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Full Length Research Paper

Analysis of electrostatically actuated microbeams using dimensionless groups

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The vibrational behavior of coupled-domain micro electro mechanical systems (MEMS) devices with electrostatic actuation and squeeze film effect is investigated in this article. π-Buckingham theory is used to extract dimensionless parameters for this problem. Using these parameters, non-dimensional equation of Euler-Bernoulli beam theory and Reynolds equation of squeeze film damping are developed. Finite difference method is applied to solve these dimensionless equations simultaneously. After validation of presented non-dimensional model through comparison with literature, pull-in voltage is calculated for different conditions of microbeam. In addition, stability of the system is investigated for different non-dimensional voltage. The results will be useful in designing coupled-domain microbeams.

Key words: Microbeam, squeeze film damping, dynamic model, dimensionless groups.

INTRODUCTION

Micro electro mechanical systems (MEMS) have a lot of application recently. Their light weight, low cost, small dimensions, low-energy consumption and durability make them quite useful. Electrostatic actuation is an essential method of driving MEMS devices because of its second order gain, its compatibility with micro-fabrication technology and its high power density. Electrostatically actuated micro-structures are widely used for various applications and tools, such as acoustic resonators, radio frequency (RF) switches, pressure sensors, electrostatic step motors and thermal loading.

Many studies have been performed by researchers on the dynamic behavior of micro-structures. Veijola et al. (1999) presented a dynamic simulation model for a vibrating fluid density sensor. The model included electrostatic excitation, two torsional and two bending vibration modes of the double-loop structure and gas film damping. Miki et al. (2003) investigated high speed micro-rotors for power MEMS application to enhance rotor dynamics performance. Vibration of an angular-rate microgyroscope was simulated by Rajendran and Liewa (2004). Also, performance analysis of a vibrating beam microgyroscope under general support motion was presented by Esmaeili et al. (2007). An electric field sensor which uses a micromachined micro-spring supported membrane as the sensing element was presented by Roncin et al. (2005). Moghimi and Ahmadian (2007) investigated on vibrational behavior and dynamics of multi-layer microplates using coupled finite element and finite difference methods. Also, nonlinear oscillations of microbeams, actuated by electrostatic force, were investigated by Moghimi et al. (2009a). In addition, they (Moghimi and Ahmadian, 2009b) simulated the vibrational behavior of electrostatically actuated microplate subjected to nonlinear squeeze film damping and in-plane electrostatic forces. Also, using homotopy perturbation method, static pull-in analysis of electrostatically actuated microbeams was presented by Mojahedi et al. (2010). On the basis of the Euler-Bernoulli hypothesis, nonlinear static and dynamic responses of a viscoelastic microbeam under two kinds of electric forces (a purely direct current (DC) and a combined current composed of a DC and an alternating current) were studied by Fu and Zhang (2009). The effect of thermoelastic coupling on a micro-machined beam resonator was investigated by Guo and Rogerson (2003). Effect of geometric non-linearity on dynamic pull-in behavior of coupled-domain

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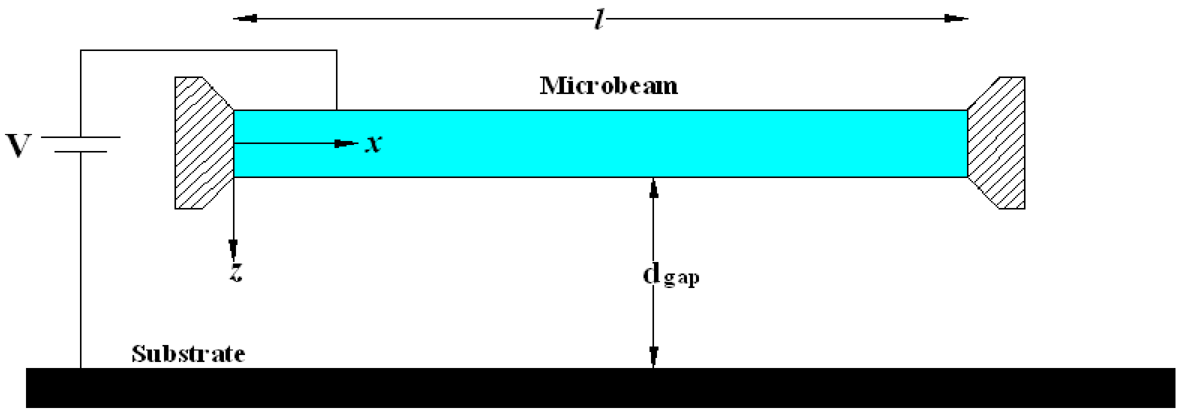

