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Full Length Research Paper

A reliable algorithm for physical problems

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This paper gives a detailed and comprehensive study of a reliable algorithm which is called the Modified Decomposition Method (MDM) and is mainly due to Geijji and Jafari to solve linear and nonlinear problems of physical nature. It has been shown that the MDM is very easy to implement and is fully compatible with the nonlinear nature of the physical problems. Moreover, this algorithm is independent of the inbuilt deficiencies of most of the previous techniques. Several examples are given to re-confirm the reliability and efficiency of the algorithm.

Key words: Modified decomposition method, partial differential equations, nonlinear problems.

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INTRODUCTION

The physical phenomena related to physics, astrophysics, telecommunications, signals and systems, magnetic dynamics, water surface, gravity waves, ion acoustic waves in plasma, electromagnetic radiation reactions, engineering and applied sciences are governed by differential equations and hence the appropriate solutions of such equations are utmost important (Abbasbandy, 2007; Abdou et al., 2005; Geijji et al., 2006; He, 2008; Ma, 2006; Mohyud-Din et al., 2009, 2010; Zhu, 2007). The through study of the literature sees the development of number of new techniques including decomposition, homotopy perturbation, homotopy analysis, polynomial spline, sink Glarkin, B-spline, perturbation, differential transform, exp-function, variation of parameters and variational iteration (Abbasbandy, 2007; Abdou et al., 2005; Geijji et al., 2006; He, 2008; Ma, 2006; Mohyud-Din et al., 2009; 2010; Zhu, 2007). Most of these used schemes are coupled with the inbuilt deficiencies like calculation of the so-called Adomian's polynomials, linearization, perturbation, limited convergence and non compatibility with the physical nature of the problems.

Moreover, these techniques involve very lengthy calculations coupled with a complicated computational procedure. In the similar context there was a dire need to develop an appropriate reliable, efficient and simple technique which could be fully compatible with the physical nature of the nonlinear problems without compromising their basic physics coupled with the suitable level of accuracy. Recently, Geijji and Jafari (Geijji et al., 2006) presented an exceptionally simple but very accurate technique which is Modified Decomposition Method (MDM) to solve nonlinear problems of diversified physical nature. It has been observed that MDM (Geijji et al., 2006; Mohyud-Din et al., 2009) is much better as compare to the above mentioned algorithms. Firstly, it does not require the small parameter assumption which is a major draw back in the traditional perturbation methods. No discretization or linearization is required and hence the scenario of getting some ill posed problems is avoided successfully.

Moreover, MDM is more reliable than homotopy analysis method (HAM) which is a generalized Taylor series method, gives an infinite series solution and is coupled with all the deficiencies and limitations of this technique to have practical examples. Moreover, such schemes (HAM) are not compatible to cope with the secular terms arising in the higher-order approximate solutions, whereas Modified Decomposition Method (MDM) gives an asymptotic solution with few terms. The MDM does not require the calculation of so-called Adomian's polynomials and hence is a better option as

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compare to the traditional Adomian's decomposition method. Moreover, MDM is brief, concise and more generalized than the above mentioned technique and does not even require any unrealistic assumptions which ruin the basic physical structure of the nonlinear problems. The basic motivation of the present paper is a detailed and comprehensive study of Modified Decomposition Method (MDM) and its further extension for physical problems. Several examples are given to reveal the complete reliability of these algorithms.

Modified Decomposition Method (MDM)

Consider the following general functional equations:

$$f(x) = 0, \tag{1}$$

To convey the idea of the Modified Decomposition Method (Geijji et al., 2006; Mohyud-Din et al., 2009), we rewrite the above equation as:

$$y = N(y) + c, \tag{2}$$

Where N is a nonlinear operator from a banach space $B \to B$ and *f* is a known function. We are looking for a solution of equation (1) having the series form:

$$y = \sum_{i=0}^{\infty} y_i. \tag{3}$$

The nonlinear operator N can be decomposed as:

$$N\left(\sum_{i=0}^{\infty} y_i\right) = N(y_0) + \sum_{i=0}^{\infty}\left\{N\left(\sum_{j=0}^{i} y_j\right) - N\left(\sum_{j=0}^{i-1} y_j\right)\right\}. \tag{4}$$

