International Journal of Physical Sciences Vol. 7(14), pp. 2229 - 2237, 30 March, 2012
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS11.1695
ISSN 1992 - 1950 ©2012 Academic Journals

Full Length Research Paper

Experiments on hydraulic relations for flow over a compound sharp-crested weir

Jung-Tai Lee[1], Hsun-Chuan Chan[2]*, Chin-Kun Huang[3] and Jan-Mou Leu[3]

[1]Sinotech Engineering Consultants, LTD. Geotechnical Engineering Department, 12F, 171, Nanking E. Rd. Sec.5, Taipei 10570, Taiwan, Republic of China.
[2]Department of Soil and Water Conservation, National Chung Hsing University, 250 Kuo Kuang Rd., Taichung 402, Taiwan, Republic of China.
[3]Department of Hydraulic and Ocean Engineering, National Cheng Kung University, Tainan 70101, Taiwan, Republic of China.

Accepted 6 March, 2012

Basic experiments were conducted in a near full-scale compound sharp-crested weir. The compound sharp-crested weir composed of a trapezoidal weir, sloping crests and a rectangular weir. Detailed velocity measurements were performed for the effects of the compound weir geometry on the flow velocity distributions and the kinetic energy correction factors. Furthermore, a theoretical discharge equation for the proposed compound weir was derived and experimentally validated. The results showed that the kinetic energy correction factors decreased with increasing the head of the weir. The weir behaves as uniform flows passing over the weir under the conditions of the $Y/Y_1 \geq 1.5$. The proposed compound weir provided a good estimation of the discharge without discontinuities. Consequently, an equation for the discharge coefficient was introduced. The experimental data indicated that the discharge coefficient could be simplified in the form of the relative head P/H_1.

Key words: Compound sharp-crested weir, velocity distributions, water discharge.

INTRODUCTION

A weir was designed as an overflow structure perpendicular to an open channel axis to measure the discharge. The weir provides a unique relationship between the upstream head and the discharge. Flow over a weir plunging the air bubbles into a downstream water pool has an impact on the amount of dissolved oxygen in a river system (Unsal, 2010). Therefore, it is often used in water supply system, wastewater system, sewage system, stormwater control system, and hydrologic watershed research. There are mainly two types of weirs: sharp-crested weirs and broad-crested weirs. The sharp-crested weirs are overflow structures whose upstream edge of thickness do not exceeds 2.0 mm and a bevel of angle greater than 45° on the downstream face edge. True sharp-crested weir flow occurs when the overflow nappe is completely free from the downstream face and no clinging flow. In general, sharp-crested weirs are used in hydraulic laboratories, industry and irrigation systems where highly accurate discharge measurements are required. Many researchers have studied the head-discharge relations for flows over sharp-crested weirs with a simple cross-section shape, such as triangular, rectangular, trapezoidal, circular, proportional weirs and others. These researchers have presented some useful empirical discharge equations for these weirs (Chow, 1959; French, 1986; Ranga, 1993; Boiten and Pitlo, 1982; Henderson, 2006). The triangular sharp-crested weir is often used for flow measurement, particularly when accurate measurement of low flow rates is required

*Corresponding author. E-mail: hcchan@nchu.edu.tw. Tel: 886-4-2284-0381/118. Fax: 886-4-22876851.

(Martínez et al., 2005). The rectangular sharp-crested weir is widely accepted at high flow rates, but available discharge formulas have limitations, originated in the minimum head limitations associated with clinging flow, when the nappe clings onto the weir plate (Zhang et al., 2010). The trapezoidal sharp-crested weir is a modification of rectangular sharp-crested weir (Bos, 1976). It can provide increased sensitivity to low flows, while maintaining measurement of high flows due to the outward slope of the weir sides. For trapezoidal weirs, the flow equation is as follows (Subramanya, 2007):

$$Q = \frac{2}{3} C_d \sqrt{2g} Y^{3/2} (L + \frac{4}{5} Y \tan\theta) \qquad (1)$$

where Q = discharge; C_d = discharge coefficient; g = gravitational acceleration; L = bottom width of the cross section; Y = head; θ = side wall angle. However, the measurement accuracy of single sharp-crested weir is abated in areas where flows exhibits great variability, ranging from very low flow in dry conditions to very high flow in extreme rainfall events. In these cases, a common solution is to construct a compound weir, with a small weir designed for low flows set into the base of a larger structure designed for high flows (Grant and Dawson, 1995). Martinez et al. (2005) proposed a compound weir composed with two triangular sharp-crested weirs. The discharge equation for the compound weir was developed by linearly combining the discharge relations of triangular sharp-crested weirs. The equation provided a good estimation of the discharge. When being compared with the single triangular sharp-crested weir, this compound weir might easily handle the higher range of discharges. From the viewpoint of practical engineering, however, much larger flows still require a compound weir whose cross section consists with rectangular and/or trapezoidal parts. Jan et al. (2006) experimentally studied four compound sharp-crested weirs, and confirmed that the linear combination method for developing the discharge equation of compound sharp-crested weirs is reasonable. However, their compound weirs are inconvenient to use. When the discharges begin to exceed the capacity of the lower weirs, thin sheets of water will begin to pass over the wide horizontal crests. The overflows cause discontinuities in the transitions between the two parts of the weirs on the discharge curves (Bergmann, 1963).