Figure 1. Schematic view of a double-clamped electrostatically actuated micro-beam.

microstructures based on classical and shear deformation plate theories is investigated by Tajalli et al. (2009). Also, analysis of one-dimensional deformations in a structural interface with micro-rotations was presented by Vasiliev et al. (2010). In this reference, the static and dynamic properties of a Cosserat-type lattice interface of finite thickness were studied. Design and analysis of a novel 2.4 GHz MEMS/NEMS voltage controlled oscillator was presented by Sreeja and Radha (2011). In this research, the variable capacitors and the inductors were designed using ANSYS and imported through data access component (DAC) in advanced design software (ADS).

The objective of this study is to develop a comprehensive model of electrostatically actuated micro-beams using dimensionless groups. Euler-Bernoulli beam theory is used to model dynamical system, while Reynolds equation is applied to model nonlinear squeeze film damping. Dimensionless model is used to verify microbeam pull-in results with the results published in the literature. Afterwards, vibrational analysis for the model is performed to compare results with literature. Using these certifications, an analysis for vibrational and dynamic behavior of dimensionless microbeams is presented.

EQUATIONS OF MOTIONS

Schematic view of electrostatically actuated doubly clamped micro-beam is depicted in Figure 1. Using Euler-Bernoulli beam theory, the nonlinear differential equation of microbeam can be expressed as (Meirovitch, 2001):

$$\rho A\frac{\partial^2 w(x,t)}{\partial t^2}+EI\frac{\partial^4 w(x,t)}{\partial x^4}=F(x,t) \tag{1}$$

In which $F(x,t)$ is the force per unit length of microbeam and contains electrostatic and squeeze film forces. The electrostatic force per unit length takes the following form (Chowdhury et al., 2005):

$$F_e(x,t)=\frac{1}{2}\frac{\varepsilon b V^2}{\left(d_{gap}-w(x,t)\right)^2}\left(1+\beta\frac{d_{gap}-w(x,t)}{b}\right) \tag{2}$$

Here, the term β accounts for the fringing fields effect due to the finite width of the beam. For a double-clamped beam $\beta=0.65$. The squeeze film damping occurs as a result of the movement of the fluid underneath the microbeam and can be modeled by compressible Reynolds equation. The Reynolds equation can be written as (Karniadakis et al, 2005):

$$\frac{\partial}{\partial x}\left[\alpha\left(d_{gap}-w\right)^3 p\frac{\partial p}{\partial x}\right]=12\mu\left[\left(d_{gap}-w\right)\frac{\partial p}{\partial t}-p\frac{\partial w}{\partial t}\right] \tag{3}$$

where α represents the rarefaction effects for air microflow and can be obtained from:

$$\alpha=1+6\frac{2-\sigma_v}{\sigma_v}Kn \tag{4}$$

So the squeeze film force per unit length can be obtained as:

$$F_s(x,t)=b\left(p_0-p(x,t)\right) \tag{5}$$

Investigation of effective parameters on microbeam vibration is difficult, because of many significant variables in Equations 1 to 5. By grouping significant quantities into dimensionless parameters, it is possible to reduce the number of variables and to make the compact results applicable to all similar situations. Using π-Buckingham theory (White, 2010), 9 dimensionless parameters are extracted for this problem. These parameters are presented in Table 1. Using these parameters, non-dimensional expression of Equations 1 and 3 can be expressed as:

$$\zeta\frac{\partial^4\Delta}{\partial X^4}+\Lambda\frac{\partial^2\Delta}{\partial T^2}=\Gamma(X,T) \tag{6}$$

Table 1. Dimensionless parameters of microbeam and squeeze film damping.