From Equations (3) and (4), Equation (2) is equivalent to:

$$\sum_{i=0}^{\infty} y_i = c + N(y_0) + \sum_{i=0}^{\infty}\left\{N\left(\sum_{j=0}^{i} y_j\right) - N\left(\sum_{j=0}^{i-1} y_j\right)\right\}. \tag{5}$$

We define the following recurrence relation:

$$\begin{cases} y_0 = c, \\ y_1 = N(y_0), \\ y_{m+1} = N(y_0 + \ldots + y_m) - N(y_0 + \ldots + y_{m-1}), \quad m = 1,2,3,\ldots, \end{cases} \tag{6}$$

Then:

$$(y_1 + \ldots + y_{m+1}) = N(y_0 + \ldots + y_m), \quad m = 1,2,3,\ldots,$$

and

$$y = f + \sum_{i=1}^{\infty} y_i,$$

if N is a contraction, that is $\| N(x) - N(y) \| \le \| x - y \|, \ 0 < K < 1$, then

$$\| y_{m+1} \| = \| N(y_0 + \ldots + y_m) - N(y_0 + \ldots + y_{m-1}) \| \le K \| y_m \| \le K^m \| y_0 \|,$$
$$m = 0,1,2,3,\ldots,$$

and the series $\sum_{i=1}^{\infty} = y_i$ absolutely and uniformly converges to a solution of Equation (1)

(Geijji et al., 2006; Mohyud-Din et al., 2009), which is unique, in view of the Banach fixed-point theorem.

Numerical applications

Here, we apply Modified Decomposition Method (MDM) to solve a wide range of physical problems.

Example 1

Consider the following homogeneous coupled Burger's equation:

$$u_t - u_{xx} - 2uu_x + (uv)_x = 0,$$
$$v_t - v_{xx} - 2vv_x + (uv)_x = 0,$$

with initial conditions:

$$u(x,0) = \sin x, \quad v(x,0) = \sin x.$$

Applying Modified Decomposition Method (MDM), we get:

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \left(\frac{\partial^2 u_n}{\partial x^2} + 2u_n (u_n)_x - (u_n v_n)_x\right) ds.$$

$$v_{n+1}(x,t) = v_n(x,t) + \int_0^t \left(\frac{\partial^2 v_n}{\partial x^2} + 2v_n (v_n)_x - (u_n v_n)_x\right) ds.$$

Consequently, following approximants are obtained:

$$\begin{cases} u_0(x,t) = c, \\ u_0(x,t) = \sin x \\ v_0(x,t) = c, \\ v_0(x,t) = \sin x, \end{cases}$$

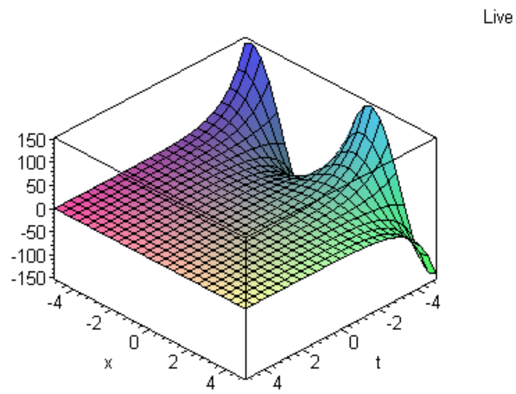

Figure 1. (u(x, t) or v(x, t)).

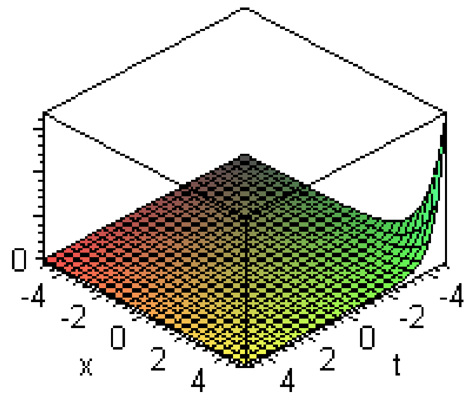

Figure 2. The series solution.

