

Full Length Research Paper

Face recognition using 3D head scan data based on Procrustes distance

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Recently, face recognition has attracted significant attention from the researchers and scientists in various fields of research, such as biomedical informatics, pattern recognition, vision, etc due its applications in commercially available systems, defense and security purpose. In this paper a practical method for face reorganization utilizing head cross section data based on Procrustes analysis is proposed. This proposed method relies on shape signatures of the contours extracted from face data. The shape signatures are created by calculating the centroid distance of the boundary points, which is a translation and rotation invariant signature. The shape signatures for a selected region of interest (ROI) are used as feature vectors and authentication is done using them. After extracting feature vectors a comparison analysis is performed utilizing Procrustes distance to differentiate their face pattern from each other. The proposed scheme attains an equal error rate (EER) of 4.563% for the 400 head data for 100 subjects. The performance analysis of face recognition was analyzed based on K nearest neighbour classifier. The experimental results presented here verify that the proposed method is considerable effective.

Key words: Face, biometrics, Procrustes distance, equal error rate, k nearest classifier.

INTRODUCTION

Perhaps face is the easiest means of identifying a person by another person. In general humans can identify themselves and others by faces in a scene without hard effort, but face recognition systems that implement these tasks are very challenging to design. The challenges are even extensive when there is a wide range of variation due to imaging situations. Both inter- and intra-subject variations are related with face images. Physical similarity among individuals is responsible for inter-subject variation whereas intra-subject variation is dependent on the following aspects such as age, head pose facial approach, presence of light and presence of other objects/people etc. However, in face recognition, it has been observed that inter-person variations are available due to variations in local geometric features. Automatic face recognition has been widely studied during the last few decades. It is an active research area spanning many disciplines such as image processing, pattern recognition,

computer vision, neural networks, artificial intelligence, and biometrics.

Many researchers from these different disciplines work toward the goal of endowing machines or computers with the ability to recognize human faces as we human beings do, effortlessly, in our everyday life (Brunelli and Poggio, 1993; Samaria, 1994; Wiskott et al., 1997; Turk and Pentland, 1991; Belhumeur et al., 1997; He et al., 2005; Wiskott et al., 1997; Lanitis et al., 1995; Cootes et al., 2001; Brunelli and Poggio, 1993; Turk, 1991; Bellhumer et al., 1997). Face recognition has a wide range of potential applications for commercial, security, and forensic purposes. These applications include automated crowd surveillance, access control, mug shot identification (e.g., for issuing driver licenses), credit card authorization, ATM machine access control, design of human computer interfaces, etc. The rapid evaluation in face recognition research can be found by the progress of systematic evaluation standards that includes the FERET, FRVT 2000, FRVT 2002, and XM2VTS protocols, and many existing software packages for example FaceIt, FaceVACS, FaceSnap Recorder,

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Cognitec, Eyematic, Viisage, and Identix, etc. Considering face representation according to Brunelli and Poggio (1993) those methods can be divided into two types: geometric feature-based approach (Samaria, 1994; Wiskott et al., 1997; Turk and Pentland, 1991a) and template-based approach (Belhumeur et al., 1997; He et al., 2005; Wiskott et al., 1997).

The first approach utilizes techniques that include elastic bunch graph matching (Lanitis et al., 1995) and active appearance model (AAM) (Cootes et al., 2001; Brunelli and Poggio, 1993) to explicit the geometric features of local features (such as the eyes, mouth and nose). In fact, perfect extraction of these features is difficult to obtain. On the contrary template-based methods use the techniques such as Eigenface (Turk and Pentland, 1991b); Fisher face (Bellhumer et al., 1997) to perform the correlation between the face and some reference templates for face recognition. In general humans, it can identify themselves and others by faces in a scene without hard effort, but the face recognition system that implements these tasks is very challenging to design. The challenges are even extensive when there is a wide range of variation due to imaging situations. Both inter- and intra-subject variations are related with face images. Physical similarity among individuals is responsible for inter-subject variation whereas intra-subject variation is dependent on the following aspects such as age, head pose facial approach, presence of light and presence of other objects/people etc. However, in face recognition, it has been observed that than inter-person variations are available due to variations in local geometric features. Recognition process in human is a high-level challenging task that uses one-to-many matching technique. It performs a comparison analysis for a specific face image against all of the images in an available database. The one which matches the closest image in the database is identified by the system image. For human and machine recognition of faces number of surveys is available in (Chellappa et al., 1995; Zhao et al. 2003). Previous approaches in the fields of face recognition have demonstrated good recognition performance under some controlled conditions without considering different head pose for variation in facial expressions. However, a practical face recognition system should also work under imaging conditions for example different head pose, variation in expressions and illumination.

