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Full Length Research Paper

A multiresolution approach for color image inpainting using an efficient generalized expectation maximization (GEM) model

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In this paper, we present a novel expectation maximization (EM) algorithm for automatic color image inpainting using a new discrete multi scale directional sparse representation called the discrete shearlet transform (DST). It is now acknowledged that the traditional wavelets are not very effective when dealing the multi dimensional signals having distributed discontinuities such as edges. To achieve a more efficient representation, one has to use basis elements with much higher directional sensitivity. Using a shearlet transform combines the power of multi scale methods with a unique ability to capture the geometry of multidimensional data and is optimally efficient in representing images with edges. The inpainting can be viewed as an interpolation or estimation problem with missing data. Towards this goal, we propose the idea of using expectation maximization (EM) algorithm in a Bayesian Framework, which is used to recover the missing samples using a sparse representation-discrete shearlet transform (DST). We first introduce an easy and efficient sparse representation-discrete shearlet transform (DST) based iterative algorithm for image inpainting. Then, we derive its convergence properties. We can demonstrate that this algorithm based on a new sparse representation-discrete shearlet transform is very competitive in image inpainting applications both in terms of performance and computational efficiency.

Key words: Sparse representation, wavelet, image inpainting, optimization.

INTRODUCTION

Image inpainting (Tony et al., 2006) refers to filling in missing or damaged regions (like cracks or scars) in images. In fine art museums, inpainting of degraded paintings is traditionally carried out by professional artists and usually very time consuming, not to mention the risk of completely destroying a precious and world-unique ancient painting due to direct retouching.

Mathematically speaking, inpainting is essentially an interpolation problem, and thus directly overlaps with many other important tasks in computer vision and image processing, including image replacement, disocclusion, zooming, super resolution and error concealment. The current work has been motivated and inspired by the error concealment application, which is to automatically recover lost packets information during transmission processes.

After the release of the new image compression standard JPEG2000, which is largely based on wavelet transforms (Fadill et al., 2007), many images are formatted and stored in terms of wavelet coefficients. In the wireless communication of these images, it could happen that certain wavelet packets are randomly lost or damaged during the transmission process. Recovering original images from their incomplete wavelet transforms is an inpainting problem. But this task remarkably differs from the classical inpainting problems in that the inpainting regions are in the wavelet domain.

Inpainting in wavelet domain (Raymond et al., 2009) or using a sparse representation (Fadill et al., 2007) is a completely different problem since there are no well

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defined inpainting regions in the pixel domain.

In the work of Fadill et al. (2007), the authors used the dictionary which contains the wavelet transform and other directional transforms (Curvelet, Ridgelet). These traditional wavelets are not effective when dealing multidimensional signals containing distributed discontinuities such as edges. This approach, however often leads to the formation of Gibbs-type artifacts around sharp discontinuities, due to the elimination of small wavelet coefficients that should have retained.

Although, new wavelet extensions such as curvelets and contourlets have a better approximation rate, they may also suffer from the same type of effects. Choosing an appropriate dictionary is a key step towards a good sparse representation hence inpainting and interpolation. Also, their work does not deal with the multivalued images. Therefore in the proposed work, a novel directional multiscale mathematical framework (Shearlets) (Kutyniok and Labate, 2009; Gao et al., 2009; Glenn et al., 2008; Glenn et al. 2009) is applied. The proposed work can also be used with the multi valued images.

TV model is the powerful tool for color image inpainting (Donald and Chengda, 1995; Leonid et al., 1992) and greatly reduce these Gibbs artifacts. But the TV models suffer from the stair case effect. That is, the smooth regions (ramp) are transformed into piece wise constant regions (stairs).

In the work of Ingrid et al. (2003), the authors used the operator ‘K’ which must be implemented sparsely. If the operator is not sparsed perfectly then iteration may be too heavy. Also, authors used variational approaches as thresholding techniques for sparsifying the wavelet expansions of noisy signals in order to remove the noise. The main drawback of these variational approaches is that they are limited to special kind of operators ‘K’.

In this paper, to overcome these deficiencies, a new generalized expectation maximization (GEM) model is introduced with the new tight frame of shearlets. A key feature is that the discrete shearlet transform has many flexible attributes that lead to better stability and reduced Gibbs type artifacts.