Notation	Definition	Notation	Definition	Notation	Definition
Δ	w/L	T	tp_0/μ	P	p/p_0
X	x/L	D	d_{gap}/L	B	b/L
ς	$EI/p_0 l^4$	Λ	$A\rho p_0/\mu^2$	Π	$\sqrt{\varepsilon/Ap_0}V$

$$\frac{\partial}{\partial X}\left[\alpha\left(D-\Delta\right)^3 P\frac{\partial P}{\partial X}\right]=12\left[\left(D-\Delta\right)\frac{\partial P}{\partial T}-P\frac{\partial \Delta}{\partial T}\right] \quad (7)$$

where

$$\Gamma\left(x,t\right)=\frac{1}{2}\frac{\Pi^2 B^2}{\left(D-\Delta\right)^2}\left(1+\beta\frac{D-\Delta}{B}\right)+B\left(1-P\right) \quad (8)$$

SOLUTION TECHNIQUE FOR DYNAMICAL ANALYSIS

The dimensionless vertical deflection Δ of the microbeam is obtained using modal superposition as:

$$\Delta=\sum_{i=1}^{N_c}\mathrm{N}\left(i,X\right)\mathrm{T}_i\left(T\right) \quad (9)$$

where $\mathrm{N}\left(i,X\right)$ is the *i*th mode shape functions of the dimensionless vertical deflection of the microbeam, $\mathrm{T}_i\left(T\right)$ is the *i*th mode time coefficients of the dimensionless vertical deflection and rotation of the microbeam and N_c is the number of modes considered. Substituting Equation 9 into Equation 6 will yield:

$$\Lambda\frac{d^2\mathrm{T}_i\left(T\right)}{dT^2}+\varsigma\beta^4\mathrm{T}_i\left(T\right)=\int_0^1\mathrm{N}\left(i,X\right)\Gamma\left(X,T\right)dX=\Xi_i\left(T\right) \quad (10)$$

Using finite difference method (FDM) the discrete form of Equation 10 is:

$$\mathrm{T}_i^{n+1}=\mathrm{T}_i^n+k\mathrm{H}_i^n$$
$$\mathrm{H}_i^{n+1}=\mathrm{H}_i^n-\frac{k\varsigma}{\Lambda}\beta^4\mathrm{T}_i^n+\frac{k}{\Lambda}\Xi_i^n \quad (11)$$

in which

$$\mathrm{H}_i\left(T\right)=\frac{d\mathrm{T}_i\left(T\right)}{dT} \quad (12)$$

and k is the time step and n represents properties of the *n*th time. Also, the discrete form of Equation 7 can be expressed as:

$$P_j^{n+1}=P_j^n+\frac{k}{12\left(D-\Delta_j^n\right)}\left[-3\alpha\frac{\Delta_{j+1}^n-\Delta_j^n}{h}\left(D-\Delta_j^n\right)^2P_j^n\frac{P_{j+1}^n-P_j^n}{h}+\alpha\left(D-\Delta_j^n\right)^3\left(\frac{P_{j+1}^n-P_j^n}{h}\right)^2\right.$$
$$\left.+\alpha\left(D-\Delta_j^n\right)^3P_j^n\frac{P_{j+1}^n-2P_j^n+P_{j-1}^n}{h^2}+P_j^n\frac{\Delta_j^{n+1}-\Delta_j^n}{k}\right] \quad (13)$$

where h is dimensionless length step and j represents properties of the *j*th node. Using Equation 13, air pressure of each node at each time step can be calculated. Also, the vertical deflection and velocity of each node at each time step can be obtained using Equations 9 and 10. A computer code developed in MATLAB software is used to simulate microbeam vibration by applying this approach.

SIMULATION RESULTS AND DISCUSSION

Mid-point deflection of undamped microbeam

Pull-in voltage is a critical voltage in which the system becomes unstable. In other words, if the applied voltage reaches this critical value, there will be no static equilibrium in the system. Using the computer code developed earlier, time history of mid point deflection of an undamped microbeam is plotted in Figure 2a. According to this figure, non-dimensional pull-in voltage ($\Pi_{pi}=\sqrt{\varepsilon/Ap_0}V_{pi}$) for this microbeam is 0.13863. Also, the results obtained by Krylov (2007) are presented in Figure 2b. Comparing these figures, show high performance of dimensionless model in simulation of microbeam vibration.