In this paper a practical method for face reorganization utilizing head cross section data based on Procrustes analysis is proposed. This proposed method relies on shape signatures of the contours extracted from face data. Thus, the need of registration due to orientation is eliminated. The shape signatures are created by calculating the centroid distance of the boundary points, which is a translation and rotation invariant signature. The shape signatures for a selected region of interest (ROI) are used as feature vectors and authentication is

done using them. After extracting feature vectors a comparison analysis is performed utilizing Procrustes distance to differentiate their face pattern from each other. The proposed scheme attains an Equal Error Rate (EER) of 4.563% for the 399 head data for 100 subjects. One head data from each subject is selected as gallery image and the remaining three are used as probe. The performance analysis of face recognition was analyzed based on K nearest neighbor classifier. The experimental results presented here verify that the proposed method is considerable effective.

POSE-INVARIANT FACE RECOGNITION ALGORITHMS

Previous research for face recognition systems has found that the biggest task is to identify people with a wide range of objects available naturally in our everyday life. One of the most common variations is in variation of face due to expression. Handling face variation, variations is extremely critical in many practical applications. The situation becomes even worse when lighting differences, occlusions and self-shadowing of facial features are present. There are different algorithms for tackling the pose variation problem. So, in order to avoid these kinds of problem we try to utilize the face detection by utilizing 3d head cross section data. In this paper, our procedure for face detection is done by four steps: (1) Head cross section data are extracted from 3d head scanner, (2) The centroid $\bar{C} \equiv (\bar{C}_x, \bar{C}_y)$ of the head cross section is calculated for the entire contour data, (3) Procrustes analysis for finding similarity for face detection. Moreover, later we used K nearest neighbor classifier to analyze our procedure performance for face detection. The invariant features extraction-based approach records some features in a face image that do not vary under pose changes, such as color or geometric invariants (Wiskott et al., 1997; Cootes et al., 2001; Bellhumer et al., 1997; Lades et al., 1993). In this paper, the Procrustes distance-based method is used to learn about similarities for face detection. The model-based procedure utilizes 2D deformable shapes to capture variations in pose whereas the multi view-based approach saves multi view images in the database to solve the pose variation problem or to synthesize new images from a given image in the database. Identification is then done using both given image and synthesized images. The synthesis algorithms generally deal with the available knowledge of faces to detect new views from one view. For face detection purpose a generic 3D model of the human under pose variations (Blanz and Vetter, 1999; Blanz and Vetter, 2003; Blanz et al., 2005). Another method that uses the linear object classes method for face detection (Vetter, 1996; Vetter and Poggio, 1997). This method represents prior face knowledge using 3d head cross

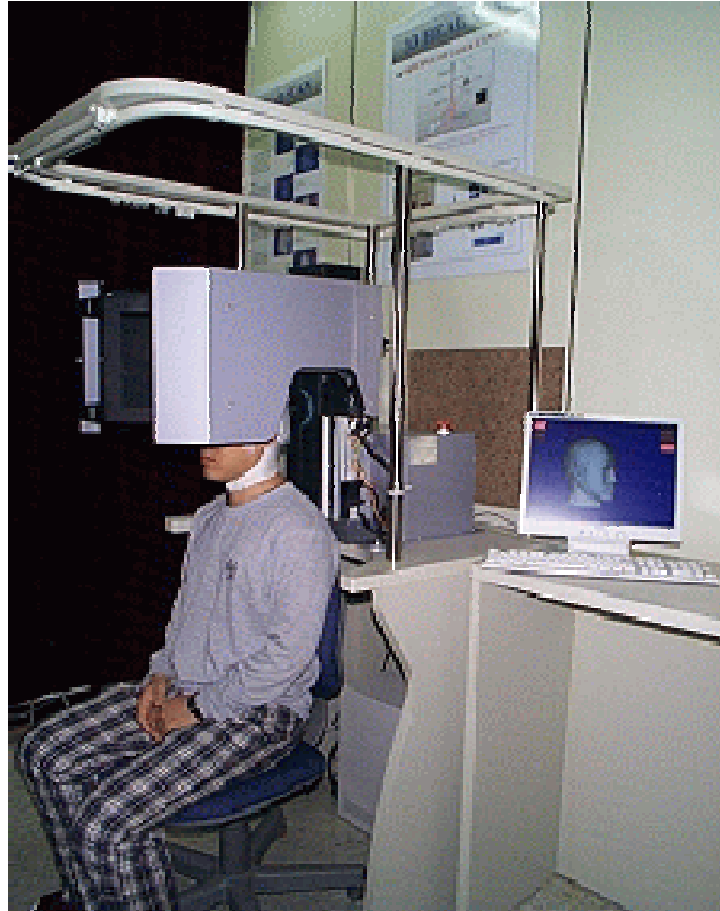


Figure 1. 3D Head scanner.

section data. These multi view-based methods are very popular solution for pose variation problem. As a result face recognition from 3D data head become an important investigation for the researchers, since a face is inherently a 3D object, a good solution would be to use 3d head cross section data. The 3D head cross section shape is invariant to changes in color or reflectance properties due to change in light. As the shape of a face is not affected due to variation in lighting or pose, the 3D face recognition approach has become an important research field. With respect to this idea in this paper we try to utilize 3d head cross section data utilizing Procrustes distance method for face recognition.

