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Full Length Research Paper

Differential transform method for solving partial differential equations with variable coefficients

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In this paper, we consider the differential transform method (DTM) for finding approximate and exact solutions of some partial differential equations with variable coefficients. The efficiency of the considered method is illustrated by some examples. The results reveal that the proposed method is very effective and simple and can be applied for other linear and nonlinear problems in mathematical physics.

Key words: Differential transform method; partial differential equations variable coefficients.

INTRODUCTION

Up to now, more and more nonlinear equations were presented, which described the motion of the isolated waves, localized in a small part of space, in many fields such as hydrodynamic, plasma physics, nonlinear optic, and others. The investigation of exact solutions of these nonlinear equations is interesting and important. In the past several decades, many authors mainly had paid attention to study solutions of nonlinear equations by using various methods, such as Backlund transformation (Ablowitz and Clarkson, 1991; Coely, 2001), Darboux transformation (Wadati et al., 1975), inverse scattering method (Gardner et al., 1967), Hirota's bilinear method (Hirota, 1971), the tanh method (Malfeit, 1992), the sine-cosine method (Yan, 1996; Yan and Zhang, 2000), the homogeneous balance method (Wang, 1996; Yan and Zhang, 2001), and the Riccati expansion method with constant coefficients (Yan, 2001). Recently, an extended tanh–function method and symbolic computation are suggested in Fan (2001) for solving the new coupled modified KdV equations to obtain four kinds of soliton solutions. This method has some merits in contrast with the tanh-function method. It not only uses a simpler algorithm to produce an algebric system, but also can pick up singular soliton solutions with no extra effort (Fan and Zhang, 1998; Hirota and Satsuma, 1981; Malfliet, 1992; Satsuma and Hirota, 1982; Wu et al., 1999). The numerical solution of Burger's equation is of great importance due to the equation's application in the approximate theory of flow through a shock wave traveling in a viscous fluid (Cole, 1951) and in the Burger's model of turbulence (Burgers, 1948). It is solved analytically for arbitrary initial conditions (Hopf, 1950). Finite element methods have been applied to fluid problems, Galerkin and Petrov-Galerkin finite element methods involving a time-dependent grid (Caldwell et al., 1981; Herbst et al., 1982). Numerical solution using cubic spline global functions were developed in (Rubin and Graves, 1975) to obtain two systems or diagonally dominant equations which are solved to determine the evolution of the system. A collocation solution with cubic spline interpolation functions used to produce three coupled sets of equations for the dependent variable and its two first derivatives (Caldwell and Hinton, 1987). Ali et al (1992) applied finite element methods to the solution of Burger's equation. The finite element approach is applied with collocation method over a constant grid of cubic spline element. Cubic spline had a resulting matrix system which is tri-diagonal and so solved by the Thomas algorithm. Soliman (2000) used the similarity reductions for the partial differential equations to develop

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a scheme for solving the Burger's equation. This scheme is based on similarity reductions of Burger's equations on small sub-domain. The resulting similarity equation is integrated analytically. The analytical solution is then used to approximate the flux vector in Burger's equation. The coupled system is derived by Esipov (1992). It is a simple model of sedimentation or evolution of scaled volume concentrations of two kinds of particles in fluid suspensions or colloids, under the effect of gravity (Nee and Duan, 1998). In this work, we aim to introduce a reliable technique in order to solve partial differential equations with variable coefficients. The technique is called differential transform method (DTM), which is based on Taylor series expansion. But, it differs from the traditional high order Taylor series method by the way of calculating coefficients. The technique and construct an analytical solution is in the form of a polynomial. The concept of differential transform was first introduced by Pukhov (1986), who solved linear and nonlinear initial value problems in electric circuit analysis. Chen and Ho (1999) developed this method for PDEs and obtained closed form series solutions for some linear and nonlinear initial value problems. Recently, Halim (Hassan, 2008) had shown that this method is applicable to a very wide range of PDEs and closed form solutions can be easily obtained. Halim (Hassan, 2008) has also been compared very well with Adomian decomposition method. The aim of this letter is to extend the DTM method proposed by (Pukhov, 1986; Chen and Ho, 1999; Hassan, 2008; Ali and Raslan, 2009) to solve partial differential equations with variable coefficients (Ali and Raslan, 2009). The structure of this paper is organized as follows: First, we begin with some basic definitions and the use of the proposed method, and we then applied the reduced differential transformation method to solve some test examples in order to show its ability and efficiency.