In this study, a compound sharp-crested weir, the combination of a trapezoidal weir, sloping crests and a rectangular weir, was experimentally studied. Laboratory experiments were performed in a near full-scale facility. The discharges and velocity distributions over the weir were measured varying the upstream head. The objectives of this study are (1) to propose a weir for a wide range of flows without discontinuities; (2) to experimentally validate the proposed theoretical discharge equation; and (3) to provide new insights into the velocity distributions of flows over a compound sharp-crested weir.

MATERIALS AND METHODS

Compound sharp-crested weir

A schematic diagram of the proposed compound sharp-crested weir is depicted in Figure 1. The cross section of the proposed compound weir consists of three areas, including areas T1, T2 and R1, as shown in Figure 2. Theoretical discharge equations for the compound sharp-crested weir are developed by linearly combining the discharge relations of trapezoidal and rectangular weirs, as shown in the following paragraphs.

If the head (Y) is lower than the height of the lower part of the compound weir (Y_1), it behaves as a single trapezoidal weir with a side wall angle (θ_1). When the head is above the lower part of the weir ($Y > Y_1$), the discharge equation can be obtained by linearly combining the discharge relations of two trapezoidal weirs with side wall angles θ_1 and θ_2, respectively:

$$\begin{aligned} Q = & \frac{2}{3} C_{d1} \sqrt{2g} Y_1^{3/2} (L_1 + \frac{4}{5} Y_1 \tan\theta_1) \\ & + \frac{2}{3} C_{d2} \sqrt{2g} (Y - Y_1)^{3/2} (L_2 + \frac{4}{5} (Y - Y_1) \tan\theta_2) \end{aligned} \qquad (2)$$

Subscripts 1 and 2 denote the trapezoidal weirs with side wall angles θ_1 and θ_2, respectively. In the case where the head occurs higher than the sloping crests, the discharge equation is developed by linearly combining the discharge relations of two trapezoidal weirs and a rectangular weir:

$$\begin{aligned} Q = & \frac{2}{3} C_{d1} \sqrt{2g} Y_1^{3/2} (L_1 + \frac{4}{5} Y_1 \tan\theta_1) \\ & + \frac{2}{3} C_{d2} \sqrt{2g} Y_2^{3/2} (L_2 + \frac{4}{5} Y_2 \tan\theta_2) \\ & + \frac{2}{3} C_{d3} \sqrt{2g} L_r (Y - Y_1 - Y_2)^{3/2} \end{aligned} \qquad (3)$$

where C_{d3}= discharge coefficient for the rectangular weir; L_r= width of the rectangular weir; Y_2 = height of the trapezoidal weir with side wall angle θ_2. The discharge coefficients on the right hand side of Equation 3 can be combined into a single global discharge coefficient (C_d), and the resulting discharge relation is:

$$Q = \frac{2}{3} C_d \sqrt{2g} \left[Y_{m1}^{3/2} (L_1 + \frac{4}{5} Y_1 \tan\theta_1) + Y_{m2}^{3/2} (L_2 + \frac{4}{5} Y_2 \tan\theta_2) + L_r (Y - Y_1 - Y_2)^{3/2} \right] \qquad (4)$$

The purpose of the global discharge coefficient is to provide correction for a number of assumptions, such as energy losses, non-uniformity of velocity distribution and streamline curvature. In the case where the head is above the lower part of the weir, the discharge coefficient is significantly dependent on the non-uniformity of velocity distribution. Moreover, in an open channel, this

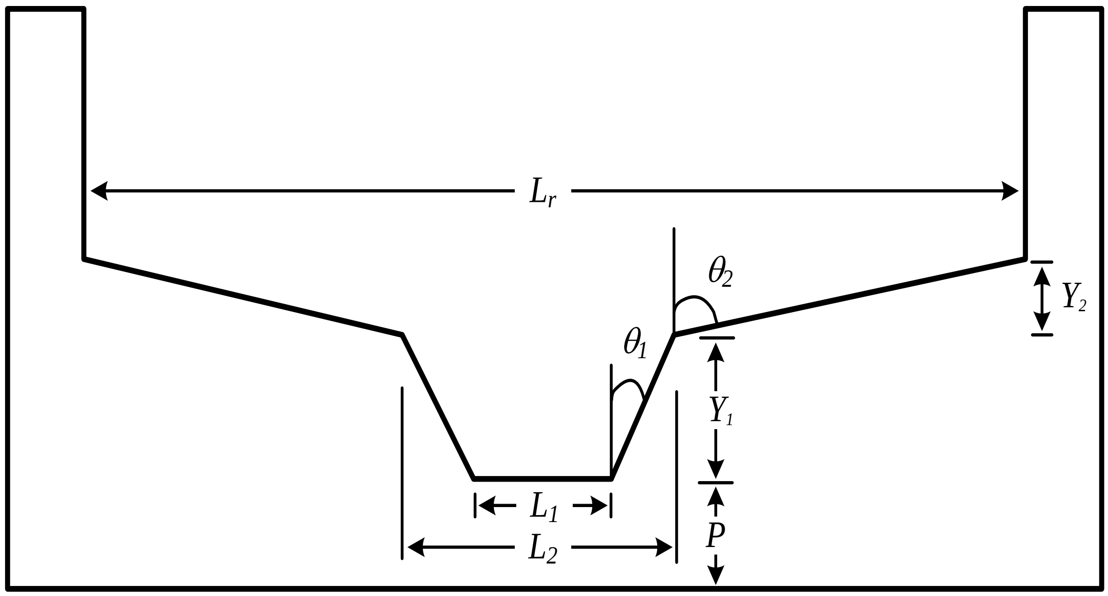

Figure 1. General definition sketch of weir geometries.