Mid-point deflection of microbeam with squeeze film air damping

Time history of a microbeam vibration is plotted in Figure 3a considering the compressible squeeze film model. This figure can be compared with Figure 3b which is the results presented by Krylov (2007). This comparison shows compatibility of presented model in this paper with the results presented by Krylov (2007) that used the Galerkin decomposition with undamped linear modes as base functions.

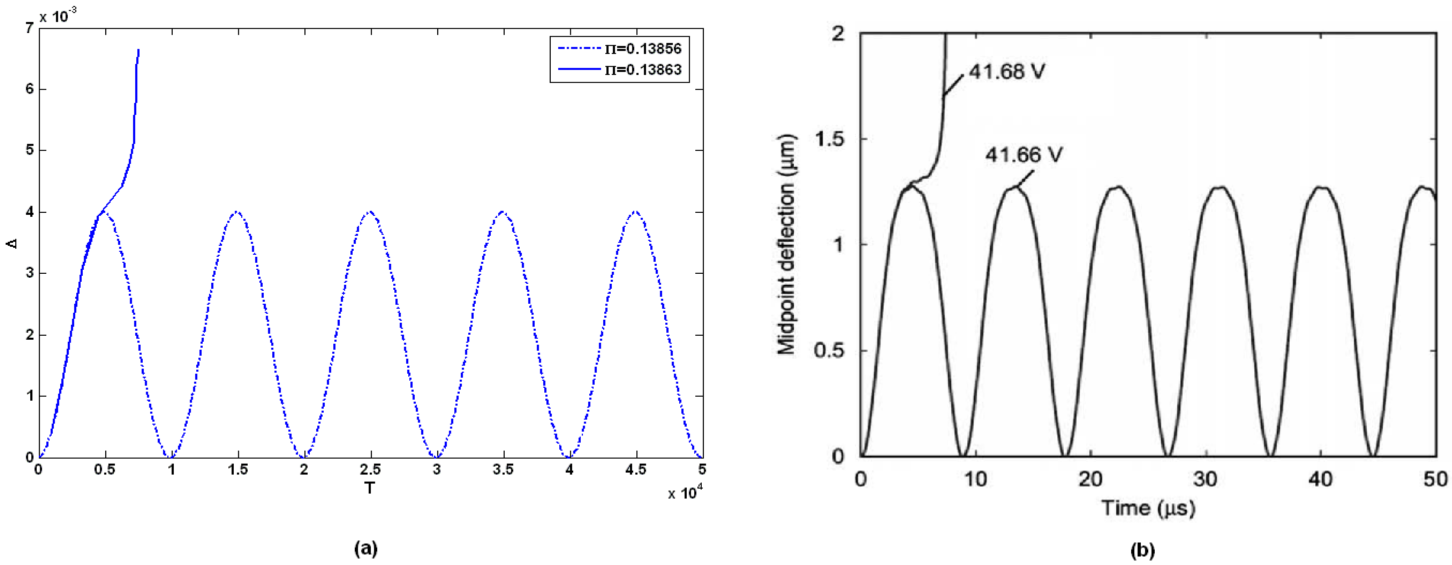

Figure 2. Midpoint deflection of undamped microbeam.