METHODOLOGY

To illustrate the basic idea of the DTM, we considered $u(x,t)$ is analytic and differentiated continuously in the domain of interest, then let

$$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=t_0}, \quad (1)$$

where the spectrum $U_k(x)$ is the transformed function, which is called T-function in brief. The differential inverse transform of $U_k(x)$ is defined as follows:

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)(t-t_0)^k, \quad (2)$$

Combining (1) and (2), it can be obtained that

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=t_0} (t-t_0)^k, \quad (3)$$

when (t_0) are taken as $(t_0 = 0)$ then Equation (3) is expressed as

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=0} t^k, \quad (4)$$

and Equation (2) is shown as

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)\, t^k, \quad (5)$$

In real application, the function $u(x,t)$ by a finite series of Equation (5) can be written as

$$u(x,t) = \sum_{k=0}^{n} U_k(x) t^k, \quad (6)$$

usually, the values of n is decided by convergence of the series coefficients. The following theorems that can be deduced from Equation (3) and Equation (4) are given as:

Theorem 1: If the original function is $u(x,t) = w(x,t) \pm v(x,t)$, then the transformed function is $U_k(x) = W_k(x) \pm V_k(x)$.

Theorem 2: If the original function is $u(x,t) = \alpha\, v(x,t)$, then the transformed function is $U_k(x) = \alpha V_k(x)$.

Theorem 3: If the original function is $u(x,t) = \dfrac{\partial^m w(x,t)}{\partial t^m}$, then the transformed function is $U_k(x) = \dfrac{(k+m)!}{k!} W_k(x)$.

Theorem 4: If the original function is $u(x,t) = \dfrac{\partial w(x,t)}{\partial x}$, then the transformed function is $U_k(x) = \dfrac{\partial}{\partial x} W_k(x)$.

Theorem 5: If the original function is $u(x,y,t) = \dfrac{\partial w(x,y,t)}{\partial y}$, then the transformed function is $U_k(x,y)=\frac{\partial}{\partial x}W_k(x,y)$.

Theorem 6: If the original function is $u(x,y,z,t)=\frac{\partial w(x,y,z,t)}{\partial z}$, then the transformed function is $U_k(x,y,z)=\frac{\partial}{\partial z}W_k(x,y,z)$.

Theorem 7: If the original function is $u(x,t)=x^m t^n$, then the transformed function is $U_k(x)=x^m\,\delta(k-n)$.

Theorem 8: If the original function is $u(x,t)=x^m t^n w(x,t)$, then the transformed function is $U_k(x)=x^m W_{k-n}(x)$.

Theorem 9: If the original function is $u(x,t)=w(x,t)v(x,t)$, then the transformed function is $U_k(x)=\sum_{r=0}^{k}W_r(x)V_{k-r}(x)$.

To illustrate the aforementioned theory, some examples of partial differential equations with variable coefficients are discussed in details and the obtained results are exactly the same which is found by varitional iteration method.

APPLICATIONS

Here, the extended differential transformation method (DTM) is used to find the solutions of the PDEs in one, two and three dimensions with variable coefficients, and compared with that obtained by other methods.

Example 1

Consider the one-dimensional heat equation with variable coefficients in the form

$$u_t(x,t)-\frac{x^2}{2}u_{xx}(x,t)=0, \tag{7}$$

and the initial condition

$$u(x,0)=x^2, \tag{8}$$

where $u=u(x,t)$ is a function of the variables x and t.

