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Deep inelastic processes and the equations of motion

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We show that the Politzer theorem on the equations of motion implies approximate constraints on the quark correlator. These, in turn, restrict considerably, for sufficiently large Q^2 , the number of independent distribution functions that characterize the internal structure of the nucleon, and of independent fragmentation functions. This result leads us to suggesting an alternative method for determining transversity. Moreover, our approach implies predictions on the Q^2 -dependence of some azimuthal asymmetries, like Sivers, Qiu-Sterman and Collins asymmetry. Lastly, we discuss some implications on the Burkhardt-Cottingham sum rule.

Key words: Equations of motion, deep inelastic.

INTRODUCTION

The problem of calculating inclusive cross-sections at high energies and high momentum transfers has become quite important in the last two decades, during which a lot of experimental data on deep inelastic processes have been accumulated. In particular, we refer to deep inelastic scattering (DIS) (Ashman et al., 1988, 1989; Adeva et al., 1998; Anthony et al., 1996a, b, 2003; Abe et al., 1997a, b, 1998; Airapetian et al., 1998; Yun et al., 2003; Zheng et al., 2004), semi-inclusive DIS (SIDIS) (Arneodo et al., 1987; Ashman et al., 1991; Adams et al., 1993; Airapetian et al., 2000, 2001, 2003, 2005a, b; Diefenthaler, 2005; Bravar et al., 1999; Alexakhin et al., 2005; Ageev et al., 2007; Bressan, 2007; Avakian et al., 2005; Alekseev et al., 2010a, b), Drell-Yan (DY) (Falciano et al., 1986; Guanziroli et al., 1988; Conway et al., 1989; McGaughey et al., 1994; Towell et al., 2001; Zhu et al., 2007) and e^+e^- annihilation into two back-to-back jets (Abe et al., 2006), while analogous experiments have been planned recently (Bunce et al., 2000; Lenisa and Rathmann, 2005; Lenisa, 2005; Afanasev et al., Jefferson, 2007; Hawranek, 2007). One of the aims of high energy physicists is to extract data from distribution and/or fragmentation functions, especially if unknown. Among them, the transversity (Ralston and Soper, 1979; Artru and Mekhfi, 1990; Jaffe and Ji, 1991a,

1992) is of particular interest, since it is the only twist-2 distribution function for which very poor information (Soffer, 1995; Anselmino et al., 2007) is available till now. But also, transverse momentum dependent (TMD) functions, especially the T-odd ones, are taken in great consideration; for instance, knowledge of the Collins (1993) fragmentation function or the Boer-Mulders (1998) function could help to extract transversity, which is chiral-odd and therefore couples only with chiral-odd functions. Moreover, TMD functions are involved in several intriguing Azimuthal asymmetries, like the already mentioned effects of Collins (1993) and Boer-Mulders (1998), or those of Sivers (1990, 1991), Qiu-Sterman (1991, 1992, 1998) and Cahn (1978, 1989), which, in part, have found experimental confirmation (Airapetian et al., 2005a,b; Diefenthaler, 2005; Bravar et al., 1999; Alexakhin et al., 2005; Bressan, 2007; Abe et al., 2006) and, in any case, have stimulated a great deal of articles (Mulders and Tangerman, 1996; Boer et al., 2000, 2003a, b; Brodsky et al., 2002a,b, 2003; Di Salvo, 2007a; Collins et al., 2006; Efremov et al., 2006a, b, 2009; Avakian et al., 2008a, b; Boffi et al., 2009; Anselmino et al., 2009a, b, 2010; Boer, 2009). Lastly, some questions remain open, among which the Parton's interpretation of the polarized structure function g_2 is given (Anselmino et al., 1995; Jaffe and Ji, 1991a). Obviously, all of these data and kinds of problems are confronted with the QCD theory and in this comparison, the short and long distance scales are included, so that the factorization

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theorems (Collins, 1998, 1989; Collins et al., 1988; Sterman, 2005) play quite an important role in separating the two kinds of effects. Strong contributions in this sense have been given by Politzer (1980), Ellis et al. (1982, 1983) (EFP), Efremov and Radyushkin (1981), Efremov and Teryaev (1984), Collins and Soper (1981, 1982), Collins et al. (1988) and Levelt and Mulders (1994) (LM).

In the present paper, we propose an approach somewhat similar to EFP's and to LM's, but we use more extensively the Politzer's (1980) theorem on equations of motion (EOM). We consider in particular, the hadronic tensor for SIDIS, DY and $e^+e^- \rightarrow \pi\pi X$. We also consider energies and momentum transfers high enough for assuming one photon approximation, but not so large that weak interactions became comparable with electromagnetic ones. As regards time-like photons, we assume to be far from masses of vector resonances, like J/Ψ , Υ or Z^0 . Lastly, we do not consider the case of active (anti-) quarks originating from gluon annihilation.

Our starting point is the "Born" (LM) approximation for the hadronic tensor, which reads, in the three aforementioned reactions as:

$$W_{\alpha\beta}(P_A, P_B, q) = C \sum_a e_a^2 \int \frac{d^4 p}{(2\pi)^4} Tr [\Phi_A^a(p) \gamma_\alpha \Phi_B^b(p') \gamma_\beta] \quad (1)$$

Here, C is due to color degree of freedom, $C = 1$ for SIDIS and $1/3$ is for DY and e^+e^- annihilation. p and p' denote the four-momenta of the active partons, such that

$$p \mp p' = q, \quad (2)$$

q represents the four momentum of the virtual photon and the $-$ sign refers to SIDIS, the $+$ to DY or to e^+e^- annihilation. Φ_A and Φ_B are correlators, relating the active partons to the (initial or final) hadrons h_A and h_B , whose four momenta are P_A and P_B , respectively. We restrict ourselves to spinless and spin-1/2 hadrons. a and b are the flavors of the active partons, with $a = u, d, s, \bar{u}, \bar{d}, \bar{s}$ and $\bar{b} = a$ in SIDIS, $b = a$ in DY and e^+e^- annihilation; e_a is the fractional charge of flavor a . In DY, Φ_A and Φ_B encode information on the active quark and antiquark distributions inside the initial hadrons. In SIDIS Φ_B is replaced by the fragmentation correlator Δ_B , describing the fragmentation of the struck quark into the final hadron h_B . In the case of e^+e^- annihilation, both correlators Φ_A and Φ_B have to be replaced by Δ_A and Δ_B , respectively.

In the approximation considered, we define the distribution correlator (commonly named correlator) as,

$$\Phi_{ij}(p; P, S) = N \int \frac{d^4 x}{(2\pi)^4} e^{ipx} \langle P, S | \bar{\psi}_j(0) \psi_i(x) | P, S \rangle. \quad (3)$$

Here, N is a normalization constant to be determined in "Zero order term: the QCD parton model". ψ is the quark[†] field of a given flavor and $|P, S\rangle$ a state of a hadron (of spin 0 or 1/2) with a given four-momentum P and Pauli-Lubanski (PL) four-vector S, while p is the quark four-momentum. The color and flavor indices have been omitted in ψ for the sake of simplicity and from now on will be forgotten, unless differently stated. On the other hand, the fragmentation correlator is defined as:

$$\Delta_{ij}(p; P, S) = N \int \frac{d^4 x}{(2\pi)^4} e^{ipx} \langle 0 | \bar{\psi}_j(0) a(P, S) a^\dagger(P, S) \psi_i(x) | 0 \rangle, \quad (4)$$

where $a(P, S) [a^\dagger(P, S)]$ is the destruction (creation) operator for the fragmented hadron, of the given four-momentum and PL four-vector.

The hadronic tensor (1) is not a color gauge invariant. Introducing a gauge link is not sufficient to fulfill this condition, but EOM suggests adding suitable contributions of higher correlators, involving two quarks and a number of gluons, so as to construct a gauge invariant hadronic tensor.

We adopt an axial gauge for the correlator of a gM/Q expansion, where g is the coupling, M the rest mass of the hadron and Q the QCD "hard" energy scale, [†]. For an antiquark, Equations (3) and (4) were slightly modified, as seen in "gauge invariant correlator" and "fragmentation correlator", which are generally assumed to be equal to $q|q^2|$. We examine in detail the first two terms of the expansion. The zero order term corresponds to the QCD parton model approximation. As regards the second term, it concerns the T-odd functions; in particular, we discuss an interesting approximation, already proposed by Collins (2002). In both cases we obtain several approximate relations among "soft" functions, which survive perturbative QCD evolution, as a consequence of EOM. Our approach allows also to determine the Q-dependence of some important azimuthal asymmetries and to draw conclusions about the Burkhardt-Cottingham (1970) sum rule.

In this paper, the gauge invariant correlator (more appropriate than the distribution correlator), whose properties are deduced with the help of EOM was discussed in detail. In particular, we derived an expansion in the powers of gM/Q , whose terms can be interpreted as Feynman-Cutkosky graphs. A prescription for writing a gauge invariant sector of the hadronic tensor which is of interest for interactions at high Q was given. Furthermore,

we study in detail the zero order term and the first order correction of the expansion, deducing approximate relations among functions which appear in the usual parameterizations of the correlator (Mulders and Tangerman, 1996; Goeke et al., 2005). Then, the fragmentation correlator was discussed. The azimuthal asymmetries involved in the three different deep inelastic processes were illustrated. Lastly, a summary of the main results was presented.

GAUGE INVARIANT CORRELATOR

The correlator (3) can be made gauge invariant, by inserting a link operator between the quark fields (Collins and Soper, 1981, 1982; Mulders and Tangerman, 1996), in the following way:

$$\Phi_{ij}(p; P, S) = N \int \frac{d^4x}{(2\pi)^4} e^{ipx} \langle P, S | \bar{\psi}_j(0) \mathcal{L}(x) \psi_i(x) | P, S \rangle. \quad (5)$$

Here,

$$\mathcal{L}(x) = \text{Pexp}[ig\Lambda_{\mathcal{I}}(x)], \quad \text{with} \quad \Lambda_{\mathcal{I}}(x) = \int_{0(\mathcal{I})}^x \lambda_a A_{\mu}^a(z) dz^{\mu}, \quad (6)$$

is the gauge link operator, “P” denotes the path-ordered product along a given integration contour \mathcal{I} , and λ_a and A_{μ}^a denotes the Gell-Mann matrices and the gluon fields respectively. The link operator depends on the choice of \mathcal{I} , which has to be fixed so as to make a physical sense. According to previous treatments (Mulders and Tangerman, 1996; Collins, 2002; Boer et al., 2003b; Bomhof et al., 2004), we define two different contours, \mathcal{I}_{\pm} , as sets of three pieces of straight lines, from the origin to $x_{1\infty} \equiv (\pm\infty, 0, 0_{\perp})$, from $x_{1\infty}$ to $x_{2\infty} \equiv (\pm\infty, x^{\pm}, x_{\perp})$ and from $x_{2\infty}$ to $x \equiv (x^{\pm}, x^{\pm}, x_{\perp})$, having adopted a frame, whose z -axis is taken along the hadron momentum, with $x_{\pm} = 1/\sqrt{2}(t \pm z)$. We remark that the choice of the path is important for the so-called T-odd[†] functions (Boer and Mulders, 1998): the path \mathcal{I}_{+} is suitable for DIS distribution functions, while \mathcal{I}_{-} has to be employed in DY (Boer et al., 2003b; Bomhof et al., 2004). For an antiquark, the signs of correlator (5) and of the four-momentum p have to be changed.