The EM algorithm formalizes the idea of replacing the missing data by estimated ones from coefficients of previous iteration, and then reestimates the new expansion coefficients from the complete formed data, and iterates the process until convergence (Jeff, 1983). We here restrict ourselves to zero-mean additive white Gaussian noise, even if the theory of the EM can be developed for the regular exponential family. In this proposed model, the expectation maximization algorithm is superior to that of TV model (Gaohang et al., 2009; Jerome and Marc, 2006; Marius and Xue, 2006; Antonon, 2004).

We shall demonstrate that our method based on combining shearlets with expectation maximization algorithm using Bayesian framework performs better than the traditional wavelet technique. Furthermore, the number of iterations is significantly reduced.

This paper is arranged as follows. Subsequently, we give a brief overview of the discrete shearlet transform (DST). Thereafter, the brief analysis of Expectation Maximization Algorithm is given. Followed by the GEM algorithm for image inpainting using a sparse representation DST. Afterward, the convergence properties of inpainting method in shearlet domain are presented. Then, numerical experiments are given to illustrate efficiency of the proposed method. Finally, the paper was concluded.

DISCRETE SHEARLET TRANSFORM (DST)

It is now widely acknowledged that traditional wavelet methods do not perform as well with multidimensional data. Indeed wavelets are very efficient in dealing with point wise singularities only. In higher dimensions, other types of singularities are usually present or even dominant, and wavelets are unable to handle them very efficiently. Images, for example, typically contain sharp transitions such as edges, and these interact extensively with the elements of the wavelet basis. As a result, many terms in the wavelet representation are needed to accurately represent these objects. In order to overcome this limitation of traditional wavelets, in this paper, a new wavelet transform is introduced, namely shearlet (Kutyniok and Labate, 2009; Gao et al., 2009; Glenn et al., 2008).

The shearlet approach which we proposed here, is particularly designed to have deal with directional and anisotropic features typically present in natural images, and has the ability to accurately and effectively capture the geometric information of edges. The number of orientations in shearlet construction doubles at each scale, while in the Curvelets case it doubles at each other scale.

The shearlets are defined on the Cartesian domain and the various directions are obtained from the action of shearing transformations. By contrast, the Curvelets are constructed in the polar domain and the orientations are obtained by applying rotations. Shearlet approach can be associated to a multiresolution analysis but the other directional transforms does not support this multiresolution analysis. When we use shearlets, there are no restrictions on the number of directions for the shearing and there are no constraints on the size of the supports for the shearing.

Continuous shearlet transform (DST)

The continuous shearlet transform is a non isotropic version of the continuous wavelet transform with a superior directional sensitivity. In dimension n=2, this is defined as the mapping.

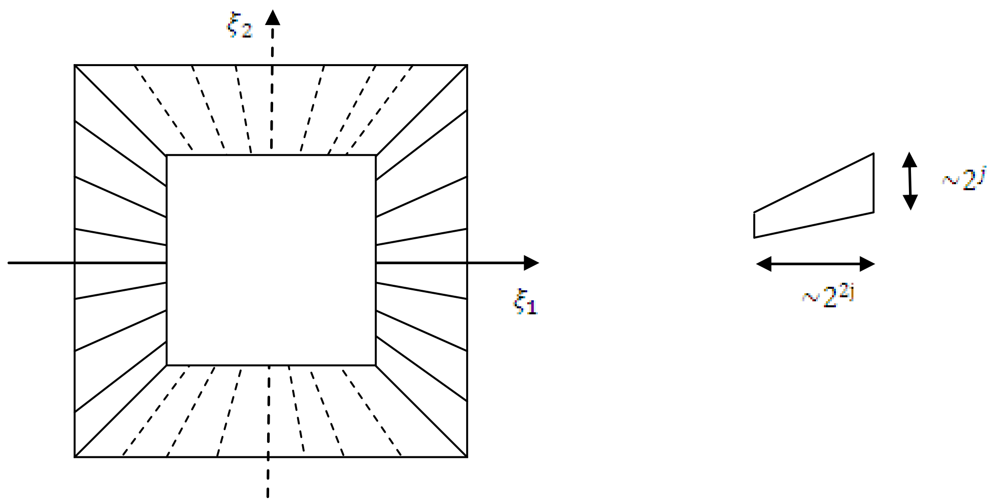

Figure 1. Frequency support of shearlets for various values of a and s.