The exact solution of this problem is $u(x,t)=x^2 e^t$. Then, by using the basic properties of the reduced differential transformation, we can find the transformed form of Equation (7) as;

$$(k+1)U_k(x)=\frac{x^2}{2}\frac{\partial^2 U_k(x)}{\partial x^2}, \tag{9}$$

using the initial condition, Equation (8), we have

$$U_0(x)=x^2. \tag{10}$$

Now, substituting Equation (10) into (9), we obtain the following $U_k(x)$ values successively

$$U_1(x)=x^2,\ U_2(x)=\frac{x^2}{2},U_3(x)=\frac{x^2}{6},U_4(x)=\frac{x^2}{24},U_5(x)=\frac{x^2}{120}$$
$$U_6(x)=\frac{x^2}{720},\ldots,U_k(x)=\frac{x^2}{k!} \tag{11}$$

Finally the differential inverse transform of $U_k(x)$ gives:

$$u(x,t)=\sum_{k=0}^{\infty}U_k(x)\,t^k=x^2\sum_{k=0}^{\infty}\frac{t^k}{k!}. \tag{12}$$

Then, the exact solution is given as

$$u(x,t)=x^2 e^t \tag{13}$$

Example 2

Consider the two-dimensional heat equation with variable coefficients as

$$u_t(x,y,t)-\frac{y^2}{2}u_{xx}(x,y,t)-\frac{x^2}{2}u_{yy}(x,y,t)=0 \tag{14}$$

where the initial condition is

$$u(x,y,0)=y^2. \tag{15}$$

Taking differential transform of Equation (14) and the initial condition Equation (15) respectively,

$$(k+1)U_k(x,y)=y^2\frac{\partial^2}{\partial x^2}U_k(x,y)+x^2\frac{\partial^2}{\partial y^2}U_k(x,y), \tag{16}$$

using the initial condition, Equation (15), we have

$$U_0(x,y)=y^2. \tag{17}$$

Now, substituting Equation (17) into (16), we obtain the following $U_k(x,y)$ values successively

$$U_1(x,y)=\mathrm{x}^2, U_2(x,y)=\frac{\mathrm{y}^2}{2}, U_3(x,y)=\frac{\mathrm{x}^2}{6}, U_4(x,y)=\frac{\mathrm{y}^2}{24}, U_5(x,y)=\frac{\mathrm{x}^2}{120}$$

$$U_6(x,y)=\frac{\mathrm{y}^2}{720}, U_7(x,y)=\frac{\mathrm{y}^2}{5040}, \ldots, U_k(x,y)=\begin{cases}\dfrac{x^2}{\mathrm{k}!} & k \text{ is odd}\\[2mm] \dfrac{y^2}{\mathrm{k}!} & k \text{ is even}\end{cases} \quad (18)$$

Finally the differential inverse transform of $U_k(x,y)$ gives:

$$u(x,y,t)=\sum_{k=0}^{\infty} U_k(x,y)\, t^k = x^2\sum_{k=0}^{\infty}\frac{t^k}{\mathrm{k}!}+y^2\sum_{k=0}^{\infty}\frac{t^k}{\mathrm{k}!} \quad (19)$$

Then, the exact solution is given by

$$u(x,y,t)=x^2\underbrace{\left(t+\frac{t^3}{3!}+\frac{t^5}{5!}+\ldots+\frac{t^{2n+1}}{(2\mathrm{n}+1)!}\right)}_{\sinh t}+y^2\underbrace{\left(1+\frac{t^2}{2!}+\frac{t^4}{4!}+\ldots+\frac{t^{2n}}{(2\mathrm{n})!}\right)}_{\cosh t}$$

$$= x^2\sinh t \quad + y^2\cosh t \quad (20)$$

which is the exact solution of Equation (14) .

Example 3

Considering three-dimensional heat equation with variable coefficient as

$$u_t-(xyz)^4-\frac{1}{36}(x^2u_{xx}+y^2u_{yy}+z^2u_{zz})=0 \quad , \quad (21)$$

and the initial condition

$$u(x,y,z,0)=0 \quad . \quad (22)$$

Taking differential transform of Equation (21) and the initial condition Equation (22) respectively,

$$(k+1)\; U_{k+1}(x,y,z)-(xyz)^4-\frac{1}{36}\Big(x^2\frac{\partial^2}{\partial x^2}U_k(x,y,z)$$
$$+y^2\frac{\partial^2}{\partial y^2}U_k(x,y,z)+z^2\frac{\partial^2}{\partial z^2}U_k(x,y,z)=0 \quad , (23)$$

