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Full Length Research Paper

Construction of some new exact structures for the nonlinear lattice equation

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In the present work we examine a generalized coth and csch functions method to construct new exact travelling solutions to the nonlinear lattice equation. The technique of the homogeneous balance method is used to handle the appropriated solutions. Some exact solutions obtained are new.

Key words: Nonlinear lattice, travelling wave solutions.

INTRODUCTION

Several methods have been developed for analytic solving of nonlinear partial differential equations. Specially, almost all of these nonlinear model equations were appeared (Wang and Li, 2008; Korteweg and Vries, 1995; Khelil et al., 2006) to give different structures to the solutions. Besides traditional methods such as auto-Backlund transformation, Lie Groups, inverse scattering transformation and Miura's transformation, a vast variety of the direct methods for obtaining explicit travelling solitary wave solutions have been found (Zerarka and Foester, 2005; Ibrahim and El-Kalaawy, 2007; Lü, 2014a,b). The availability of symbolic computation packages can be facilitating many direct approaches to establish solutions to non-linear wave equations (Xu and Zhang, 2007; Özis and Yıldırım, 2008). Various extension forms of the sine-cosine and tanh methods proposed by Malfliet and Wazwaz have been applied to solve a large class of nonlinear equations (Malfliet 1996a,b; Wazwaz 2004; He and Wu, 2006a,b). More importantly, another mathematical treatment is established and used in the analysis of these nonlinear problems, such as Jacobian elliptic function expansion method, the variational iteration method, pseudo spectral method, the averaging method, and many others powerful methods (Odibat and Momani, 2006; Rafei and Ganji, 2006; Yu, 2007; Zhu, 2007a,b; Lü and Peng, 2013a,b,c; Lü, 2013; Lü et al., 2010; Jia et al., 2014; Liu and Qian, 2011). The aim of this work is to propose an efficient approach to examine new developments in a direct manner without requiring any additional condition on the investigation of exact solutions with the coth and csch functions method for a lattice system. We expect that the presented method could lead to construct successfully many other solutions for a large variety of other nonlinear evolution equations.

ANALYSIS OF THE PROBLEM

We consider the following nonlinear problem for the lattice equation as:

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$$R(u,u_t,u_x,u_y,u_z,u_{xy},u_{yz},u_{xz},u_{xx},...)=0, \quad (1)$$

Here the subscripts represent partial derivatives, and $u(t,x,y,z,...)$ is an unknown function to be determined. We take the following transformation for the new wave variable as:

$$\xi=\sum_{i=0}^{p}\alpha_i\chi_i+\delta, \quad (2)$$

χ_i are distinct variables, and when $p=1$, $\xi=\alpha_0\chi_0+\alpha_1\chi_1+\delta$, the quantities α_0,α_1 are called the wave pulsation ω and the wave number k respectively if χ_0,χ_1 are the variables t and x respectively. In the discrete case for the position x and with continuous variable for the time t, ξ becomes with some modifications $\xi_n=nd+ct+\delta$ and n is the discrete variable. d and δ are arbitrary constants and c is the velocity. We use the traveling wave reduction transformation for Equation (1) as:

$$u(\chi_0,\chi_1,...)=U(\xi), \quad (3)$$

and the chain rule

$$\frac{\partial}{\partial\chi_i}(.)=\alpha_i\frac{d}{d\xi}(.),\ \frac{\partial^2}{\partial\chi_i\partial\chi_j}(.)=\alpha_i\alpha_j\frac{d^2}{d\xi^2}(.),\cdots, \quad (4)$$

Upon using Equations (3) and (4), the nonlinear problem (1) becomes an ODE like

$$Q(U,U_\xi,U_{\xi\xi},U_{\xi\xi\xi},U_{\xi\xi\xi\xi},...)=0 \quad (5)$$

APPLICATIONS

The one-dimensional lattice equation (Zhu, 2007, 2008) is written as:

$$\frac{du(n,t)}{dt}-(a+bu(n,t)+u^2(n,t))[u(n+1,t)-u(n-1,t)]=0, \quad (6)$$

We first combine the independent variables, into a wave variable using ξ_n as

$$\xi_n=nd+ct+\delta \quad (7)$$

and we take the travelling wave solutions of the system (6) using Equation (7) as $u(n,t)=U(\xi_n)$. By using the chain rule (4), the system (6) can be obtained as follows:

$$cU_{\xi_n}(\xi_n)-(a+bU(\xi_n)+U^2(\xi_n))(U(\xi_{n+1})-U(\xi_{n-1}))=0, \quad (8)$$

Where subscript denotes the differential with respect to ξ_n.