$$\begin{cases} u_1(x,t) = N\,u_0(x,t), \\ u_1(x,t) = \sin x - t\sin x, \\ v_1(x,t) = N\,v_0(x,t), \\ v_0(x,t) = \sin x - t\sin x, \end{cases}$$

$$\begin{cases} u_2(x,t) = N\left(u_0(x,t)+u_1(x,t)\right) - N\,u_0(x,t), \\ u_2(x,t) = \sin x - t\sin x + \dfrac{t^2}{2!}\sin x, \\ v_2(x,t) = N\left(v_0(x,t)+v_1(x,t)\right) - N\,v_0(x,t), \\ v_2(x,t) = \sin x - t\sin x + \dfrac{t^2}{2!}\sin x, \end{cases} \quad \vdots$$

The series solutions are given by:

$$u(x,t) = \sin x\left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + - \frac{t^4}{4!} + \cdots\right),$$

$$v(x,t) = \sin x\left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + - \frac{t^4}{4!} + \cdots\right),$$

and the closed form solutions are given as (Figure 1):

$$u(x,t) = \exp(-t)\sin x,$$
$$v(x,t) = \exp(-t)\sin x.$$

Example 2

Consider the following telegraph equation:

$$u_{xx} = u_{tt} + u_t - u,$$

with boundary conditions:

$$u(0,t) = e^{-2t}, \quad u_x(0,t) = e^{-2t},$$

and the initial conditions:

$$u(x,0) = e^{x}, \quad u_t(x,0) = -2e^{x}.$$

Applying Modified Decomposition Method (MDM), we get:

$$u_{n+1}(x,t) = e^{-2t} + xe^{-2t} + \int_0^t\int_0^t \left(\frac{\partial^2 u_n}{\partial t^2} + \frac{\partial u_n}{\partial t} - u_n\right) ds\,ds.$$

Consequently, the following approximants are obtained:

$$u_0(x,t) = c,$$
$$u_0(x,t) = e^{-2t}(1+x),$$
$$u_1(x,t) = N\,u_0(x,t),$$
$$u_1(x,t) = \left(1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3\right)e^{-2t},$$
$$u_2(x,t) = N\left(u_0(x,t)+u_1(x,t)\right) - N\,u_0(x,t),$$
$$u_2(x,t) = \left(1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5\right)e^{-2t},$$
$$u_3(x,t) = N\left(u_0(x,t)+u_1(x,t)+u_2(x,t)\right) - N\left(u_0(x,t)+u_1(x,t)\right),$$
$$u_3(x,t) = \left(1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 + \frac{1}{6!}x^6 + \frac{1}{7!}x^7\right)e^{-2t}, \ \vdots$$

The series solution is given by (Figure 2):

$$u(x,t) = \left(1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 + \frac{1}{6!}x^6 + \frac{1}{7!}x^7 + \frac{1}{8!}x^8 + \frac{1}{9!}x^9 + \cdots\right)e^{-2t},$$

and the closed form solution is given as:

$$u(x,t)=e^{x-2t}.$$

Example 3

Consider the following nonlinear system of partial differential equations:

$$u_t+v_xw_y-v_yw_x=-u,$$
$$v_t+w_xu_y+w_yu_x=v,$$
$$w_t+u_xv_y+u_yv_x=-w,$$

with initial conditions:

$$u(x,y,0)=e^{x+y},\qquad v(x,y,0)=e^{x-y},\qquad w(x,y,0)=e^{-x+y}.$$

Applying Modified Decomposition Method (MDM), we get:

$$u_{n+1}(x,y,t)=e^{x+y}-\int_0^t\left(\left(\frac{\partial v_n(x,y,\xi)}{\partial x}\right)\left(\frac{\partial w_n(x,y,\xi)}{\partial y}\right)\right)d\xi+\int_0^t\left(\left(\frac{\partial v_n(x,y,\xi)}{\partial y}\right)\left(\frac{\partial w_n(x,y,\xi)}{\partial x}\right)+u_n\right)d\xi,$$

$$v_{n+1}(x,y,t)=e^{x-y}-\int_0^t\left(\left(\frac{\partial u_n(x,y,\xi)}{\partial y}\right)\left(\frac{\partial w_n(x,y,\xi)}{\partial x}\right)\right)d\xi-\int_0^t\left(\left(\frac{\partial u_n(x,y,\xi)}{\partial x}\right)\left(\frac{\partial w_n(x,y,\xi)}{\partial y}\right)-v_n\right)d\xi,$$

$$w_{n+1}(x,y,t)=e^{-x+y}-\int_0^t\left(\left(\frac{\partial u_n(x,y,\xi)}{\partial x}\right)\left(\frac{\partial v_n(x,y,\xi)}{\partial y}\right)\right)d\xi-\int_0^t\left(\left(\frac{\partial v_n(x,y,\xi)}{\partial x}\right)\left(\frac{\partial u_n(x,y,\xi)}{\partial y}\right)+w_n\right)d\xi.$$