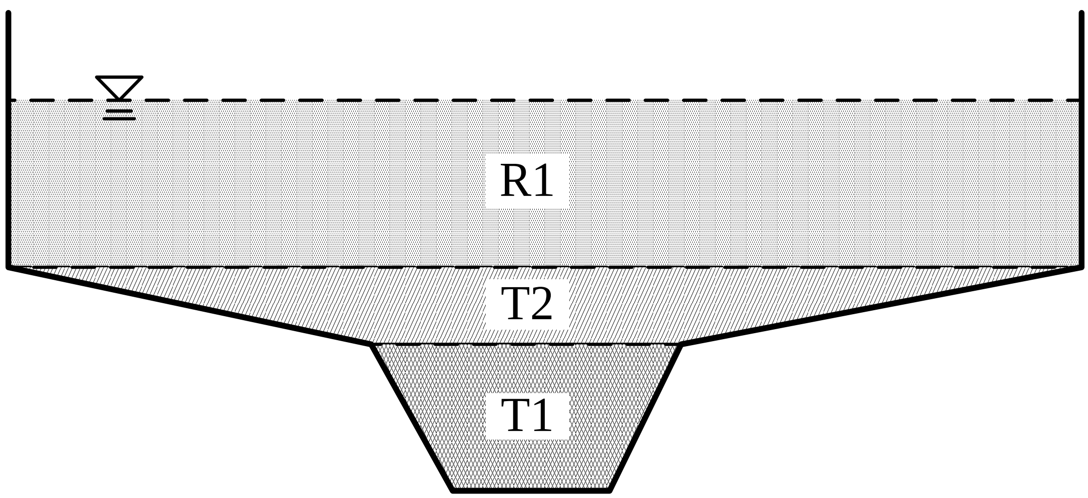

Figure 2. Weir definition sketch to show the sections.

non-uniformity is evaluated by separating the entire cross-section into various sub-sections and estimating the following kinetic energy correction factor (α):

$$\alpha = \frac{\int v^3 da}{V^3 A} = \frac{\sum v^3 \Delta a}{V^3 A} \tag{5}$$

where *V*= mean velocity of the entire cross-section; *A*= area of the entire cross-section; *v* = velocity of the discrete sub-section ; *a* = area of the discrete sub-section. The kinetic energy correction factor is equal to or greater than unity. To the authors' knowledge, only little information on the hydrodynamic properties of the kinetic energy correction factor is available for the compound weir in the previous studies. When the compound channels are straight, prismatic and uniform, the compound channel is geometrically similar to the compound weir in a local cross-section. A number of aforementioned investigations examined the factors that influence the values of the kinetic energy correction factor in compound channels. The values of the kinetic energy correction factor reach very high values in compound channels (Subramanya, 2007). The higher the non-uniformity of velocity distribution, the greater the factor value will be.

Laboratory experiments

Three compound sharp-crested weirs were tested in this study. The Y_1 / Y_2 ratio was used to define the geometry of the compound weirs, and the experiments focused on the influences of Y_1 / Y_2 ratio on the non-uniformity of velocity distribution. Tests were

Table 1. Important characteristics of flow and compound sharp-crested weirs.

Case	*Q* (m^3/s)	Y_1 (cm)	Y_2 (cm)	θ_1	θ_2	*P*(cm)	L_1 (cm)	L_2 (cm)	L_s (cm)	α
1	0.0265	12	2	30°	86.78°	20	15	28.86	100	1.325
2	0.0393	12	2	30°	86.78°	20	15	28.86	100	1.250
3	0.0413	12	2	30°	86.78°	20	15	28.86	100	1.234
4	0.0460	12	2	30°	86.78°	20	15	28.86	100	1.227
5	0.0460	8.6	2	30°	86.78°	23.4	18.4	28.86	100	1.115
6	0.0460	2.6	2	30°	86.78°	29.4	24.4	28.86	100	1.024

conducted for weirs with Y_1/Y_2 equal to 6.0, 4.3 and 1.3, respectively. Moreover, the compound sharp-crested weir with Y_1/Y_2 = 6.0 was tested in order to experimentally determine the values of the discharge coefficient. Table 1 shows a summary of the model characteristics and test conditions. The experiments were conducted at the River Engineering Laboratory in National Cheng Kung University, Taiwan. The flume utilized for this study was 25.72 m long, 1.72 m wide and 0.7 m deep. A constant head supply tank 5 m above the flume was used to supply steady flow under gravity by continuous feed and overflow systems. A standard triangular weir was installed in the upstream flume to measure the discharge. The compound weirs were installed near the end of the flume. All weirs were made of stainless of 3.0 mm thick with the crest thickness of 1.5 mm and the downstream edge beveled 60°. A drop downstream of the compound weir ensured that the tailwater did not affect the flow. The side wall angle (θ_1) was 30° in the first trapezoidal weir and θ_2 was 86.78° in the second one. The height (Y_2) of the trapezoidal weir with side wall angle θ_2 was 2 cm, the width (L_r) of the rectangular weir was 1.0 m and the bottom width (L_2) of the trapezoidal weir with side wall angle (θ_2) was 28.86 cm. For compound weirs with Y_1/Y_2 equal to 6.0, 4.3 and 1.3, the bottom width (L_1) of the trapezoidal weir with side wall angle θ_1 were selected and equal to 15, 18.4 and 24.4 cm, respectively.