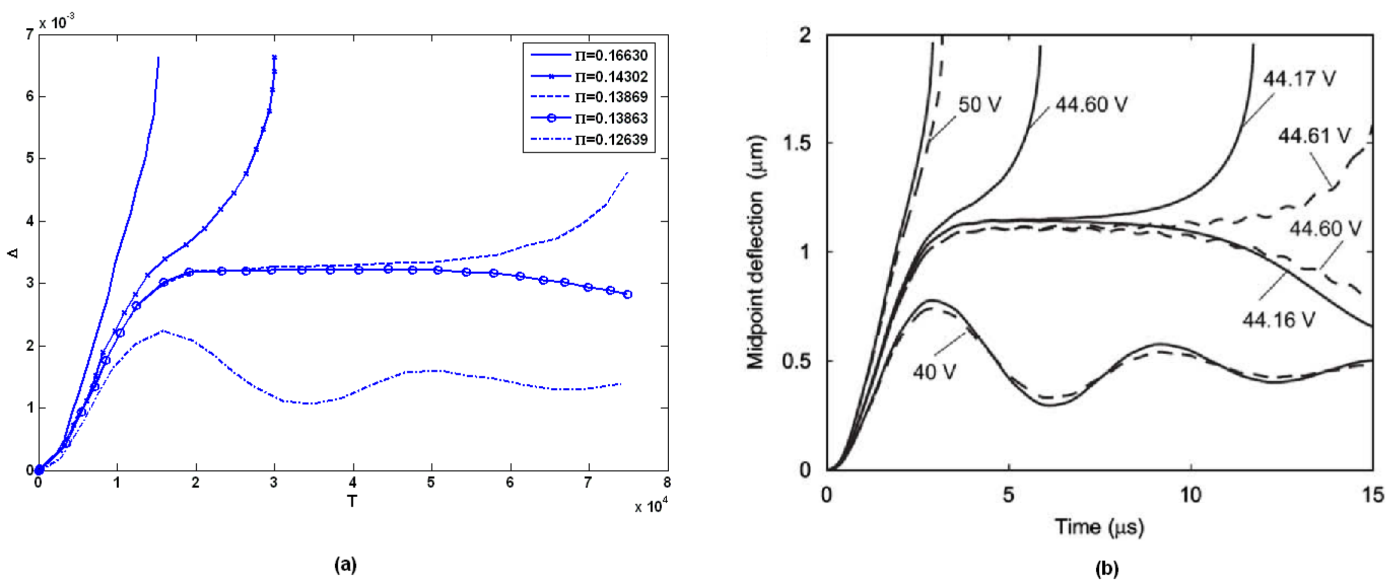

Figure 3. Midpoint deflection of microbeam with squeeze film air damping.

The pull-in analysis of microbeam

After validation of presented model through comparison with literature, vibrational analysis of the microbeam for different applied voltage is performed and pull-in voltage is calculated. Variations of non-dimensional pull-in voltage (Π_{pi}) versus non-dimensional microbeam width ($B = b/L$) for different initial air gaps ($D = d_{gap}/L$) are presented in Figure 4.

The phase portrait is plotted for non-dimensional voltage ($\Pi = 0.12639$) in Figure 5 for several cycles. As shown in this figure, the system is stable in this voltage. In addition, the phase portrait for an input ($\Pi = 0.13869$) is plotted in Figure 6. This figure shows dynamic instability in this voltage.

Conclusion

In this paper, the nonlinear dynamic responses of a dimensionless electrically actuated microbeam under

