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Analytical investigation of convective heat transfer of a longitudinal fin with temperature-dependent thermal conductivity, heat transfer coefficient and heat generation

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In this article, the heat transfer through a longitudinal fin is studied. The heat transfer coefficient, thermal conductivity and heat generation are variables and supposed to be temperature-dependent. The temperature distribution in fin with longitudinal rectangular profile was carried out by using the differential transformation method (DTM) which is an analytical solution technique. For validation of the analytical solution, the heat equation is solved numerically. The temperature distribution is shown for different values of the embedding parameters. The DTM results indicate that the fin tip temperature increases with an increase in the heat generation gradient. Results reveal that DTM is very effective and convenient. Comparison of the results (DTM and numerical) was shown that the analytical method and numerical data are in a good agreement with each other.

Key words: Fins, temperature dependent thermal properties, heat generation, analytical solutions, differential transformation method (DTM).

INTRODUCTION

Extended surfaces (also known as fins) are used to augment heat dissipation from a hot surface through its convective, radiative, or convective-radiative surface. In particular, fins are used extensively in various industrial applications such as the cooling of computer processors, air conditioning and oil carrying pipe lines. Several studies were performed on heat transfer using fins. Domairry and Fazeli (2009) solved the nonlinear straight fin differential equation to evaluate the temperature distribution and fin efficiency. Also, temperature distribution for annual fins with temperature-dependent thermal conductivity was studied by Ganji et al. (2011). The effects of temperature-dependent thermal conductivity of a moving fin and added radiative component to the surface heat loss have been studied by Aziz and Khani (2011). They applied the homotopy analysis method (HAM) to solve governing equations. Hatami et al. (2014) studied the temperature distribution

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for a fully wet, semi-spherical porous fin. Heat transfer and temperature distribution for circular convective-radiative porous fins was studied by Hatami and Ganji (2013). Hatami and Ganji (2014a) studied temperature distribution and refrigeration efficiency for fully wet circular porous fins with variable sections. Ghasemi et al. (2014) solved the nonlinear temperature distribution equation in a longitudinal fin with temperature-dependent internal heat generation and thermal conductivity using differential transformation method (DTM). Heat transfer and temperature distribution equations for longitudinal convective-radiative porous fins are solved by Hatami and Ganji (2014b). Heat transfer through porous fins with temperature-dependent heat generation was studied by Hatami et al. (2013). They employed DTM, collocation method (CM) and least square method (LS) for solving governing equations. Sharqawy and Zubair (2007) applied the analytical method for the annular fin with combined heat and mass transfer. Arslanturk (2005) and Rajabi (2007) obtained efficiency and fin temperature distribution by Adomian decomposition method (ADM) and the homotopy perturbation method (HPM) with temperature-dependent thermal conductivity. An analytical method for determining the optimum thermal design of convective longitudinal fin arrays is presented by Franco (2009). Lin and Lee (1999) investigated boiling on a straight fin with linearly varying thermal conductivity.

The concept of DTM was first introduced by Zhou (1986) and it was used to solve both linear and nonlinear initial value problems. This method can be applied directly to linear and nonlinear differential equation without requiring linearization, discretization, or perturbation and this is the main benefit of this method. Abbasov and Bahadir (2005) employed DTM to obtain approximate solutions of the linear and nonlinear equations related to engineering problems and they showed that the numerical results are in good agreement with the analytical solutions. Rashidi et al. (2010) solved the problem of mixed convection about an inclined flat plate embedded in a porous medium by DTM; they applied the Pade approximant to increase the convergence of the solution. Ghafoori et al. (2011) used the DTM for solving the nonlinear oscillation equation. Abdel-Halim (2008) has applied the DTM for different systems of differential equations and discussed the convergency of this method in several examples of linear and nonlinear systems of differential equations. Joneidi et al. (2009) used DTM for the analytical solution of convective straight fins with temperature-dependent thermal conductivity and comparing results with exact and numerical one. Their results reveal the capability, effectiveness, convenience and high accuracy of this method.

