

Full Length Research Paper

Application of weighted average on modal parameters for damage detection algorithms: Case study on steel beam

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This study verifies the use of the proposed weighting method by Fayyadh and Abdul Razak (2011a) for damage detection algorithms applied on cracked steel beam. The results of the proposed weighting method for two damage severity algorithms are presented. That is, one based on the natural frequencies and the other based on the mode shape vectors. In addition, the stiffness deterioration based on the mathematical calculation of the change in the second moment of inertia is presented. In order to demonstrate the significance and capability of this new method, the magnitude of damage was calculated from an experimental model on a simply supported solid steel beam. Modal parameters were first obtained for the undamaged state by performing modal testing on the un-cracked steel beam. Subsequently, damage was created by cutting the beam section at mid-span from the tension zone to induce different degrees of flexural damage. The crack was created with different depth such as 2, 5, 10 and 20 mm with a constant width of 2 mm. At each level of damage, the Eigen value analysis was repeated to obtain the modal parameters relevant to the degree of damage induced. Stiffness change based on frequency and mode shapes were obtained; the proposed weighting method was applied and compared to the normal averaging method. Dynamic algorithms were compared to the change in the flexural rigidity as based on the change in the second moment of inertia of the beam cross-section at the crack zone. Proposed method returns one value of stiffness deterioration based on the considered set of modes, without affecting the sensitivity of damage detection algorithms. Flexural rigidity results showed higher sensitivity compared to dynamic algorithms.

Key words: Proposed weighting method, modal parameters, modal assurance criteria (MAC), flexural rigidity.

INTRODUCTION

Many engineering structures when exposed to various external loads such as earthquakes, traffic, explosion and vibration during their lifetime, suffer damage and deterioration over the years. Their performance is affected and may lead to consequent structural failure. Hence structural health monitoring of structures for damage detection are of utmost importance in high hazard areas. Tools such as dynamic testing have become increasingly popular and important for this purpose. Damage inspection of structures is important in order to formulate a strategic plan for repair and maintenance works. Vibration test has been used for damage detection since the 1970s and in the offshore oil

industry since early 1980s (Vandiver, 1975; Begg et al., 1976; Coppolino and Rubin, 1980). The basic idea behind this approach is that modal parameters, that is, natural frequency, mode shape and modal damping, are functions of physical properties of structures namely mass, damping, and stiffness. Therefore, any change in the physical properties will result in detectable changes in the modal parameters. Considerable research has been carried out using the change in natural frequencies for damage detection (Salawu, 1997). The alternative to using natural frequency as damage identification is the use of mode shape, with Modal Assurance Criteria (MAC) determining the level of correlation between modes from the control beam and the modes from the damaged beam (Doebeling et al., 1998). MAC was first used by West (1984) to locate structural damage without the prior use of finite element model. Modal parameters of lower

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modes were found to have satisfactory precision in detecting the crack position and depth (Ruotolo and Surace, 1997). The trend in the natural frequencies was found to be sensitive to the corrosion deterioration state of RC beams (Abdul Razak and Choi, 2001). Frequencies were found to be affected by whether the loading configuration was symmetrical or asymmetrical, with odd modes affected more by the symmetrical configuration and the even modes affected more by the asymmetrical configuration. Though the MAC factor was found to be less sensitive than frequencies, it is able to give an indication of the symmetrical or asymmetrical nature of damage (Ndambi et al., 2002). Modal parameters were found to underestimate the damage severity than the actual damage size (Kim and Stunns, 2002). However, modal parameters turned out as good indicators using developed direct stiffness calculation to assess damage in RC structures from experimental natural frequencies, mode shapes and its derivatives (Johan, 2003). The investigation by Douka et al. (2004) found there was a shift in the anti resonances of the cracked beam depending on the location and size of the cracks; this can be used as additional information carrier for crack identification in double cracked beams. The number of required measured frequencies adequate in predicting the location of the multi-cracks cases was found to be equal to twice the number of cracks (Patil and Maiti, 2005). The natural frequencies were found to decrease to a larger extent as the crack size increased, with the change varying based on the mode number (Choubey et al., 2006).