Consequently, following approximants are obtained:

$$\begin{cases}u_0(x,t)=c,\\ u_0(x,y,t)=e^{x+y},\\ v_0(x,t)=c,\\ v_0(x,y,t)=e^{x-y},\\ w_0(x,t)=c,\\ w_0(x,y,t)=e^{-x+y},\end{cases}$$

$$\begin{cases}u_1(x,t)=N\,u_0(x,t),\\ u_1(x,y,t)=e^{x+y}(1-t),\\ v_1(x,t)=N\,v_0(x,t),\\ v_1(x,y,t)=e^{x-y}(1+t),\\ w_1(x,t)=N\,w_0(x,t),\\ w_1(x,y,t)=e^{-x+y}(1+t),\end{cases}$$

$$\begin{cases}u_2(x,t)=N\left(u_0(x,t)+u_1(x,t)\right)-N\,u_0(x,t),\\ u_2(x,y,t)=e^{x+y}\left(1-t+\dfrac{t^2}{2!}\right),\\ v_2(x,t)=N\left(v_0(x,t)+v_1(x,t)\right)-N\,v_0(x,t),\\ v_2(x,y,t)=e^{x-y}\left(1+t+\dfrac{t^2}{2!}\right),\\ w_2(x,t)=N\left(w_0(x,t)+w_1(x,t)\right)-N\,w_0(x,t),\\ w_2(x,y,t)=e^{-x+y}\left(1+t+\dfrac{t^2}{2!}\right),\end{cases}$$

$$\begin{cases}u_3(x,t)=N\left(u_0(x,t)+u_1(x,t)+u_2(x,t)\right)-N\left(u_0(x,t)+u_1(x,t)\right),\\ u_3(x,y,t)=e^{x+y}\left(1-t+\dfrac{t^2}{2!}-\dfrac{t^3}{3!}\right),\\ v_3(x,t)=N\left(v_0(x,t)+v_1(x,t)+v_2(x,t)\right)-N\left(v_0(x,t)+v_1(x,t)\right),\\ v_3(x,y,t)=e^{x-y}\left(1+t+\dfrac{t^2}{2!}+\dfrac{t^3}{3!}\right),\\ w_3(x,t)=N\left(w_0(x,t)+w_1(x,t)+w_2(x,t)\right)-N\left(w_0(x,t)+w_1(x,t)\right),\\ w_3(x,y,t)=e^{-x+y}\left(1+t+\dfrac{t^2}{2!}-\dfrac{t^3}{3!}\right),\\ \vdots\end{cases}$$

The series solution is given by:

$$\begin{cases}u(x,y,t)=e^{x+y}\left(1-t+\dfrac{t^2}{2!}-\dfrac{t^3}{3!}+\dfrac{t^5}{5!}-\dfrac{t^7}{7!}+\cdots\right),\\ v(x,y,t)=e^{x-y}\left(1+t+\dfrac{t^2}{2!}+\dfrac{t^3}{3!}+\dfrac{t^5}{5!}+\dfrac{t^7}{7!}+\cdots\right),\\ w(x,y,t)=e^{-x+y}\left(1+t+\dfrac{t^2}{2!}-\dfrac{t^3}{3!}+\dfrac{t^5}{5!}-\dfrac{t^7}{7!}+\cdots\right).\end{cases}$$

The closed form solution is given as (Figures 3 to 5):

$$(u,v,w)=(e^{x+y-t},e^{x-y+t},e^{-x+y+t}).$$

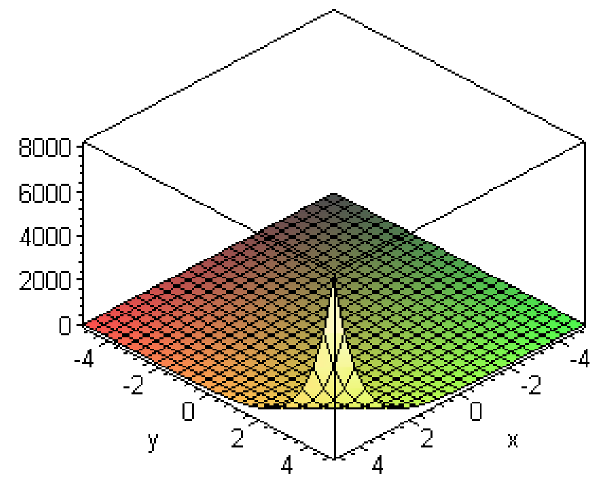

Figure 3. (U, t = 1).

