powerful mathematical tool for solving systems of nonlinear partial differential equations, which appears in mathematical modeling of different phenomena. These models have been solved by homotopy perturbation method and by Adomian's method. DTM method in comparison with ADM and HPM has the advantage of overcoming the difficulty arising in calculating ADM and HPM.

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