THE COTH FUNCTION METHOD

Suppose that Equation (8) has the following solution:

$$U(\xi_n)=\sum_{j=0}^{M}A_j\coth^j(\xi_n), \quad (9)$$

Where M is an undetermined integer and A_j are coefficients to be determined later. In order to determine values of the parameter M, we balance the linear term of highest order in Equation (8) with the highest order nonlinear term. By simple calculation, we have $2M=M+1$ and the solution (9) takes the form

$$U(\xi_n)=A_0+A_1\coth(\xi_n), \quad (10)$$

Substituting the solution (10) into Equation (8), and equating to zero the coefficients of all powers of $\coth^j(\xi_n)$ yields a set of algebraic equations for A_0, A_1 and c as:

$$\begin{aligned} bA_1+2A_0A_1&=0\\ 2(a+bA_0+A_0^2)&=c\coth(d)\\ -2A_1^2\coth(d)&=c \end{aligned} \quad (11)$$

Solving the system of algebraic equations with the aid of Mathematical, we obtain

$$\begin{aligned} A_0&=-\frac{b}{2},\\ A_1&=\pm\frac{\tanh(d)}{2}\sqrt{b^2-4a},\\ c&=\frac{\tanh(d)}{2}(4a-b^2), \end{aligned} \quad (12)$$

and the two travelling wave solutions of the problem of interest follow

$$U_\pm(\xi_n)=-\frac{b}{2}\pm\frac{\tanh(d)}{2}\sqrt{b^2-4a}\coth\left[nd+\frac{\tanh(d)}{2}(4a-b^2)t+\delta\right] \quad (13)$$

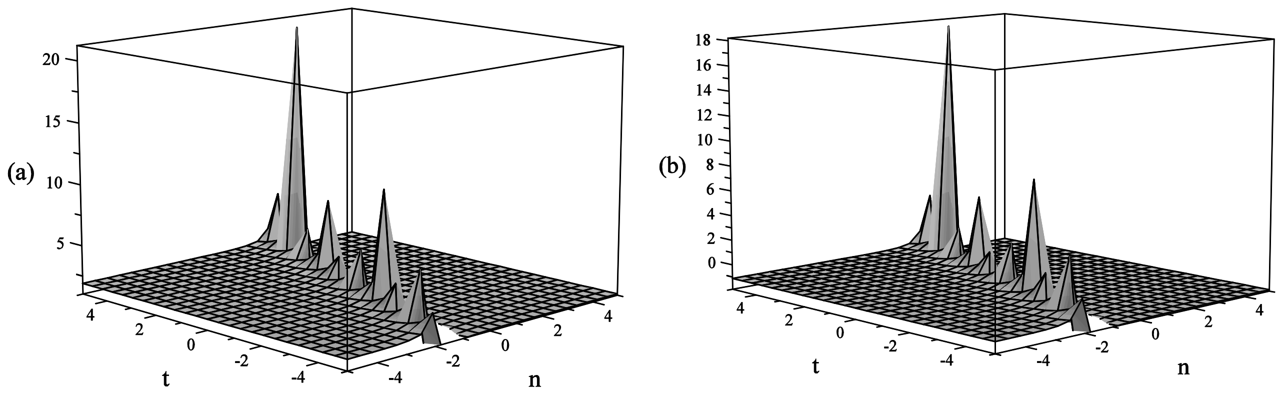

Figure 1. The graphs show the wave solutions of $u_{\pm}(n,t)=U_{\pm}(\xi_n)$ in Equation (13): (a) solution $u_{+}(n,t)=U_{+}(\xi_n)$, (b) solution $u_{-}(n,t)=U_{-}(\xi_n)$. For both curves: $a=2, b=3, d=1, \delta=0$.

Where d and δ are arbitrary constants.

Figure 1(a) and (b) show the physical waves $u_{+}(n,t)=U_{+}(\xi_n)$ and $u_{-}(n,t)=U_{-}(\xi_n)$ in Equations (13).