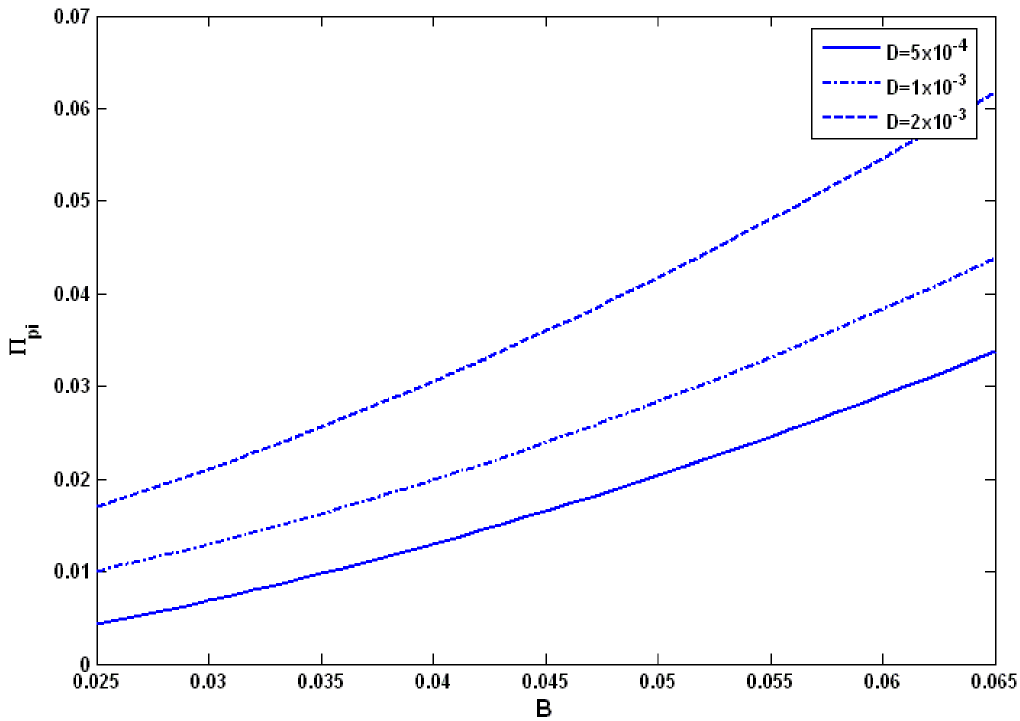

Figure 4. Non-dimensional pull-in voltage versus non-dimensional microbeam width.

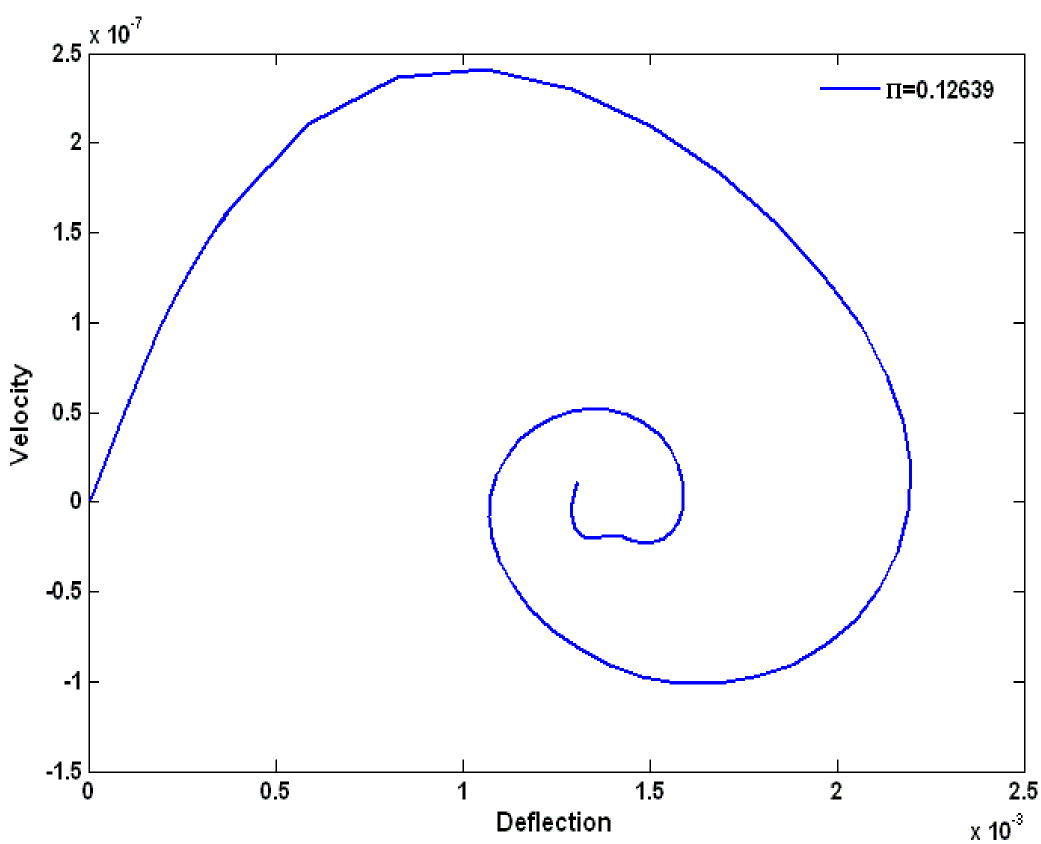

Figure 5. The phase portrait for non-dimensional voltage $\Pi = 0.12639$.