Using the initial condition Equation (22), we have

$$U_0(x,y,z,0)=y^2 \quad , \quad (24)$$

Now, substituting Equation (24) into (23), we obtain the following $\mathrm{U_k(x,y,z)}$ values successively

$$U_1(x,y,z)=\mathrm{x}^4\,\mathrm{y}^4\,\mathrm{z}^4, U_2(x,y,z)=\frac{1}{2}\mathrm{x}^4\,\mathrm{y}^4\,\mathrm{z}^4, U_3(x,y,z)=\frac{1}{6}\mathrm{x}^4\,\mathrm{y}^4\,\mathrm{z}^4,$$
$$U_4(x,y,z)=\frac{1}{24}\mathrm{x}^4\,\mathrm{y}^4\,\mathrm{z}^4, U_5(x,y,z)=\frac{1}{120}\mathrm{x}^4\,\mathrm{y}^4\,\mathrm{z}^4, U_6(x,y,z)=\frac{1}{720}\mathrm{x}^4\,\mathrm{y}^4\,\mathrm{z}^4$$
$$, \ldots, U_k(x,y,z)=\frac{1}{\mathrm{k}!}\mathrm{x}^4\,\mathrm{y}^4\,\mathrm{z}^4. \quad (25)$$

Finally the differential inverse transform of $U_k(x,y,z,)$ gives:

$$u(x,y,z,t)=\sum_{k=0}^{\infty}U_k\ t^k=(U_0+U_1t+U_2t^2+..U_kt^k) \quad , (26)$$

Then the exact solution is given by

$$u(x,y,z,t)=x^4\ y^4\ z^4(t+\frac{t^2}{2!}+\frac{t^3}{3!}+\frac{t^4}{4!}+\frac{t^5}{5!}+.....+\frac{t^k}{k!}) , \quad (27)$$

$$u(x,y,z,t)=x^4\ y^4\ z^4(e^t-1) , \quad (28)$$

Example 4

Considering the one-dimensional wave equation with variable coefficient as

$$u_{tt}-\frac{x^2}{2}u_{xx}(x,t) \quad =0 , \quad (29)$$

with an initial condition

$$u(x,0)=x, \quad u_t(x,0)=x^2 , \quad (30)$$

Taking differential transform of Equation (29)

$$(k+1)(k+2)U_k(x)-\frac{x^2}{2}\frac{\partial^2}{\partial x^2}U_k(x)=0 , \quad (31)$$

using the initial condition, Equation (30), we have

$$U_0(x)=x, U_1(x)=x^2 . \quad (32)$$

Now, substituting Equation (32) into (31), we obtain the following $U_k(x)$ values successively

$$U_k(x)=0, k=2,4,6,\cdots\cdots$$

$$U_3(x)=\frac{1}{6}x^2, U_5(x)=\frac{1}{120}x^2, U_7(x)=\frac{1}{5040}x^2,\cdots \quad (33)$$

Finally the differential inverse transform of $U_k(x)$ gives:

$$u(x,t)=\sum_{k=0}^{\infty}U_k\ t^k=x^2(1+t+\frac{t^2}{2}+\cdots\cdots+\frac{t^k}{k!}) , \quad (34)$$

Thus, the exact solution is given in the closed form as

$$u(x,t)=x^2\ e^t . \quad (35)$$

Example 5

Considering the two-dimensional wave equation with variable coefficient as

$$u_{tt}(x,y,t)-\frac{x^2}{12}u_{xx}(x,y,t)-\frac{y^2}{12}u_{yy}(x,y,t)=0 , \quad (36)$$

with the initial condition

$$u(x,y,0)=x^2, u_t(x,y,0)=y^4 , \quad (37)$$

Taking differential transform of Equation (36) and the initial condition Equation (37) respectively,

$$(k+1)U_{k+1}(x,y)-\frac{x^2}{12}\frac{\partial^2}{\partial x^2}U_k(x,y)-\frac{y^2}{12}\frac{\partial^2}{\partial y^2}U_k(x,y)=0 \quad (38)$$

using the initial condition, Equation (37), we have

$$U_0(x,y)=x^2, U_1(x,y)=y^4 . \quad (39)$$

Now, substituting Equation (39) into (38), we obtain the following $U_k(x)$ values successively

$$U_2(x,y)=\frac{1}{2}x^4\ U_3(x,y)=\frac{1}{6}x^4, U_4(x,y)=\frac{1}{24}x^4, U_5(x,y)=\frac{1}{120}y^4, U_6(x,y)=\frac{1}{720}y^4,.... \quad (40)$$