Subsequently, we investigate some properties of the correlator.

T-even and T-odd correlator

We set (Boer et al., 2003b)

$$\Phi_{E(O)} = \frac{1}{2} [\Phi_{+} \pm \Phi_{-}], \quad (7)$$

where Φ_{\pm} corresponds to the contour \mathcal{I}_{\pm} in Equation (6), while Φ_E and Φ_O select the T-even and the T-odd “soft” functions respectively. These two correlators contain the link operators $\mathcal{L}_E(x)$ and $\mathcal{L}_O(x)$, respectively where

$$\mathcal{L}_{E(O)}(x) = \frac{1}{2} \text{P} \left\{ \exp[ig\Lambda_{\mathcal{I}_{+}}(x)] \pm \exp[ig\Lambda_{\mathcal{I}_{-}}(x)] \right\} \quad (8)$$

and $\Lambda_{\mathcal{I}_{\pm}}(x)$ are defined by the second Equation (6). Equations (7) and (8) imply that the T-even functions are independent of the contour (\mathcal{I}_{+} or \mathcal{I}_{-}), while the T-odd ones change sign according to whether they are involved in DIS or in DY (Collins, 2002; Boer et al., 2003b). In this sense, such functions are not strictly universal (Collins, 2002), as already stressed. It is convenient to consider an axial gauge,

$$A^{\dagger} = 0, \quad (9)$$

More precisely, one should speak of “naive T”, consisting of reversing all momenta and angular momenta involved in the process, without interchanging initial and final states (DeRujula, 1971; Bilal et al., 1991; Sivers, 2006) with antisymmetric boundary conditions (Mulders and Tangerman, 1996):

$$A^{\mu}(-\infty, x^{\pm}, x_{\perp}) = -A^{\mu}(+\infty, x^{\pm}, x_{\perp}). \quad (10)$$

Here, we have adopted the shorthand notation A^{μ} for $\lambda^a A_{\mu}^a$. In this gauge, it was proposed for the first time by Kogut and Soper (1970) and named KS gauge in the following:

$$\Lambda_{\mathcal{I}_{+}}(x) = -\Lambda_{\mathcal{I}_{-}}(x) = \int_{x_1}^{x_2} dz_{\mu} A^{\mu}(z), \quad (11)$$

where x_i is a shorthand notation for $x_{i,+}$, $i = 1, 2$. Therefore, in the KS gauge,

$$\mathcal{L}_E(x) = \text{Pcos}[g\Lambda_{\mathcal{I}_{+}}(x)], \quad \mathcal{L}_O(x) = \text{Psin}[g\Lambda_{\mathcal{I}_{+}}(x)] \quad (12)$$

and the T-even (T-odd) part of the correlator consists of a series of even (odd) powers of g , each term being endowed with an even (odd) number of gluon legs. As a consequence, the zero order term is T-even, while the first order correction is T-odd. This confirms that no T

odd terms occur without interactions among partons, as claimed also by other authors (Brodsky et al., 2002a, b, 2003; Collins, 2002). Gauge invariance of the correlator implies that these conclusions hold true in any axial gauge, such that condition (9) is fulfilled. From now on we shall work in such a type of gauge (Ji and Yuan, 2002; Belitsky et al., 2003).

Power expansion of the correlator

We consider Φ_+ , which is explained before as DIS. As regards DY, the T-odd terms will change sign, as seen from the choice of the path - l_- instead of l_+ - and from the first Equation (11) and second Equation (12). We rewrite $L(x)$ as

$$\mathcal{L}(x) = \sum_{n=0}^{\infty} (ig)^n \Lambda_n(x). \tag{13}$$

Here, $\Lambda_0(x) = 1$, while for $n \geq 1$ we have the following equation in the KS gauge:

$$\Lambda_n(x) = \int_{x_1}^{x_2} dz_1^{\mu_1} \int_{x_1}^{z_1} dz_2^{\mu_2} \dots \int_{x_1}^{z_{n-1}} dz_n^{\mu_n} [A_{\mu_n}(z_n) \dots A_{\mu_2}(z_2) A_{\mu_1}(z_1)], \tag{14}$$

where $z_i \equiv (\infty, z_i^+, z_{i\perp})$, $i = 1, 2, \dots, n$, are points in the space-time along the line through x_1 and x_2 . Substituting Equation (13) into Equation (5), we have the following expansion of Φ in powers of g :

$$\Phi = \sum_{n=0}^{\infty} (ig)^n \Gamma_n, \tag{15}$$

with

$$(\Gamma_n)_{ij} = N \int \frac{d^4x}{(2\pi)^4} e^{ipx} \langle P, S | \bar{\psi}_j(0) \Lambda_n(x) \psi_i(x) | P, S \rangle. \tag{16}$$

As noticed already, Γ_n is T-even for even n and T-odd for odd n .

Now, we invoke the Politzer's (1980) theorem, concerning EOM. This states that if we consider the matrix element between two hadronic states of a given composite operator, constituted by quark and/or gluon fields, each of such field fulfils EOM, despite the fact that the parton is off-shell and/or renormalized. We show in Appendix A that owing to the Politzer theorem, the term Γ_0 fulfils the Dirac homogeneous equation,

$$(\not{p} - m)\Gamma_0 = 0, \tag{17}$$

where m is the quark rest mass. The corresponding Feynman-Cutkosky graph is represented in Figure 1. For $n \geq 1$ we have instead

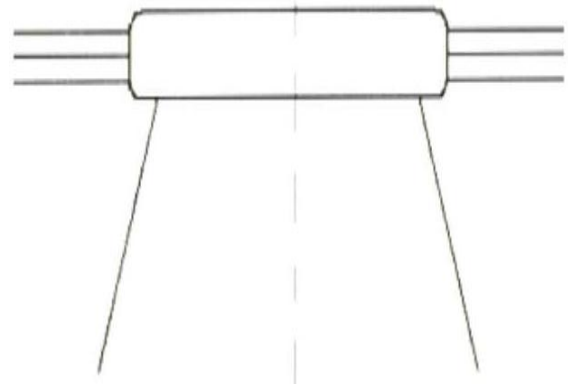


Figure 1. Feynman-Cutkosky graph for zero order term of expansion (15).

$$(ig)^n \Gamma_n = N \int d\Omega_n S^{\mu_1 \dots \mu_n} \Phi_{\mu_1 \dots \mu_n}^{(n)}(p, k_1, k_2 \dots k_n). \tag{18}$$

Here, we have a set of equations:

$$d\Omega_n = \prod_{l=1}^n \frac{d^4 k_l}{(2\pi)^4}, \tag{19}$$

$$S^{\mu_1 \dots \mu_n} = \frac{ig}{\not{p} - m + i\epsilon} i\gamma^{\mu_1} \frac{ig}{\not{p} - \bar{k}_1 - m + i\epsilon} i\gamma^{\mu_2} \dots \times \frac{ig}{\not{p} - \bar{k}_{n-1} - m + i\epsilon} i\gamma^{\mu_n}, \tag{20}$$

$$\bar{k}_l = \sum_{r=1}^l k_r. \tag{21}$$

The k_r ($r = 1, 2, \dots, n$) are the four-momenta of the n gluons involved in the quark-gluon correlator $\Phi_{\mu_1 \dots \mu_n}^{(n)}$. This is defined as

$$[\Phi_{\mu_1 \dots \mu_n}^{(n)}(p, k_1, k_2 \dots k_n)]_{ij} = N \int \frac{d^4x}{(2\pi)^4} e^{i(p - \bar{k}_n)x} \times \langle P, S | \bar{\psi}_j(0) P' [B_{\mu_n}(k_n) \dots B_{\mu_1}(k_1)] \psi_i(x) | P, S \rangle, \tag{22}$$

with

$$B_\mu(k) = \hat{A}_\mu(k) + \tilde{A}_\mu(k), \tag{23}$$

$$\hat{A}_\mu(k) = \int \frac{d^4z}{(2\pi)^4} A_\mu(z) e^{ikz}, \tag{24}$$

$$\tilde{A}_\mu(k) = \delta(k^+) \lim_{M \rightarrow \infty} \int d\kappa e^{-i\kappa M} \hat{A}_\mu(k^-, \kappa, k_\perp). \tag{25}$$

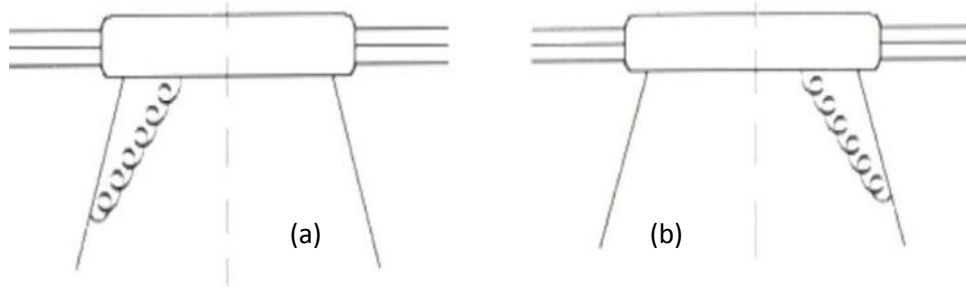


Figure 2. Feynman-Cutkosky graph for first order correction in the coupling.

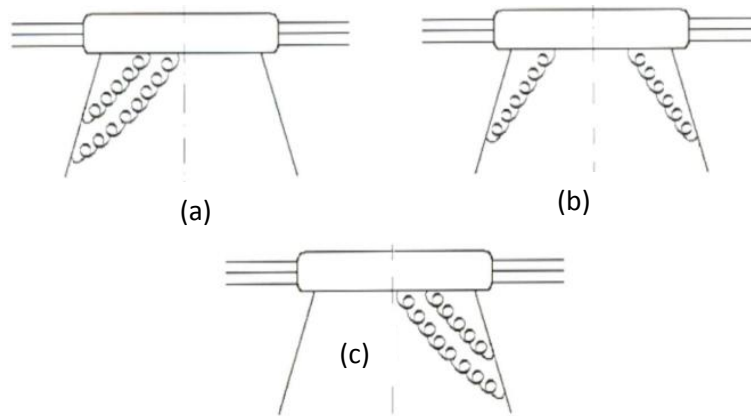


Figure 3. Feynman-Cutkosky graph for second order correction.

