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Full Length Research Paper

Using linear goal programming in surveying engineering for vertical network adjustment

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In engineering, especially in Surveying Engineering, network adjustment is made to find out the definite values of the unknows and the measurements. Generally Least Square method is used for vertical network adjustment. By using Linear Goal Programming method, as opposed to the Least Squares method used in network adjustment, similar results are reached. By this study, Linear Goal Programming method, proposed as an alternative method to Least Squares method for network adjustment, is explained with an example, and results are compared. Results, found by using these two methods, are close to each other. It is shown that Linear Goal Programming methods can be used in vertical network adjustment.

Key words: Linear goal programming, least squares, network adjustment.

INTRODUCTION

The aim of the network adjustment is to find out the optimum values of the unknows and the measurements, performed much more than needed. The sensitiveness and reliance of the surveys, certain values and their functions are set. In other words, the statistical analysis of mathematical model which is used is made. During the network adjustment, measurements are accepted as normally distributed. To perform this aim least squares (LS) method of Gauss is applied. This method is widely used in Surveying Engineering. The sensitiveness of the adjustment values which are obtained by the end of this method is more than the sensitiveness of the measurements.

The application of an alternative method proposed in network adjustment, of goal programming (GP) is set up by Charnes and Cooper (1955, 1961). GP is widely spread out by Iijiri (1965), Lee (1972) and Ignizio (1976). Nowadays, GP is one of the most important methods of Multi-Criteria Optimization methods. Main idea in GP is to determine targets for each constraint (Steuer, 1986).

Main idea in GP is to find solutions for one or more goal functions of predetermined targets. GP is a method of searching to find out the ways of reaching the target values according to priority and weight predetermined for every purpose by the goals of decision maker's. The decision maker's decide which of goals are premtitive (Deb, 2004; Levin et al., 1982).

If goal and absolute constraints and goal functions are linear, there is a Linear Goal Programming model. Linear goal programming (LGP) is easly solved by the reconstruction of Simplex Method (Alp, 2008).

To see the application in Surveying Engineering by the application of LGP similar results, which are observed in LS method, are observed. This shows that LGP may be used as an alternative method in Surveying Engineering.

There have been a number of works on LS. Samples are: Estimating Regional Deformation (Dong et al., 1998), Smooting and Differentiation (Steinier et al.,1972) and Global Positioning (Adhikary, 2003).

There have been a number of works on LGP as LS. Samples are: Tranportation (Ahern and Anandarajah, 2007), Planning of Energy (Linares and Romero, 2002), Planning of Medical Services (Giokas, 2002), Ferland (2001), Project Selection (Lee and Kim, 2000), Portfolio Selection (Cooper et al., 1997), Aouni et al. (2005), Financial Management (Lin and O'Leary, 1993), Planning of Investment (Hajidimitriou and Georgiou, 2002), Agricultural Planning (Aromolaran and Olayemi, 1999).

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NETWORK ADJUSTMENT ACCORDING TO LEAST SQUARE

Mathematical model in network adjustment is formed of two constituents. One of them is called functional model and the other is called stochastic model (Teunissen, 1999). These constituents form the base of balance account. These models are formed before the adjustment.

After the adjustment we can say that exact values, determined by condition:

$$[Pvv] = \min \quad (1)$$

are the best values.

In geodesy science, accepting, h_{ik} values as elevation differences between two points and H_i values as adjusted values in a levelleng net, correction equations that give functional relation between measured elevation differences and elevations, are as follows:

$$h_{ik} + v_{ik} = H_k - H_i \quad (2)$$

v_{ik} values are used as correction amounts for levelling observations. By using the above equation, functional model, that gives relation between observations and unkowns, is as follows:

$$v_{ik} = -dh_i + dh_k - l_{ik} \quad (3)$$

$-l_{ik}$ values are constant terms in above equation. These constant terms are calculated as follows:

$$-l_{ik} = H_k^{\circ} - H_i^{\circ} - h_{ik} \quad (4)$$

H_k and H_i values are approximate elevations of the points.