Moradi and Ahmadikia (2010) applied the DTM to solve the energy equation for a fin with three different profiles and temperature-dependent thermal conductivity. Balkaya et al. (2009) applied the DTM to analyze the vibration of an elastic beam supported on elastic soil. Borhanifar and Abazari (2011) employed DTM on some partial differential equations (PDEs) and their coupled versions.

The DTM is used to solve a wide range of physical problems. This method provides a direct scheme for solving linear and nonlinear deterministic and stochastic equations without linearization and yield convergent series solution rapidly.

The goal of this study is obtaining an analytical solution for temperature distribution of a fin with temperature-dependent thermal conductivity, heat transfer coefficient and heat generation. The effect of the range of values of heat transfer parameters on the temperature distribution is shown. Also, the DTM is applied to solve nonlinear problem analytically. To validate analytical results, the obtained DTM results are compared with numerical data.

PROBLEM DESCRIPTION

Consider a one-dimensional longitudinal fin, with an arbitrary profile $F(X)$ and cross-section area A_c as shown in Figure 1. The periphery of the fin is denoted by *P* and its length by *L*. The fin is attached to a fixed base surface of temperature T_b and extend to an ambient fluid of temperature T_a. The fin thickness is given by δ and the base thickness is δ_b. The $1\text{-}D$ steady state energy equation for the fin with internal heat generation can be expressed as:

$$\frac{\partial}{\partial X}\left(\frac{\delta_b}{2}F(X)K(T)\frac{\delta T}{\delta X}\right)-\frac{P}{A_c}H(T)\left(T-T_a\right)+q^*=0, \quad 0\le X\le L. \qquad (1)$$

Where K, H and q^* are the non-uniform thermal conductivity, heat transfer coefficients and heat generation depending on the temperature, *T* is the temperature distribution and X is the spatial variable. An insulated fin at one end with the base temperature at the other implies boundary condition which is given by (Kraus, 2001):

$$T(L)=T_b \qquad , \ \frac{\partial T}{\partial X}\Big|_{X=0}=0. \qquad (2)$$

Because of the heat generation varying with temperature, so we have:

$$q^*=q_a^*\left(1+\varepsilon\left(T-T_a\right)\right) \qquad (3)$$

For simplifying the above equations, some dimensionless parameters are introduced as follows:

$$x=\frac{X}{L},\ \ \theta=\frac{T-T_a}{T_b-T_a},\ \ h=\frac{H}{h_b},\ \ k=\frac{K}{k_a},\ \ M^2=\frac{Ph_bL^2}{A_ck_a},\ \ R=\frac{q_a^*A_c}{h_bP(T_b-T_a)}, \qquad (4)$$

$$\varepsilon_R=\varepsilon(T_b-T_a),\ \ f(x)=\frac{\delta_b}{2}F(X)$$

Equation 1 reduces to:

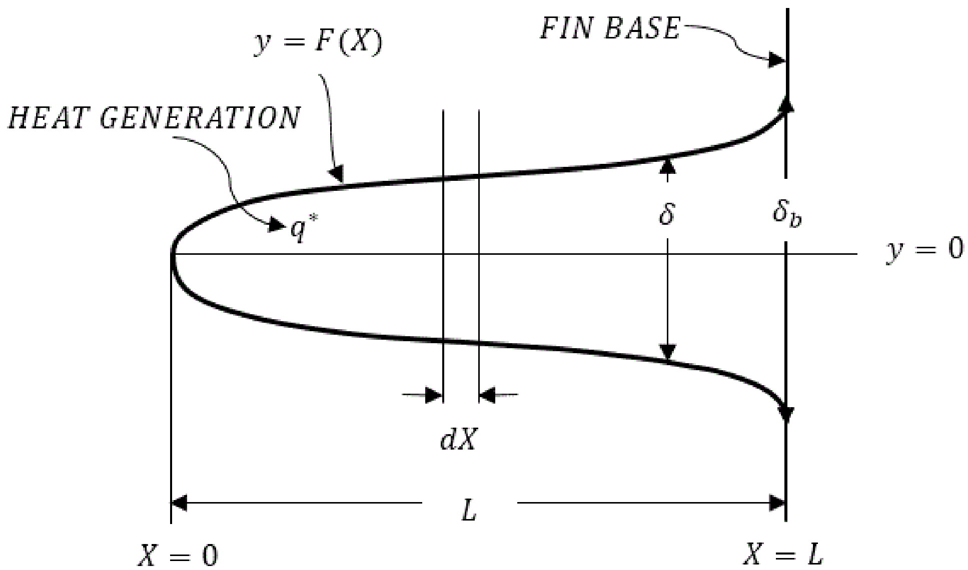

Figure 1. Schematic representation of a longitudinal fin with arbitrary profile *F(X)*.