Moreover the operator product P' is defined according to the following rules:

- any $\hat{A}_\mu(k)$ is at the left of any $\tilde{A}_\mu(k)$;
- the $\tilde{A}_\mu(k)$ are ordered as $\tilde{A}_{\mu_1}(k_1)\tilde{A}_{\mu_2}(k_2)\dots\tilde{A}_{\mu_l}(k_l)$;
- the $\hat{A}_\mu(k)$ are ordered as $\hat{A}_{\mu_m}(k_m)\dots\hat{A}_{\mu_2}(k_2)\hat{A}_{\mu_1}(k_1)$.

Lastly, the quark-gluon correlators $\Phi_{\mu_1\dots\mu_n}^{(n)}$ fulfill the following homogeneous equation:

$$(\not{p} - \not{k}_n - m)\Phi_{\mu_1\dots\mu_n}^{(n)}(p, k_1, k_2\dots k_n) = 0. \tag{26}$$

Each term of expansion (15) – which is somewhat similar to the one obtained by Collins and Soper (1981, 1982) – may be interpreted as a Feynman-Cutkosky graph. It corresponds to an interference term between the amplitude

$$\text{”nucleon} \rightarrow \text{quark} + \text{spectator partons”} \tag{27}$$

without any rescattering, and an analogous one, where n gluons are exchanged between the active quark and

the spectator partons. In particular, the interference term is such that the gluons (for $n > 0$) are attached to the left quark leg (Figures 2a and 3a). An important result, deduced at the end of Appendix A, is that such a term turns out to correspond to any interference term between two amplitudes, such that k and $n - k$ gluons are respectively exchanged between the active quark and the spectator partons, with $0 \leq k \leq n$. The situation is illustrated in Figures 2 and 3 for $n = 1$ and 2.

It is worth noting that a radiation ordering similar to the one established here is found in semiinclusive processes at large x (Catani et al., 1991a) and in totally inclusive DIS at small x (Catani et al., 1991b).

Moreover, the terms (22) consist of quark-gluon-quark correlations, analogous to the one introduced by Efremov and Teryaev (1984) and by Qiu and Sterman (1991, 1992, 1998).

As a consequence of the Politzer’s theorem, formulae (15) to (22) hold for renormalized fields, provided we take into account the scale dependence of the coupling of the quark mass m and of the correlators $\Phi_{\mu_1\dots\mu_n}^{(n)}(p, k_1, k_2\dots k_n)$ (Rogers, 2007). Moreover one has to observe that the four-momenta appearing in the propagators are highly off-shell: p^2 and $(p - k_r)^2$ are of



Figure 4. Graphs for "hard" amplitudes interfering coherently, first order correction in the coupling.

order Q^2 (Collins and Soper, 1982; Levett and Mulders, 1994), because the un-certainty principle demands hard interactions to occur in a very limited space-time interval, corresponding to the condition

$$|p^2| \gg M^2. \tag{28}$$

Therefore we have $p^2 \approx 2p^+p^-$ and $p^+ = O(Q)$, whence

$$|p^-| = O(Q) \tag{29}$$

and it follows that the coefficients Γ_n are of order Q^{-n} , up to QCD corrections, consisting of terms of the type $g^{2k}(\ln Q)^m$, with k and m integers and $k \geq m$ (Dokshitzer et al., 1980). For the same reason, the coupling g , which appears in expansion (15), assumes small values, corresponding to short distances and times.

To summarize, we have found that the T-even and the T-odd correlators, given by equations (7), may be written as expansions in gM/Q ,

$$\Phi_E(p) = \sum_{n=0}^{\infty} \left(\frac{igM}{Q}\right)^{2n} \bar{\Gamma}_{2n}, \Phi_O(p) = \sum_{n=0}^{\infty} \left(\frac{igM}{Q}\right)^{2n+1} \bar{\Gamma}_{2n+1}, \tag{30}$$

where $\bar{\Gamma}_n = \Gamma_n Q^n / M^n$ has a relatively weak Q -dependence, as told above. Moreover, as already explained, Φ_O changes sign when involved in DY. Stated differently, T-odd terms present an odd number of quark propagators. See equation (20) for odd n : in the limit of negligible quark mass, quark four-momenta in DIS are space like, whereas in DY they are time like (Boer et al., 2003b).

The first two terms of expansion (15) will be studied in detail in "Zero order term: the QCD parton model" and "First Order Correction" respectively.

HADRONIC TENSOR

Here, we refer indifferently to the hadronic tensor of one of the three processes introduced. To be precise, among

these, only DY involves two correlators of the type illustrated in asymmetries, whereas SIDIS and e^+e^- annihilation include respectively one and two fragmentation correlators. However, as we shall see in fragmentation correlator, this object requires only minor modifications with respect to correlator (5).

If we substitute this correlator into the hadronic tensor (1), this latter does not fulfill the requirement of electromagnetic gauge invariance: only the term of zero order in the coupling satisfies this condition. In order to get a complete gauge invariance at any order, we have to recall the interpretation given above of the correlator. For example, at first order in the coupling in SIDIS, we see that the "hard" scattering amplitude $q\gamma^* \rightarrow q'\tilde{g}$ where we have denoted by q and q' the initial and final quark and by \tilde{g} a gluon - consists not only of the graph of Figure 4a, encoded in the first order term of the correlator, but also of the one represented in Figure 4b, which interferes coherently with it. This guarantees electromagnetic gauge invariance for the first order graph (Berger and Brodsky, 1979). Furthermore, convoluting "hard" graphs with the "soft" factors, these two amplitudes give rise, among other objects, to asymmetric Feynman-Cutkosky graphs (Figure 5), related to interference terms. These are observables - necessarily gauge invariant - and therefore assume real values. This procedure, already suggested by LM, can be generalized to the three kinds of hadronic tensors considered in the present article, at any order in g , so as to obtain sets of graphs corresponding to observable, and therefore gauge invariant, quantities. We show how to construct them at any order n , corresponding to the overall number of gluons exchanged between active quarks and spectator partons. The procedure consists in the following steps, for a given n :

- Consider the $n + 1$ possible combinations of gluons occurring in the hadronic tensor (1), say, s for hadron A and $n - s$ for hadron B, with $s = 0, 1 \dots n$.
- For a given s ($n-s$), consider all possible correlators, according to the definition given in "Power Expansion of the Correlator" as seen in the summary of this study, where $s+1$ ($n-s+1$) correlators equal to Γ_s (Γ_{n-s}).

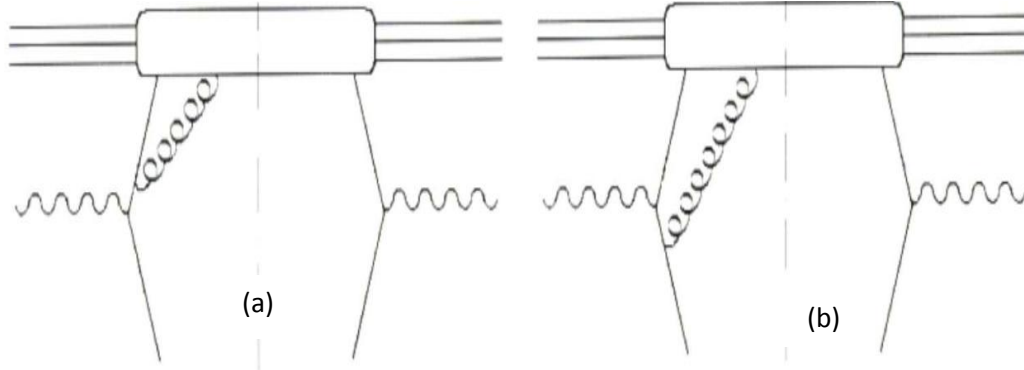


Figure 5. Feynman-Cutkosky graphs corresponding to "hard" amplitudes of Figure 4. Also the complex conjugate graphs, which amount to mirror images of these two, contribute to first order corrections.

-Add each of such correlator to those graphs whose "hard" parts interfere coherently with it, as shown in Figure 5. In practice, one has to do this for the correlator gluons are attached to the "left" quark leg and to multiply by the number of gluons of each correlator.

Then we have, up to QCD corrections at each order of the expansion,

$$W_{\alpha\beta}(q) = \sum_{n=0}^{\infty} W_{\alpha\beta}^{(n)}(q), \tag{31}$$

$$W_{\alpha\beta}^{(n)} = C \int \frac{d^4p}{(2\pi)^4} \int d\Omega_n \sum_{r=0}^n \sum_{s=0}^n Tr M_{\alpha\beta}^{(n)}, \tag{32}$$

with

$$M_{\alpha\beta}^{(n)} = \sum_{s=0}^n (s+1)(n-s+1) [\tilde{\Gamma}_{\alpha}^{(s,0)} \Phi_A^{s,0} \tilde{\Gamma}_{\beta}^{(n,s)} \Phi_B^{n,s}], \tag{33}$$

$$\tilde{\Gamma}_{\rho}^{(l,r)} = \sum_{m=r}^l S_r^m \gamma_{\rho} S_m^l. \tag{34}$$

Here we have used the following shorthand notations:

$$S_r^m = S^{\mu_{r+1}, \mu_{r+2}, \dots, \mu_m}, \quad \Phi^{n,s} = \Phi_{\mu_{s+1}, \mu_{s+2}, \dots, \mu_n}^{n,s} \tag{35}$$

$S^{\mu_{r+1}, \mu_{r+2}, \dots, \mu_m}$ and $\Phi_{\mu_{s+1}, \mu_{s+2}, \dots, \mu_n}^{n,s}$ are defined analogously to Equations (20) and (22): the matrix product starts from μ_{r+1} and from μ_{s+1} respectively, rather than from μ_1 . In particular, $\Phi^{n,0}$ coincides with the definition (20). Last, we have set $S_r^r = 1$.

For each term of expansion (31) we have to take into account three kinds of effects:

a) gluon radiation by scattered partons;

b) perturbative QCD corrections;

c) higher correlators, such that the active quarks exchange gluons with quark- antiquark pairs or gluon pairs or triplets belonging to spectator partons.

The first two effects may be calculated according to the algorithm suggested by Collins and Soper (1981, 1982). As to the contributions c), they can be included in the basic term of expansion (31), since they have the same (T-even or T-odd) behavior. Lastly, we recall that unless we integrate over some final transverse momentum [of the lepton pair in the case of DY, of a final hadron in SIDIS or e^+e^- annihilation], the phase space of the final gluons emitted undergoes a restriction (Dokshitzer et al., 1980), expressed by a doubly logarithmic form factor; this is more and more sizable at increasing energy, resulting in the well-known Sudakov-like damping (Collins and Soper, 1981; Boer, 1999).