Preccision of the elevation differences are reverse proportioned with square root of levelling distances. According to this, we can describe the stochastic model using levelling distances P_{ik} as follows:

$$P_{ik} = \frac{1}{S_{ik}} \quad (5)$$

Normal equations can be formed using Equations (3) and (5).

$$A^T PA\ X - A^T Pl = 0 \quad (6)$$

In this equation, A: coefficient matrix of unknowns, X : vector of unknowns, L : vector of constant terms (Teunissen, 1999; Dong et al., 1998).

Equation (6) is used for the solution of normal equations using moderned Gauss algorithm:

$$X = (A^T PA)^{-1} A^T Pl \quad (7)$$

Thus, Equation (7) is used to calculate adjusted elevations of unknown points:

$$H_i = H_i^{\circ} + dh_i \quad (8)$$

Corrections for the measured elevation differences are calculated as follows:

$$v_{ik} = -dh_i + dh_k - l_{ik} \quad (9)$$

As a result, adjustment values of the elevation differences ar calculated as follows:

$$\overline{h}_{ik} = h_{ik} + v_{ik} \quad (10)$$

By comparing these values with differences between the adjusted elevation values, the adjusted values of elevation differences are calculated as follows:

$$\overline{h}_{ik} = H_k + H_i \quad (11)$$

Equations (10) and (11) must give the same result (Peng et al., 2006).

LINEAR GOAL PROGRAMMING MODEL

In solution of LGP models, performed to minimize the deviation of determined target according to priority and weight coefficients defined by decision maker's are carried.

Goal programming method is not only a technique to minimize the sum of all deviations, but also a technique to minimize priorty deviations as much as possible (Alp, 2008).

The general form of a multi objective programming is as follows:

$$x^T = (x_1, x_2, x_3, ..., x_n) \quad (12)$$

$$\max/\min f_t(x) \qquad t = 1, 2, ..., s \quad (13)$$

$$g_i(x)\begin{Bmatrix}\le\\=\\\ge\end{Bmatrix} b_i \qquad i=1,2,...,m \tag{14}$$

x^T: Vector of decision variables, b_i: the right side value belonging to the i'nd constraint s: Count of goal functions, m: Count of absolute constraints.

As seen in Equation (14) the absolute constraints, $g_i(x)$, the right side belonging to these constraints b_i can be less equal, equal or more equal (Ignizio, 1985).

Decision variables (also known as control or structural variables) are those problem variables over which one can exercise some control. In mathematical programming, an objective is a function that we seek to optimite, via changes in the decisin variables (Ignizio, 1985).

For each goal function desired target value is to be determined in order to solve a multi objective mathematical model by a goal programming. The target value for each function is shown as G_t

Goal constraints are obtained by equalizing every goal function to target values. The inequalizations of the goal constraints observed are transferred to equalizations by adding derivation variables to the left side of them.

The difference between the realized values and the target values is assigned to deviation variables. (n_i, p_i) shows if the values of deviation variables by the solution attain the target values.Deviation variables show the distance of deviation from the target if not attained (Lee, 1973).

The mathematical formulation of goal programming model is as follows:

$$\left.\begin{array}{ll} Minimize \left[\sum_{t=1}^{s}\left(n_i+p_i\right)^r\right]^{1/r} & ,r\ge 1\\ Subject\,to & \\ g_i(\underline{x})\le b_i & ,i=1,2,...,m\\ f_t(\underline{x})+n_i-p_i=G_t & ,t=1,2,...s\\ n_i,p_i\ge 0 & ,\forall t\\ n_i\times p_i=0 & ,\forall t\\ x_j\ge 0 & ,j=1,2,...,n \end{array}\right\} \tag{15}$$

"r", variable in goal function is used in determining the related significance of deviation variables. If *"r"* is 1 related significance of all deviation variables are equal to each other. If *"r"* value increases, related significance of higher deviation variables increase, too. $n_i\times p_i=0$ fixed constraint shows that n_t or p_t or both are zero.