The crack locations and sizes notably influenced the natural frequencies and mode shapes of the cracked beams especially when the cracks are located at the step parts of the beams (Kisaa and Gurelb, 2007). The structure becomes weaker than its previous condition when the crack size increases in the course of time (Orhan, 2007). The effect of the temperature variation on the natural frequencies was evident (Kim et al., 2007). Jassim et al. (2010) found that both modal parameters were affected by the crack regardless of position or size. Lower modes were found to be more sensitive to the change in the support conditions (Fayyadh and Abdul Razak, 2010; Fayyadh et al., 2011a). The slope of the first mode shape was used for damage detection and showed good results; the only concern was that even a small error in identifying the mode shape can result in a significant error in the damage detection results (Zhua et al., 2011). A new damage detection index based on the combination between the mode shape vectors and their curvature was developed and verified to have higher sensitivity than existing algorithms (Fayyadh and Abdul Razak, 2011b). Modifications to two of the existing damage location algorithms were proposed by Fayyadh and Abdul Razak (2011c). A new damage severity algorithm was proposed by Fayyadh et al. (2011b) which based on the combination of both natural frequencies and

mode shape and it was proven to be better sensitive than exist damage severity algorithms. Fundamental mode shapes and static deflection were used for damage detection and were found to have good sensitivity (Cao et al., 2011).

Previous studies have shown that both frequency and mode shape were used for damage detection, which means that any proposed weighting method has to be validated for both parameters. It was also observed that the change in the modal parameter affected by the presence of the damage varied based on the mode number. Moreover, the modal parameters were also found to be affected by whether the loading configuration was symmetrical or asymmetrical. Odd modes were more affected by the symmetrical configuration while the even modes were more affected by the asymmetrical configuration. The need to develop a reliable weighting method which considers the different sensitivities of various modes and which is able to return one stiffness deterioration value was evident, hence, the new weighting method suggested by Fayyadh and Abdul Razak (2011a). This study will verify the use of the proposed weighting method on an experimental model using cracked solid steel beam.

WEIGHTING METHODS

The normal averaging method (NAM) always results in a constant weight for all the modes, which is equal to $1/n$, where 'n' is the total adopted modes. The proposed weighting method by Fayyadh and Abdul Razak (2011a) results in different weights for different modes as based on the area under the curve of the mode shapes. As described in the paper by Fayyadh and Abdul Razak (2011a), and based on the relationship between the work and the bending stiffness as shown in Equation 1, it is suggested that the area under the mode shape curve of each specific mode can be used as a weighting for that specific mode to the change in the bending stiffness EI . It is expected that the more the work that is needed for the formation of the mode shape, the higher the weighting that the mode shape would have.

$$W = U = \frac{\int M^2 dx}{2EI} \quad (1)$$

where 'W' is the external work, 'U' is the strain energy, 'M' is the bending moment, 'I' is the second moment of inertia and 'E' is the elasticity modulus.

The area under the curves of the mode shapes will be calculated for each mode as A_i , where i is the mode number, and the total area (A_t) is the summation of the areas of the adopted modes. The Proposed Weighting (PW_i) of each specific model and the average index value of any adopted set of modes are as describe in Equations 2 and 3. Figure 1 show the area under the curve for the first four bending modes for a case when four modes are adopted.