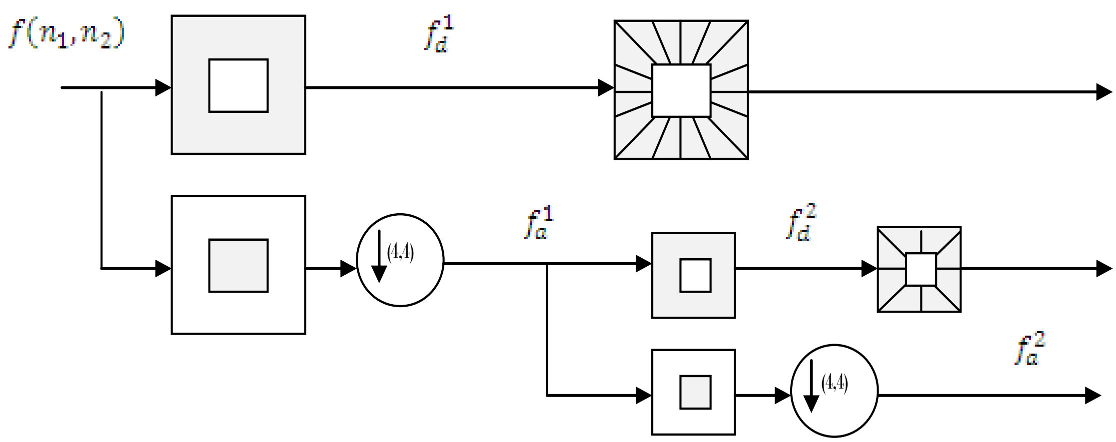

Figure 2. The figure illustrates the succession of Laplacian pyramid and directional filtering.

$$SH_{\psi} f(a,s,t) = \langle f, \psi_{a,s,t} \rangle \tag{1}$$

Each analyzing elements $\psi_{a,s,t}$ called shearlets has a frequency support on a pair of trapezoids, at various scales, symmetric with respect to the origin and oriented along a line of slope s. The support becomes increasingly thin as $a \to 0$. As a result, the shearlets form a collection of well-localized waveforms at various scales, orientations and locations, controlled by a, s and t, respectively. The frequency supports of some representative shearlets are illustrated in Figure 1.

Discrete shearlet transform (DST)

By sampling, the continuous shearlet transform on appropriate discretizations of the scaling, shear, and translation parameters a, s, t one obtains a discrete transform which is associated to a Parseval (tight) frame for $L^2 R^2$ (Peter and Edward, 1983). The construction of shearlet transform procedure is illustrated on Figure 2 and Table 1.

GENERALIZED EXPECTATION MAXIMIZATION (GEM) ALGORITHM

Let us now turn to the missing data case and let us write $Y = (Y_{obs}, Y_{miss})$ with $Y_{miss} = \{y_i\}_{i \in I_0}$ is the missing data, and $Y_{obs} = \{y_i\}_{i \in I_0}$. The incomplete

Table 1. Construction of Discrete Shearlet Transform.

S/N	Algorithm 1
1	Apply the laplacian pyramid scheme to decompose f_a^{j-1} into a low pass image f_a^j and a high pass image f_d^j
2	Compute $P.\ f_d^j$ on a pseudo polar grid.
3	Apply a band pass filtering to the matrix $P.\ f_d^j$
4	Directly re-assemble the Cartesian sampled values and apply the inverse two-dimensional FFT

observations do not contain all information to apply standard methods to solve the problem equations. Nevertheless, the EM algorithm can be applied to iteratively reconstruct the missing data and then solve the equation for the new estimate. The estimates are iteratively refined until convergence. Recall that the EM algorithm is a means of obtaining MAP/PMLE estimates (of which maximum likelihood is a particular case) of a parameter in cases where the PMLE is hard to obtain. The (Bayesian) EM algorithm will then produce a sequence of estimates alternating between two steps:

E-step: Computes the conditional expectation of the log likelihood of the complete data, given the observed data and the current estimate $\theta^{(t)}$, by defining the surrogate function:

$$Q\left(\theta/\theta^{(t)}\right)=E\left[\ell\ell\left(y/\theta\right)Y_{obs,}\theta^{(t)}\right]-\lambda\psi\left(\alpha\right) \tag{2}$$

M-step: Updates the estimates according to:

$$\theta^{(t+1)}=\underset{\theta\in\Theta}{\arg\min}-Q\left(\theta/\theta^{(t)}\right) \tag{3}$$