FACE SHAPE SIGNATURE ANALYSIS

As the 3D modeling system creates a computerized human model that has the correct body size and properties, for our purpose a 3-D head scanner system (I-Ware Laboratory Co., Ltd.) was used to obtain 3d head cross section data. The starting point of the feature extraction method is the noise elimination embedded in the 3D head (face) surface geometry data. The marker coordinates and information from the marker protocol is used for this purpose. After the post processed head data is obtained, the cross-section

contours along z axis is established. Next, the nose tip is identified to select the region of interest (ROI). The centroid distance shape signature is calculated from the selected contours. To eliminate the effect of data point shift on the contour surface Discrete Fourier Transform is performed and final feature vectors are obtained. The whole feature extraction process is done with Matlab.

Segmentation of head data using marker positions

Figure 1 shows 3D Head Scanner. Head shape was measured while the subjects put on an elastic cap and measurement points were marked. The 3D surface data can be read in '.ply' format which is a triangular surface mesh, constructed from the points on the surface. The surface points are coloured according to the height of the points. The face colour of the triangles is interpolated. From the colour information the marker locations can be identified as the opistocranium (O1), vertex (V2), middle point (M3) from O1 to V2, nuchale (N4), middle point (M5) from N4 to D6 and depression neck (D6). The virtual head model with landmarks is shown in Figure 2. The colour code of marker can be used to separate the head data from the rest of the data.

Selection of region of interest: Nose tip identification

From the points on the surface, cross-sections along z axis is

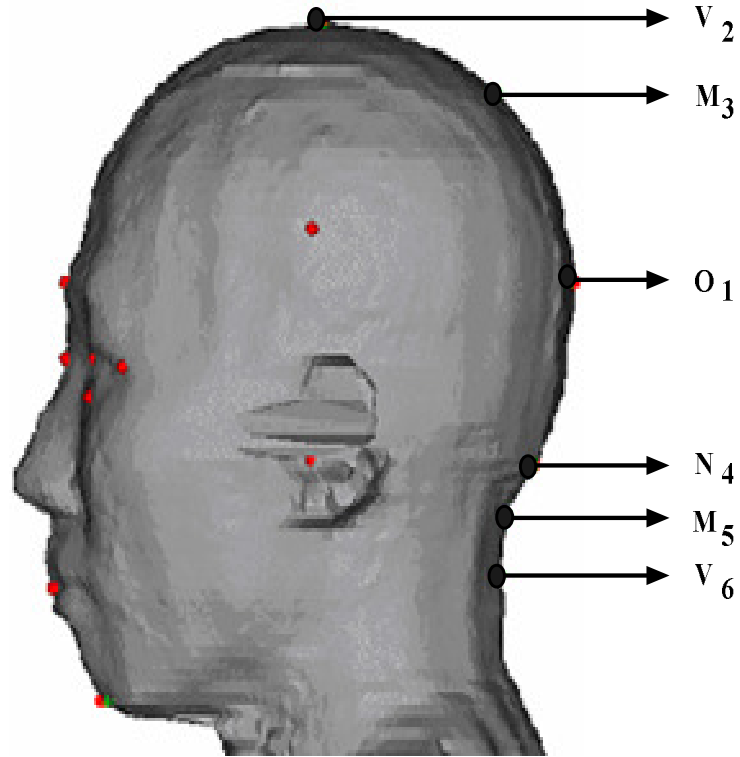


Figure 2. Marker landmarks on 3D face.

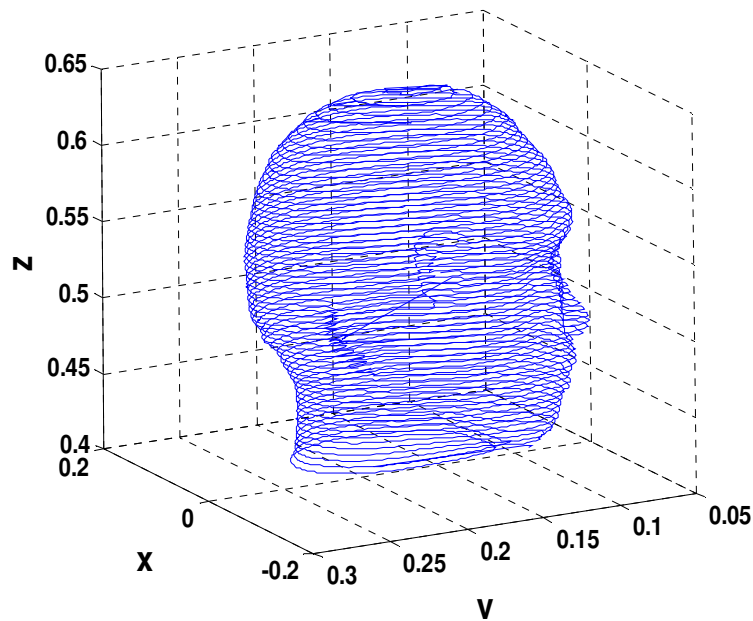


Figure 3. Cross-section (along z axis) contours extracted for figure 2.