The head was obtained by using a point gage with 0.1 mm accuracy to measure the water-surface level upstream from the weir. The streamwise velocity distributions above the weirs were measured by an electro-magnetic micro-velocimeter (Union Engineering, UECM-200A) at a rate of 10 Hz. The velocimeter was attached to a roller carriage on the top railway of the flume. Measurements were conducted at just one half cross-section of the weir to determine the streamwise velocity distributions within the whole weir cross-section. Because the whole cross-section has symmetry in the passing flow, it was referred to as the simplified geometric cross-section. Measurements were conducted at the vertical lines of the one half cross-section. Streamwise velocity records were taken over the entire flow depth at heights separated by 0.5 to 1.0 cm. Each sampling point was instantaneous at the measuring sampling frequency for 3 min. The instantaneous velocities were statistically determined to get the averaged velocities.

RESULTS AND DISCUSSION

Figure 3a to c investigate the effect of the head on the behavior of the contour plots of streamwise velocity. An important consequence of the velocity distribution is the understanding of the dominant region of the discharge capacity. The velocities on the sloping crest are lower than that in the central part of the weir. A region of relatively high velocity exists near the central bottom. The aforementioned velocity distribution is often referred to as a loose gradient when the velocity contours are far apart. Large velocity gradients are present at the interface region, called mixing zone, between the sloping crest and the central part of the weir. They are expected to cause exchanges of mass and momentum. The discontinuities of the compound weir discharge equation are usually associated with the exchanges of mass and momentum. With the increase of the head, the velocities of the mixing zone increase and the gradients of the velocity contour in the mixing zone decrease. For the highest discharge (0.0413 m^3/s), the velocity contours are nearly flat at the interface region and extend along the sloping crest. The effect of the weir geometry on the velocity distribution is investigated by comparing the velocity contours obtained from the experiments with varying Y_1/Y_2 ratio and constant discharge. The distributions of the velocity contour for three different Y_1/Y_2 ratios are plotted as shown in Figure 4a to c. In the region above the sloping crest, the velocities increase as the Y_1/Y_2 ratio decreases. The velocity gradient of the interface region increases as the Y_1/Y_2 ratio increases. In the central bottom part, the high velocity part plays an important role. As the Y_1/Y_2 ratio decreases, this dominant mechanism of the discharge capacity gradually vanishes. Velocity contours of the $Y_1/Y_2 = 1.3$ case show uniform flows existing at central part of the weir and over the sloping crest.

The kinetic energy correction factors (α) are obtained from the detailed calculation of the mean velocities with the head as shown in Figure 5. The head is non-dimensionalised using the height of the lower part of the compound weir (Y_1). The kinetic energy correction factor decreases as the head of the weir increases. These results agree well with the qualitative observation from