The E step: for regular exponential families, it is known that the E step involves finding the expected values of the sufficient statistics of the complete data Y given observed data Y_{obs} and the estimate of $\alpha^{(t)}$ and σ^2 (Fadill et al., 2009; Donald and Chengda, 1995). Then, as the noise is zero-mean white Gaussian, the E-step reduces to calculating the conditional expected values and the conditional expected squared values of the missing data, that is:

$$y_i^{(t)}=E\left(y_i/\phi,Y_{obs,},\alpha^{(t)},\sigma^{2(t)}\right)=\begin{cases}y_{obs_i}, & for\ observed\ data, i\in I_0\\ \phi_i^T\alpha^{(t)}, & for\ missing\ data, i\in I_m\end{cases} \tag{4}$$

and

$$E\left(y_i^2/\phi,Y_{obs},\alpha^{(t)},\sigma^{2(t)}\right)=\begin{cases}y_{obs_i}^2 & i\in I_0\\ \left(\phi_i^T\alpha^{(t)}\right)^2+\sigma^{2(t)}, & i\in I_0\end{cases} \tag{5}$$

The M step consists in maximizing the penalized surrogate function with the missing observations replaced by their estimates in the E step at iteration t, that is:

$$\theta^{(t+1)}=\underset{\alpha\sigma^2}{\arg\min}\ \frac{1}{2\sigma^2}\left\|Y^{(t)}-\phi\alpha\right\|_2^2+\lambda\psi\left(\alpha\right)+\frac{n}{2}\log 2\pi\sigma^2 \tag{6}$$

Thus, the M step updates $X^{(t+1)}$ and σ^2 according to:

$$X^{(t+1)}=\phi D\phi^{+}Y^{(t)} \tag{7}$$

$$\sigma^{2(t+1)}=\frac{1}{n}\left[\Sigma_{i\in i_0}\left(y_i-x_i^{(t)}\right)^2+\left(n-n_0\right)\sigma^{2(t)}\right] \tag{8}$$

Where $n_0=trM=CardI_0$ is the number of observed pixels. D denotes whichever estimation operation, associated to the penalty function $\psi(\alpha)$, applied to the expansion coefficients in ϕ. Note that at convergence, we have the noise variance inside the mask (that is, with observed pixels). If the noise variance is known in advance, the re-estimation of σ^2 in the M step can be ignored.

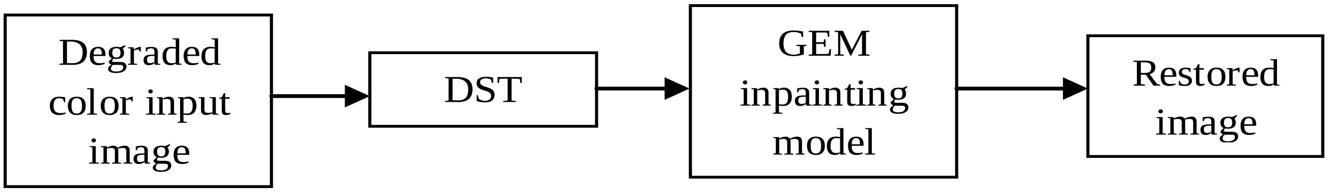

Figure 3. The framework of our proposed scheme (DST + GEM Model for Inpainting).

A very important feature of the M step is that it involves a denoising operation depending on the choice of thepenalty function. For example, under the l_1 norm penalization and an orthogonal dictionary $\alpha^{(t+1)}$ can be simply estimated using the well known soft thresholding scheme. This can also be extended to redundant representations as we will see later. Other prior penalties will lead to different estimation rules in the M-Step.

PROPOSED SCHEME: DST BASED GEM INPAINTING MODEL

The frame work of our proposed scheme is illustrated in Figure 3. An image with continuous color and intensity levels can be treated as a two dimensional object. Colors are treated as fluid that flow (or) diffuse from the surrounding areas into the empty region.

In order to deal with color images, RGB images are turned into a Luma/Chrominance representation e.g., YC_rC_b or Lab, and each channel is processed independently. The received degraded color input image is subjected with discrete shearlet transform (DST). Then, the shearlet transformed image is inpainted using generalized expectation maximization (GEM) model. The algorithmic view of the whole model is described in Table 2.