segmented. Simple indexing process is used for this purpose. Figure 3 shows the extracted contours for the face shown in Figure 2. The presented method does not need to register data to a standard orientation. But as it works with cross-section contours, it

must be able to do one-to-one correspondence between two data sets. That means a cross-section on the nose region for a gallery image will not match with a cross-section on the eye region of a probe image. To select the correct contours for authentication, a

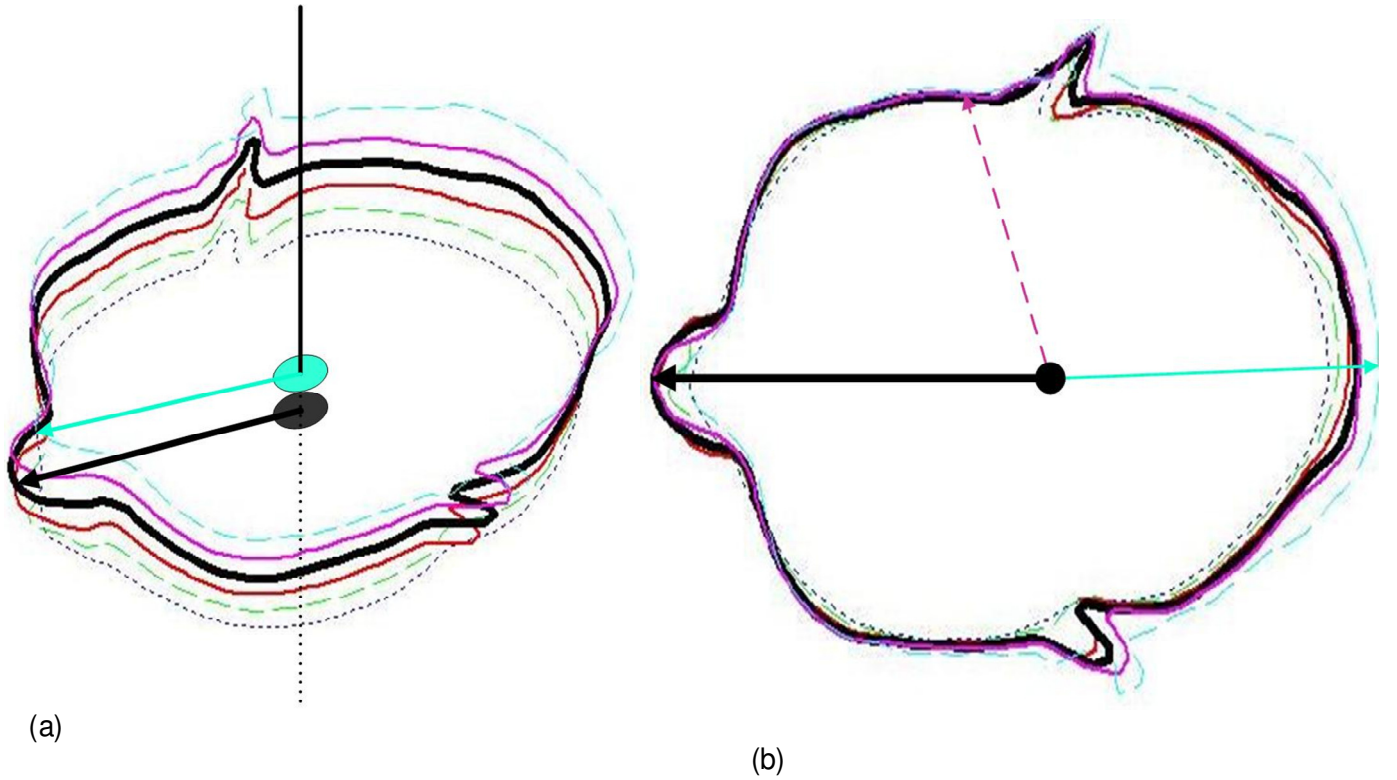


Figure 4. (a) Demonstration of the nose tip identification process: 25° angle view. (b) Demonstration of the nose tip identification process: Top view. (The thick black contour corresponds to the one containing nose tip).

common region of interest (ROI) is selected. Using the daily life knowledge, it can be said that the region between lower part of nose and eyebrows (just under forehead) is probably going to show most discriminate features. In this case the contour containing the nose tip is identified first. And to make things simplified 10 contours above and below that one is selected. Thus, a total of 21 contours are selected for feature extraction.