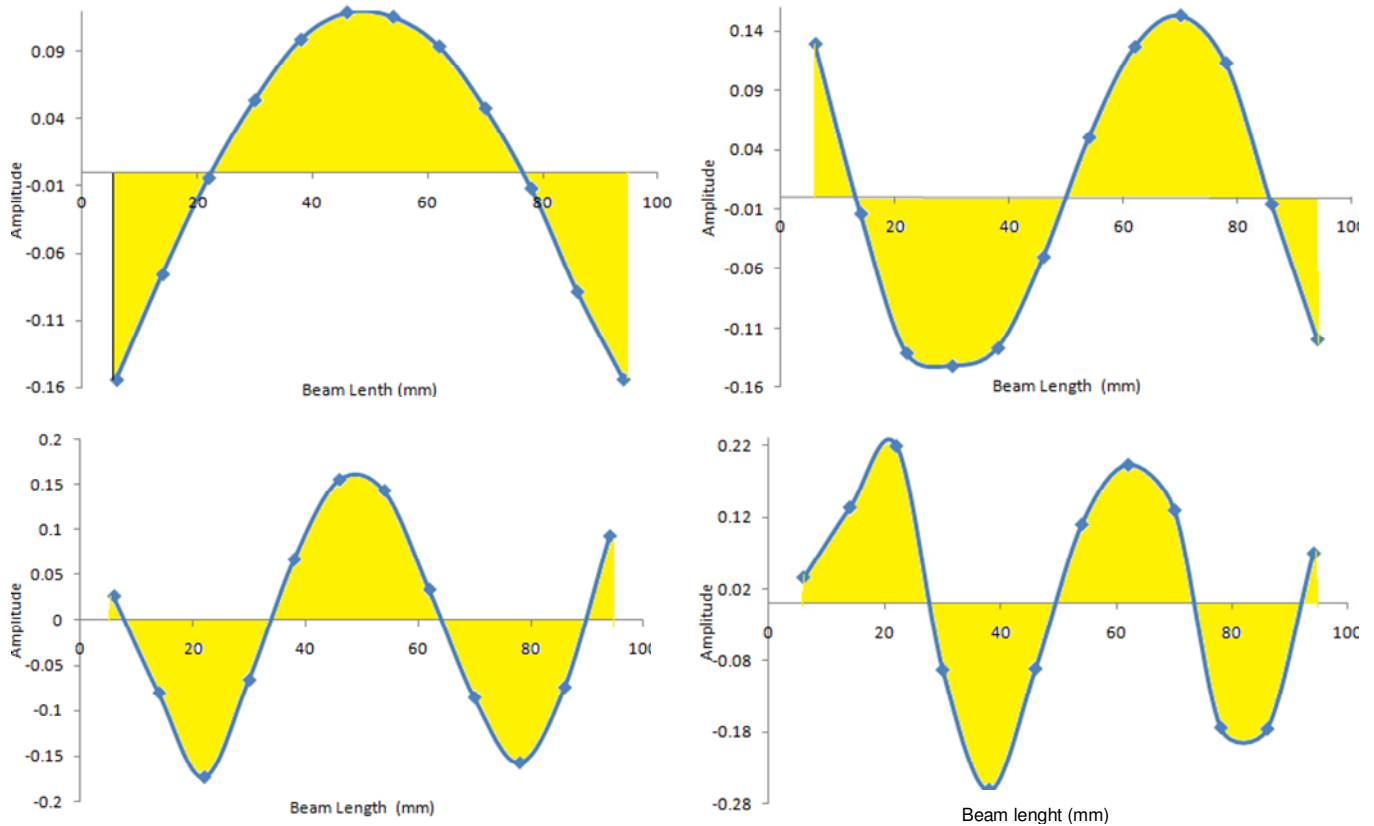


Figure 1. Area under the curve for each specific mode shape.

$$PW_i = \frac{A_i}{A_t} \tag{2}$$

$$Avergae_{DSA} = \sum_{i=1}^n DSA_i * PW_i \tag{3}$$

where *i* is the mode number and *DSA_i* is the damage severity algorithm at mode *i*.

DAMAGE SEVERITY ALGORITHMS

The proposed weighting method will be examined with two damage severity algorithms in order to verify its capability. One of the algorithms is based on the natural frequencies and the other on the mode shapes.

Algorithm based on natural frequency

The natural frequency, *f*, for transverse free vibration of a simply supported beam, as suggested by Demeter (1973) and used by Abdul Razak and Choi (2001) is given by:

$$f = \frac{n^2 \pi}{2} \sqrt{\frac{EI}{mL^4}} \tag{4}$$

where *n* the mode number, *m* the mass per unit length,

and *L* the span length, is proportional to the square root of its flexural rigidity, *EI*. From Equation (4), the following is derived:

$$f \propto \sqrt{EI} \tag{5}$$

Equation (5) can be rewritten as:

$$f_n^2 \propto (EI) \tag{6}$$

By introducing a constant,

$$f_n^2 = A(EI) \tag{7}$$

where

$$A = 2 f_n \frac{\delta f_n}{\delta (EI)} \tag{8}$$

Equation (8) can be rearranged as:

$$2 f_n \delta f_n = A \delta (EI) \tag{9}$$

By substituting Equation (7) into Equation (9), the following relationship is derived:



Figure 2. Test setup for the solid steel beam.

$$2 \left(\frac{1}{f} \right) \partial f = \left(\frac{1}{EI} \right) \partial (EI) \quad (10)$$

which implies that a change in flexural rigidity (EI) doubles the change in natural frequency. The stiffness change based on frequency is defined as:

$$\text{Stiffness change based on frequency} = 2 \cdot \left(1 - \frac{f_{i,d}}{f_{i,c}} \right) \cdot 100 \quad (11)$$

where $f_{i,c}$ and $f_{i,d}$ are the natural frequency at i^{th} mode for the control and damaged beam, respectively.