CONVERGENCE PROPERTIES OF GEM INPAINTING MODEL IN SHEARLET DOMAIN

In Jeff (1983) there is no simple general result for the EM that guarentees convergence to a local or global minimum without further assumptions. For instance, if the penalty function is convex, then the penalized log likelihood will be strictly convex and the EM algorithm will be guaranteed to converge to the unique maximum penalized likelihood value and a unique optimal image. But, if the penalty function is not convex then the sequences of the Bayesian EM estimates will only converge to a stationary point. The image at convergence will depend on the initialization of the algorithm.

As noted in the work of Fadill et al. (2009), the convergence rate (at least when the initial position is not too far from the true image) is linear and governed by the fraction of missing information, that can be evaluated from the Fisher information matrix. Thus, here we decide to use the observed part of the image Y_{obs} as an initial estimate.

Many interesting penalties that produce sparse solutions are non-convex or even non-smooth. Unfortunately, their use will be at the price of no guaranty to converge to a global or even to a local minimum. To circumvent this major drawback, we use the same heuristic argument as that of the MCA, for which we give a statistical interpretation.

Indeed, one can consider the penalized log likelihood functional of the M-step as Gibbs energy, where the regularization parameter parallels the temperature in the same spirit as for simulated annealing. Then, we start at a high temperature (that is, λ) and then decrease λ according to a prescribed schedule (e.g. exponential or linear). For each value of λ, we run one iteration of the GEM inpainting Algorithm. This algorithm has flavor of a stochastic version of the EM.

PERFORMANCE COMPARISON

In previous work, the inpainting algorithm based on p-Laplacian (Zhang et al., 2007; Shibo 2001) with TV model was applied to several synthetic and real degraded images, from which we present few examples. Figure 4 depicts an example on real old degraded photograph. The dictionary contained the shearlet transform and the convex l_1 penalty was used. The threshold parameter was fixed to the classical value 3σ.

From the resulting pictures shown in Figure 4, we can see that out shearlet inpainting model based on the p-Laplacian operator can dramatically improve the image quality better than TV wavelet inpainting model, especially in the large number of damaged coefficients.

We illustrate the performance of the proposed algorithm for image inpainting in shearlet domain using GEM model and compare it with the image inpainting method proposed by Liu et al. (2007). Our codes are written in MATLAB 2008a.

We test our inpainting systems on a number of standard color images from the USC-SIPI image data base and Kodak image library. Some results are

Table 2. Shearlet domain inpainting with GEM Model.

Algorithm 2
1. Get the degraded color image as an input image
2. Using the input image obtained in step (1), obtain a Discrete Shearlet Transformed image
3. Apply the GEM Inpainting Model by using the following steps to get a restored image
Require : Observed image Y_{obs} and a mask M , Convergence threshold δ
3.1 Repeat
3.2 E Step
3.3 Update the image estimate
$Y^{(t)} = Y_{obs} + (I - M)X^{(t)}$
3.4 M Step
3.5 For Each transformation k in the dictionary do
Calculate the transform coefficients
$\phi_k^{+}\left(Y^{(t)} - \sum_{k' \neq k} \phi_{k'} \alpha_{k'}^{t}\right)$
3.6 Apply the estimation operator (soft thresholding) to these coefficients and get $\alpha_k^{(t+1)}$
3.7 End for
3.8 Update $X^{(t+1)} = \sum_k \phi_k \alpha_k^{(t+1)}$
3.9 Update $\sigma^{2^{(t+1)}}$ according to (3.8).
3.10 Until Convergence $\frac{\left\| X^{(t+1)} - X^{(t)} \right\|_2}{\left\| X^{t} \right\|_2} \leq \delta$