The identification of nose tip is performed by calculating the centroid $C \equiv (C_x, C_y)$ of each of the available cross-section contours assuming them as closed polygons. Then, the average value of centroid $\bar{C} \equiv (\bar{C}_x, \bar{C}_y)$ is calculated for the entire contour data. A vertical axis through this average centroid point is located and the distance of the boundary points from this average is calculated. The maximum distance corresponds to the nose tip. Figure 4 (a) and (b) demonstrates the idea just described. The term centroid actually means the geometric center of the object's shape, as above, which is its center of mass or the center of gravity, depending on the context. The centroid of a non-overlapping closed polygon defined by N vertices (x_i, y_i) can be calculated in terms of its area A and vertices as below. The area A can be represented by

$$A = \frac{1}{2} \sum_{i=0}^{N-1} (x_i y_{i+1} - x_{i+1} y_i) \quad (1)$$

and its centroid is $C \equiv (C_x, C_y)$ where,

$$C_x = \frac{1}{6A} \sum_{i=0}^{N-1} (x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i) \quad (2)$$

$$C_y = \frac{1}{6A} \sum_{i=0}^{N-1} (y_i + y_{i+1})(x_i y_{i+1} - x_{i+1} y_i) \quad (3)$$

Note that, the polygon has to be closed, that is, the vertex (x_N, y_N) is assumed to be the same as (x_0, y_0) .

Feature extraction

The feature extracted from the contour data is the centroid distance, which has been used in shape retrieval applications. The centroid distance for a contour j is defined as follow

$$r_{ji} = \sqrt{(x_{ji} - C_{jx})^2 + (y_{ji} - C_{jy})^2} \quad j=1,2,3,\dots;21 \quad (4)$$

Where, $C_j \equiv (C_{jx}, C_{jy})$ is the centroid of j^{th} contour and (x_{ji}, y_{ji}) is a point on its boundary. Figure 5 shows some example of calculated centroid distance. Although, this centroid distances can act as feature vectors but further processing is necessary as the number of boundary points may not be equal for

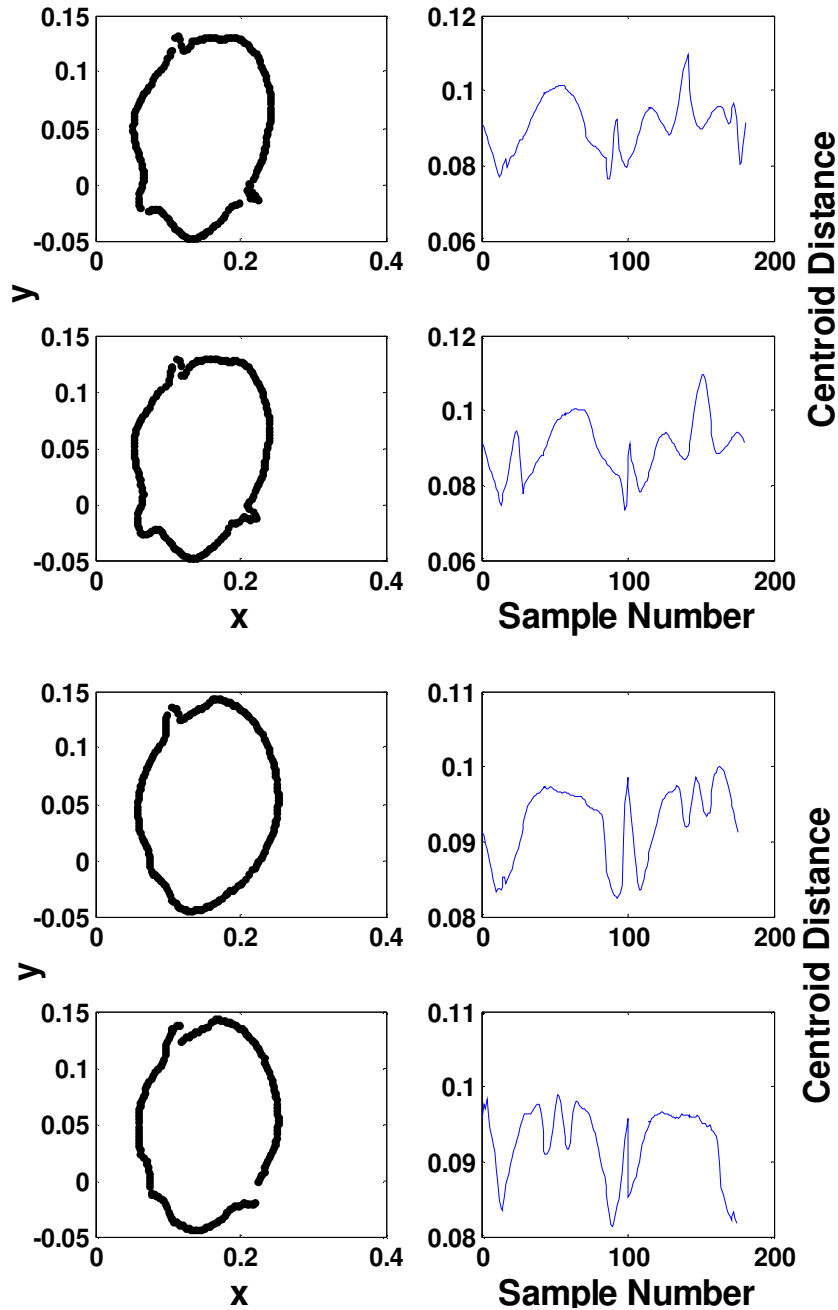


Figure 5. Extracted features for the selected four contours.

gallery and probe images. So, first the centroid distance vectors are interpolated to equally spaced 256 points on the boundary using spline interpolation. Furthermore, magnitude DFT of these vectors are taken to account for any position variation of the boundary points. Thus, the final features are obtained for authentication.