Finally the differential inverse transform of $U_k(x,y)$ gives:

$$u(x,y,t)=\sum_{k=0}^{\infty}U_k(x,y)\ t^k=x^4\sum_{k=0,2,4,}^{\infty}U_k(x,y)\ t^k+y^4\sum_{k=1,3,5,}^{\infty}U_k(x,y)\ t^k \quad (41)$$

$$u(x,y,t)=x^4\left(1+\frac{t^2}{2!}+\frac{t^4}{4!}+\cdots\right)+y^4\left(t+\frac{t^3}{3!}+\frac{t^5}{5!}+\cdots\right)$$

Hence, the exact solution is

$$u(x,y,t)=x^4\cosh(t)+y^4\sinh(t) \quad (42)$$

Example 6

Considering the three-dimensional wave equation with variable coefficient as

$$u_{tt}-(x^2+y^2+z^2)-\frac{1}{2}(x^2u_{xx}+y^2u_{yy}+z^2u_{zz})=0, \quad (43)$$

with the initial condition

$$u(x,y,z,0)=0,\ u_t(x,y,z,0)=x^2+y^2-z^2, \quad (44)$$

Taking differential transform of Equation (43) and the initial condition Equation (44) respectively,

$$(k+1)(k+2)U_{k+2}(x,y,z)-\frac{x^2}{12}\frac{\partial^2}{\partial x^2}U_k(x,y,z)-\frac{y^2}{12}\frac{\partial^2}{\partial y^2}U_k(x,y,z)=0, \quad (45)$$

using the initial condition, Equation (44), we have

$$U_0(x,y,z)=0,\quad U_1(x,y,z)=x^2+y^2-z^2, \quad (46)$$

Now, substituting Equation (46) into (45), we obtain the following $U_k(x,y,z)$ values successively

$$U_2(x,y,z)=\frac{1}{2}(x^2+y^2+z^2),\ U_3(x,y,z)=\frac{1}{6}(x^2+y^2-z^2),\ U_4(x,y,z)=\frac{1}{24}(x^2+y^2+z^2),$$
$$U_5(x,y,z)=\frac{1}{120}(x^2+y^2-z^2),\ U_6(x,y,z)=\frac{1}{720}(x^2+y^2-z^2),\cdots\ldots \quad (47)$$

Finally the differential inverse transform of $U_k(x,y,z)$ gives:

$$u(x,y,z,t)=\sum_{k=0}^{\infty}U_k(x,y,z)\,t^k$$
$$=(x^2+y^2+z^2)\sum_{k=0,2,4,}^{\infty}U_k(x,y,z)\,t^k+(x^2+y^2-z^2)\sum_{k=1,3,5,}^{\infty}U_k(x,y,z)\,t^k \quad (48)$$

$$u(x,y,t)=(x^2+y^2)\left(t+\frac{t^2}{2!}+\frac{t^3}{3!}+\cdots\right)+z^2\left(-t+\frac{t^2}{2!}-\frac{t^3}{3!}+\cdots\right)$$

Then, the exact solution is given in the closed form by

$$u(x,y,z,t)=(x^2+y^2)e^t+z^2e^{-t}-(x^2+y^2+z^2) \quad (49).$$

Example 7

Considering the linear Klein-Gordon equation in the form

$$u_{tt}(x,t)-u_{xx}(x,t)-u(x,t)=0, \quad (50)$$

with an initial condition

$$u(x,0)=1+\sin x,\ u_t(x,0)=0, \quad (51)$$

Taking differential transform of Equation (50) and the initial condition Equation (51) respectively, we have

$$(k+1)(k+2)U_k(x)-\frac{\partial^2}{\partial x^2}U_k(x)-U_k(x)=0, \quad (52)$$

using the initial condition, Equation (51), we have

$U_0(x)=1+\sin x$, $U_1(x)=0$ (53)

substituting (53) into (52), we obtain the following $U_k(x)$ values successively

$$U_k(x) = 0 \quad k = 1,3,5,\cdots$$

$$U_2(x) = \frac{1}{2},\ U_4(x) = \frac{1}{24},\ U_6(x) = \frac{1}{720},\cdots\cdots \tag{54}$$