$$\frac{\partial}{\partial x}\left(f(x)k(\theta)\frac{\delta\theta}{\delta x}\right)-M^2h(\theta)\theta+M^2R\left(1+\varepsilon_R\theta\right)=0,\quad 0\le x\le 1. \tag{5}$$

And the boundary conditions become

$$\theta(1)=1 \qquad , \frac{\partial\theta}{\partial x}\Big|_{x=0}=0. \tag{6}$$

Where *M* is the thermogeometric fin parameter, θ is the dimensionless temperature, *X* is the dimensionless spatial variable, q_a^* is the internal heat generation at temperature T_a, *k* is the dimensionless thermal conductivity, k_a is the thermal conductivity of the fin at ambient temperature, h_b is the heat transfer coefficient at the fin base. For most industrial application, the heat transfer coefficient maybe given as a power law (Unal, 1987):

$$H(T)=h_b\left(\frac{T-T_a}{T_b-T_a}\right)^n \tag{7}$$

Where *n* and h_b are constants. The constant n may vary between -6.6 and 5. However, in most practical applications it lies between -3 and 3 (Unal, 1987). The exponent n represents laminar film boiling or condensation when $n=-1/4$, laminar natural convection when $n=1/4$, turbulent natural convection when $n=1/3$, nucleate boiling when $n=2$, radiation when *n* = 3 and *n* = 0 implies a constant heat transfer coefficient. Exact solutions may be constructed for the steady-state one-dimensional differential equation describing temperature distribution in a straight fin when the thermal conductivity is a constant and the exponent of the heat transfer coefficient is given by $n=-1,0,1$ or 2 (Unal, 1987).

In dimensionless variables we have $h(\theta)=\theta^n$. Also, for many engineering applications, the thermal conductivity may depend linearly on temperature, that is;

$$K(T)=k_a\left[1+\gamma(T-T_a)\right] \tag{8}$$

The dimensionless thermal conductivity given by the linear function of temperature is $k(\theta)=1+\beta\theta$, where the thermal conductivity gradient is $\beta=\gamma(T_b-T_a)$.

As such, the governing equation is given by:

$$\frac{\partial}{\partial x}\left(f(x)(1+\beta\theta)\frac{\delta\theta}{\delta x}\right)-M^2\theta^{n+1}+M^2R\left(1+\varepsilon_R\theta\right)=0,\ 0\le x\le 1 \tag{9}$$

FUNDAMENTAL OF DIFFERENTIAL TRANSFORMATION METHOD (DTM)

Let $x(t)$ be analytic in a domain *D* and let $t=t_i$ represent any point in *D*. The function $x(t)$ is then represented by one power series whose center is located at t_i. The Taylor series expansion function of $x(t)$ is in form of:

$$x(t)=\sum_{k=0}^{\infty}\frac{\left(t-t_i\right)^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=t_i} \qquad \forall t\in D \tag{10}$$

The particular case of Equation 10 when $t_i=0$ is referred to as the Maclaurin series of $x(t)$ and is expressed as:

Table 1. The fundamental operations of the differential transform method.