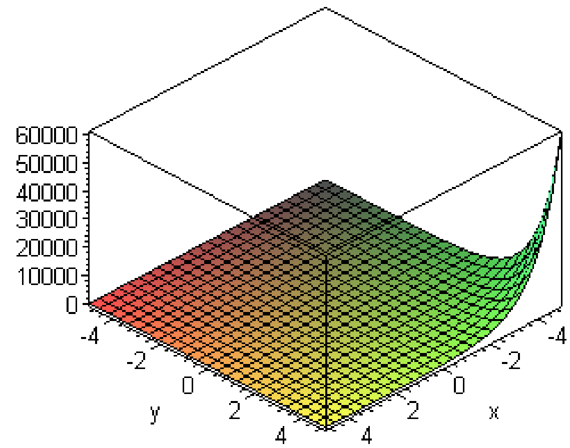

Figure 5. (W, t = 1).

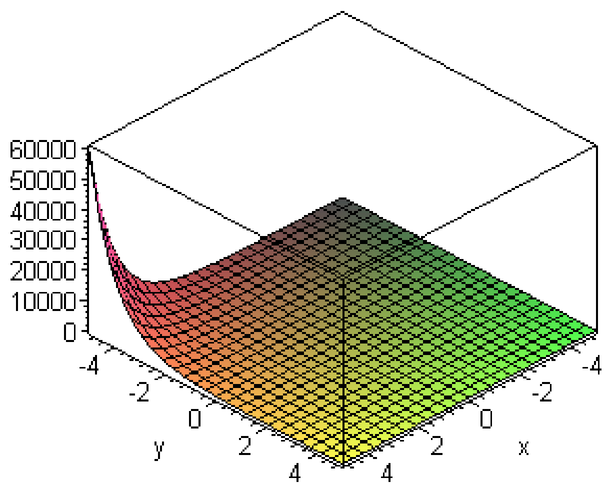

Figure 4. (V, t = 1).

Example 4

Consider the following fourth-order singular parabolic partial differential equation:

$$\frac{\partial^2 u}{\partial t^2}+\left(\frac{x}{\sin x}-1\right)\frac{\partial^4 u}{\partial x^4}=0, \qquad 0<x<1, \qquad t>0$$

with initial conditions:

$$u(x,0)=x-\sin x, \qquad 0<x<1,$$

$$\frac{\partial u}{\partial t}(x,0)=-(x-\sin x), \qquad 0<x<1$$

and the boundary conditions:

$$u(0,t)=0, \qquad u(1,t)=e^{-t}(1-\sin 1), \quad t>0,$$

$$\frac{\partial^2 u}{\partial x^2}(0,t)=0, \quad \frac{\partial^2 u}{\partial x^2}(1,t)=e^{-t}\sin 1, \quad t>1.$$

Applying Modified Decomposition Method (MDM), we get:

$$u_{n+1}(x,t)= u_0(x,t)-\int_0^t\int_0^t\left(\left(\frac{x}{\sin x}-1\right)\frac{\partial^4 u_n}{\partial x^4}\right)ds.$$

Consequently, following approximants are obtained:

$$u_0(x,t)=c,$$
$$u_0(x,t)=x-\sin x,$$
$$u_1(x,t)=N\,u_0(x,t),$$
$$u_1(x,t)=-(x-\sin x)t,$$
$$u_2(x,t)=N\left(u_0(x,t)+u_1(x,t)\right)-N\,u_0(x,t),$$
$$u_2(x,t)=(x-\sin x)\left(\frac{t^2}{2!}-\frac{t^3}{3!}\right),$$
$$u_3(x,t)=N\left(u_0(x,t)+u_1(x,t)+u_2(x,t)\right)-N\left(u_0(x,t)+u_1(x,t)\right),$$
$$u_3(x,t)=(x-\sin x)\left(\frac{t^4}{4!}-\frac{t^5}{5!}\right), \vdots$$

The solution is given as:

$$u(x,t)=(x-\sin x)\left(1-t+\frac{t^2}{2!}-\frac{t^3}{3!}+\frac{t^4}{4!}-\frac{t^5}{5!}+\cdots\right)=(x-\sin x)e^{-t},$$

which is the exact solution. (Figure 6).