Before writing the mathematical formulation of the goal programming containing the priority and weight coefficients, the priority level is to be explained. Priority level is to determine in which order the targets are to be minimized. The significance levels of priority decreasing from left to right is explained as follows:

$$P_1 >>> P_2 >>> ... >>> P_k \tag{16}$$

$P_j >>> P_{j+1}$ for $\forall j$, P_j level is prior to P_{j+1}. No values of "w" can maintain $w\cdot P_{j+1} > P_j$

By getting "r" in Equation (15) as 1 and using the priority levels the mathematical formulation of goal programming model is as follows (Alp, 2008):

$$\left.\begin{array}{ll} \min\left[P_1w_1\left(\underline{n},\underline{p}\right),P_2w_2\left(\underline{n},\underline{p}\right),...,P_kw_k\left(\underline{n},\underline{p}\right)\right] & \\ Subject\,to & \\ g_i(\underline{x})\le b_i & ,i=1,2,...,m\\ f_t(\underline{x})+n_t-p_t=G_t & ,t=1,2,...,s\\ n_t,p_t\ge 0 & ,\forall t\\ n_t\times p_t=0 & ,\forall t\\ x_j\ge 0 & ,j=1,2,...,n \end{array}\right\} \tag{17}$$

$w_i(\underline{n},\underline{p}),\ (i=1,2,...,k)$ are the linear functions of deviation variables.

If all the functions of the general goal programming model are linear there is a linear goal programming.

APPLICATION

Problem

The surveys made in levelling network to determine the height of the points of the earth is given in Table 1 and appoximate elevations is given in Table 2.

A software developed by Yavuz (2002) is used to solve the problem according to LS adjustment and WinQSB is used to solve the problem according to LGP.

Using the least square in network adjustment

In Table 3, the given correction equations listed have been formed by using the data in Table 1.

Using the linear goal programming in network adjustment

The aim of network adjustment is to find out the optimum

Table 1. Level differences (*Fixed level differences).

	From point	To point	h_{ik} (m)
1	1	0	6.782
2	2	1	5.116
3	2	3	3.553
4	3	4	2.944
5	4	0	5.402
6	2	0	11.907
7	3	0	8.364
8*	3	1	1.575
9*	4	2	6.500

Table 2. Appoximate elevations (*Fixed elenation).

Point	H^0_i (m)
0*	40.000
1	46.780
2	51.900
3	48.360
4	45.400

Table 3. Correction equations.

	v_{ij}		$-l_{ij}$
1	$v_{1,0}$	$+d_0-d_1$	$+H^0_0-H^0_1-h_{1,0}$
2	$v_{2,1}$	$+d_1-d_2$	$+H^0_1-H^0_2-h_{2,1}$
3	$v_{2,3}$	$+d_3-d_2$	$+H^0_3-H^0_2-h_{2,3}$
4	$v_{3,4}$	$+d_4-d_3$	$+H^0_4-H^0_3-h_{3,4}$
5	$v_{4,0}$	$+d_0-d_4$	$+H^0_0-H^0_4-h_{4,0}$
6	$v_{2,0}$	$+d_0-d_2$	$+H^0_0-H^0_2-h_{2,0}$
7	$v_{3,0}$	$+d_0-d_3$	$+H^0_0-H^0_3-h_{3,0}$

V_{ij} : Corrections, d_{ij} : corrections unknows, H_i^0 : appoximate elevations, h_{ij} : elevation differences.

values by distributing the corrections of the measurements in a proper way. To implement this aim generally LS method is used.

In this study network adjustment is made by using both LS and LGP methods. The height values of 5 points in the network are accepted as decision variables (X_i) in network adjustment by LGP. By LGP model, values of decision variables, in other words adjusted height values of points in network, are obtained.

7 level differences surveyed between two points (h_{ij}) in network are accepted as goal values (G_i).The level differences between the two surveyed points are equalized to an measured level difference accepted as the goal value. In the survey; the error values appearing are accepted as deviation variables (n_t, p_t), 7 goal constraints are obtained by adding the deviation variables to the equalities and 2 absolute constraints are given.