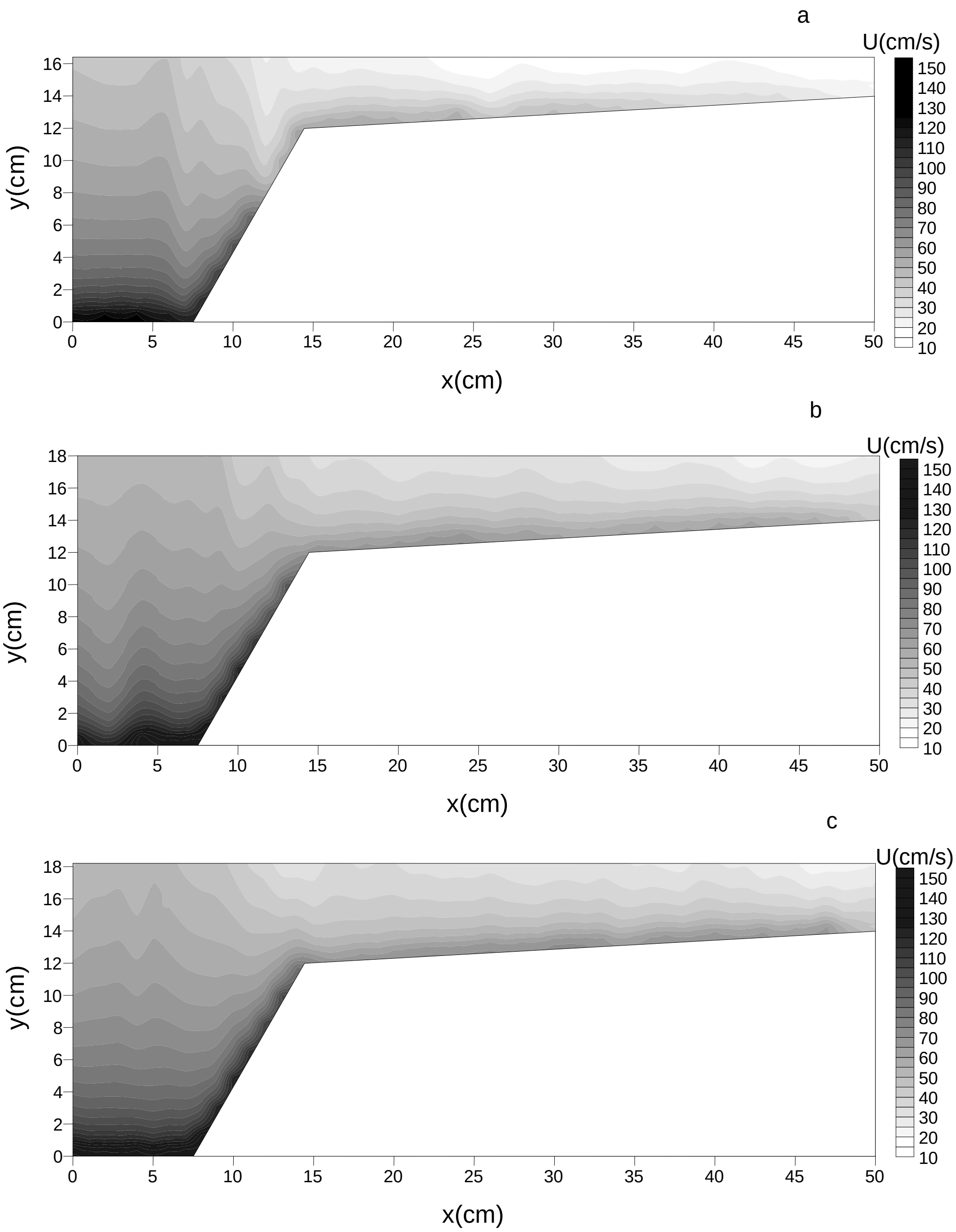

Figure 3. Effect of head on contours of streamwise velocity: (a) case 1; (b) case 2; (c) case 3.

the previous velocity distributions. Examination of the data in Figure 5 shows that for $Y/Y_1 < 1.5$ the non-uniformity of the flow gives rise to a slow growth in α with the increase of the head, and for $Y/Y_1 \geq 1.5$ the weir behaves as uniform flows passing over the weir where α approaches 1.0 quickly. Additional data of a rectangular compound channel with horizontal sloping crests adapted from Subramanya (2007) are also plotted in Figure 5, to provide a reference data set for comparison. While similar trend is observed in the measured data of the compound weir, their values remain higher than that of the compound channel. This difference may be due to the fact that the acceleration of the flow over the weir causes significant vertical variation of the velocities.

The variation of measured discharge with head for the compound sharp-crested weir with Y_1/Y_2 = 6.0 is as shown in Figure 6. The continuity of flow over the compound weirs for variation of head was checked using