THE COTH-CSCH FUNCTION METHOD

The solutions of Equation (8) can be expressed in the form

$$U(\xi_n)=\alpha+\sum_{j=1}^{M} A_j \coth^j(\xi_n)+B_j \operatorname{csch}^j(\xi_n), \tag{14}$$

Where α, A_j and B_j are parameters to be determined. The parameter M is found by balancing the highest-order linear term with the nonlinear terms, we obtain $M=1$, and $U(\xi_n)$ becomes

$$U(\xi_n)=\alpha+A\coth(\xi_n)+B\operatorname{csch}(\xi_n), \tag{15}$$

Substituting Equation (15) into the relevant nonlinear differential Equation (8) and with the help of Mathematical we get a system of algebraic equations with respect to c, α, A and B.

$$\begin{aligned} Ac+2A\left(A^2+B^2\right)\coth(d)+4AB^2\operatorname{csch}(d)&=0,\\ 4A^2B\coth(d)+Bc\coth(d)^2-2B\left(\alpha^2-B^2+a+\alpha b\right)\operatorname{csch}(d)&=0,\\ -(Bc)-4A^2B\coth(d)-2B\left(A^2+B^2\right)\operatorname{csch}(d)&=0,\\ 2A(2\alpha B+Bb)&=0,\\ 2A(2A\alpha+Ab)\coth(d)+2B(2\alpha B+Bb)\operatorname{csch}(d)&=0,\\ 2A\left(\alpha^2-B^2+a+\alpha b\right)-Ac\coth(d)&=0 \end{aligned} \tag{16}$$

After some algebra, and with the help of Mathematical, the following values for the parameters c, α, A, and B are obtained:

First set

$$\begin{aligned} c&=\frac{1}{2}\left(4a-b^2\right)\tanh(d),\\ \alpha&=-\frac{b}{2},\\ A&=\pm\frac{1}{2}\sqrt{b^2-4a}\,\tanh(d),\\ B&=0, \end{aligned} \tag{17}$$

and the travelling solutions of Equation (17) are obtained as:

$$U_{\pm}(\xi_n)=-\frac{b}{2}\pm\frac{\tanh(d)}{2}\sqrt{b^2-4a}\,\coth(\xi_n), \tag{18}$$

Where $\xi_n=nd+\frac{\tanh(d)}{2}(4a-b^2)t+\delta$. The solutions (18) are similar to those obtained by the coth-function method Equation (13).

Second set

$$\begin{aligned} c&=\frac{1}{2}\left(4a-b^2\right)\sinh(d),\\ \alpha&=-\frac{b}{2},\\ A&=0,\\ B&=\pm\frac{1}{2}\sqrt{b^2-4a}\,\sinh(d), \end{aligned} \tag{19}$$

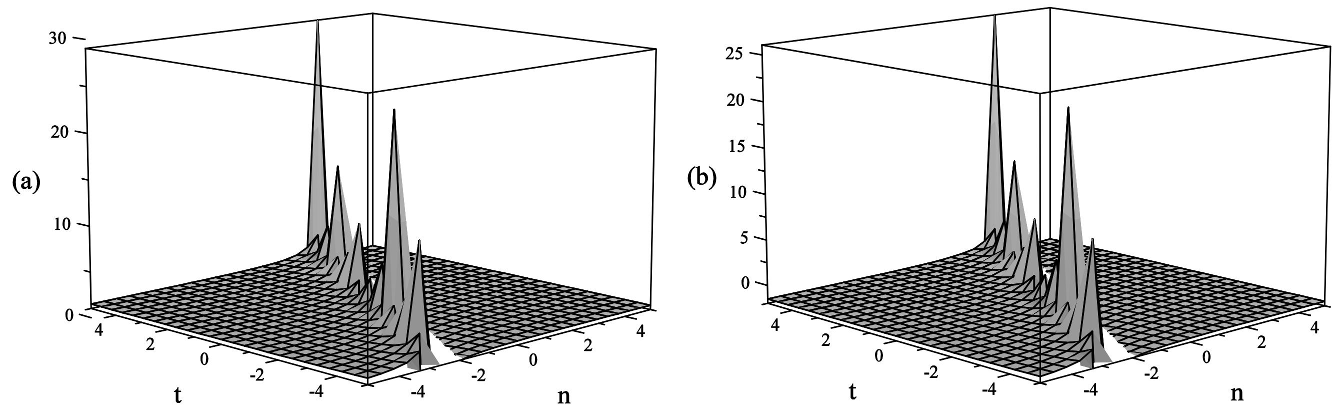

Figure 2. The graphs show the wave solution of $U_{\pm}(\xi_n)$ in (20): (a) solution $u_{+}(n,t)=U_{+}(\xi_n)$, (b) solution $u_{-}(n,t)=U_{-}(\xi_n)$. For both curves: $a=2, b=3, d=1, \delta=0$.

and the travelling solutions of Equation (19) are obtained as

$$U_{\pm}(\xi_n)=-\frac{b}{2}\pm\frac{\sinh(d)}{2}\sqrt{b^2-4a}\,\operatorname{csch}(\xi_n), \qquad (20)$$

Where $\xi_n = nd + \frac{\sinh(d)}{2}(4a-b^2)t+\delta$. The portraits of solutions (20) for $U_{\pm}(\xi_n)$ are displayed in Figure 2(a) and (b).