Algorithm based on mode shape

The method used to ascertain configuration errors between experimental mode shapes and eigen vectors predicted from the finite element model is called Modal Assurance Criterion (MAC) (Ewins, 2000). It is a correlation between experimental mode shapes and curve-fitted mode shapes with the correlation for the i^{th} element given by the following formula:

$$[\text{MAC}_{\tilde{x}x}]_{ii} = \frac{(\{\Phi_i^x\}^T \{\Phi_i^x\})^2}{(\{\Phi_i^x\}^T \{\tilde{\Phi}_i^x\})(\{\tilde{\Phi}_i^x\}^T \{\Phi_i^x\})} \quad (12)$$

where $[\tilde{\Phi}^x] = [\Phi^A] \cdot [C] = [\Phi^A][\Phi^A]^+[\Phi^x]$. Matrix $[\Phi^A]$ contains the analytical model mode shapes, $[\Phi^A]^+$ is the pseudo-inverse of the matrix, and $[C]$ is the curve-fitting

matrix. The values of the diagonal elements of the MAC matrix give the curve-fitting results.

Utilizing the concept in the previous paragraph, the stiffness deterioration indicator can be considered as the reduction in MAC values for the damage cases based on the datum cases such as that shown in Equation 13. The stiffness change based on MAC is defined as:

Stiffness change based on MAC =

$$1 - \frac{|\sum_{i=1}^n \Phi_{i,c} \cdot \Phi_{i,d}|^2}{(\sum_{i=1}^n \Phi_{i,c} \cdot \Phi_{i,c})(\sum_{i=1}^n \Phi_{i,d} \cdot \Phi_{i,d})} \quad (13)$$

and $\Phi_{i,c}$ and $\Phi_{i,d}$ are the mode shapes at i^{th} mode for control and damaged beam, respectively.

MATERIALS AND METHODS

To demonstrate the significance and capability of this new method, experimental work on solid steel beam was prepared. The clear span length of the beam is 1000 mm with cross sectional dimensions of 75 mm by 180 mm (width*depth). Figure 2 shows the solid steel beam during test setup. The physical and material properties of the steel were Poisson's ratio of 0.3, mass density of 7850 kg/m³, Young's modulus of 200,000 MPa and Von Mises yield value of 420 MPa.

MODAL TESTING

Modal testing was carried out on the solid steel beam using the transfer function technique. Accelerometers with a sensitivity of 100 mV/g were used to pick up the response on the tested beam under forced excitation. The beam was randomly excited using an electromagnetic shaker with a white noise signal source from an output channel of the digital signal analyzer via a power amplifier.

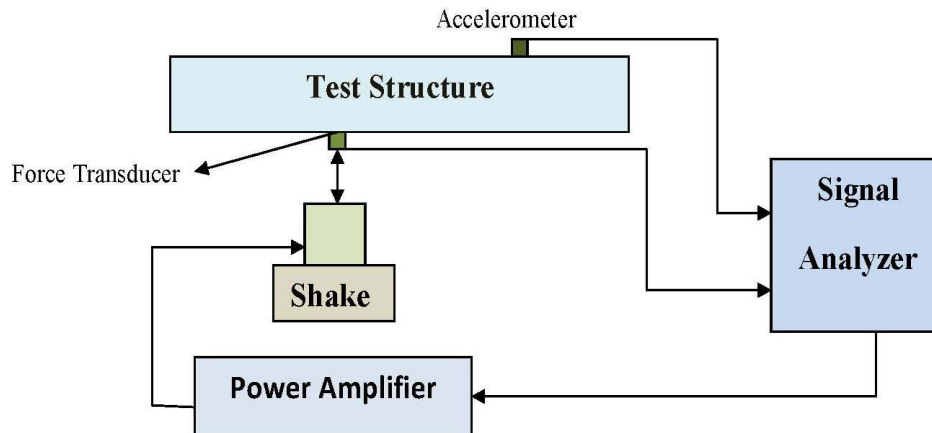


Figure 3. Modal test setup detail.

Table 1. Details of damage levels.

Damage level	Crack depth (mm)	Crack width (mm)	Crack location
Control	-	-	-
DL 1	2	2	Mid-Span
DL 2	5	2	Mid-Span
DL 3	10	2	Mid-Span
DL 4	20	2	Mid-Span

Table 2. Area under the curve and propose weighting for each mode.

Mode no.	Area under the curve of the mode shape	PWM	NAM
Mode 1	5.13	0.17	0.25
Mode 2	6.03	0.2	0.25
Mode 3	8.75	0.29	0.25
Mode 4	10.25	0.34	0.25
Total	30.16 square unit	1.00	1

The vibration was transferred from the shaker to the beam via a plunger with a force transducer having a sensitivity 11.24 mV/N attached to the top to measure the input force. Both the input and output signals were fed to the signal analyzer for real-time data acquisition and for processing in the frequency domain. Figure 3 shows the modal test setup.