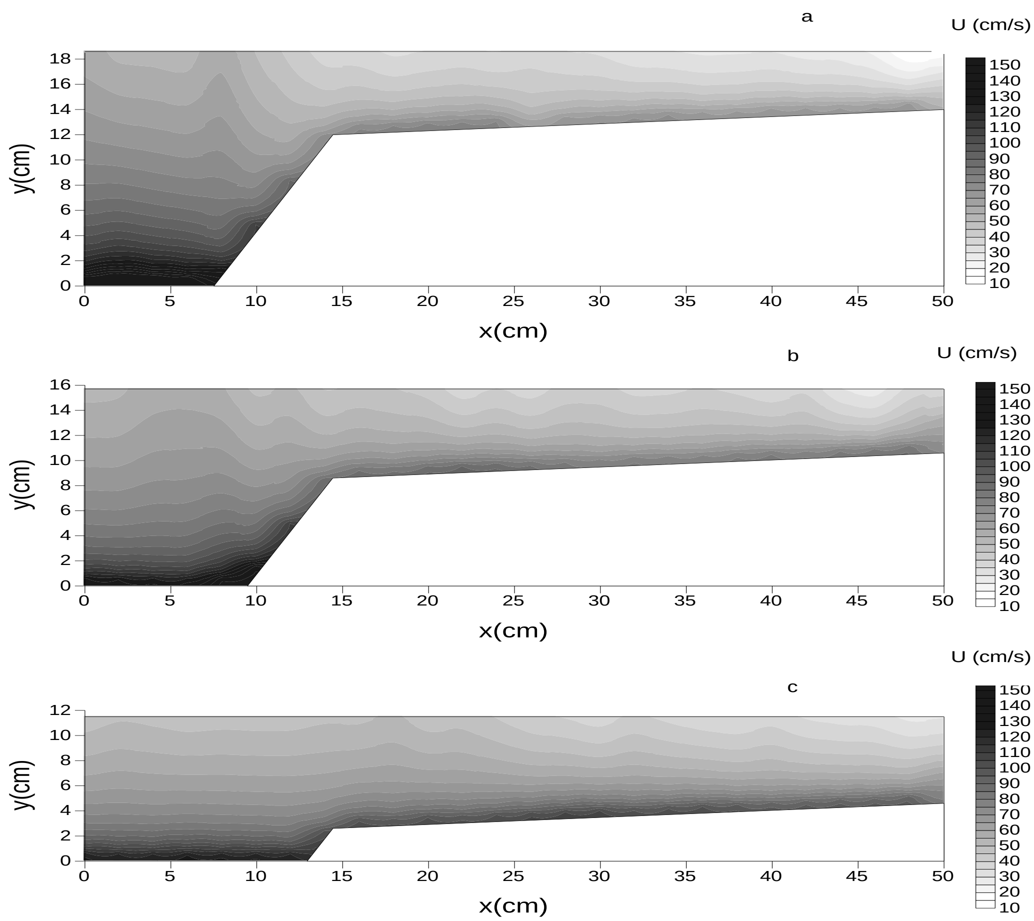

Figure 4. Effect of Y_1 / Y_2 on contours of streamwise velocity: (a) case 4; (b) case 5; (c) case 6.

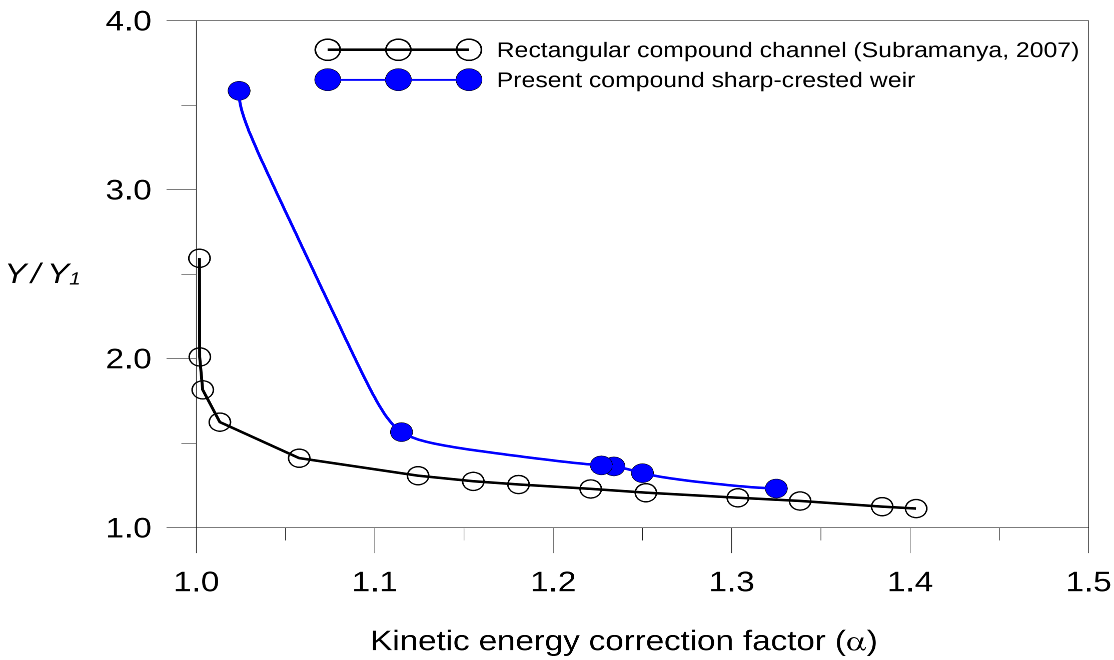

Figure 5. Variation of kinetic energy correction factor (α) with Y / Y_1.

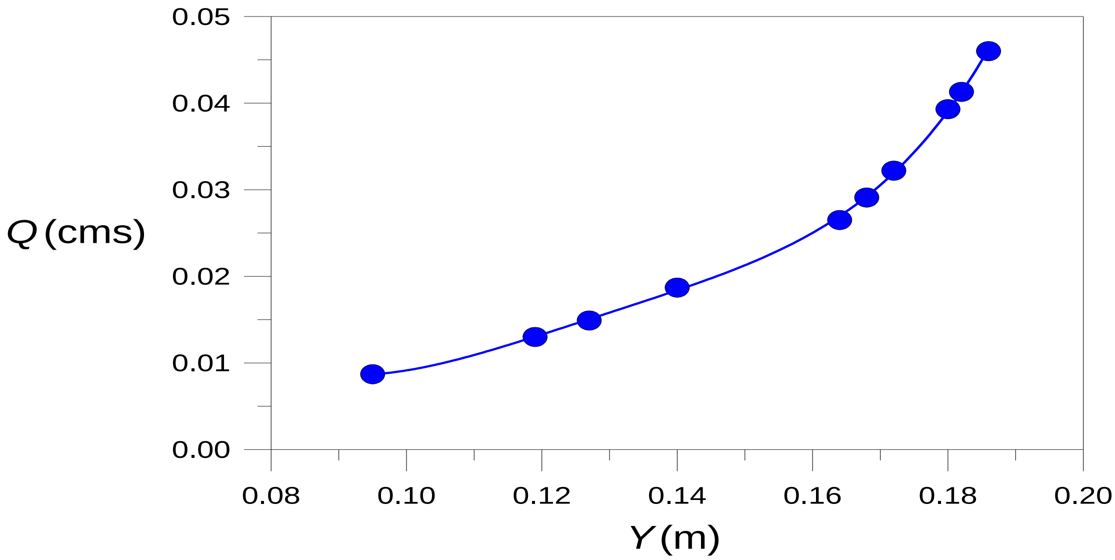

Figure 6. Variation of discharge (Q) with head (Y).