ZERO ORDER TERM: THE QCD PARTON MODEL

Here and subsequently, we elaborated the first two terms of the expansion of the hadronic tensor. To this end, we defined a suitable reference frame, such that the momentum P_B of the hadron B has an opposite direction to the momentum P_A of the hadron A, $|P_A|$ and $|P_B|$ are of order Q and the z-axis is along P_A . Moreover we focus on the hadronic tensor for DY process.

However, as shown previously, our results can be trivially extended to SIDIS and e^+e^- annihilation; the main difference, concerning the fragmentation function, will be discussed in "fragmentation correlator".

Let us consider the hadronic tensor (32) at zero order,

$$W_{\alpha\beta}^{(0)} = C \int \frac{d^4p}{(2\pi)^4} Tr [\gamma_{\alpha} \Gamma_0^A(p) \gamma_{\beta} \Gamma_0^B(p')]. \tag{36}$$

Here the Γ_0 's are given by Equation (16), for $n = 0$, and fulfil the homogeneous Dirac equation (17). Incidentally, they are T-even and gauge invariant at zero order in g . Moreover p' is defined by Equation (2). The tensor (36), T-even itself, can be calculated, once we know the "soft" functions involved in the parameterizations of the correlators Γ_0 's. We show in Appendix B that

$$\Gamma_0(p) = \frac{N}{4\mathcal{P}}(\not{p} + m) [f_1(p) + \gamma_5 \not{p}'_{\parallel} g_{1L}(p) + \gamma_5 \not{p}'_{\perp} h_{1T}(p)] 2p^+ \delta(p^2 - m^2). \quad (37)$$

Here $f_1(p)$, $g_{1L}(p)$ and $h_{1T}(p)$ are functions of the four-momentum p of the active quark, which, in this case, is on shell: $p \equiv (E, \mathbf{p})$, with $E = \sqrt{m_q^2 + \mathbf{p}^2}$. S_{\parallel}^q and S_{\perp}^q are the components of the quark PL vector, respectively parallel and perpendicular to the hadron momentum. Moreover we have set

$$\mathcal{P} = \frac{1}{\sqrt{2}} p \cdot n_{\pm}, \quad (38)$$

having defined the dimensionless, light-like four-vectors n_{\pm} in such a way that

$$n_+ \cdot n_- = 1 \quad (39)$$

and such that their spatial components are along (+) or opposite (-) to the hadron momentum. It is important to notice that, if integrated over p^- , the expression obtained for the zero order correlator turns out to be proportional to the density matrix of a quark confined in a finite volume, but free of interactions with other partons (Di Salvo, 2007b). Therefore we fix the normalization constant N so as to obtain, after integration, just the density matrix,

$$N = 2\mathcal{P}. \quad (40)$$

Lastly, it is convenient to express S_{\parallel}^q and S_{\perp}^q in terms of the components of the PL vector of the hadron. As shown in Appendix B, one has

$$S_{\parallel}^q = \lambda \left(\frac{\bar{p}}{m} \right) - \bar{\eta}_{\perp} + O(\bar{\eta}_{\perp}^2), \quad S_{\perp}^q = S_{\perp} + \bar{\lambda}_{\perp} \frac{\bar{p}}{m} + O(\bar{\eta}_{\perp}^2). \quad (41)$$

Here

$$\lambda = -S \cdot \frac{n_+ + n_-}{\sqrt{2}}, \quad S_{\perp} = S - \lambda \frac{n_+ + n_-}{\sqrt{2}}, \quad (42)$$

$$\bar{p} \equiv (|p|, E\hat{p}), \quad \hat{p} = p/|p|, \quad \bar{\eta}_{\perp} = p_{\perp}/\mathcal{P}, \quad (43)$$

$$\bar{\lambda}_{\perp} = -S \cdot \bar{\eta}_{\perp}, \quad p_{\perp} \equiv (0, 0, p_{\perp}) \quad (44)$$

and p_{\perp} is the transverse momentum of the active quark with respect to the hadron momentum. Equation (37) has important consequences on TMD T-even functions, as will be illustrated in Twist-2, T-even Correlator and Twist-3, "Hybrid" Correlator. To this end, we compare that equation with the naive parameterization of the TMD correlator in terms of Dirac components, without introducing any dynamic conditions (Mulders and Tangerman, 1996; Boer et al., 2000; Goeke et al., 2005). We give such a parameterization in Appendix C, up to and including twist-3 terms. The twist-2, T-even sector corresponds to quark distribution functions which survive when interactions with gluons are turned off.

As regards the twist-3 functions, we distinguish among the T-even, the T-odd and the "hybrid" ones, these last deriving contributions both from T-even and T-odd terms.

Twist-2, T-even Correlator

If quark-gluon interactions are neglected, the correlator includes just twist-2, T-even terms. We show in Appendix C that it can be parameterized as

$$\Phi_E^f = \frac{\mathcal{P}}{\sqrt{2}} \{ f_1 \not{n}_+ + (\lambda g_{1L} + \lambda_{\perp} g_{1T}) \gamma_5 \not{n}_+ + \frac{1}{2} h_{1T} \gamma_5 [\not{S}_{\perp}, \not{n}_+] + \frac{1}{2} (\lambda h_{1L}^{\perp} + \lambda_{\perp} h_{1T}^{\perp}) \gamma_5 [\not{\eta}_{\perp}, \not{n}_+] \} 2p^+ \delta(p^2 - m^2). \quad (45)$$

Here we have adopted the usual notations for the non-perturbative functions (Kotzinian, 1995; Tangerman and Mulders, 1995); the indices f and E of Φ denote respectively the feature of "free" and "T-even". The Dirac operators considered are purely T-even, as can be checked; moreover

$$\eta_{\perp} = p_{\perp}/\mu_0, \quad \lambda_{\perp} = -S \cdot \eta_{\perp} \quad (46)$$

and μ_0 is an undetermined energy scale, introduced for dimensional reasons, in such a way that all functions embodied in the parameterization of Φ have the dimensions of a probability density. This scale (Kotzinian, 1995) determines the normalization of the functions which depend on η_{\perp} . In particular, as is well-known, the 6 twist-2 functions, which appear in the parameterization (45), are interpreted as TMD probability densities: f_1 is the unpolarized quark density, g_{1L} the longitudinally polarized density in a longitudinally polarized (spin 1/2) hadron, g_{1T} the longitudinally polarized density in a transversely polarized hadron, h_{1L}^{\perp} the transversity in a longitudinally polarized hadron and

$$h_{1T}^{\perp} = h_{1T} + |\eta_{\perp}^2| h_{1T}^{\perp} \quad (47)$$

is the TMD transversity in a transversely polarized hadron.

Now we compare the parameterization (45) with the correlator (37). To this end we consider projections of both matrices over the various Dirac components, for a given Dirac operator Γ ,

$$\Phi^\Gamma = \frac{1}{2} \text{Tr} \Gamma \Phi, \quad (48)$$

taking into account Equation (41) wherever necessary.

The function h_{1T}^l is known as "pretzelosity" (Avakian et al., 2008b). First of all, $\Gamma = \gamma_5 \gamma^+$ and $\gamma_5 \gamma^+ \gamma_i$ are approximately in the limit of

$$h_{1L}^\perp \approx -\frac{\mu_0}{\mathcal{P}} g_{1L}, \quad g_{1T} \approx \frac{\mu_0}{\mathcal{P}} h_{1T}, \quad h_{1T}^\perp \approx \frac{\mu_0^2}{\mathcal{P}^2} h_{1T}. \quad (49)$$

These relations hold up to terms of order $(gM/Q)^2$, since, as we have seen, the T-even Dirac components of Φ derive contributions only from even powers of gM/Q . Moreover, the Politzer theorem implies that the relations are not modified by renormalization effects, and therefore hold also taking into account QCD evolution.

In order to determine μ_0 , we observe that the functions involved in both sides of Equation (49) are independent of \mathcal{P} . Therefore we must set $\mu_0 = C_0 \mathcal{P}$, C_0 being a dimensionless numerical constant, independent of momentum. But since these functions are quark densities, they should be normalized adequately, setting $C_0 = 1$. Then, neglecting the quark mass,

$$\mu_0 = \mathcal{P} = \frac{1}{\sqrt{2}} p \cdot n_-. \quad (50)$$

This result differs from the treatments of previous authors (Mulders and Tangerman, 1996; Goeke et al., 2005), who assume $\mu_0 = M$. Some mismatches have been shown, as consequences of this choice (Bacchetta et al., 2008); these could be eliminated by taking into account result (50).

By comparing CLAS (Avakian et al., 2005) and HERMES (Airapetian et al., 2005b) results, at not too high values of Q^2 (1.5 to 3 GeV) the first relation (49), together with equation (50), is verified for $x < 0.35$ (Di Salvo, 2007b), discrepancies at larger x being attributed to higher twist contributions.

Twist-3, "Hybrid" Correlator

Now we consider a sector of the correlator which, as explained in the foregoing, has both T-even and T-odd

contributions. In particular, here we focus on that part of "hybrid" correlator which comes from the so-called "kinematic" twist-3 terms. In Appendix C we find, according to the usual notations (Mulders and Tangerman, 1996; Goeke et al., 2005),

$$\begin{aligned} \Phi_H^f &= \left\{ \frac{1}{2} (f^\perp + \lambda g_L^\perp \gamma_5 + \lambda_\perp g_T^\perp \gamma_5) \not{p}_\perp + \frac{1}{4} \lambda_\perp h_T^\perp \gamma_5 [\not{s}_\perp, \not{p}_\perp] \right. \\ &+ \left. \frac{1}{2} x M \left(e + g_T' \gamma_5 \not{s}_\perp + \frac{1}{2} (\lambda h_L + \lambda_\perp h_T) \gamma_5 [\not{p}_-, \not{p}_+] \right) \right\} 2p^+ \delta(p^2 - m^2). \end{aligned} \quad (51)$$

Comparing the operator (51) with the correlator (37), and considering, in particular, the projections over $\Gamma = \gamma_i$ ($i = 1, 2$) of such operators, the approximate relation is yielded:

$$f^\perp \approx f_1, \quad (52)$$

which corresponds to the Cahn (1978, 1989) effect and is approximately verified for sufficiently large Q^2 and small x (Anselmino et al., 2007). Also, this equation, like Equations 49, survives QCD evolution. As we shall observe in the "First Order Correction", Equation 52 holds the terms of order gM/Q , since f^\perp derives also T-odd contributions from one-gluon exchange. The projections of the same operators over $\Gamma = \gamma_5 \gamma_i$ ($i = 1, 2$) yield (after integration over p_\perp)

$$g_T(x) \approx \frac{m}{xM} h_1(x). \quad (53)$$

Here

$$g_T(x) = \int d^2 p_\perp g_T'(x, p_\perp^2) \quad (54)$$

and

$$h_1(x) = \int d^2 p_\perp \left[h_{1T}(x, p_\perp^2) + |\eta_\perp^2| h_{1T}^\perp(x, p_\perp^2) \right]. \quad (55)$$

This last equation has been obtained from equation (47). The contribution of the QCD parton model to $g_T(x)$ is very small: m is negligible for u- and d-quarks, while for s- quarks h_1 is presumably small, because the sea is produced mainly by annihilation of gluons, whose transversity is zero in a nucleon. Therefore the contribution of quark-gluon interactions, neglected in the approximation considered, becomes prevalent in this case, as well as for $\Gamma = 1$ and $\gamma_5 \gamma^+ \gamma_-$, corresponding respectively to the functions e and h_L in

equation (51). The effect of such interactions will be discussed in “First Order Correction”.