Original function	Transformed function
$w(t)=\alpha u(t)\pm\beta v(t)$	$W(k)=\alpha U(k)\pm\beta V(k)$
$w(t)=\frac{d^m u(t)}{dt^m}$	$W(k)=\frac{(k+m)!}{k!}U(k+m)$
$w(t)=u(t)v(t)$	$W(k)=\sum_{l=0}^{k}U(l)V(k-l)$
$w(t)=t^m$	$W(k)=\delta(k-m)=\begin{cases}1, & if\ k=m\\ 0, & if\ k\neq m\end{cases}$
$w(t)=\exp(t)$	$W(k)=\frac{1}{k!}$

$$x(t)=\sum_{k=0}^{\infty}\frac{t^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0} \qquad \forall t\in D \tag{11}$$

As explained in Zhou (1986) and Abdel-Halim (2004), the differential transformation of the function $x(t)$ is defined as follows:

$$X(k)=\sum_{k=0}^{\infty}\frac{H^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0} \tag{12}$$

Where $x(t)$ is the original function and $X(k)$ is the transformed function. The differential spectrum of $X(k)$ is confined within the interval $t\in[0,H]$, where *H* is a constant. The differential inverse transform of $X(k)$ is defined as follows:

$$x(t)=\sum_{k=0}^{\infty}\left(\frac{t}{H}\right)^k X(k) \tag{13}$$

It is clear that the concept of the differential transformation is based upon the Taylor series expansion. The values of function $X(k)$ at values of argument *k* are referred to as discrete, that is, $X(0)$ is known as the zero discrete, $X(1)$ as the first discrete, etc. the more discrete available, the more precise it is possible to restore the unknown function. The function $x(t)$ consists of *T*-function $X(k)$, and its value is given by the sum of the *T*-function with $(t/H)^k$ as its coefficient. In real applications, at the right choice of constant *H*, the larger values of argument k the discrete of spectrum reduce rapidly. The function $x(t)$ is expressed by a finite series and Equation 13 can be written as:

$$x(t)=\sum_{k=0}^{n}\left(\frac{t}{H}\right)^k X(k) \tag{14}$$

Mathematical operations performed by differential transform method are listed in Table 1.

ANALYTICAL SOLUTION

By 1-D transform of Equation 9 considered by using the related definition in Table 1, we have the following:

(1) Rectangular profile ($f(x)=1$), case $n=1$

$$(K+1)(K+2)\Theta(K+2)+\beta\left(\sum_{i=0}^{K}\Theta(i)(K+1-i)(K+2-i)\Theta(K+2-i)\right)+\beta\left(\sum_{i=0}^{K}(i+1)\Theta(i+1)(K+1-i)\Theta(K+1-i)\right)$$
$$-M^2\left(\sum_{i=0}^{K}\Theta(i)\Theta(K-i)\right)+M^2R\left(\delta(K)+\varepsilon_R\Theta(K)\right)=0, \tag{15}$$

(2) Rectangular profile ($f(x)=1$), case $n=2$

$$(K+1)(K+2)\Theta(K+2)+\beta\left(\sum_{i=0}^{K}\Theta(i)(K+1-i)(K+2-i)\Theta(K+2-i)\right)+\beta\left(\sum_{i=0}^{K}(i+1)\Theta(i+1)(K+1-i)\Theta(K+1-i)\right)$$
$$-M^2\left(\sum_{j=0}^{K}\Theta(K-j)\sum_{i=0}^{j}\Theta(i)\Theta(j-i)\right)+M^2R\left(\delta(K)+\varepsilon_R\Theta(K)\right)=0, \tag{16}$$

In the above equations $\Theta(K)$ is transformed function of $\Theta(x)$. The transformed boundary condition takes the form:

$$\Theta(1)=0 \tag{17}$$

$$\sum_{i=0}^{\infty}\Theta(i)=1 \tag{18}$$

Supposing that $\Theta(0)=a$ and using Equations 17 and 18, another value of $\Theta(i)$ can be calculated.