The height of the zero point is accepted fixed and the result is transferred as absolute constraint.

$$x_0 = 40,000 \tag{17}$$

The goal constraints are shown in Table 4 and absolute constraints are shown in Table 5.

The goal function in mathematical model is arranged as below:

$$Min\sum_{i=1}^{7}(n_i - p_i) \tag{18}$$

Table 4. Goal constraint.

	Points	Goal constraint
1	(1-0)	$x_0 - x_1 + n_1 - p_1 = 6.782$
2	(2-1)	$x_1 - x_2 + n_2 - p_2 = 5.116$
3	(2-3)	$x_3 - x_2 + n_3 - p_3 = 3.553$
4	(3-4)	$x_4 - x_3 + n_4 - p_4 = 2.944$
5	(4-0)	$x_0 - x_4 + n_5 - p_5 = 5.402$
6	(2-0)	$x_0 - x_2 + n_6 - p_6 = 11.907$
7	(3-0)	$x_0 - x_3 + n_7 - p_7 = 8.364$

Table 5. Absolute constraint.

	Points	Absolute constraint
1	(3-1)	$x_1 - x_3 = 1.575$
2	(4-2)	$x_2 - x_4 = 6.500$
3	0	$x_0 = 40.500$

Table 6. Levelling adjustment with LS and LGP.

Point	Approximate H_0 (m)	LS		LGP		Differences	
		Corrections	Adjusted H_E (m)	Corrections	Adjusted H_D (m)	LS-LGP m	LS-LGP mm
0	6.782	-0.001	6.783	-0.007	6.789	-0.006	-6
1	5.116	-0.008	5.124	0.003	5.113	0.011	11
2	51.9	-0.007	51.907	-0.002	51.902	0.005	5
3	48.36	0.002	48.358	-0.004	48.364	-0.006	-6
4	3.553	0.004	3.549	0.015	3.538	0.011	11

H_0 : approximate heights of the points, H_E : adjusted heights of the points with LS, H_D : adjusted heights of the points with LGP.

If the goal function is to be written in detail:

$$Min[n_1+p_1+n_2+p_2+n_3+p_3+n_4+p_4+n_5+p_5+n_6+p_6+n_7+p_7]$$

Compare of between linear goal programming and least square

Taking the height of point (0) fixed, by the help of the measured heights of 5 points, adjusted heights are obtained in both methods. The conclusions obtained in both methods are given in Table 6.

Corrections, related to measured level differences, received by both methods, have been obtained in mm sensitivitivity. Level differences for both methods are obtained by adding the correction amount to the levelling measurements.

For both methods; comparing level differences between 7 different measured points and adjusted heights of the points, the result control of the adjustment in both methods is done.

Conclusion

The functional and the stochastic models of the adjustment which are used are appropiate according to LS approach.

Close values to the values obtained by LS method a re

Table 7. Compare of between LS and LGP results.

	Level differences H_0 (m)	LS		LGP		Differences	
		Corrections	Adjusted H_E (m)	Corrections	Adjusted H_D (m)	LS-LGP m	mm
(1-0)	6.782	-0.001	6.783	-0.007	6.789	-0.006	-6
(2-1)	5.116	-0.008	5.124	0.003	5.113	0.011	11
(2-3)	3.553	0.004	3.549	0.015	3.538	0.011	11
(3-4)	2.944	-0.007	2.951	-0.019	2.963	-0.012	-12
(4-0)	5.402	-0.005	5.407	0.001	5.401	0.006	6
(2-0)	11.907	0	11.907	0.005	11.902	0.005	5
(3-0)	8.364	0.006	8.358	0	8.364	-0.006	-6

received by using LGP method which is a minimizing of deviation of the targets whose aims are determined. The highest difference between results of two methods is 12 mm. In LGP method, different from LS method (Table 7), without using the approximate height values of the points, but using only the level differences between two points, the height of the points are calculated close to the height values obtained according to LS.

REFERENCES

Adhikary KR (2003). Global Positioning System on Cadastral Survey of Nepal, Nepalese J. Geoinformatics, 2: 58-63.