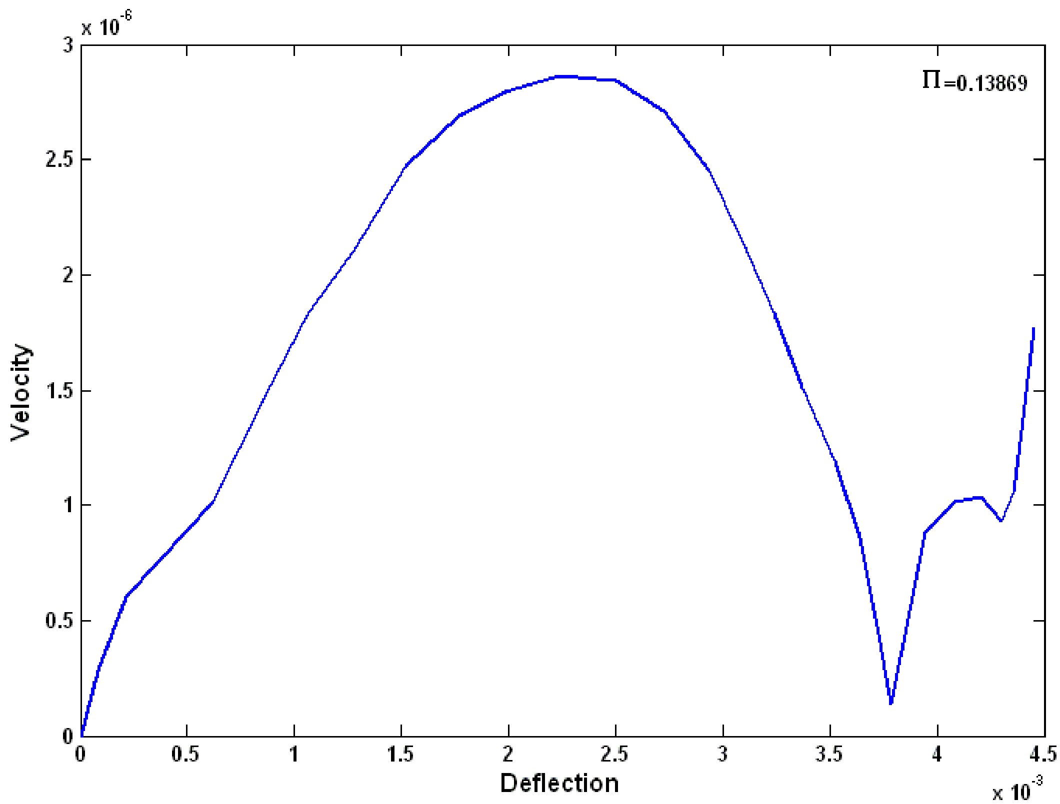

Figure 6. The phase portrait for non-dimensional voltage $\Pi = 0.13869$.

suddenly applied DC voltage have been studied. The model has been developed to consider nonlinear electrostatic actuation and nonlinear squeeze film damping. Using the developed model in this article, pull-in analysis of microbeams is performed and results are in good agreement with literature. Considering nonlinear dimensionless Reynolds equation and electrostatic actuation, effect of non-dimensional microbeam width and initial air gap on the non-dimensional pull-in voltage has been studied. Also, vibrational behavior of microbeams has been investigated and pull-in voltage is calculated for different conditions of microbeam. This model will be useful in designing coupled-domain microbeams.

Nomenclature: ρ, Microbeam density; A, microbeam cross-sectional area; E, Young's module of elasticity; I, moment of inertia; w, microbeam deflection; L, microbeam length; d_{gap}, initial air gap; b, micro-beam width; P_0, ambient pressure; P, squeez film air pressure; μ, coefficient of the air viscosity; ε, electrical permittivity; V, applied actuating voltage; Kn, Knudsen number; σ_v, tangential momentum accommodation coefficient.

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