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Full Length Research Paper

A remark on the classifications of rhotrices as abstract structures

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This paper presents additional classifications of rhotrices as abstract structures of Ring, Field, Integral Domain, Principal Ideal Domain and Unique Factorization Domain, as a sort of an additional work to our earlier classifications of rhotrices as algebraic structures of Groups, Semi groups, Monoids and Boolean Algebra. Rhotrix is a new paradigm of matrix theory, concerned with representing arrays of real numbers in mathematical rhomboid form, as an extension of ideas on matrix-tertions and matrix noitrets proposed by Atanassov and Shannon.

Key words: Rhotrices, ring, field, integral domain, principal ideal domain, unique factorization domain.

INTRODUCTION

The interest to stimulate ideas on how to imagine new forms of mathematical rhomboid arrays and their classify-cations as abstract structures of Groups, Semi-groups, Monoids, and Boolean Algebra was first proposed by Mohammed (2007a) as part of a note on enrichment exercises through extension to rhotrices. Thus, the purpose of this paper is to outlines development of abstract structures of Ring, Field, Integral Domain (ID), Principal Ideal Domain (PID) and Unique Factorization Domain (UFD) of rhotrices, as a remark to the earlier work discussed in Mohammed (2007a)

The initial concept and analysis of rhotrix was introduced by Ajibade (2003), as objects which are, in some ways, between (2x2) –dimensional and (3x3)-dimensional matrices, defined by

$$R = \left\{ \left\langle \begin{array}{ccc} & a & \\ b & c & d \\ & e & \end{array} \right\rangle : a, b, c, d, e \in \Re \right\} \tag{1.1}$$

Where; $h(R) = c$ is called the heart of R. Extension in the size of R was considered possible but of interest (Ajibade, 2003) is three dimensional rhotrix. It is worthy noticing that an n-dimensional rhotrix R, denoted by R_n, will have entries as $card(R_n)$, where, $card(R_n) = \frac{1}{2}(n^2+1)$, and $n \in 2Z+1$. This implies that all rhotrices are of odd dimension. For example, a rhotrix S of dimension 7 is given by

$$S = \left\langle \begin{array}{ccccccc} & & & a & & & \\ & & b & c & d & & \\ & e & f & g & h & i & \\ j & k & l & h(S) & n & o & p \\ & q & r & s & t & u & \\ & & v & w & x & & \\ & & & y & & & \end{array} \right\rangle$$

The binary operations of addition (+), scalar multiplication, and multiplication (o) defined in (Ajibade, 2003) (respectively, recorded below) that were adopted in Mohammed (2007a) will also be adopted in this paper for the development of the additional abstract structures:

Let $Q = \left\{ \left\langle \begin{array}{ccc} & f & \\ g & h(Q) & j \\ & k & \end{array} \right\rangle : f, g, h(Q), j, k \in \Re \right\}$ be as defined (Ajibade, 2003) and then it follows that

$$R + Q = \left\langle \begin{array}{ccc} & a & \\ b & h(R) & d \\ & e & \end{array} \right\rangle + \left\langle \begin{array}{ccc} & f & \\ g & h(Q) & j \\ & k & \end{array} \right\rangle = \left\langle \begin{array}{ccc} & a+f & \\ b+g & h(R)+h(Q) & d+j \\ & e+k & \end{array} \right\rangle, \tag{1.2}$$

$$\propto R = \propto \left\langle \begin{array}{ccc} & a & \\ b & h(R) & d \\ & e & \end{array} \right\rangle = \left\langle \begin{array}{ccc} & \propto a & \\ \propto b & \propto h(R) & \propto d \\ & \propto e & \end{array} \right\rangle \quad (1.3)$$

and

$$RoQ = \left\langle \begin{array}{ccc} & ah(Q)+fh(R) & \\ bh(Q)+gh(R) & h(R)h(Q) & dh(Q)+jh(R) \\ & eh(Q)+kh(R) & \end{array} \right\rangle \quad (1.4)$$