The value of a can be calculated using Equation 18. Thus, we end up having the following:

(1) Rectangular profile, case $n=1$

$$\Theta(2)=-\frac{1}{2}\frac{M^2\left(-a^2+R+R\varepsilon_R a\right)}{1+\beta a}$$
$$\Theta(3)=0$$
$$\Theta(4)=-\frac{1}{24}\frac{M^4\left(-a^2+R+R\varepsilon_R a\right)\left(2a-\beta a^2+3\beta R+2\beta R\varepsilon_R a-R\varepsilon_R\right)}{(1+\beta a)^3} \tag{19}$$

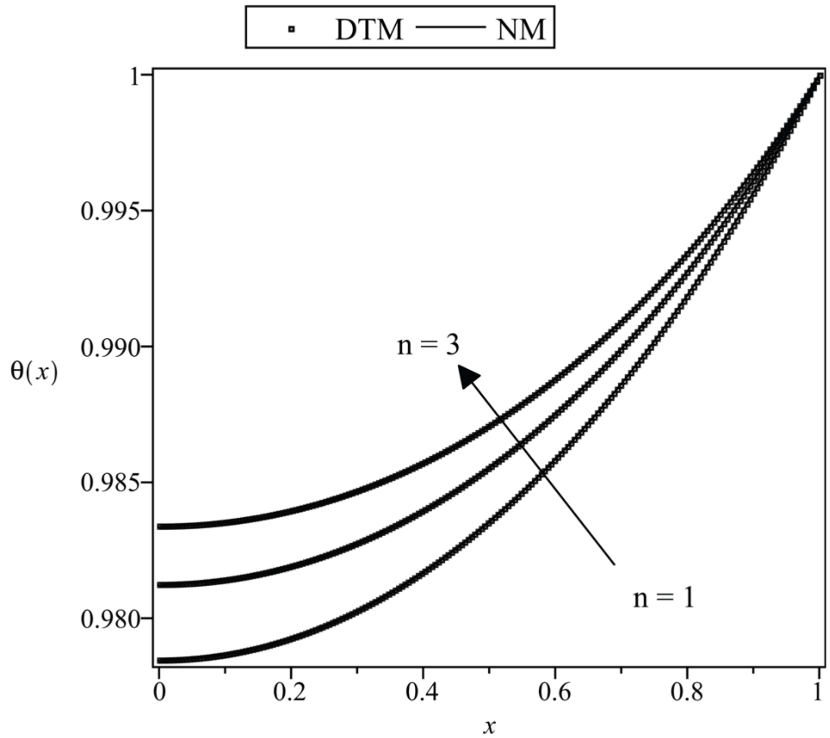

Figure 2. Comparison of (x) obtained by the DTM with numerical solution in a longitudinal rectangular fin when M = 1, β = 1, R = 0.8, ε_R = 0.1.

(2) Rectangular profile, case $n=2$

$$\Theta(2)=-\frac{1}{2}\frac{M^2\left(-a^3+R+R\varepsilon_R a\right)}{1+\beta a}$$
$$\Theta(3)=0$$
$$\Theta(4)=-\frac{1}{24}\frac{M^4\left(-a^3+R+R\varepsilon_R a\right)\left(3a^2+3\beta R+2\beta R\varepsilon_R a-R\varepsilon_R\right)}{(1+\beta a)^3} \quad (20)$$

From the above continuing process, substituting Equation 19 in Equation 14 for *H* = 1, we can obtain the closed form of the solution:

(1) Rectangular profile, case $n=1$

$$\theta(x)=a-\frac{1}{2}\frac{x^2M^2\left(-a^2+R+R\varepsilon_R a\right)}{1+\beta a} \quad (21)$$
$$-\frac{1}{24}\frac{x^4M^2\left(-a^2+R+R\varepsilon_R a\right)\left(2a-\beta a^2+3\beta R+2\beta R\varepsilon_R a-R\varepsilon_R\right)}{(1+\beta a)^3}+\ldots$$

In order to obtain the value a, we used Equation 18. Then, we will have:

$$\theta(1)=a-\frac{1}{2}\frac{M^2\left(-a^2+R+R\varepsilon_R a\right)}{1+\beta a} \quad (22)$$
$$-\frac{1}{24}\frac{M^2\left(-a^2+R+R\varepsilon_R a\right)\left(2a-\beta a^2+3\beta R+2\beta R\varepsilon_R a-R\varepsilon_R\right)}{(1+\beta a)^3}+\ldots=1$$

Solving Equation 22 by Maple software gives the value of a. For other cases the same process is used to obtain the value of a and temperature distribution.