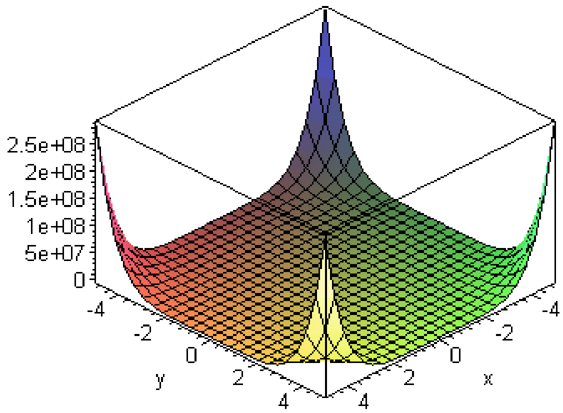

Figure 6. Depicts the series solution.

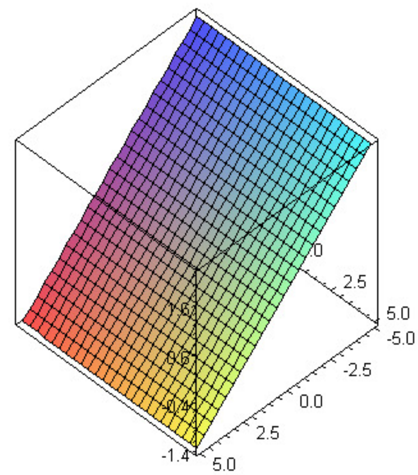

Figure 7. $t=1$ and $z=1$.

Example 5

Consider the following three-dimensional initial boundary value problem:

$$u_{tt}=\frac{1}{45}x^2u_{xx}+\frac{1}{45}y^2u_{yy}+\frac{1}{45}z^2u_{zz}-u, \qquad 0<x,y<1, \qquad t<0,$$

subject to the Neumann boundary conditions:

$$u_x(0,y,z,t)=0, \qquad u_x(1,y,z,t)=6y^6z^6\sinh t, \qquad u_y(x,0,z,t)=0,$$
$$u_y(x,1,z,t)=6x^6z^6\sinh t, \qquad u_z(x,y,0,z,t)=0, \qquad u_x(1,y,z,t)=6y^6z^6\sinh t,$$

and the initial conditions:

$$u(x,y,z,0)=0, \qquad u_t(x,y,z,0)=x^6y^6z^6.$$

The exact solution for this problem is:

$$u(x,y,z,t)=x^6y^6z^6\sinh t.$$

Applying Modified Decomposition Method (MDM), we get:

$$u_{n+1}(x,y,z)=x^6y^6z^6t+\frac{1}{45}\int_0^t\int_0^t\left(x^2\frac{\partial^2u_n}{\partial x^2}+y^2\frac{\partial^2u_n}{\partial x^2}+z^2\frac{\partial^2u_n}{\partial x^2}-u_n\right)ds\,ds.$$

Consequently, following approximants are obtained:

$$u_0(x,t)=c,$$
$$u_0(x,y,z,t)=x^6y^6z^6t,$$
$$u_1(x,t)=N\,u_0(x,t),$$
$$u_1(x,y,z,t)=x^6y^6z^6\left(t+\frac{t^3}{3!}\right),$$
$$u_2(x,t)=N\left(u_0(x,t)+u_1(x,t)\right)-N\,u_0(x,t),$$
$$u_2(x,y,z,t)=x^6y^6z^6\left(t+\frac{t^3}{3!}+\frac{t^5}{5!}\right),$$
$$u_3(x,t)=N\left(u_0(x,t)+u_1(x,t)+u_2(x,t)\right)-N\left(u_0(x,t)+u_1(x,t)\right),$$
$$u_3(x,y,z,t)=x^6y^6z^6\left(t+\frac{t^3}{3!}+\frac{t^5}{5!}+\frac{t^7}{7!}\right),\ \vdots$$