Initially, the frequency response function (FRF) spectrum within a 4 kHz frequency span were obtained by executing fast Fourier transform (FFT) on the time domain signals acquired. A total of 12 measurement points on the top surface of the beam were covered by roving the accelerometers while keeping the excitation point permanently fixed. The 12 measuring points were located within regular gaps where the first point was located at 6 cm from the left support and the subsequent points with 8 cm gaps from previous point until the last point (point 12) at 94 cm from the left support and 6 cm from the right support. The transfer functions for all the points are derived by dividing the Fourier transforms of the output (acceleration response) with the input (excitation force). Modal parameter extraction process is done by curve fitting the set of

transfer function measurements in order to obtain the natural frequencies, mode shapes and damping ratios. In this study, the first four flexural modes were identified so as to establish the effect of bond action on the bending action.

Initially, modal analysis was performed so that modal parameters for the control beam could be obtained. Next, the damage was introduced by creating a crack on the tension zone at the mid-span of the beam in order to induce three different degrees of flexural damage. First damage level was by creating a crack with depth of 2 mm. The same procedure was applied to the other damage levels by increasing the crack depth as shown in Table 1. At each damage level, modal testing was repeated to obtain the modal parameters relevant to the degree of damage induced. Subsequently the damage severity algorithms based on frequency and MAC were calculated for the first four bending modes. Lastly, the PW_i for each mode was calculated based on the area under the curve of the mode shapes, and then the average value was calculated for each algorithm. The proposed weightage of each mode are as shown in Table 2 which also presents the modes weightage based on the

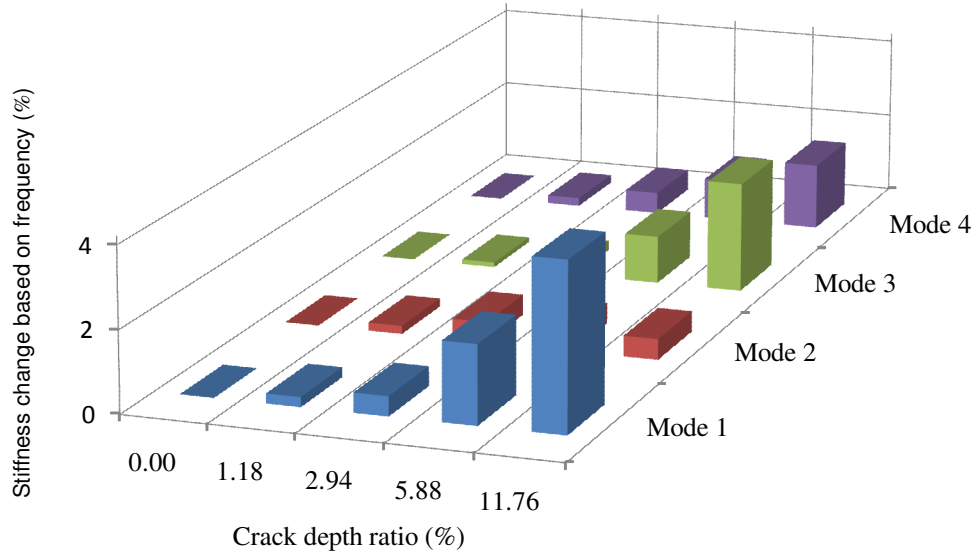


Figure 4. Stiffness change based on frequency for the first four bending modes.