RESULTS AND DISCUSSION

In this paper, the steady-state heat transfer in a longitudinal rectangular fin was studied. The dependence of the thermal conductivity, heat transfer coefficients and heat generation on the temperature rendered the problem highly nonlinear. The effects of the thermogeometric fin parameter (M), thermal conductivity gradient (β), heat generation gradient (ε_R) and *R* are investigated on the temperature distribution. To validate the analytical results, the temperature distribution through the longitudinal rectangular fin is compared with the numerical solution. The results are well matched with the results carried out by numerical solution as shown in Figure 2 and Table 2. In this table, error is introduced as follows:

$$\%Error=\left|\frac{\theta(x)_{NM}-\theta(x)_{DTM}}{\theta(x)_{NM}}\right|\times 100.$$

This accuracy gives high confidence in the validity of this

Table 2. Comparison between DTM and numerical results of θ(x) for rectangular profile when M=1, β=1, R=0.8 and $\varepsilon_R = 0.1$

$f(x)=1$		*n = 1*			*n = 2*			*n = 3*	
x	**NM**	**DTM**	**Error (%)**	**NM**	**DTM**	**Error (%)**	**NM**	**DTM**	**Error (%)**
0	0.9784318	0.9784798	0.0048984	0.9812073	0.9812627	0.0056476	0.9833487	0.9834069	0.005912
0.1	0.9786395	0.9786799	0.0041301	0.9813846	0.9814303	0.0046542	0.9835028	0.9835497	0.0047697
0.2	0.9792382	0.9792822	0.0044861	0.9818847	0.9819353	0.0051584	0.983928	0.983981	0.0053895
0.3	0.9802475	0.9802919	0.0045252	0.982733	0.9827848	0.005271	0.9846536	0.9847085	0.0055756
0.4	0.9816774	0.981718	0.0041318	0.9839427	0.9839903	0.004838	0.985695	0.9857458	0.0051467
0.5	0.9835368	0.9835732	0.0036963	0.9855253	0.9855683	0.0043627	0.9870656	0.9871118	0.0046793
0.6	0.9858406	0.9858738	0.0033713	0.9875004	0.9875404	0.0040514	0.988788	0.9888318	0.0044207
0.7	0.9886144	0.9886403	0.0026174	0.9899023	0.9899337	0.0031732	0.9909028	0.9909375	0.0034959
0.8	0.9918745	0.991897	0.0022701	0.9927527	0.9927809	0.0028364	0.9934361	0.993468	0.003207
0.9	0.9956481	0.9956725	0.0024481	0.9960891	0.9961211	0.0032124	0.9964326	0.9964702	0.0037711
1	1	1	0	1	1	0	1	1	0