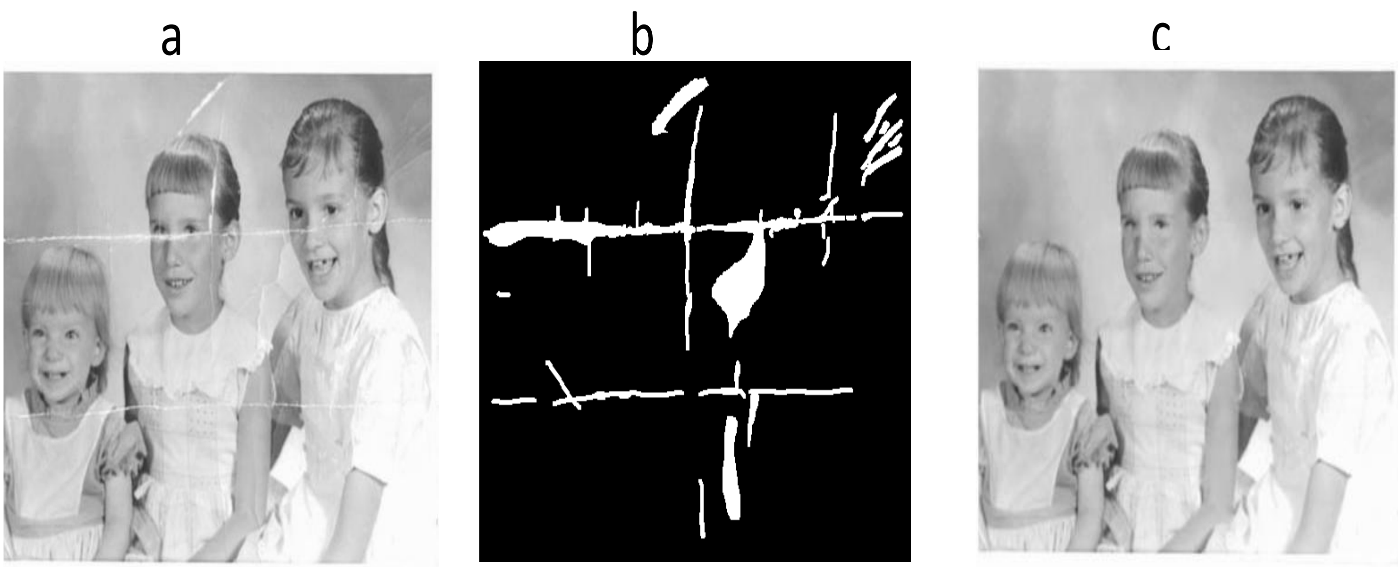

Figure 4. (a) Children (b) Separated inpainting region (c) Inpainted image with the TV + p-Laplacian.

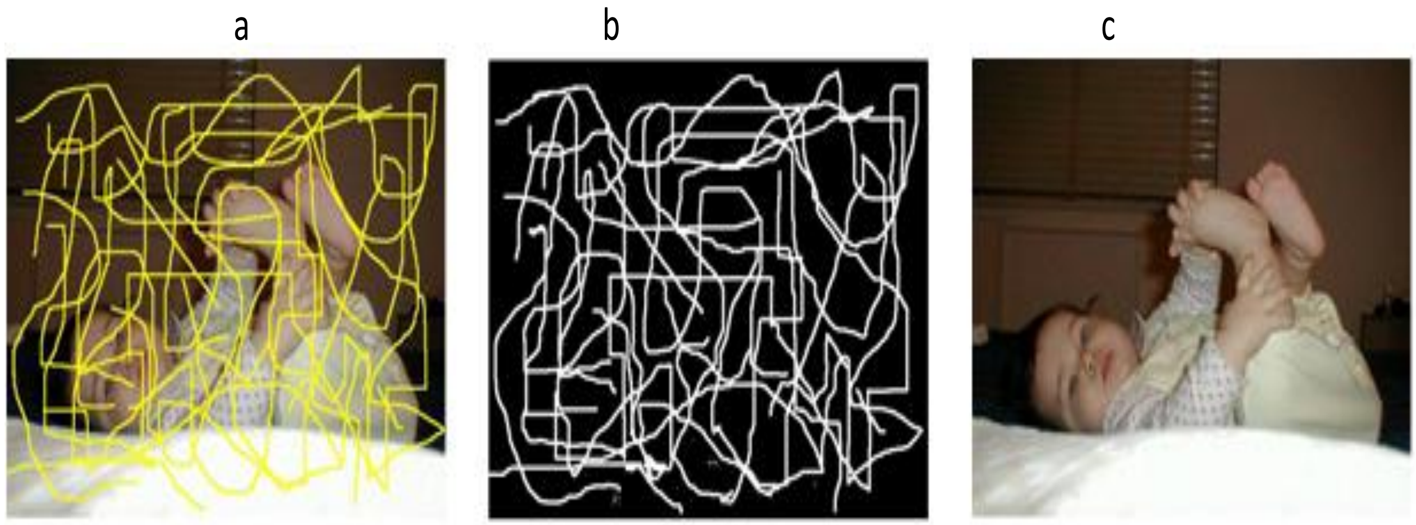

Figure 5. (a) Baby (b) Separated Inpainting Region (c) Inpainted image with the GEM Model in Shearlet Domain.