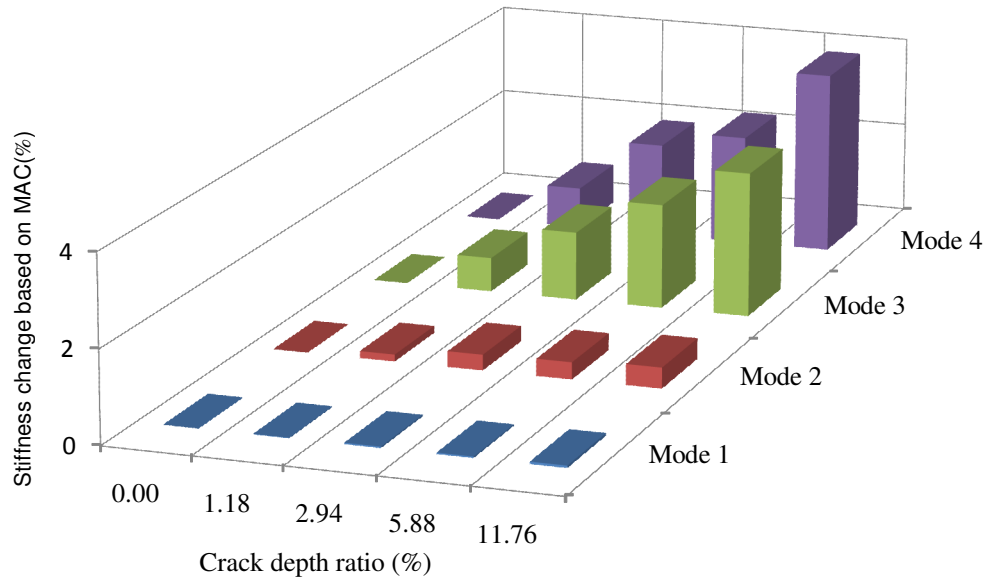


Figure 5. Stiffness change based on MAC for the first four bending modes.

normal averaging method (NAM).

RESULTS AND DISCUSSION

The results from the experimental work carried on the cracked solid steel beam used in present study are presented here. The stiffness change indices based on frequency and MAC were calculated for the first four bending modes of the control-beam model and at four different crack depths of 2, 5, 10 and 20 mm.

Damage severity algorithm

Figures 4 and 5 show the comparison of the stiffness change based on frequency and MAC, respectively. The stiffness was drawn corresponding to the crack depth ratio which was calculated by dividing the crack depth (h_c) by the beam depth (h), which gives four ratios levels as 1.18, 2.94, 5.88 and 11.76%.

The results show that the first and the third modes are the most sensitive to detect damage severity for the stiffness change based on frequency, while modes three

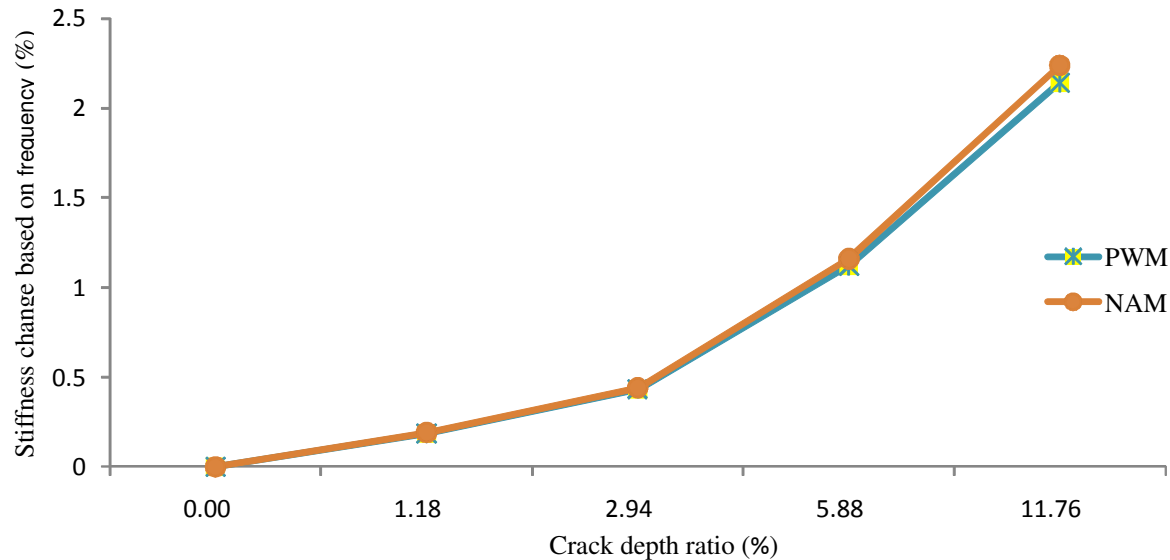


Figure 6. Comparison between PWM and NAM for stiffness change based on frequency.