Procrustes analysis

Procrustes shape analysis is a particularly popular method in directional statistics (Kent, 1992; Mardia and Jupp, 2000). It is used for comparing shapes of objects; the word generalized is being

used when there are more than 2 objects involved. The shape of an object is defined in a mathematical context as all the geometrical information that remains after translation, scale and rotational effects are filtered out; that is, an object's shape is invariant under the Euclidean similarity transformations of translation, scaling and rotation. For analysis purposes shape is described by a finite number of points on each object's surface called by landmarks. So, a landmark is a point of correspondence on each object that matches between and within populations.

In this paper Procrustes shape analysis is intended for coping with two-dimensional head cross sections and provides a good method to find similarity between two shapes. A shape in 2D space

Table 1. Relative ‘d’ values for subject 2.

Cross section	d values						
	Subject 1	Subject 3	Subject 4	Subject 5	Subject 6	Subject 7	Subject 8
Nose	0.1399	0.1315	0.1920	0.2004	0.0923	0.1032	0.1860
Jaw	0.1151	0.6301	0.3346	0.1610	0.0271	0.0898	0.3046
Lips	0.0451	0.1256	0.1259	0.0597	0.0979	0.1330	0.2234
Eyes	0.0674	0.0643	0.1164	0.1016	0.0151	0.0711	0.0575
Upper forehead	0.1148	0.0727	0.0917	0.2578	0.2113	0.1662	0.3756
Nose tip	0.0725	0.1621	0.0510	0.0555	0.0271	0.0858	0.0799

can be described by a vector of k complex numbers, $z = [z_1, z_2, \dots, z_k]^T$, called a configuration. For two shapes, z_1 and z_2 , if their configurations are equal through a combination of translation, scaling and rotation, we may consider, they are the same shape.

$$z_1 = \alpha z_k + \beta z_2 \tag{5}$$

Where, $\beta = |\beta|e^{i\angle\beta}$ $\alpha, \beta \in C$.

That is, αz_k translates z_2 , and $|\beta|$ and $\angle\beta$ scale and rotate z_2 . It is very convenient to center shapes through defining the centered configuration $u = [u_1, u_2, \dots, u_k]^T$, $u_i = z_i - \bar{z}$, and $\bar{z} = \sum_{i=1}^k z_i$.

Full generalized Procrustes analysis is a way of finding the similarity transforms which need to be applied to two or more configuration vectors u_i so that they are as close to each other as possible in terms of the Euclidean norm. It may be thought of in 2D as rotating, scaling and translating all the u_i so that the sum of the areas of the smallest circles which enclose each set of corresponding vertices is at a minimum; or more algebraically, as minimizing the sum of the Euclidean distances/norms of each shape from every other shape. It is a generalization of full Ordinary Procrustes Analysis (OPA) which finds the similarity transformations to be applied to one configuration vector u_1 which minimize its Euclidean distance from a second configuration vector u_2 . The minimum Procrustes distance between two configurations can be defined by

$$d(u_1, u_2) = 1 - \frac{|u_1^T u_2|}{\|u_1\|^2 \|u_2\|^2} \tag{6}$$

To measure the similarity between two shapes we can conclude that the smaller d is, the more similar the two shapes are.

EXPERIMENTAL RESULTS AND DISCUSSION

The proposed method uses the 3D head scanner to obtain 3D data for each subject of experiment. The 100 voluntary subjects were chosen from public having different age and sex. The 3D head data were collected with 4 different postures. As mentioned earlier, the cross-sectional (on a horizontal plane) data consisting of X and Y co-ordinate values were treated as a complex

configuration vectors. At this point we have to take necessary measures to make the configuration points equal for all the cross-sections. Several cross-sections were taken for each subject. For each of the cross sections we calculated the Procrustes distance to compare the available data among subjects. Specifically, one subject is compared with others on the basis of ‘d’ for each cross section layer, which will indicate a clear similarity measure for them. Ideally, a sufficiently higher value of ‘d’ indicates high dissimilarity and sufficient lower values will correspond high similarity. In Table 1, a small subset of the whole data range is provided for demonstration. This table contains information about 6 chosen cross-section layers which the authors consider as illustrative for first eight subjects among 100 subjects. They are identified as numeric integer numbers as subject 1, subject 2 and so on. The d values are relative to subject 2. After having got the relative d values for all the subjects, for all available cross-sections we can build up a robust data bank using those d values indexed by each individual subjects and sub-indexed by cross-section layers. In this paper relative d values refer to a subject’s Procrustes distance from all other subjects.