The series solution is given by:

$$u(x,y,z,t)=\lim_{n\to\infty}u_n(x,y,z,t)=x^6y^6z^6\left(t+\frac{t^3}{3!}+\frac{t^5}{5!}+\cdots+\frac{t^{2n+1}}{(2n+1)!}+\cdots\right),$$

and the closed form solution is obtained as (Figure 7):

$$u(x,y,z,t)=x^6y^6z^6\sin ht.$$

Figure 7 depicts the series solution at $t=1,\ z=1.$

Example 6

Consider the following Helmholtz equation:

$$\frac{\partial^2u(x,y)}{\partial^2x^2}+\frac{\partial^2u(x,y)}{\partial^2y^2}+8u(x,y)=0,$$

with initial conditions:

$$u(0,y)=\sin(2y), \qquad u_x(0,y)=0.$$

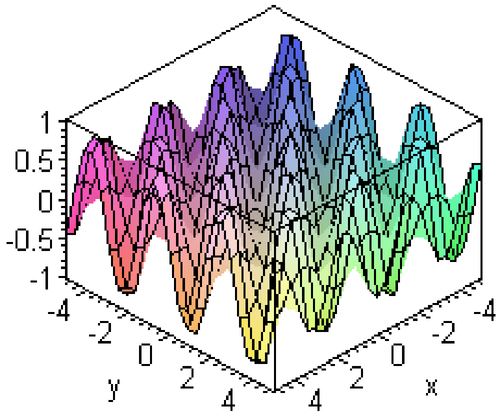

Figure 8. Depicts the series solution.

The exact solution for this problem is:

$$u(x, y) = \cos(2x)\sin(2y).$$

Applying Modified Decomposition Method (MDM), we get:

$$u_{n+1}(x,t) = \sin 2y + \int_0^t \int_0^t \left(\left(\frac{\partial^2 u_n}{\partial x^2} \right) + \left(\frac{\partial^2 u_n}{\partial y^2} \right) + 8(u_n) \right) ds\, ds.$$

Consequently, following approximants are obtained (Figure 8):

$$u_0(x,t) = c,$$
$$u_0(x, y) = \sin(2y),$$
$$u_1(x,t) = N u_0(x,t),$$
$$u_1(x, y) = \sin(2y) - 2x^2 \sin(2y),$$
$$u_2(x,t) = N(u_0(x,t) + u_1(x,t)) - N u_0(x,t),$$
$$u_2(x, y) = \sin(2y) - 2x^2 \sin(2y) + \frac{2}{3} x^4 \sin(2y),$$
$$u_3(x,t) = N(u_0(x,t) + u_1(x,t) + u_2(x,t)) - N(u_0(x,t) + u_1(x,t)),$$
$$u_3(x, y) = \sin(2y) - 2x^2 \sin(2y) + \frac{2}{3} x^4 \sin(2y) - \frac{4}{45} x^6 \sin(2y), \vdots$$
$$u(x, y) = \lim_{n\to\infty} u_n(x, y) = \sin(2y)\cos(2x),$$

which is the exact solution. Figure 8 depicts the series solution.

Example 7

Consider the following Helmholtz equation:

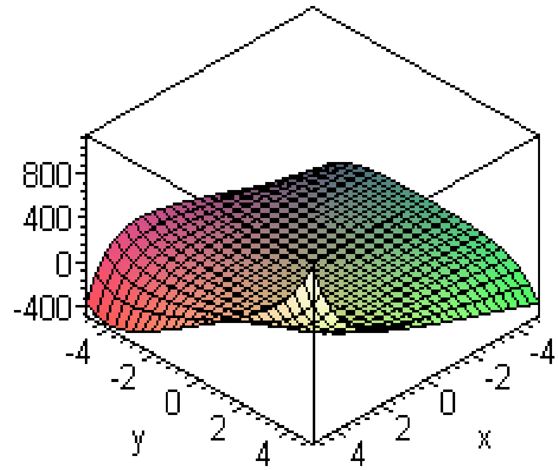

Figure 9. Depicts the series solution.