Finally the differential inverse transform of $U_k(x)$ gives:

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)\, t^k = \sum_{k=0}^{\infty} U_k(x)\, t^k$$

$$= (1+\sin x) + \left(\frac{t^2}{2!} + \frac{t^4}{4!} + \frac{t^6}{6!} + \cdots\right), \tag{55}$$

Then, the exact solution is given by

$$u(x,t) = \sin x + \cosh t \tag{56}$$

Example 8a

Considering the nonlinear partial differential equation

$$u_t(x,t) + u_x(x,t) + u^2(x,t) = 0, \tag{57}$$

with an initial condition

$$u(x,0) = \frac{1}{2x}, \tag{58}$$

Taking differential transform of Equation (57) and the initial condition Equation (58) respectively, we have

$$(k+1)U_{k+1}(x) + \frac{\partial U_k(x)}{\partial x} + \sum_{r=0}^{k} U_r(x) U_{k-r}(x) = 0, \tag{59}$$

using the initial condition Equation (59), we have

$$U_0(x) = \frac{1}{2x}, \tag{60}$$

Now, substituting Equation (60) into Equation (61), we obtain the following $U_k(x)$ values successively

$$U_1(x) = \frac{1}{4x^2}, U_2(x) = \frac{1}{8x^3}, U_3(x) = \frac{1}{16x^4}, U_4(x) = \frac{1}{32x^5}, U_5(x) = \frac{1}{64x^6}, \ldots\ldots\ldots\ldots \tag{61}$$

Finally the differential inverse transform of $U_k(x)$ gives:

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)\, t^k = \frac{1}{2x}\left(1 + \frac{t}{2x} + \frac{t^2}{4x^2} + \frac{t^3}{8x^3} + \frac{t^4}{16x^4} + \cdots\right)$$

Then, the exact solution is given by

$$u(x,t) = \frac{1}{2x-t}. \tag{62}$$

Example 8b

Consider the nonlinear partial differential equation

$$u_t(x,t) + u_x(x,t) + u^2(x,t) = 0, \tag{63}$$

with an initial condition

$$u(0,t) = \frac{-1}{t}, \tag{64}$$

Taking differential transform of Equation (63) and the initial condition Equation (64) respectively, we have

$$(k+1)U_{k+1}(t) = \frac{\partial U_k(t)}{\partial t} + \sum_{r=0}^{k} U_r(t) U_{k-r}(t), \tag{65}$$

using the initial condition Equation (64), we have

$$U_0(t) = \frac{-1}{t}, \tag{66}$$

Now, substituting Equation (66) into Equation (65), we obtain the following $U_k(x)$ values successively

$$U_1(t) = \frac{2}{t^2},\ U_2(t) = \frac{-4}{t^3}, U_3(t) = \frac{8}{t^4}, U_4(t) = \frac{-16}{t^5},\ U_5(t) = \frac{32}{t^6}, U_6(t) = \frac{-64}{t^7}, \cdots \tag{67}$$

Then, the exact solution is given by

$$u(x,t) = \sum_{k=0}^{\infty} U_k(t)\, x^k = \frac{-1}{t}\left(1 + \frac{2x}{t} + \frac{4x^2}{t^2} + \frac{8x^3}{t^3} + \cdots\cdots\right)$$

$$= \frac{1}{2x-t} \tag{68}$$

Both operator yield distinct series which converge to the same solution.

Conclusion

The differential transform method has been successfully applied for solving partial differential equations with variable coefficients. The solution obtained by differential transform method is an infinite power series for appropriate initial condition, which can in turn express the exact solutions in a closed form. The results show that the differential transform method is a powerful mathematical tool for solving partial differential equations with variable coefficients. The reliability of the differential transform method and the reduction in the size of computational domain give this method a wider applicability. Thus, we conclude that the proposed method can be extended to solve many PDEs with variable coefficients which arise in physical and engineering applications.

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