To investigate the proposed 3D face authentication method, the Equal Error Rate (EER) is analyzed with the 399 head data samples after the Procrustes distance analysis. A total of 400 data samples for 100 subjects were collected; one of those samples was discarded due to poor quality. One sample of each subject was taken as gallery data and the rest were used as probe. Biometric verification or authentication can be considered as a detection task, involving a tradeoff between two types of errors. Type 1 error, denoted as False Rejection (FR), false non-match or miss (detection), occurs when a genuine (or authorized) user is rejected by the system. On the other hand, type II error, known as False Acceptance (FA), false match or false alarm, occurs when an impostor is accepted as being a true user. These two types of errors can be traded off against each other by varying the decision threshold. A more secure system aims for low FA at the expense of higher FR, while a more convenient system aims for low FR at the expense of higher FA. Generally, the complete tradeoff curve over many operating points (threshold values) is

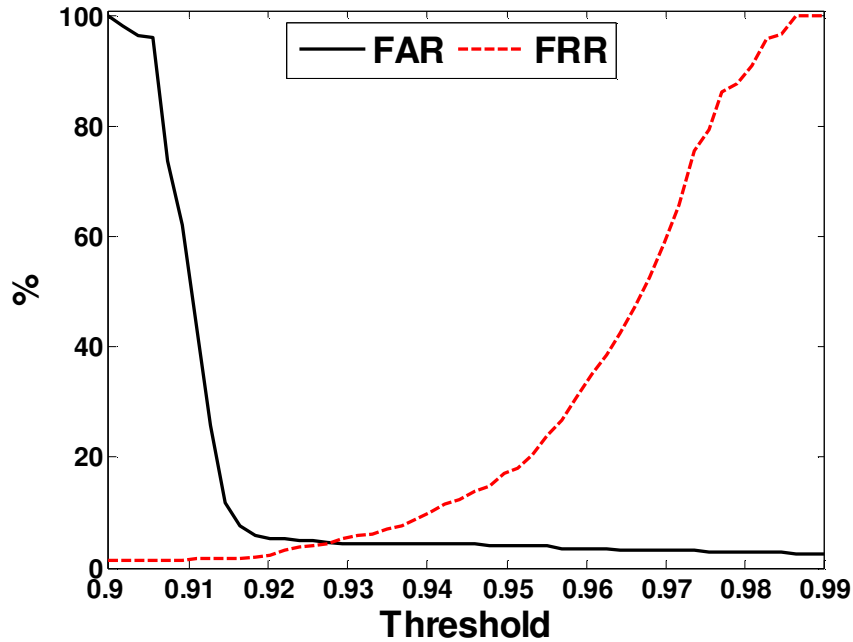


Figure 6. Imposter and genuine distribution for the data set for different thresholds.