$$\frac{\partial^2 u(x, y)}{\partial^2 x^2} + \frac{\partial^2 u(x, y)}{\partial^2 y^2} - u(x, y) = 0,$$

with initial conditions:

$$u(0, y) = y, \qquad u_x(0, y) = y + \cosh(y).$$

The exact solution for this problem is:

$$u(x, y) = ye^x + x\cosh(y).$$

Applying Modified Decomposition Method (MDM), we get:

$$u_{n+1}(x,t) = \sin 2y + \int_0^t \int_0^t \left(\left(\frac{\partial^2 u_n}{\partial x^2} \right) + \left(\frac{\partial^2 u_n}{\partial y^2} \right) - (u_n) \right) ds\, ds.$$

Consequently, following approximations are obtained (Figure 9):

$$u_0(x, y) = y + xy + x\cosh(y),$$
$$u_1(x, y) = y + xy + x\cosh(y) + \frac{1}{6} yx^3 + \frac{1}{2} yx^2,$$
$$u_2(x, y) = y + xy + x\cosh(y) + \frac{1}{6} yx^3 + \frac{1}{2} yx^2 + \frac{1}{24} yx^4,$$
$$u_3(x, y) = y + xy + x\cosh(y) + \frac{1}{6} yx^3 + \frac{1}{2} yx^2 + \frac{1}{24} yx^4 + \frac{1}{120} yx^5, \vdots$$
$$u(x, y) = \lim_{n\to\infty} u_n(x, y) = ye^x + x\cosh(y),$$

which is the exact solution. Figure 9 depicts the series solution.

CONCLUSION

In this paper, we applied Modified Decomposition Method (MDM) to solve a wide range of physical problems related to physics and applied sciences. The method is applied in a direct way without using linearization, perturbation, transformation, discretization or restrictive assumptions. It may be concluded that the MDM is very powerful and efficient in finding the analytical solutions for a wide class of boundary value problems. The method gives more realistic series solutions that converge very rapidly in physical problems. It is worth mentioning that the method is capable of reducing the volume of the computational work as compare to the classical methods while still maintaining the high accuracy of the numerical result. The fact that the MDM solves nonlinear problems without using the Adomian's polynomials is a clear advantage of this technique over the decomposition method.

REFERENCES

Abbasbandy S (2007). A new application of He's variational iteration method for quadratic Riccati differential equation by using Adomian's polynomials. J. Comput. Appl. Math., 207: 59-63.

Abbasbandy S (2007). Numerical solutions of nonlinear Klein-Gordon equation by variational iteration method. Int. J. Numer. Meth. Eng., 70: 876-881.

Abdou MA, Soliman AA (2005). New applications of variational iteration method. Phys. D., 211(1-2): 1-8.

Abdou MA, Soliman AA (2005). Variational iteration method for solving Burger's and coupled Burger's equations. J. Comput. Appl. Math., 181: 245-251.

Geijji VD, Jafari H (2006). An iterative method for solving nonlinear functional equations. J. Math. Anal. Appl., 316: 753-763.

He JH (2008). An elementary introduction of recently developed asymptotic methods and nanomechanics in textile engineering. Int. J. Mod. Phys. B., 22(21): 3487-4578.

Ma WX, Chen M (2006). Hamiltonian and quasi- Hamiltonian structures associated with semi-direct sums of Lie algebras. J. Phys. A: Math. Gen., 39: 10787-10801.

Mohyud-Din ST (2009). Solution of nonlinear differential equations by exp-function method. Wd. Appl. Sci. J., 7: 116-147.

Mohyud-Din ST, Hosseini MM, Yildirim A, Usman M (2010). An iterative algorithm for higher-dimensional IBVPs with variable co-efficient. Wd. Appl. Sci. J., 11(2): 159-164.

Mohyud-Din ST, Noor MA, Noor KI (2009). Some relatively new techniques for nonlinear problems. Math. Prob. Eng. 2009, Article ID 234849, p. 25, doi:10.1155/2009/234849.

Mohyud-Din ST, Noor MA, Noor KI (2009). Travelling wave solutions of seventh-order generalized KdV equations using He's polynomials. Int. J. Nonlin. Sci. Numer. Sim., 10(2): 223-229.

Mohyud-Din ST, Usman M, Yildirim A (2010). An iterative algorithm for boundary value problems using Padé approximants. Wd. Appl. Sci. J., 10(6): 637-644.

Mohyud-Din ST, Usman M, Yildirim A (2010). An iterative algorithm for boundary value problems using Padé approximants. Wd. Appl. Sci. J., 10(6): 637-644.