Table 3. PSNR values of our scheme compared to edge based inpainting and p-Laplacian inpainting.

Original image	Edge based inpainting	p-Laplacian inpainting	Our scheme
Baby	24.34	28.76	35.56
Seawater	34.67	35	45.67
Festival	12.23	15.5	25.5
Kodim01	45.78	45.99	56.67
Kodim03	34.87	35	48.9
Kodim22	22.13	24.23	32.1
Kodim23	62.11	62.11	65.23
Kodim08	18	18	21.23
Yard	50	51.1	65.12
girl	25.65	25.7	37.8

presented here to evaluate the peak signal to noise ratio (PSNR) as well as reconstructed quality of our scheme. In all testes, we use shearlet base GEM model inpainting. Peak signal to noise ratio (PSNR) is also used to measure the quality of the restored images. It is defined as follows:

$$PSNR = 10\log_{10}\left(\frac{\max_I^2}{MSE}\right)$$

Where max_I is the maximum possible pixel value of the image. Figure 5 shows test image baby and corresponding results of our proposed system. Compared with the p- Laplacian operator inpainting and block based inpainting (Ignacio and Jung, 2007), the proposed GEM model gives good PSNR values with similar visual quality levels. Table 3 summarizes the PSNR values for different images which ensure the reproducibility of the results. Figures 6, 7 and 8 illustrate different kinds of features found in actual photographs.

Figure 6 shows a 640×480-pixel photograph exhibiting uncorrelated high frequencies represented by the leaves of the trees. It was superimposed with a textual mask (18 pt font size) covering 18.77% of its original area. The restored image, obtained in 6.37 s, essentially recovers all details of the original picture. Notice, for instance, the children playing in the back.

Figure 7 shows a 640 × 480 - pixel image containing very few high contrast edges, but with 14.54% of its area scratched. The image shown on its right was recovered in 5.87 s. Finally, Figure 8 shows a Seawater scene (512 × 384 pixels) containing a large number of high contrasts edges and superimposed with a mask covering 16.19% of its area. Figure 7 (right) was reconstructed in 4.06 s.

Figure 6. Yard: Image containing uncorrelated high frequency with text covering 18.77% of its area. Right: restored image obtained with our algorithm.

Figure 7. Festival: Image containing few high contrast edges. Mask covers 14.54% of its area. Right: restored image.

Figure 8. Seawater. Left: Image containing many high contrast edges, with text covering 16.19% of its area. Right: restored image.

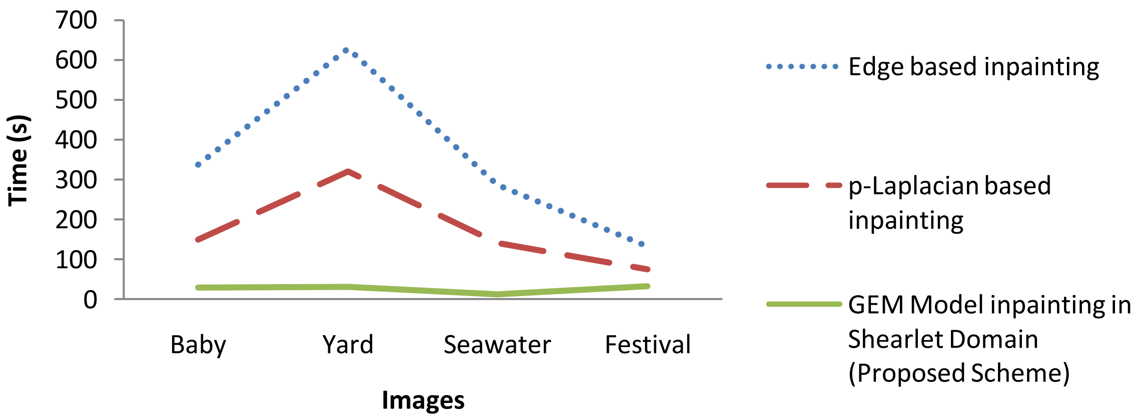

Figure 9. Plot of time (s) required for implementing algorithm vs. Images.