Third set

$$\begin{aligned} c &= \left(4a-b^2\right)\tanh(\frac{d}{2}),\\ \alpha &= -\frac{b}{2},\\ A &= \pm\frac{1}{2}\sqrt{b^2-4a}\,\tanh(\frac{d}{2}),\\ B &= \pm\frac{1}{2}\sqrt{b^2-4a}\,\tanh(\frac{d}{2}), \end{aligned} \qquad (21)$$

Finally, third set admits the following two types:

$$U_{1\pm}(\xi_n)=-\frac{b}{2}-\frac{1}{2}\tanh(\frac{d}{2})\sqrt{b^2-4a}(\coth(\xi_n)\pm\operatorname{csch}(\xi_n)), \qquad (22)$$

and

$$U_{2\pm}(\xi_n)=-\frac{b}{2}+\frac{1}{2}\tanh(\frac{d}{2})\sqrt{b^2-4a}(\coth(\xi_n)\pm\operatorname{csch}(\xi_n)), \qquad (23)$$

Where $\xi_n = nd + \left(4a-b^2\right)\tanh(\frac{d}{2})t+\delta$. The behaviors of solutions (22) and (23) for $U_{1+}(\xi_n)$ and $U_{2+}(\xi_n)$ are shown in Figure 3(a) and (b) respectively. The solutions given for the second and the third sets appear to be new.

CONCLUSION

The basic goal of this work, is to provide a new trial travelling solution to build the exact solutions to the nonlinear lattice equation. Two types of functions are used to find the exact solutions, which are named the coth-function and the coth-csch function methods. Eight variants of travelling wave solutions are obtained. The present method provides a reliable technique that requires less work if compared with the difficulties arising from computational aspect. The main advantage of this method is the flexibility to give exact solutions to nonlinear problems without linearization. We may conclude that, this method can also be extended to other high-dimensional nonlinear phenomena. It will be then interesting to study more general systems. These points will be investigated in a future research.

Conflict of Interest

The authors have not declared any conflict of interest.

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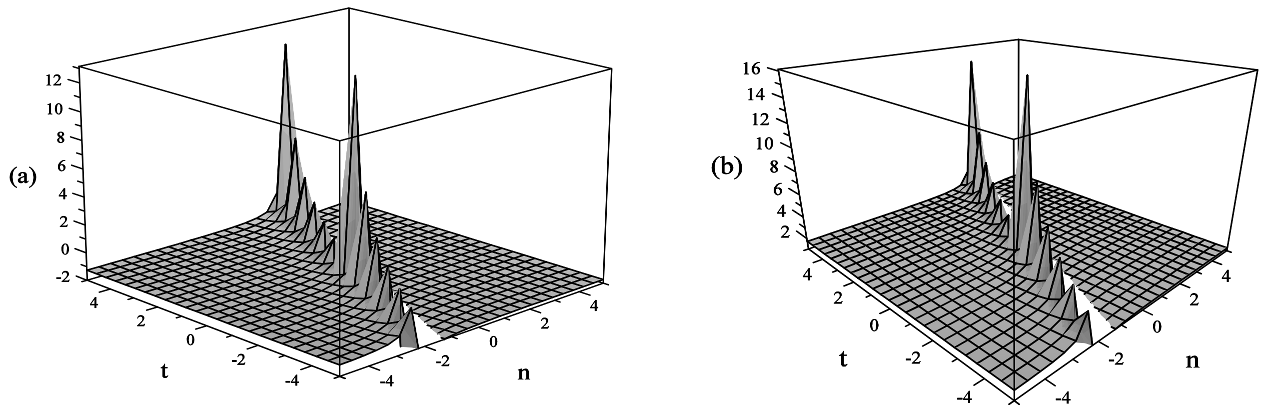

Figure 3. The graphs show the wave solution of $U_{1\pm}(\xi_n)$ and $U_{2\pm}(\xi_n)$ in (22) and (23) respectively: (a) solution $u_{1+}(n,t)=U_{1+}(\xi_n)$, (b) solution $u_{2+}(n,t)=U_{2+}(\xi_n)$. For both curves: $a=2, b=3, d=1, \delta=0$.

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