Remarks

To conclude the analysis of the “Zero order term: the QCD parton model”, we sketch some consequences of our theoretical results.

A) In expression (47) or (55) for transversity, the second term is due to a relativistic effect. To illustrate this, consider a transversely polarized hadron. The longitudinal polarization of the quark, due in this case to the transverse momentum, is magnified by the boost from the quark rest frame. This additional polarization, along the quark momentum, has again a transverse component with respect to the nucleon momentum.

B) Equation 55, together with the last two equations (49), suggests a method for determining approximately the nucleon transversity. Indeed, g_{1T} can be conveniently extracted from double spin asymmetry (Kotzinian and Mulders, 1996; Di Salvo, 2002, 2003) in SIDIS with a transversely polarized target. This asymmetry is expressed as a convolution of the unknown function with the usual, well-known fragmentation function of the pion. Therefore, the method appears complementary to the one usually proposed (Airapetian et al., 2000; Anselmino et al., 2007), based on the Collins (1993) effect in single spin SIDIS asymmetry; in this latter case one is faced with the convolutive product of h_{1T} with the Collins function, which is poorly known (Efremov et al., 2006a,b).

C) Equation 53 establishes a relation between transversity and transverse spin. Indeed, the two quantities are related to each other. But, unlike transversity, the transverse spin operator is chiral even and does not commute with the free Hamiltonian of a quark (Jaffe and Ji, 1991a): in QCD parton model it is proportional to the quark rest mass, which causes chirality flip.

D) We note that g_{1T} , h_{1L}^\perp and h_{1T}^\perp are associated with “twist-2” Dirac operators (Jaffe and Ji, 1991a, 1992), and yet, in our treatment, they are multiplied by inverse powers of Q, as results from Equations (45) and (50): Q^{-1} for the first two functions, Q^{-2} for the third one. This would be unacceptable for common distribution functions; but, when transverse momentum is involved, also the orbital angular momentum plays a role. To illustrate this point, we recall that the quark distribution functions may be regarded as the absorptive parts of u-channel quark-hadron amplitudes (Soffer, 1995).

For example, g_{1T} corresponds to an amplitude of the type (+ + | - +), denoting by $|\Lambda\lambda\rangle$ a state in which the nucleon and quark helicities are, respectively, Λ and λ . The amplitudes corresponding to the functions in question involve a change $\Delta L = 1$ (for g_{1T} and h_{1L}^\perp) or

$\Delta L = 2$ (for h_{1T}^\perp) in the orbital angular momentum; therefore they are of the type

$$A = A(\sin\theta)^{\Delta L}, \tag{56}$$

where $\theta = \arcsin|p_\perp|/|p|$ is the angle between the nucleon momentum and the quark momentum, while A is weakly energy dependent. But $|p|$ is of order Q and $|p_\perp|$ of order M. Therefore Equation 56 reproduces the Q-dependence of the coefficients relative to the above mentioned functions.

FIRST ORDER CORRECTION

The first order correction was done in g of the hadronic tensor (Equations 32 and 33).

$$W_{\alpha\beta}^{(1)} = -2gC \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} Tr N_{\alpha\beta}. \tag{57}$$

Here we have set

$$N_{\alpha\beta} = 2[h_\alpha^\mu(p, p', k)\Phi_{A\mu}^{(1)}(p, k)\gamma_\beta\Gamma_0^B(p') + \gamma_\alpha\Gamma_0^A(p)h_\beta^\mu(p', p, k)\Phi_{B\mu}^{(1)}(p', k)] \tag{58}$$

and

$$h_\alpha^\mu(p, p', k) = \gamma_\alpha \frac{1}{\not{p} - m + i\epsilon} \gamma^\mu + \gamma^\mu \frac{1}{\not{p}' - \not{k} - m + i\epsilon} \gamma_\alpha. \tag{59}$$

Furthermore, the $\Phi_\mu^{(1)}$ are given by Equation (22) for $n = 1$ and fulfill the homogeneous Dirac equation

$$(\not{p} - \not{k} - m)\Phi_\mu^{(1)}(p, k) = 0. \tag{60}$$

Therefore, in the gauge adopted, this function is parameterized as

$$\Phi_\mu^{(1)}(p, k) = \Psi_\mu(p, k)\delta\left(p_1^- - \frac{m^2 + P_{1\perp}^2}{2p_1^+}\right). \tag{61}$$

Here we have set

$$p_1 = p - k, \quad \text{with} \quad p_1^2 = m^2 \tag{62}$$

and

$$\Psi_\mu(p, k) \approx \frac{1}{2}(p_1 + m)\hat{L}[C_\mu + \Delta C_\mu\gamma_5\hat{S}_\parallel^q + \Delta_T C_\mu\gamma_5\hat{S}_\perp^q + \Delta_T C'_\mu\gamma_5\hat{S}_\perp]. \tag{63}$$

This observation is the fruit of a stimulating discussion with Nello Paver.

This is a consequence of the Politzer theorem, as shown in Appendix B. The quantities C_μ , ΔC_μ , $\Delta_T C_\mu$ and $\Delta_T C'_\mu$ are correlation functions of p and k . In particular, we have (Appendix B.2):

$$C_\mu = p_{1\perp\mu} C_1 + \epsilon_{\mu\nu\rho\sigma} n_-^\nu (C_2 \lambda n_+^\rho p_{1\perp}^\sigma + C_3 M S_\perp^\rho n_+^\sigma), \quad (64)$$

$$\Delta C_\mu = \Delta C_{p_{1\perp\mu}}, \quad (65)$$

$$\Delta_T C_\mu = \Delta_T C_{p_{1\perp\mu}}, \quad (66)$$

$$\Delta_T C'_\mu = \Delta_T C'_{p_{1\perp\mu}}. \quad (67)$$

Here the C_i ($i = 1, 2, 3$) are unpolarized. ΔC ($\Delta_T C$) is a longitudinally (transversely) polarized function in a longitudinally (transversely) polarized nucleon. $\Delta_T C$ is a transversely polarized correlation function in an unpolarized nucleon: it is connected to quark-gluon interaction, for example, to a spin-orbit coupling (Brodsky et al., 2002a,b, 2003).

Last, we have set in Equation (63)

$$\sqrt{|p_\perp^2|} \bar{S}_{\perp\alpha} = \epsilon_{\alpha\beta\rho\sigma} n_+^\beta n_-^\rho p_\perp^\sigma, \quad (68)$$

$$\hat{L} = \sqrt{\frac{\mathcal{P}}{\mathcal{P}_1}} \left[\cosh\varphi + \gamma_0 \gamma_3 a \frac{\sinh\varphi}{2\varphi} \right]. \quad (69)$$

Here $\mathcal{P}_1 = p_1^+ / \sqrt{2}$ while ϕ and a are defined in Appendix B.

Approximate factorization

The second term of Equation 59 is not factorizable, in agreement with the observations of various authors (Brodsky et al., 2002a,b, 2003; Peigné, 2002; Collins and Qiu, 2007), who have shown failures of universality (Peigné, 2002; Collins and Qiu, 2007) at large transverse momentum. However, for sufficiently large Q , and adopting an axial gauge, this term is negligibly small (Berger and Brodsky, 1979) in comparison with the first one, which instead is factorizable. In fact, the gluon corresponding to the first term has a smaller offshellness than the one involved in the second term. This approximation is especially acceptable, even for relatively small Q , provided we limit ourselves to small transverse momenta (Collins, 2002) of the initial hadrons with respect to the direction of the momentum of the virtual photon in the center of mass of the DY pair. However, as already explained in ‘‘Gauge Invariant

Correlator’’, also in the case when factorization is approximately satisfied, the T-odd distribution functions change sign from SIDIS to DY. We shall illustrate phenomenological implications of this change of sign in ‘‘Asymmetries’’.

In this approximation the tensor (57) amounts to

$$W_{\alpha\beta}^{(1)} = -4gC \int \frac{d^4 p}{(2\pi)^4} \gamma_\alpha [\Gamma_1^A(p) \gamma_\beta \Gamma_0^B(p') + \Gamma_0^A(p) \gamma_\beta \Gamma_1^B(p')], \quad (70)$$

where

$$\Gamma_1(p) = \frac{1}{\not{p} - m + i\epsilon} \gamma^\mu \int \frac{d^4 k}{(2\pi)^4} \Phi_\mu^{(1)}(p, k) \quad (71)$$

and Γ_0 is given by Equation(37). Then the tensor $W_{\alpha\beta}^{(1)}$ assumes a form similar to $W_{\alpha\beta}^{(0)}$ giving rise to an approximate (Brodsky et al., 2002a) factorization of T-odd functions. Our conclusion is quite analogous to the one drawn by Collins (2002) and presents some similarity with the Qiu-Sterman (1991) assumption about the quark-gluon-quark correlation functions. In particular, as regards the factors $\Gamma_1(p)$, defined by Equation(71), we have to take into account Equations (61) to (67), together with eqns. (41). These induce for Γ_1 the following parameterization, at twist-3 approximation:

$$\Gamma_1(p) \approx \frac{2p^+}{\pi(p^2 - m^2 + i\epsilon)} \frac{1}{2} \gamma_- \gamma_+ [\not{p}_\perp f_o^\perp + \gamma^i \epsilon_{i\nu\rho\sigma} n_-^\nu (\lambda p_\perp^\rho n_+^\sigma f_L^\perp + M n_+^\sigma S_\perp^\rho g_{T,o}^\perp) + \gamma_5 \not{p}_\perp \not{p}_\perp h_{T,o} + \gamma_5 \not{p}_\perp \not{p}_\perp h' + \lambda \gamma_5 \not{p}_\perp g_{L,o}^\perp]. \quad (72)$$