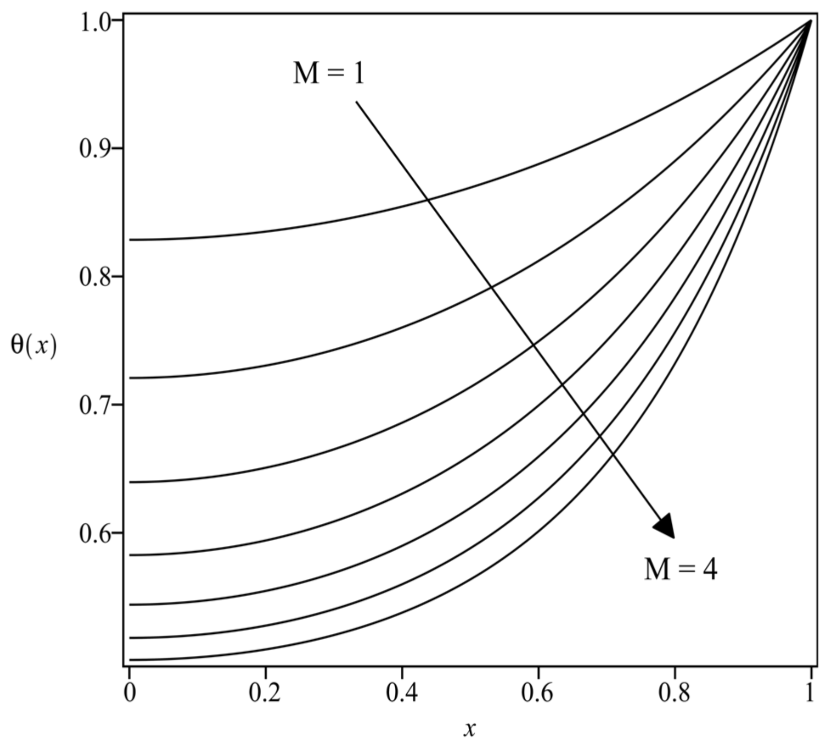

Figure 3. Temperature distribution in a longitudinal rectangular fin for varying values of thermo-geometric parameter (M) when n = 1, β = 0.5, *R* = 0.2, ε_R = 0.2.

problem, and reveals an excellent agreement in engineering accuracy.

The effect of thermo-geometric parameter (M) on temperature distribution is shown in Figure 3. It is illustrated that the magnitude of temperature is increased with decreasing the thermo-geometric parameter (M). Note that the thermo-geometric fin parameter $M=(Bi)^{1/2}E$, where $Bi=h_b\delta/k_a$ is the Biot number and $E=L/\delta$ is the aspect ratio or the extension factor. Undoubtedly, small values of M correspond to the relatively short and thick fins of high conductivity and high values of M correspond to longer and thin fins of poor conductivity (Mills, 1995). A fin is an excellent squanderer at small values of M. As M increases the convective heat loss increases and the temperature profile becomes steeper reflecting high base heat flow rates. In Figure 4,

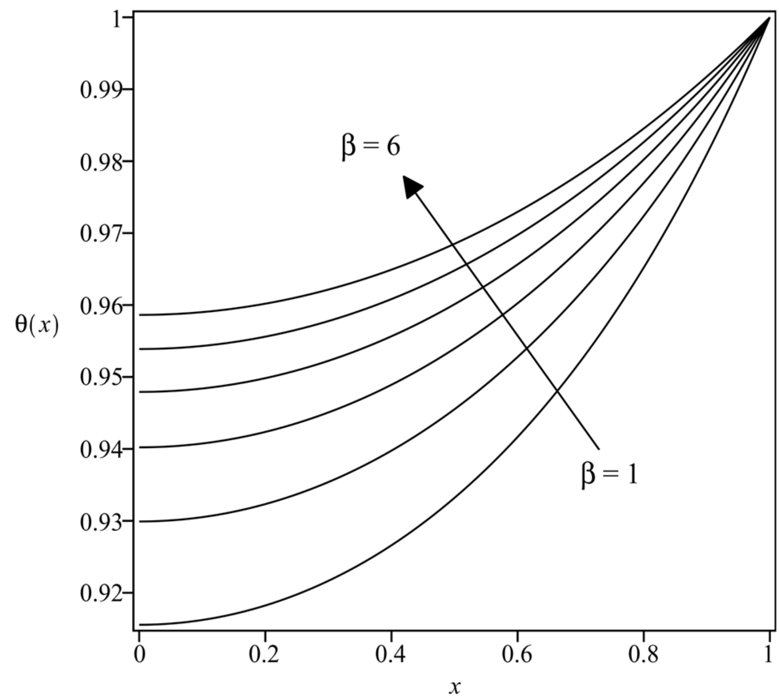

Figure 4. Temperature distribution in a longitudinal rectangular fin for varying values of β when n = 1, M = 2, R = 0.5, ε_R = 0.6.