In addition to these binary operations, the identity and inverse of the rhotrix R were also defined, respectively, by

$$I = \left\langle \begin{array}{ccc} & 0 & \\ 0 & 1 & 0 \\ & 0 & \end{array} \right\rangle \quad (1.5)$$

and

$$R^{-1} = \frac{-1}{(h(R))^2} \left\langle \begin{array}{ccc} & a & \\ b & -h(R) & d \\ & e & \end{array} \right\rangle \quad (1.6)$$

satisfying the properties $R \circ I = I \circ R = R$ and $R \circ R^{-1} = R^{-1} \circ R = I$, provided, $h(R) \neq 0$. It is simple to show that $\langle R, + \rangle$ is a commutative group with

$$0 = \left\langle \begin{array}{ccc} & 0 & \\ 0 & 0 & 0 \\ & 0 & \end{array} \right\rangle \quad (1.7)$$

The multiplicative operation (◦) defined (Ajibade, 2003) was noted with concern in the paper that multiplication of rhotrices could be defined in many ways. Following this, Sani (2004) proposed an alternative method for multiplication of rhotrices based on its rows and columns vectors, as comparable to matrices. This alternative multiplication (◦) approach was used in the paper to establish some relationships between rhotrices and matrices through an isomorphism. Thus, two methods for multiplication of rhotrices having the same dimension are currently available in the literature. Each method provides enabling environment to explore the usefulness of rhotrices as an applicable tool for the creation of algebraic structures (Mohammed, 2007a) enrichment of matrix theory (Mohammed 2007a; Sani, 2007, 2008), and development of special series and polynomial equations (Mohammed, 2007b, 2008).

It is noteworthy to mention that, the development of all the further abstract structures we present in this paper, are based on multiplication (◦) defined (Ajibade, 2003). Also, without any loss in generality, rhotrices of dimension three, considered to be the base rhotrices, will be used to present our work, except where ever necessary. Thus, all the results stated in this paper are true for high dimensional rhotrices.

The following definitions may be noted as they will serve on our discussions in section 3.

Definition 1: A rhotrix R is called a *real rhotrix* if all its entries belong to the set of Real Numbers.

Definition 2: A rhotrix R is called an *integer rhotrix* if all its entries belong to the set of Integer Numbers

The classifications of rhotrices as abstract structures

In this section, we draw attention of our reader to the paper (Mohammed, 2007a) that first presented the ideas on classifications as abstract structures of Groups, Semigroups, Monoids, and Boolean Algebra, as part of the discussions, in the note on enrichment exercises through extension to rhotrices.

Remark

In this section, we apply the definitions of relevant abstract structures (Seymour and Marc, 2002) in the development of the following new abstract structures of rhotrices with respect to the binary operations of addition (+) and multiplication (◦) given by equations (1.2) and (1.4) respectively.

Ring of rhotrices: Let $R^* = \langle R, +, \circ \rangle$ be an abstract structure consisting of the set R of all real rhotrices of the same dimension as given by equation (1.1), together with the operations of addition (+) and multiplication (◦) then R is a commutative ring of rhotrices.

The unity element of the ring R^* is the identity rhotrix I, given by equation (1.5). The zero element of ring R^* is the zero rhotrix 0 given by equation (1.7). The units elements in the ring R^* are the nonzero heart rhotrices (that is the invertible rhotrices) given by the set:

$$U_1 = \left\{ \left\langle \begin{array}{ccc} & a & \\ b & c & d \\ & e & \end{array} \right\rangle : a,b,c,d,e \in \Re \text{ and } c \neq 0 \right\} \quad (1.8)$$

In addition, to the above properties of the ring R^*, we define a set ZD, consisting of all non-zero *zero divisors* in the ring R^*, by

$$ZD=\left\{\left\langle \begin{array}{ccc} & a & \\ b & 0 & d \\ & e & \end{array}\right\rangle : a,b,d,e,0 \in \Re \text{ and at least one of a,b,c,d} \neq 0\right\} \tag{1.9}$$