Here we have defined

$$f_o^\perp = - \int d\tilde{\Omega} \mathcal{C}_1, \quad f_L^\perp = \int d\tilde{\Omega} (\mathcal{C}_2 + r \Delta \mathcal{C}), \quad g_{T,o}^\perp = \int d\tilde{\Omega} \mathcal{C}_3, \quad (73)$$

$$h_{T,o} = - \int d\tilde{\Omega} \Delta_T \mathcal{C}, \quad h' = \int d\tilde{\Omega} \Delta_T \mathcal{C}', \quad g_{L,o}^\perp = \int d\tilde{\Omega} \mathcal{C}_2. \quad (74)$$

Moreover

$$d\tilde{\Omega} = \pi \frac{d^3 \tilde{k}}{(2\pi)^4} \frac{p^- p_1^+}{2p^+} L^{(-)}, \quad r = \frac{k^- \bar{p}_0^+}{p_1^+ p^-} \frac{L^{(+)}}{L^{(-)}}, \quad (75)$$

$$L^{(\pm)} = \frac{\mathcal{P}}{\mathcal{P}_1} \left[\cosh\varphi \pm a \frac{\sinh\varphi}{2\varphi} \right] \quad \text{and} \quad d^3 \tilde{k} = 2p_1^+ d^4 p_1 \delta(p_1^2 - m^2). \quad (76)$$

Lastly p_1 is defined by Equations (62) and

$$\bar{p}_0 \equiv \left(|p|, p \frac{\sqrt{p^2 + m^2}}{|p|} \right). \quad (77)$$

The notations for the functions are somewhat similar to those introduced by Mulders and Tangerman (1996) and Goeke et al. (2005). The suffix "o" in f_o^\perp , $g_{T,o}$, $g_{L,o}^\perp$ and $h_{T,o}$, denotes T-odd contribution to these three functions. They have T-even counterparts, as explained in "Zero order term: the QCD parton model", Equation (51), where we introduced "hybrid" functions. The T-odd functions are normalized coherently with their T-even counterparts, as can be seen from the factor in front of Γ_1 , Equation (72): indeed, considering the case of an approximately on-shell quark, we have

$$[\pi(p^2 - m^2 + i\epsilon)]^{-1} \rightarrow -i\delta(p^2 - m^2). \tag{78}$$

Furthermore the (-i)-factor in (78) is compensated by the i-factor present in the term with n = 1 in expansion (15), but absent in the term with n = 0; therefore also the phase of the T-odd functions is in agreement with the one of the T-even counterparts. It follows from such observations that the factor (78) in expression (72) automatically fixes also the normalization and the phase of the remaining functions included in Γ_1 .

Lastly, as already noticed in connection with correlation functions, the function h' describes a quark transverse polarization induced by quark-gluon interactions: this polarization, present also in spinless or unpolarized hadrons, is somewhat similar to the Boer-Mulders (1998) function, although it is twist-3 and not twist-2.

Twist-3, T-odd correlator

As explained in "Approximate Factorization", $\Gamma_1(p)$, Equation 72, yields, in the approximation discussed above, the contribution to the quark correlator of quark-gluon interactions, at Q^{-1} approximation. We compare this expression with the purely kinematic parameterization of the twist-3, interaction dependent correlator, as given in appendix C. In this way we obtain several approximate relations among the "soft" functions involved in that parameterization. This last reads

$$\Phi^i = \Phi_H^i + \Phi_o^i. \tag{79}$$

Here Φ_H^i is obtained from Equation 51, by substituting $\delta(p^2 - m^2)$ by $[\pi(p^2 - m^2 + i\epsilon)]^{-1}$, according to the rule just stated at the end of "Approximate Factorization". On the other hand, from Appendix C we get

$$\begin{aligned} \Phi_o^i &= \frac{2p^+}{\pi(p^2 - m^2 + i\epsilon)} \{ \epsilon_{ij} S_\perp^i (p_\perp^j e_T^\perp + M\gamma^j f_T) + \epsilon_{ij} \bar{S}_\perp^i p_\perp^j e_T'^\perp + \gamma_5(xMe_L\lambda \\ &+ e_T p_\perp \cdot S_\perp + e_T' p_\perp \cdot \bar{S}_\perp) + \epsilon_{ij} \gamma_i p_\perp^j (f_L^\perp \lambda + f_T^\perp \lambda_\perp + \gamma_5 g^\perp) \\ &+ \gamma_5 \not{p}_\perp \not{S}_\perp h' + \frac{1}{2} \gamma_5 [\gamma_+, \gamma_-] p_\perp \cdot \bar{S}_\perp h'^\perp \}. \end{aligned} \tag{80}$$

Comparison between parameterization (79) and result (72), component by component, yields the following approximate relations:

$$g^\perp \approx f_o^\perp, \quad f_L^\perp \approx g_{L,o}^\perp, \quad f_T \approx g_{T,o}' \tag{81}$$

$$e_T \approx -e_T^\perp \approx h_{T,o}^\perp \approx h_{T,o}, \tag{82}$$

$$e_T' \approx -e_T'^\perp \approx h'^\perp \approx h', \tag{83}$$

$$e_L \approx f_T^\perp \approx g_{T,o}^\perp \approx e_o \approx h_{L,o} \approx 0. \tag{84}$$

Also these equations survive QCD evolution, like Equations (49) and (52). Aside from that, it is important to notice that the second Equation (81) implies, together with the second Equation (73) and with the third Equation (74),

- a) that $\Delta C = 0$;
- b) that Γ_1 includes 5 independent functions in all.

Remarks

- A) Some of the functions, which appear in the equalities (81) to (83), are longitudinally (g^\perp , $g_{L,o}^\perp$) or transversely (h'^\perp , h') polarized in an unpolarized nucleon. Conversely, other functions are unpolarized in a longitudinally (f_L^\perp) or transversely (f_T and the "e"-functions) polarized nucleon. This is a consequence of the spin-orbit coupling (Brodsky et al., 2002a) in gluon-quark interactions. Furthermore, unlike previous authors (Boer and Mulders, 1998; Boer et al., 2000; Goeke et al., 2005), we k_{fT} is known as the Sivers (1990, 1991) function find that such functions are associated to the same inverse power of Q, independent of the kind of polarization (longitudinal or transverse) of the quark or of the nucleon.
- B) Among Equations (81) to (84), those which concern only T-odd functions hold up to terms of order $(gM/Q)^2$. On the contrary, those which involve "hybrid" functions - including Equation(52) - hold up to terms of order gM/Q . Analogous approximate relations of this latter type have been found by Avakian et al. (2008a) and by Efremov et al.(2009).
- C) By integrating the correlator (72) over the transverse

momentum of the quark, we obtain interesting results as regards twist-3 common functions. First of all, the fourth equation (equation 84) implies that $e(x)$ derives just T-even contributions, and therefore, apart from the (negligible) term illustrated in “Zero order term: the QCD parton model”, it is essentially of order $(gM/Q)^2$. On the contrary, the main contributions to g_T and h_L are of order gM/Q and are T-odd; therefore they change sign according as to whether they are involved in DIS or DY reaction. These last predictions could be tested by confronting the DIS double spin asymmetry (Anthony et al., 1996a,b, 2003) with the DY one (Di Salvo, 2001; Soffer and Taxil, 1980). In the case of DY one has to integrate over the transverse momentum of the virtual photon; moreover, if possible, it may be more promising to detect $\tau^+\tau^-$ pairs, whose polarization is perhaps less problematic to determine (Kodaira and Yokoya, 2003).

D) Lastly, the twist-2 T-odd functions h_1^\perp , corresponding to transverse polarization in an unpolarized nucleon, and the unpolarized distribution function f_{1T}^\perp , (Boer and Mulders, 1998) in a transversely polarized nucleon find no place in parameterization (72).

Consequences of g_1 and g_2

Now we examine some consequences of our results on the DIS structure functions $g_1(x)$ and $g_2(x)$, whose properties have been studied by various authors (Anselmino et al., 1995; Jaffe and Ji, 1991b; Bluemlein and Tkablaze, 1999). To this end, here, we re-introduce the flavor indices, dropped out in formula (1), in order to recover the usual definitions of those functions. Moreover, we recall that

$$g_i(x) = \sum_a e_a^2 [g_i^a(x) + \bar{g}_i^a(x)] \quad (i = 1, 2; \quad a = u, d, s), \tag{85}$$

where e_a is the fractional charge of the flavour a and the barred quantities refer to antiquarks. On the other hand,

$$g_T = g_1(x) + g_2(x) = g_{T,e}(x) + g_{T,o}(x). \tag{86}$$

Here we have defined

$$g_{T,e(o)}(x) = \sum_a e_a^2 \int d^2 p_\perp [g_{T,e(o)}^a(x, \mathbf{p}_\perp^2) + \bar{g}_{T,e(o)}^a(x, \mathbf{p}_\perp^2)]. \tag{87}$$

But Equation (53) implies

$$g_{T,e}(x) = \sum_a e_a^2 \frac{m_a}{xM} [h_1^a(x) + \bar{h}_1^a(x)] + O(M^2/Q^2). \tag{88}$$

As discussed in “Twist-3, Hybrid Correlator”, $g_{T,e}$ is negligibly small for a nucleon. Therefore our result is in contrast with the Burkhardt-Cottingham (1970) (BC) sum rule that is,

$$\int_0^1 g_2(x) dx = 0. \tag{89}$$

Indeed, integrating both sides of Equation (86) between 0 and 1, and assuming relation (89), implies

$$\int_0^1 g_1(x) dx \approx \int_0^1 g_{T,o}(x) dx. \tag{90}$$

But this result is unacceptable, since a twist-2, T-even function like $g_1(x)$ has a priori relation with $g_{T,0}$, which is twist-3 and T-odd.

Furthermore, Equation 89 implies, together with the operator product expansion (Anselmino et al., 1995),

$$g_1(x) + g_2(x) = \int_x^1 \frac{dy}{y} g_1(y) + g_T^{(3)}, \tag{91}$$

where $g_T^{(3)}$ is the twist-3 contribution to g_T (Anselmino et al., 1995), to be identified, according to our results, with $g_{T,0}$. Then Equation 86 would yield

$$\int_x^1 \frac{dy}{y} g_1(y) = g_{T,e}(x) + O(M^2/Q^2), \tag{92}$$

which appears in contrast with the data of $g_1(x)$ (Ashman et al., 1988, 1989; Airapetian et al., 1998), enforcing arguments against the BC rule (See Anselmino et al. (1995) and articles cited therein). An experimental confirmation of the violation of the BC rule was found years ago in a precision measurement of $g_2(x)$ (Anthony et al., 2003).