Ahern A, Anandarajah G (2007). Railway Projects Prioritisation for Investment : Application of Goal Programming. Transp. Pol., 14(1): 70-80.

Alp S (2008). An Application and Modelling of Intercity Passenger Transportation by Bus with Linear Goal Programming. PhD Thesis. Marmara University, Institute of Social Sciences: Istanbul.

Aromolaran AB, Olayemi JK (1999). Multiple Objective Farm Planning for Food Crop Farmers: A Goal Programming Approach. Disco. Innovation, 11(1-2): 91-103.

Aouni B, Abdelaziz FB, Martel JM (2005). Decision-Maker's Preferences Modeling in the Stochastic Goal Programming. Eur. J. Operational Res., 162(1): 610-618.

Charnes A, Cooper WW (1961). Management Models and Industrial Applications of Linear Programming. Wiley & Sons. New York.

Charnes A, Cooper WW, Ferguson RO (1955). Optimal Estimation of Executive Ompensation by Linear Programming. Manage. Sci., 1: 138–151.

Cooper WW, Lejans V, Sueyoshi T (1997). Goal Programming Models and Their Duality Relations for Use in Evaluating Security Portfolio and Regression Relations. Eur. J. Operational Res., 98(2): 431-443.

Deb K (2004). Multi-Objective Optimization Using Evolutionary Algorithms. John Wiley & Sons.

Dong D, Herring TA, King RW (1998). Estimating Regional Deformation from a Combination of Space and Terrestrial Geodetic Data. J. Geodesy, 72: 200-214.

Ferland JA (2001). Generalized Assignment Type Goal Programming Problem: Application to Nurse Scheduling. J. Heuristics, 7(1): 391-413.

Giokas D (2002). The Use of Goal Programming, Regression Analysis and Data Envelopment Analysis for Estimating Efficient Marginal Costs of Hospital Services. J. Multicr. Decision Anal., 11(4-5): 261-268.

Ijiri Y (1965). Management Goals and Accounting for Control. North-Holland Pub.

Hajidimitriou YA, Georgiou AC (2002). A Goal Programming Model For Partner Selection Decisions In International Joint Ventures. Eur. J. Operational Res., 138(1): 649-662.

Ignizio JP (1985). Introduction to Linear Goal Programming. Second Edition. Duxbury Pres.

Ignizio JP (1976) Goal Programming and Extensions. Lexington Books.

Lee JW, Kim SH (2000). Using Analytic Network Process And Goal Programming For Interdependent Information System Project Selection. Comp. Opera. Res., 27(1): 367-382.

Lee SM (1973) Goal Programming for Decision Analysis of Multiple Objectives. Sloan Manage. Rev., 14(2): 11-24.

Lee SM (1972). Goal Programming for Decision Analysis. Auerback.

Levin RI, Kirkpatrick CA, Rubin DS (1982). Quantitative Approaches To Managemet. Fifth Edition. McGraw-Hill International Book Company.

Lin TW, O'Leary DE (1993). Goal Programming Applications in Financial Managemet. Adv. Math. Prog. Fin. Planning, 3(1): 211-230.

Linares P, Romero C (2002). Aggregation of Preferences in An Environmental Economics Context: A Goal Programming Approach. Int. J. Manage. Sci. (Omega), 30(2): 89-95.

Peng J, Chunxi G, Zang H (2006). An Aggregate Constraint Method for Inequality-Constrained Least Squares Problems. J. Geodesy, 79: 705-713.

Steinier J, Termonia Y, Deltour J (1972). Smoothing and Differentiation of Data by Simplified Least Square Procedure. Anal. Chem., 44(11): 1906-1909.

Steuer RE (1986). Multiple Criteria Optimization: Theory, Computation and Application. John Wiley & Sons Inc.

Teunissen PJG (1999). Adjustment Theory. Delft University Presss.

Yavuz E (2002). Stochastic Model Search in Horizontal Control Network. PhD Thesis. İstanbul Technical University, Institute of Science and Technology: Istanbul.