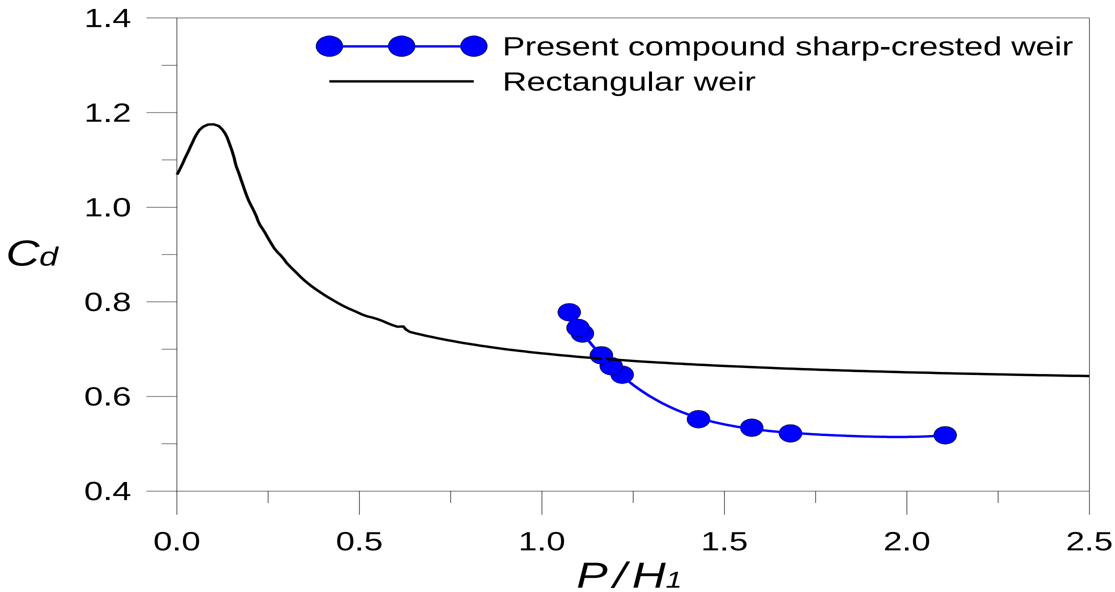

Figure 7. Variation of global discharge coefficient (C_d) with relative head (P/H_1).

the experimental data. An important consequence of the difference in head-discharge relation is the continuity of the present weir as compared to a situation in which the crests of the compound weir are taken as horizontal. The comparison reveals that there is no discontinuity over the whole range of flows. This is not the case with compound weirs having horizontal crests, where the head-discharge relation is not continuous. As a result, when the head is just over the horizontal crests, the flow measurements are found to be inaccurate (Bergmann, 1963; Martínez et al., 2005). Such difficulties have been overcome by using the compound weir with the sloping crests as suggested in this study.

Discharges over the compound sharp-crested weir were measured by using the standard triangular weir. The values of global discharge coefficient (C_d) have been calculated from the experimental flow and head data with Equation 4. The C_d versus head represented by the relative head P/H_1 $(H_1 = Y - P)$ is represented in Figure 7. The data are presented with the dimensionless parameter P/H_1 identified as a variable that strongly affects the discharge coefficient. Figure 7 reveals that C_d increases slightly for $1.25 < P/H_1 < 2.25$ where the head is lower than the sloping crest. The discharge coefficient increases significantly and linearly as P/H_1 increases for $P/H_1 \geq 2.25$. The scatter observed in the data values is attributed to the effect of the lateral contraction. Past researcher plotted the C_d for a rectangular sharp-crested weir as a function of P/H_1 (Rehbock, 1929). The C_d of a rectangular sharp-crested weir are also plotted against P/H_1, as shown in Figure 7. The discharge coefficients of compound sharp-crested weir are found to have much higher values than those of rectangular weir for flows over the sloping crests. In other words, the discharge over compound sharp-crested weir is greater than that over an equivalent rectangular weir. Empirical correlation to predict discharge coefficient C_d is developed for the compound sharp-crested weir with Y_1/Y_2, according to the experimental results. The resulting correlation is given in Equation 6.

$$C_d = 7.83 - 15.75\frac{P}{H_1} + 12.83\left(\frac{P}{H_1}\right)^2 - 4.68\left(\frac{P}{H_1}\right)^3 + 0.64\left(\frac{P}{H_1}\right)^4 \quad (6)$$

To evaluate the accuracy of the estimates produced with nonlinear equations, the correlation coefficient (R) criteria are used. The R coefficient shows the degree to which two variables are linearly related. The correlation coefficient (R) value for Equation 6 is 0.99. Very good agreements are obtained between the measured values and the computed values.