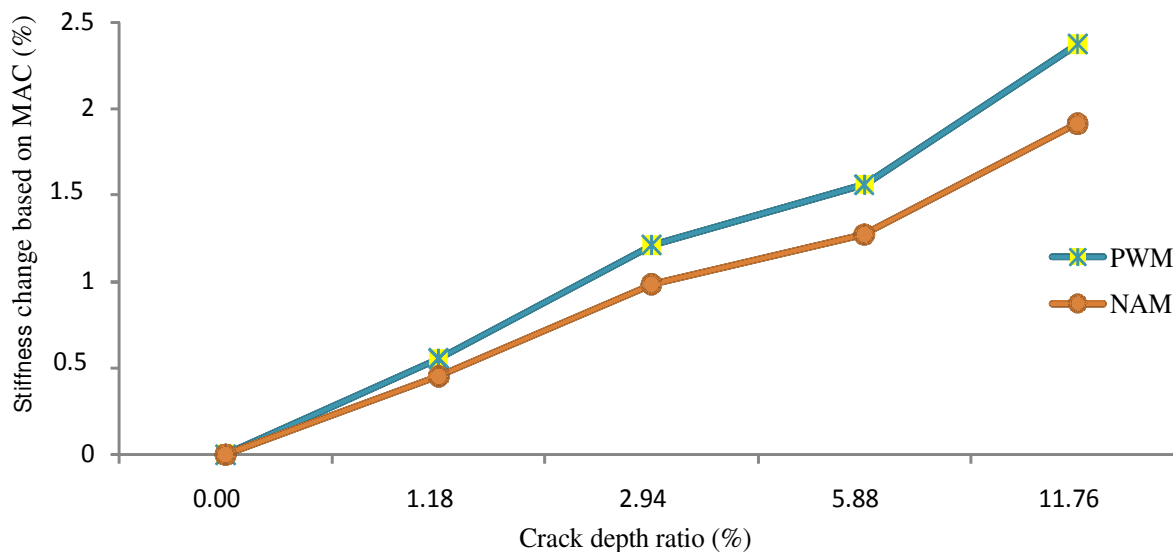


Figure 7. Comparison between PWM and NAM for stiffness change based on MAC.

and four are the most sensitive for the stiffness change based on MAC. The higher values for all the modes were at the highest crack depth ratio of 11.76% for both algorithms. The first mode showed the highest value for stiffness change based on frequency, while mode four had the highest value for stiffness change based on MAC. For stiffness change is based on frequency, the second mode showed very small sensitivity, while for the stiffness change based on the MAC the first mode showed significantly small sensitivity. It is difficult to judge the stiffness deterioration quantitatively based on both modal parameters algorithms because there is a variance

in sensitivity between different modes.

Application of proposed weighting method

In order to come out with one stiffness deterioration value at each crack depth ratio for both algorithms, the proposed weighting method (PWM) is used and is compared with the normal averaging method (NAM). The comparison results are shown in Figures 6 and 7 for stiffness change based on frequency and MAC, respectively.

The results showed that the PWM returns one value at

Table 3. Change in the second moment of inertia corresponding to crack depths.

Crack depth (mm)	Crack depth ratio (%)	Change in I (%)
Datum	0.00	0.00
2	1.18	3.49
5	2.94	8.57
10	5.88	16.63
20	11.76	31.30

each damage level for both algorithms, which makes it easier to derive the stiffness deterioration. The proposed method did not affect the sensitivity of the algorithms, and only helped to average its values for the set of the considered modes in a more reliable method. For stiffness change based on frequency, the PWM showed slightly smaller value than the NAM, where maximum difference was 4% at the higher crack depth ratio of 11.76%. This can be due to the fact that for stiffness change based on frequency, the first mod was the highest sensitive mode and according to PWM its weightage was only 0.17 while according to the NAM its weightage was 0.25. Whilst for stiffness change based on MAC, the PWM showed higher values than NAM for all the crack depth ratios. The higher difference was 24% at the higher crack depth ratio of 11.76%. This can be due to the fact that for stiffness change based on MAC, the fourth mode had the highest sensitivity and its weightage according to PWM was 0.34 while according to NAM it was 0.25. Although the PWM showed slightly smaller value than NAM at higher crack depths for frequency based stiffness change, it is still able to return a single stiffness deterioration value based on the sensitivity of each individual mode in more reliable ways than the NAM, and even higher value for the algorithm based on MAC. Based on the PWM, the sensitivity of mode shape, in term of MAC, is higher than the sensitivity of frequency. For the 11.76% crack depth ratio, there is a change of 2.14% in the stiffness based on frequency and 2.37% in the stiffness based on mode shapes.

Assessment of dynamic algorithms

The relationship between the dynamic and static properties, that is, the natural frequency and the stiffness of the structural elements is expressed in the equation for transverse free vibration of a simply supported Bernoulli-Euler beam given in Equation 14:

$$f = \frac{1}{2L} \sqrt{\frac{EI}{m}}$$

(14)

where f is the natural frequency, n is the mode number, m is the mass per unit length, and L is the span length. Rewriting Equations 14, and replacing the flexural rigidity,

EI with the symbol K and assuming that the mass and length are constant, the expression below is obtained,

$$K \propto \sqrt{f} \tag{15}$$

implying that a change in flexural rigidity causes changes in natural frequency.