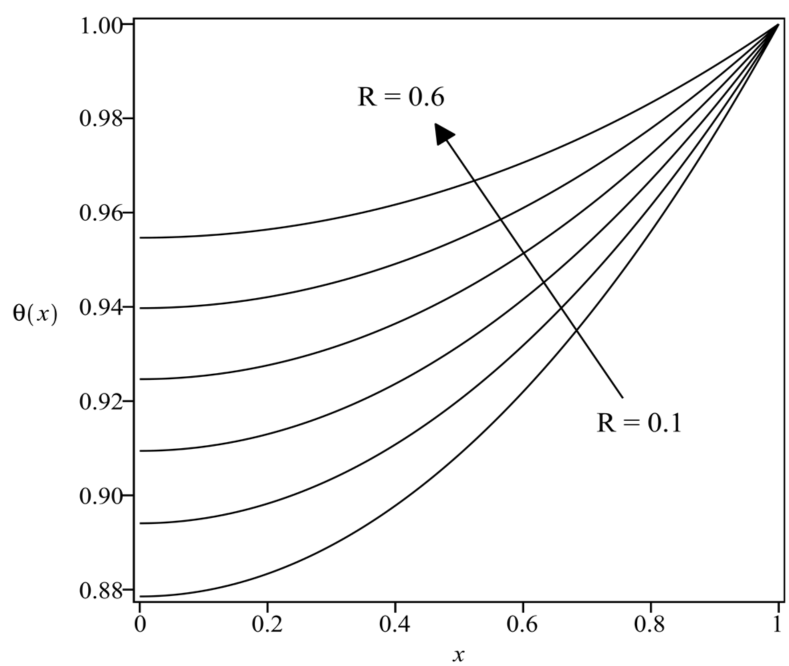

Figure 5. Temperature distribution in a longitudinal rectangular fin for varying values of R when n = 1, M = 1, β = 2, ε_R = 0.1.

we observe that the temperature in the fin increases with the increasing values of the thermal conductivity gradient (β). Figure 5 illustrates the variation of the temperature distribution with R. In addition, the effect of the heat generation gradient on the temperature distribution is shown in Figure 6. With a decrease in the ε_R, the losing

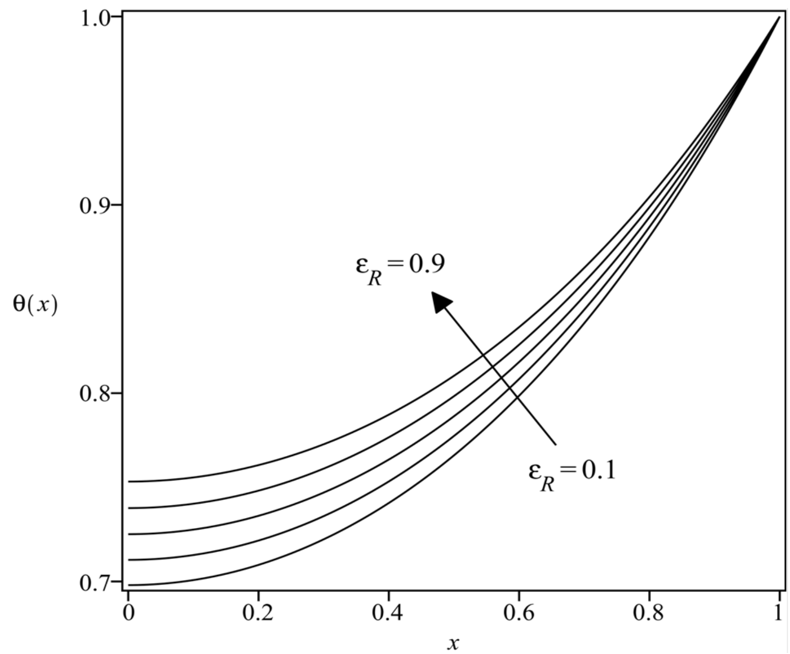

Figure 6. Temperature distribution in a longitudinal rectangular fin for varying values of ε_R when n = 1, M = 2, β = 1.5, R = 0.2.

heat from fin surface becomes stronger, thus the fin temperature decreases.

Conclusion

In this study, the DTM was applied to solve the heat transfer problem in a rectangular fin with temperature-dependent thermal conductivity, heat transfer coefficient and heat generation. Also, this problem is solved by a numerical method and some conclusions were summarized as follows:

(a) The DTM is a powerful approach for solving nonlinear differential equation, such as this problem. Also, it can be observed that there is good agreement between the present and numerical results.
(b) Increasing thermo-geometric parameter leads to a decrease in temperature distribution.
(c) By increasing thermal conductivity and heat generation gradient, temperature distribution increase.

Conflict of Interest

The author(s) have not declared any conflict of interest.

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