The optimal weir was determined by designer's purpose. The compound sharp-crested weir shows good performance of handling the wild range of flow discharges. However, for the compound sharp-crested weir with horizontal crests, the head-discharge relation is directly affected by the discontinuities of the flow. The relation is not accurate for the head just over the horizontal crests. Therefore, continuous measurement of flow is also important for weir construction. Considering these aspects, this study concluded that a compound sharp-crested weir with sloping crests provide accurate measurement for a wide range of flows without discontinuities.

Conclusions

In this study, a compound sharp-crested weir that composed of a rectangular weir and two trapezoidal weirs has been proposed. A series of laboratory experiments were conducted to investigate the effect of the compound weir geometry on the flow velocity distributions and the kinetic energy correction factors. Furthermore, a theoretical discharge equation for this type of weir has been derived and experimentally validated. From the analysis of the experimental results, the following conclusions can be stated:

1) For a given geometry condition (fixed Y_1/Y_2), the velocities of the mixing zone tend to increase, but the velocity gradients of the mixing zone tend to decrease at a increasing head. For the highest discharge (0.0413 m^3/s), the velocity contours are nearly flat at the interface region and extend along the sloping crest.

2) For a given discharge, as the ratio of Y_1/Y_2 decreases, the corresponding velocities increase in the region above the sloping crest. The high velocity of the central bottom part of the lower weir has a considerable effect on discharge capacity. This dominant mechanism of the discharge capacity gradually vanishes as the Y_1/Y_2 ratio decreases.

3) The kinetic energy correction factors (α) decreases with increase of the head. The result shows that the non-uniformity of the flow give rise to a slow growth in α with the increase of the head for $Y/Y_1 < 1.5$ and the weir behaves as uniform flows passing over the weir as α approaches 1.0 quickly for $Y/Y_1 \geq 1.5$.

4) It has been validated experimentally that the theoretical equation provides a good estimation of the discharge. The discharge coefficient data presented can be simplified in the form of the relative head P/H. For practical hydraulic engineers, the proposed compound sharp-crested weir can be selected due to the accurate measurements for a wide range of flow without discontinuity.

List of symbols

A: area of the entire cross-section (cm^2)
a: Area of the discrete sub-section (cm^2)
C_d: Discharge coefficient
C_{d1} : Discharge coefficient for the trapezoidal weir with side wall angle θ_1 (–)
C_{d2} : Discharge coefficient for the trapezoidal weir with side wall angle θ_2 (–)
C_{d3}: Discharge coefficient for the rectangular weir (–)
g: Gravitational acceleration (m^2/s)
H_1: Upstream total head measured above the crest (cm)
L: Bottom width of the cross section (cm)
L_1: Bottom width of the trapezoidal weir with side wall angle θ_1 (cm)
L_2: Bottom width of the trapezoidal weir with side wall angle θ_2 (cm)
L_r: Width of the rectangular weir (cm)
P : Height of weir crest (cm)

Q : Discharge (m^3/s)
V: Mean velocity of the entire cross-section (cm/s)
v : Velocity of the discrete sub-section (cm/s)
Y : FLOW depth (cm)
Y_1 : Height of the trapezoidal weir with side wall angle θ_1 (cm)
Y_2 : Height of the trapezoidal weir with side wall angle θ_2 (cm)
α : Kinetic energy correction factor (–)
Θ: Side wall angle of the trapezoidal weir (°)
θ_1 : Side wall angle of the lower trapezoidal weir (°)
θ_2 : Side wall angle of the upper trapezoidal weir (°)

REFERENCES

Bergmann JM (1963). Compound Weir Study, U. S. Department of the Interior, Bureau of Reclamation, Denver, United States.
Boiten W, Pitlo HR (1982). The V-shape broad-crested weir. J. Irrig. Drain. Div., 108(2): 142–160.
Bos MG (1976). Discharge measurement structures, International Institute for Land Reclamation and Improvement, Wagemingen, The Netherlands.
Chow VT (1959). Open-channel hydraulics, McGraw-Hill, New York.
French RH (1986). Open-channel hydraulics, McGraw-Hill, New York.
Grant DM, Dawson BD (1995). Isco Open Channel Flow Measurement Handbook, 4th Edition, Isco Environmental Division, Lincoln, NE.
Henderson FM (2006). Open channel flow, Macmillan, New York.
Jan CD, Chang CJ, Lee MH (2006). Discussion of 'Design and calibration of compound sharp-crested weir', J. Martinez. J. Hydraul. Eng., 132(8): 868–871.
Martinez J, Reca J, Morillas MT, López JG (2005). Design and calibration of a compound sharp-crested weir. J. Hydraul. Eng., 131(2): 112–116.
Ranga-Raju KG (1993). Flow through open channel, McGraw-Hill, New York.
Rehbock T (1929). Discussion of precise weir measurements by E.W. Schoder and K.B.Turner, Trans ASCE 93: 1143–1162
Subramanya K (2007). Flow in open channels, Tata McGraw-Hill publishing Co., New Delhi.
Unsal M (2010). Modeling of penetration depth of air bubbles entrained by sharp crested weirs using ANFIS. Int. J. Phys. Sci., 5(5): 507–517.
Zhang X, Yuan L, Peng R, Chen Z (2010). Hydraulic Relations for Clinging Flow of Sharp-Crested Weir. J. Hydraul. Eng., 136(6): 385–390.