Notice that such a reconstruction is mostly fine, except for some disconnected branches on the top right. Due to the relatively small scale of some of the masked branches, p-Laplacian inpainting techniques are also likely to fail to connect these edges.

From Figure 9 we can easily observe that the proposed scheme gives best results and the results produced by this algorithm are comparable to those obtained by edge based inpainting and p-Laplacian inpainting but faster, that is, required too little time to produce the results.

Conclusion

In this paper, a new discrete shearlet transform domain inpainting model based on generalized expectation maximization (GEM) for restoring arbitrary number of coefficients in arbitrary locations of shearlet coefficients for images with noise is presented. Comparing the proposed model to TV wavelet p Laplacian inpainting model, the better inpainting quality with much less computing time is achieved, especially with large number of damaged wavelet coefficients.

Experimental results using many standard color images validate the ability of our proposed scheme in achieving higher PSNR values while preserving good visual quality. Compared to edge based inpainting and p -Laplacian inpainting, at the similar visual quality levels, maximum PSNR value of 65.23 can be acquired by our approach.

REFERENCES

Antonon C (2004). An algorithm for total variation minimization and applications. J. Math. Imaging Vision, 20: 89-97.

Jeff CFW (1983). On the Convergence Properties of the EM algorithm. Anal. Stat., 11(1): 95-103.

Donald G, Chengda Y (1995). Nonlinear image recovery with half quadratic regularization. IEEE Trans. Image process., 4(7): 932-946.

Liu D, Sun X, Wu F, Li S, Zhang YQ (2007). Image compression with edge based inpainting. IEEE Trans circuits Syst. video Technol., 17(10): 1273-1280.

Kutyniok G, Labate D (2009). Resolution of the wavefront set using continuous shearlets. Trans. Am. Math. Soc., 361: 2719-2754

Gaohang Yu, Liqun Qi, Yuhong D (2009). On nonmonotone chambolle gradient projection algorithms for total variation image restoration. J. Math. Imaging Vis., 35: 143-154.

Glenn E, Demetrio L, Wang QL (2008). Sparse directional image representation using discrete shearlet transform. Appl. Comput.Harmon. Analy., 25(1): 25-46.

Glenn R.Easley, Demetrio Labate, and Flavia Colonna (2009). Shearlet based total variation diffusion for denoising. IEEE Trans. Image Process., 18(2): 260-269.

Ingrid D, Michel D, Christine DM (2003). An iterative Thresholding algorithm for linear inverse problems with a sparsity constraints. J. Math. FA. V2:1-30.

Jerome D, Marc S (2006). Image restoration with discrete constrained total variation part I: Fast and exact optimization. J. Math. Imaging, 26: 261-276.

Gao K, Labate D, Lim W (2009). Edge analysis and identification using the continuous shearlet transform. Appl. Comput. Harmon. Anal., 27(1): 24-46.

Leonid IR, Stanley O, Emad F (1992). Non linear total variation based noise removal algorithms. Physica D, 60:259-268.

Fadill MJ, Starck JL, Murtagh F (2007). Inpainting and Zooming Using Sparse Representation. Comput. J. Adv. Access Published. 1:16.

Fadill MJ, Starck JL, Elad M, Donoho DL (2009). MCA Lab: Reproducible Research in Signal and Image Decomposition and Inpainting. J. Comput. Sci. Eng. 12(1): 44-63.

Marius L, Xue CT (2006). Iterative image restoration combining total variation minimization and a second order functional. Int. J. Comput. Vision, 66(1): 5-18.

Peter JB, Edward HA (1983). The lapalcian pyramid as a compact image code. IEEE Trans. Commun., 31(4): 532-540.

Raymond HC, You-Wei W, Andy MY (2009). A Fast optimization transfer algorithm for image inpainting in wavelet domain. IEEE Trans. Image Process., 18(7): 1467-1477.

Shibo Liu (2001). Existence of solutions to a superlinear p laplacian equation. Electron. J. Differential Equations, 2001(66): 1-6.

Tony FC, Jianhong S, Hao MZ (2006). Total variation wavelet inpainting. J. Math. Imaging Visual, 25: 107-125.

Ignacio UA, Jung CR (2007). Block based image inpainting in the wavelet domain. J. Visual Comput., 23: 733-741.

Zhang Hong Ying, Peng Qi cong, WU Yang dong (2007). Wavelet inpainting based on laplacian operator. Automatica Sinica. 33(5): 546-549.