For the case of this study and since the only damage induced is the crack at a specific location along the beam length, the only change in the flexural rigidity is the change in the second moment of inertia (I) of the cross section at the crack location. The flexural stiffness (K) change is only related to the change in the second moment of inertia (I).

$$K \propto I \tag{16}$$

The change in the second moment of inertia (I) is calculated by dividing I value of the cracked section (considering the new depth of the beam without the crack depth) by the un-cracked beam depth as expressed in Equation 17.

$$\text{Change in } I = \frac{h_{cr}^3 - h^3}{h^3} \tag{17}$$

where h is the un-cracked beam depth and h_{cr} is the cracked beam depth. For the adopted crack depths of this study, Table 3 shows the change in the second moment of inertia (I) corresponding to adopted crack depths.

In order to compare the damage severity based on the dynamic algorithms with the damage severity based on the change in the flexural rigidity, Figure 8 shows the stiffness change based on the dynamic algorithms, frequency and MAC, and flexural rigidity.

The results showed that both damage severity dynamic algorithms were less sensitivity than the change in the flexural rigidity where at higher crack depth ratio of 11.76%, the flexural rigidity indicated a deterioration of 31.3% while the both dynamic algorithms detected a deterioration of less than 2.5%. This highlights the dynamic algorithms using modal parameters, frequency and mode shape, as underestimating algorithms and there is a need to develop a new algorithm which may be

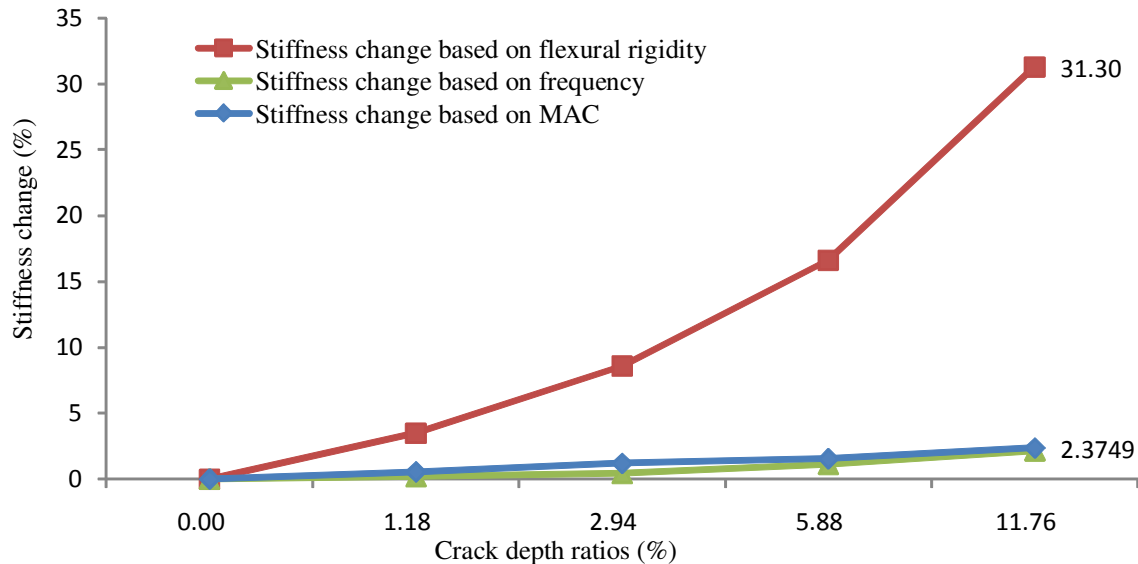


Figure 8. Stiffness change based on frequency, MAC and flexural rigidity corresponding to the adopted crack depth ratios.

based on the mix between both parameters in order to improve the dynamic algorithm's sensitivity.

Conclusions

Based on the results obtained from the experimental modal testing on the solid steel beam that was subjected to deterioration in stiffness by inducing of cracks at certain location with specific depths, the following conclusions are drawn:

1. Different modes have different sensitivity to the deterioration level based on the mode number and the modal parameter used.
2. The simple calculation of the proposed method helped to return one value with regards to the stiffness deterioration based on the adopted set of modes.
3. The proposed method did not affect the sensitivity of the damage algorithms based on the natural frequencies or mode shapes; it helps to apply simple and reliable calculations for each specific mode.
4. The use of the flexural rigidity showed higher sensitivity compared to the dynamic algorithms based on modal parameter.
5. Existing dynamic algorithms based on the modal parameters were found to be under estimating the value.

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