Also the Efremov-Leader-Teryaev (ELT) sum rule, according to the version given by Anselmino et al. (1995) that is,

$$\int_0^1 dx x [g_1(x) + 2g_2(x)] = 0, \tag{93}$$

is in contrast with our result. Indeed, it gives rise, together with Equations (86) and (88), to the approximate relation

$$\int_0^1 dx x g_1(x) \approx \int_0^1 dx 2x g_{T,o}(x), \tag{94}$$

which, again, relates a T-even function to a T-odd one.

However, it is worth noting that the ELT sum rule was successively reformulated (Efremov et al., 1997) by suitably redefining g_1 and g_2 .

FRAGMENTATION CORRELATOR

Fragmentation correlator (4) can be made gauge invariant analogously to the distribution correlator that is for a quark,

$$\Delta_{ij}(p; P, S) = 2P \int \frac{d^4x}{(2\pi)^4} e^{ipx} \langle 0 | \mathcal{L}(x) \bar{\psi}_j(0) a(P, S) a^\dagger(P, S) \psi_i(x) | 0 \rangle, \tag{95}$$

where $L(x)$ is given by Equation (6).

The object (95) may be treated analogously to the distribution correlator, described previously. Indeed, also in this case, for an antiquark one has to change the four-momentum from p to $-p$ and to put a minus sign in front of the correlator. Moreover one has to choose the path $l+$ for quark fragmentation from e^+e^- annihilation, whereas the path $l-$ refers to fragmentation in SIDIS. The only important difference with the distribution correlator is that one has to take into account also the nonperturbative interactions among the final hadrons produced. However, as we shall see in a moment, this does not involve any change in the parameterization.

We treat only the case of pions, adopting for T-odd terms an approximation analogous to the one discussed in "Approximate Factorization", valid for small transverse momenta of the final hadron with respect to the fragmenting quark. Under this condition, we have

$$\Delta(p) = 2p^+ \{ \bar{\Delta}^{(f)}(p) \delta(p^2 - m^2) + \bar{\Delta}^{(i)}(p) [\pi(p^2 - m^2 + i\epsilon)]^{-1} \}, \tag{96}$$

$$\bar{\Delta}^{(f)}(p) = \frac{1}{2} (\not{p} + m) D_\pi, \tag{97}$$

$$\bar{\Delta}^{(i)}(p) = \gamma_- \gamma_+ [\not{p}_\perp D_\pi^\perp + \gamma_5 \not{p}_\perp \bar{S}_\perp H']. \tag{98}$$

Here D_π is the common fragmentation function of the pion; D_π^\perp , defined according to Mulders and Tangerman (1996), is the analog of f^\perp ; last, H' assumes the role of the Collins (1993) function, describing the asymmetry of a pion fragmented from a transversely polarized quark, the so-called Collins asymmetry (see also Leader, 2004).

Final state interactions give rise to terms which decrease as inverse powers of Q , independent of the nature of the interactions themselves. As an example, we re-consider the interactions which produce the aforementioned Collins asymmetry from a different point

of view. Analogously to the distribution functions illustrated in remark D, such an asymmetry may be connected to the absorptive part of an amplitude of the type $\langle +|- \rangle$, where \pm denotes the helicity of the fragmenting quark. This kind of amplitude - a typical helicity flip one - behaves as

$$\langle +|- \rangle = B \sin\theta, \tag{99}$$

where B is a given function, weakly dependent on the quark momentum. Then, similarly to Equation (56), we conclude that the effect of the final state interaction between the fragmenting quark and the fragmented hadron decreases like Q^{-1} . This confirms our previous result, but independent of the nature of the interaction.

More generally, we examine the interactions that the fragmented hadron, say hadron B , undergoes with other final hadrons. These cause in the momentum P_B of B a change ΔP_B which depends weakly on Q , since the multiplicity of the hadrons produced in inclusive reactions increases only logarithmically with energy. Moreover, for sufficiently large Q and not too small fractional momenta z of B with respect to the fragmenting quark, the ratio

$$\mathcal{R} = \frac{|\Delta P_B|}{|P_B|} \tag{100}$$

is quite small. Then, under such conditions, R decreases approximately like Q^{-1} . Our result agrees with the approach by Collins and Soper (1981), who do not include "soft" final state interaction in the leading term of (almost) back-to-back fragmentation in e^+e^- annihilation.

ASYMMETRIES

Here, we consider some important azimuthal and single spin asymmetries, which, as is well known, may be produced by coupling two chiral-even or two chiral-odd TMD distribution or fragmentation functions. More precisely, the terms of the hadronic tensor which give rise to asymmetries are written as convolutive products of two "soft" functions times a suitable weight function (Boer et al., 2000; Di Salvo, 2007a) which changes from asymmetry to asymmetry. These last depend on some azimuthal angle ϕ , relative to the final hadron (for SIDIS and e^+e^- annihilation), or to the final muon pair (for DY). Some of these asymmetries arise from the first order correction of the hadronic tensor, while others belong to the second order one, whose complete parameterization is not considered in this paper.

Cahn effect

This effect, pointed out for the first time by Cahn (1978), has been exhibited by Anselmino et al. (2007) examining

some SIDIS data (Arneodo et al., 1987; Ashman et al., 1991; Adams et al., 1993) (see also Anselmino et al., 2006). We consider the asymmetry corresponding to the "product"

$$A_C \propto f^\perp \otimes D_\pi + f_1 \otimes D_\pi^\perp. \quad (101)$$

This asymmetry is proportional to $\cos\phi$ and decreases like Q^{-1} . To the extent that f_\perp and D_π^\perp can be approximated by f_1 and D_π respectively, one speaks properly of Cahn effect (Anselmino et al., 2007): this amounts to neglecting quark-gluon interactions, see Equation (52) for distribution functions, an analogous equation holding for unpolarized fragmentation functions. This approximation is acceptable for relatively large Q and at small x , as shown by Anselmino et al. (2007). However, one has to observe that both f^\perp and D_π^\perp are "hybrid" functions and in general their T-odd contributions cannot be neglected. It is worth considering also the "product"

$$A_{C2} \propto f^\perp \otimes D_\pi^\perp, \quad (102)$$

which generates a $\cos 2\phi$ asymmetry decreasing like Q^{-2} , hardly distinguishable from another one, arising from the "product" of two chiral-odd functions, as we shall see in a moment. Under the approximation just discussed, we predict a sort of "second order" Cahn effect.

Qiu-Sterman effect

An important transverse single spin asymmetry is the one predicted by Qiu and Sterman (1991, 1992, 1998) (QS) (Efremov and Teryaev, 1984, 1985; Boer et al., 1998, 2003b). This can be observed both in SIDIS and in DY, by integrating over the transverse momentum of the final hadron detected (SIDIS) or of the final pair (DY). This is described by the "products"

$$A_{QS} \propto g_T' \otimes D_\pi \quad (\text{in SIDIS}) \quad \text{and} \quad \propto g_T' \otimes \bar{f}_1 + c.c. \quad (\text{in DY}), \quad (103)$$

the "bar" indicating the antiquark function and c.c. "charge conjugated". A similar effect could be observed in e^+e^- annihilation, if one of the final hadrons observed is spinning. This asymmetry decreases like Q^{-1} . Moreover, since g_T' is prevalently T-odd, while f_1 , \bar{f}_1 and D_π are T-even, the asymmetry is expected to assume an opposite sign in SIDIS and DY.

Sivers effect

The Sivers (1990, 1991) single transverse spin

asymmetry is described by the "product"

$$A_{SIV} \propto f_T \otimes D_\pi \quad (\text{in SIDIS}) \quad \text{and} \quad \propto f_T \otimes \bar{f}_1 + c.c. \quad (\text{in DY}). \quad (104)$$

This asymmetry was detected by HERMES (Airapetian et al., 2005b; Diefenthaler, 2005) and COMPASS (Alexakhin et al., 2005) experiments. It is T-odd, since it consists of the "product" of a T-odd function (f_T) times a T-even function (f_T , D_π and \bar{f}_1). Therefore the asymmetry is predicted to change sign (Collins, 2002; Collins et al., 2006; Anselmino et al., 2009), according as to whether it is observed in SIDIS or DY, similar to the QS effect. However the T-odd character of f_T leads us to conclude that the Sivers asymmetry decreases like Q^{-1} , in disagreement with the current literature (Boer and Mulders, 1998; Efremov et al., 2006b; Anselmino et al., 2007).

Furthermore the third Equation (81) that is, $f_T \approx g_{T,o}'$, implies, together with Equations (103) and (104), that the Sivers and QS asymmetries are related to each other, although the weight functions (Boer et al., 2000; Di Salvo, 2007a) involved in the two "products" are different. This analogy was already noticed by other authors (Boer et al., 2003b; Ji et al., 2006a, b, c; Koike et al., 2008).

Collins and Boer-Mulders effect

In the framework of chiral-odd functions, an important single spin asymmetry is produced by combination of two transversities. In particular, single transverse polarization gives rise to an asymmetry described by the "product"

$$A_{COL} \propto h_{1T} \otimes H' \quad (\text{in SIDIS}), \quad (105)$$

or

$$A_{BM} \propto h_{1T} \otimes \bar{h}' + c.c. \quad (\text{in DY}). \quad (106)$$

The asymmetry A_{COL} predicted by Collins (1993) and exhibited by HERMES data (Airapetian et al., 2005b; Diefenthaler, 2005) - decreases like Q^{-1} according to our treatment. It has been studied recently by Leader (2004), Anselmino (2009, 2010) and Boer (2009).

We have also the following azimuthal, $\cos 2\phi$ asymmetries:

$$A_{CL2} \propto h' \otimes H' \quad (\text{in SIDIS}), \quad (107)$$

or

$$A_{BM2} \propto h' \otimes h' \quad (\text{in DY}), \quad (108)$$

or

$$A_{CL3} \propto H' \otimes \bar{H}' \quad (\text{in } e^+e^- \text{ annihilation}), \quad (109)$$

which decrease like Q^{-2} . Therefore, as in the case of the Siverts asymmetry, we obtain a Q^2 dependence of asymmetries (105) to (109) which differs from other authors (Boer and Mulders, 1998; Efremov et al., 2006a; Burkardt and Hannafious, 2008). Our prediction for the Boer-Mulders asymmetry A_{BM2} is supported (Di Salvo, 2007a) by DY data (Falciano et al., 1986; Guanziroli et al., 1988; Conway et al., 1989). On the other hand, the Q^2 dependence of the Collins and Siverts asymmetries might be tested in new planned experiments at higher energies (Afanasev et al., 2007).

SUMMARY

In the present paper, we studied the gauge invariant quark-quark correlator, which we have expanded in powers of the coupling and split into a T-even and a T-odd part. Working in the KS gauge, the Politzer theorem on EOM has allowed us to interpret each term of the expansion according to Feynman-Cutkosky graphs, involving higher correlators and corresponding to the powers of gM/Q . We have also elaborated an algorithm for writing a gauge invariant sector of the hadronic tensor in deep inelastic processes, like SIDIS, DY and e^+e^- annihilation. This gives rise to a rather long and complicated sum of terms. However, in the gauge considered, and especially at small transverse momenta, the "Born" terms of the type (1) prevail over the remaining ones, as we have shown explicitly for first order correction in gM/Q .