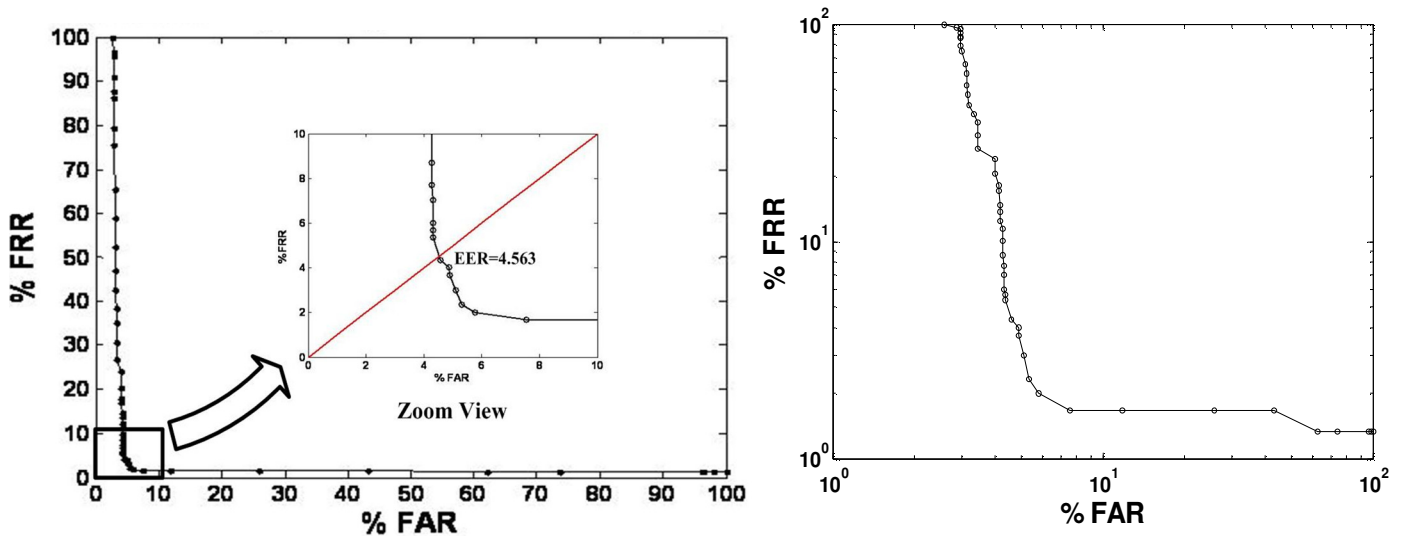


Figure 7. (a) Receiver operating characteristics (ROC), (b) Decision error tradeoff (DET).

often used to characterize biometric performance. ‘False acceptance rate (FAR)’ and ‘false rejection rate (FRR)’ is defined as follows.

$$FAR = \frac{\text{Number of accepted imposters}}{\text{total number of imposters}} \times 100 \% \quad (7)$$

$$FRR = \frac{\text{Number of rejected genuines}}{\text{total number of genuines}} \times 100 \% \quad (8)$$

The distribution of FAR and FRR with different threshold is showed in Figure 6. The FAR rapidly decreases around threshold value 0.91. The Receiver Operating Characteristics is shown in Figure 7a. The Decision Error Tradeoff curve is also shown in 7(b) for better understanding. A zoom view of ROC reveals that the Equal Error Rate (EER), which is the minimum operating value for FAR and FRR, is around 4.563%. The presented method utilizes a data set of 100 individuals and a total data of 399. This is a high number compared to most of the previous studies. Whereas, most of the

Table 2. Comparison between the proposed and other reported methods.

Author	No. of subjects in data set	No. of images / probes	3 D face data	Performance (%)
Hasher et al. (2003)	37	222	RI	97
Lee et al. (2003)	35	70	Feature vector	94 at rank 5
Medioni and Waupotitsch (2003)	100	700	Point set	98
Moreno et al. (2003)	60	420	Feature vector	78
Pan et al. (2003)	30	360	Point set, range image	3-5, 5-7 EER
Lee and Shim (2004)	42	84	Range, curvature	98
Russ et al. (2004)	200 FRGC v1	468	Range I	98
Lu et al. (2004)	18	113	Point set	98
Xu et al. (2004)	120	720	Point set, feature vector	72
Bronstein et al. (2005)	30	220	Point set	100
Chang et al. (2005)	466 FRGC v2	4007	Point set	92
Lee et al. (2005)	100	200	Feature vector	96
Lu and Jain (2005)	100	196	Feature vector	89
Russ et al. (2005)	200 FRGC v2	398	Range image	98.5
Proposed Scheme	100	399	Surface geometry+feature vector	4.563 EER

previous work was for identification, this work is mainly done for authentication. The EER is 4.563% which is impressive considering the size of the data. Authentication based on geometric surface is expected to have lesser error rate, but that was not the case in practice. The reason behind that is the presence of spike and holes in the data. These noises are the prime reason of relatively higher EER. In this work no attempt was made to minimize the effect of such noise. Table 2 contains a summary of results of other reported identification/authentication methods. These information are shown from (Bowyer et al., 2004).

Now we can test our recognition method with various classification methods. There are few of them available like k-nearest neighbours (KNN) classifier and exemplar nearest neighbours (ENN) classifier. The KNN classifier is employed with $k = 1$ in this paper. Using the 'Leaving-one-out-cross-validation' method one example is left from a sample data and the rest is used as training. Then, it is trained with the others to classify. For our case we can take out one index of our data bank and train it with others for each of the sub-indices. The same can be done for other indices and each time we can successfully discriminate them on basis of d .

Further, we can extend our work space for recognition on the basis of previous knowledge. As we have a data bank already set up if we have new data coming along and we can recognize that subject's availability on our data bank by two steps. First, we have to calculate relative d values for that subject for the data bank and secondly we can track the person using the classifier. This promises a higher success rate for recognition. For the classification purpose we used matlab[®] knnclassify subroutine in statistics toolbox.

Conclusion

Face recognition is still a very challenging topic after three decades of research. A large number of face recognition algorithms, along with their modifications are available during over the past decades. In this paper, we proposed a new face recognition method framework using 3d head scan data utilizing Procrustes distance. Based on this framework, we present a new algorithm for recognition tasks. The algorithm applied to face recognition is implemented and experiments are also carried out to evaluate this method. The performance analysis is done using KNN classifier. We have shown that our proposed method can provide an efficient and accurate approach to the problem of face recognition subject to facial expressions, illumination effects and head variations. The proposed method utilizes the cross-section contour data to extract pose invariant features for authentication. An EER of 4.563% has been achieved, which is good for the 399 data set of 100 individuals. Future research challenges will be to analyze performance of such method under different head poses.

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