Further more, the sets

$$J=\left\{\left\langle \begin{array}{ccc} & a & \\ b & 0 & d \\ & e & \end{array}\right\rangle : a,b,d,e,0 \in \Re\right\} \tag{2.0}$$

and

$$k=\left\{\left\langle \begin{array}{ccc} & 0 & \\ 0 & 0 & 0 \\ & 0 & \end{array}\right\rangle\right\}$$

together with the operations of subtraction '-' and 'o', are some of the ideals of ring R^*.

However, if R is a ring of integer rhotrices, then the only units in R is given by the set

$$U_2=\left\{\left\langle \begin{array}{ccc} & a & \\ b & c & d \\ & e & \end{array}\right\rangle : a,b,c,d,e \in \mathrm{Z} \text{ and } \mathrm{c} \in \{1,-1\}\right\}$$

The irreducible elements in a ring of integer rhotrices is given by the set of non-units integer rhotrices

$$E_2=\left\{\left\langle \begin{array}{ccc} & a & \\ b & c & d \\ & e & \end{array}\right\rangle : a,b,c,d,e \in \mathrm{Z} \text{ and } \mathrm{c} \notin \{1,-1\}\right\}$$

ID of rhotrices: Let R and ZD are as defined by equation (1.1) and equation (1.9) respectively and let $D^*=\langle (R-ZD),+,\circ\rangle$ be an abstract structure consisting of some real rhotrices of the same dimension, together with the operations of addition (+) and multiplication ($\circ$) then D^* is an Integral Domain.

Field of rhotrice: Let U_1 be as given by equation (1.8) and let

$$F^*=\left\langle U_1 \cup \left\{\left\langle \begin{array}{ccc} & 0 & \\ 0 & 0 & 0 \\ & 0 & \end{array}\right\rangle\right\},+,\circ\right\rangle$$

be an abstract structure consisting of some real rhotrices of the same dimension, together with the operations of addition (+) and multiplication ($\circ$) then F^* is a Field of real rhotrices.

PID of rhotrices: Let

$$P=\left\{\left\langle \begin{array}{ccc} & a & \\ b & 1 & d \\ & e & \end{array}\right\rangle : a,b,d,e \in Z\right\} \cup \left\{\left\langle \begin{array}{ccc} & a & \\ b & -1 & d \\ & e & \end{array}\right\rangle : a,b,d,e \in Z\right\} \cup \left\{\left\langle \begin{array}{ccc} & 0 & \\ 0 & 0 & 0 \\ & 0 & \end{array}\right\rangle\right\}$$

be a set of some integer rhotrices and let $P^*=\langle P,+,\circ\rangle$ be an abstract structure consisting of the set P, together with the operations of addition (+) and multiplication ($\circ$) then P^* is a Principal Ideal Domain of rhotrices.

UFD of rhotrices: Let

$$U=\left\{\left\langle \begin{array}{ccc} & a & \\ b & c & d \\ & e & \end{array}\right\rangle : a,b,c,d,e \in Z \text{ and } \mathrm{c} \neq 0\right\} \cup \left\{\left\langle \begin{array}{ccc} & 0 & \\ 0 & 0 & 0 \\ & 0 & \end{array}\right\rangle\right\}$$

be a set of some integer rhotrices and let $U^*=\langle U,+,\circ\rangle$ be an abstract structure consisting of the set U, together with the operations of addition (+) and multiplication ($\circ$) then U^* is a Unique Factorization Domain.

Conclusion

We have presented additional classifications of rhotrices as abstract structures. Rhotrices serve as an efficient and reliable tool in the study of infinite abstract structures as demonstrated in this paper. The search for the application of rhotrices in the study of finite abstract structures offered an amazing area of interest for research.

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