The zero order term and the first order correction of the expansion have been examined in detail. In both cases the Politzer theorem produces a considerable reduction of independent functions with respect to the naive parameterization in terms of Dirac components, giving rise to approximate (up to powers of gM/Q) relations among "soft" functions. These relations survive QCD evolution. One such relation has been approximately verified against experimental data (Airapetian et al., 2005b; Avakian et al., 2005), another one suggests a method for determining transversity, while others could be checked in next experiments (Bunce et al., 2000; Adams et al., 1993). Also an energy scale, introduced in the naive parameterization for dimensional reasons, has been determined in our approach, leading to predictions on Q^2 dependence of various azimuthal asymmetries. One of these predictions finds confirmation in unpolarized DY data (Falciano et al., 1986; Guanziroli et al., 1988; Conway et al., 1989).

The hierarchy of TMD functions in terms of inverse powers of Q is established taking into account not only

the Dirac operators, as in the case of common functions (Jaffe and Ji, 1991a, 1992), but also the p_{\perp} dependence, since in this case the orbital angular momentum plays a role as well as spin.

Moreover a relation is found among g_T , the QS asymmetry and the Siverts asymmetry; in particular, both g_T and the two asymmetries are found to change their sign according to whether they are observed in SIDIS or in DY. We draw also some conclusions about the structure of function $g_2(x)$, and in particular against the BC sum rule.

Quark fragmentation involves "soft" interactions among final hadrons, but this does not imply a substantial difference with the distribution correlator. Rather, a caveat should be kept in mind for timelike photons, in DY and e^+e^- annihilation, when Q approaches the energy of a vector boson resonance, like the Υ or the Z^0 . Since such a resonance interferes with the photon, one has to take into account its offshellness, quite different from Q^2 . Particular attention has to be paid also to the case when the active quark (or antiquark) comes from gluon annihilation, as seen, for example, in DY from proton to proton collisions. This may give rise to T-odd Feynman-Cutkosky graphs, in which the (anti)-quark propagator is only slightly off-shell. These two situations deserve a separate treatment.

As a conclusion, we stress that although other authors, like Efremov and Teryaev (1984) have already proposed EFP and LT years ago as a decomposition of the hadronic tensor in terms of Feynman-Cutkosky graphs, our deduction, based on EOM, leads to strong constraints on the parameterization of the "soft" parts of the graphs.

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Appendix A

We deduce a recursion formula for the terms of the expansion of the correlator.

Our starting point is the Politzer (1980) theorem, which implies

$$\langle P, S | \bar{\psi}_j(0) \mathcal{L}(x) (i\mathcal{D} - m)_{il} \psi_l(x) | P, S \rangle = 0. \quad (1)$$

Here, $|P, S\rangle$ denotes the state of a hadron (for instance, but not necessarily, a nucleon) with four-momentum P and PL four-vector S . ψ is the quark field, of which we omit the color and flavor index. $D_\mu = \partial_\mu - igA_\mu$ is the covariant derivative, adopting for the gluon field the shorthand notation A_μ for $A_\mu^a \lambda_a$. For the sake of simplicity, color and flavor indices of the quark field have been omitted. Moreover

$$\mathcal{L}(x) = \sum_{n=0}^{\infty} (ig)^n \Lambda_n(x), \quad (A. 2)$$

where g is the strong coupling, while $\Lambda_0(x) = 1$. We have, for $n \geq 1$, in the KS gauge,

$$\Lambda_n(x) = \int_{x_1}^{x_2} dz_1^{\mu_1} \int_{x_1}^{z_1} dz_2^{\mu_2} \dots \int_{x_1}^{z_{n-1}} dz_n^{\mu_n} [A_{\mu_1}(z_1) A_{\mu_2}(z_2) \dots A_{\mu_n}(z_n)]. \quad (A. 3)$$

Here we have adopted the reference frame and the notations and definitions introduced in sect. 2. In particular, x_2 is related to x : $x_2 \equiv (\pm\infty, x^\pm, x_\perp)$, $x \equiv (x^-, x^\pm, x_\perp)$. It is worth observing that

$$\partial_\mu \Lambda_n = A_\mu(x_2) \Lambda_{n-1}. \quad (A. 4)$$

Substituting expansion (A. 2) into Equation (A. 1), we get

$$\sum_{n=0}^{\infty} (ig)^n \{ \bar{\psi}_j(0) \Lambda_n(x) (i\mathcal{D} - m)_{il} \psi_l(x) - i \bar{\psi}_j(0) \Lambda_{n-1}(x) [i\mathcal{A}(x)]_{il} \psi_l(x) \} = 0, \quad (A. 5)$$

with

$$\Lambda_{-1}(x) = 0 \quad \text{and} \quad \Lambda_0(x) = 1. \quad (A. 6)$$

Equation (A. 5) is an operator equation, to be intended in a weak sense: it holds when calculated between hadronic states. All equations of this Appendix will be of this type from now on.

Looking for a perturbative solution for the correlator in powers of g , we set each term of the series (A. 5) equal to zero that is,

$$(i\mathcal{D} - m) \mathcal{O}_n(x) = i\mathcal{A}(x) \mathcal{O}_{n-1}(x), \quad (A. 7)$$

where

$$[\mathcal{O}_n(x)]_{ij} = \bar{\psi}_j(0) \Lambda_n(x) \psi_i(x). \quad (A. 8)$$

By Fourier transforming both sides of Equation (A. 7), and recalling relation (A. 4), we get

$$(\not{p} - m) \tilde{\mathcal{O}}_n(p) = i\gamma_\mu \int \frac{d^4x}{2\pi^4} e^{ipx} [A^\mu(x_2) \mathcal{O}_{n-1}(x) + \mathcal{O}_{n-1}(x) A^\mu(x)], \quad (A. 9)$$

where

$$\tilde{\mathcal{O}}_n(p) = \int \frac{d^4x}{2\pi^4} e^{ipx} \mathcal{O}_n(x). \quad (A. 10)$$

Equation (A. 9) can be rewritten as

$$(\not{p} - m) \tilde{\mathcal{O}}_n(p) = i\gamma_\mu \int \frac{d^4k}{2\pi^4} [\tilde{A}^\mu(k) \tilde{\mathcal{O}}_{n-1}(p-k) + \tilde{\mathcal{O}}_{n-1}(p-k) \hat{A}^\mu(k)], \quad (A. 11)$$

where

$$\hat{A}^\mu(k) = \int \frac{d^4x}{2\pi^4} e^{ikx} A^\mu(x), \quad (A. 12)$$

$$\tilde{A}^\mu(k) = \delta(k^+) \lim_{M \rightarrow \infty} \int d\kappa e^{-i\kappa M} \hat{A}_\mu(k^-, \kappa, \mathbf{k}_\perp). \quad (A. 13)$$

Equation (A. 11) is a recursion formula for $\tilde{\mathcal{O}}_n(p)$, eqns. (A. 6) constituting the first steps. This formula implies Equations (17) (for $n = 0$) and (18) (for $n \geq 1$) in the text. In particular, as regards Equation (18), the quantity Γ_n results in

$$\Gamma_n = N \langle P, S | \tilde{\mathcal{O}}_n(p) | P, S \rangle, \quad (A. 14)$$

where N is a normalization constant. The operator $\tilde{\mathcal{O}}_n(p)$ in Equation (A. 11) corresponds to a graph endowed with n gluons, such that the n -th gluon leg is attached to the quark leg on the left side of the graph (Figures 2a and 3a).

Taking into account the hermitian character of $\hat{A}^\mu(k)$ and the relation $[\tilde{\mathcal{O}}_n(p)]^\dagger = \gamma_0 \tilde{\mathcal{O}}_n(p) \gamma_0$ Equation (A. 11) implies

$$\tilde{\mathcal{O}}_n(p) (\not{p} - m) = -i \int \frac{d^4k}{2\pi^4} [\tilde{\mathcal{O}}_{n-1}(p-k) \hat{A}^\mu(k) + \hat{A}^\mu(k) \tilde{\mathcal{O}}_{n-1}(p-k)] \gamma_\mu. \quad (A. 15)$$

In this case $\tilde{\mathcal{O}}_n(p)$ corresponds again to a graph with n gluons, but such that the n -th gluon is attached to the quark leg on the right side of the graph. This last result

implies that Γ_n represents any graph with n gluons, each gluon leg being attached to the left or right quark leg.

Appendix B

Here we deduce the parameterizations of the quark-quark correlator at zero order and of the quark-gluon-quark correlation, arising from first order correction.

B.1. The Zero Order Quark-Quark Correlator

The matrix $\Gamma_0(p)$, defined by

$$(\Gamma_0)_{ij} = N \int \frac{d^4x}{(2\pi)^4} e^{ipx} \langle P, S | \bar{\psi}_j(0) \psi_i(x) | P, S \rangle, \tag{B. 1}$$

fulfils the homogeneous Dirac equation

$$(\not{p} - m)\Gamma_0(p) = 0, \tag{B. 2}$$

where m is the rest mass of the quark. As shown in Appendix A, this is a consequence of the Politzer theorem. This implies, at zero order in the coupling,

$$(\not{p} - m)\psi(x) = 0. \tag{B. 3}$$

Therefore, in the approximation considered, the quark can be treated as if it were on shell (see also Qiu, 1990). Then, initially, we consider the Fourier expansion of the unrenormalized field of an on-shell quark that is,

$$\psi(x) = \int \frac{d^3\tilde{p}}{(2\pi)^{3/2}} \frac{1}{\sqrt{2P^+}} e^{-ipx} \sum_s u_s(p) c_s(p). \tag{B. 4}$$

Here $s = \pm 1/2$ is the spin component of the quark along a given direction in the quark rest frame, u its four-spinor, c the destruction operator for the flavor considered and

$$d^3\tilde{p} = d^4p \delta\left(p^- - \frac{m^2 + \mathbf{p}_\perp^2}{2p^+}\right), \quad P = p^+/\sqrt{2}. \tag{B. 5}$$

As regards the normalization of u_s and c_s , we assume

$$\bar{u}_s u_s = 2m, \quad \langle P, S | c_s^\dagger(p') c_s(p) | P, S \rangle = (2\pi)^3 \delta^3(\tilde{p}' - \tilde{p}) q_s(p), \tag{B. 6}$$

where

$$|\tilde{p} \equiv (p^+, \mathbf{p}_\perp) \tag{B. 7}$$

and $q_s(p)$ is the probability density to find a quark with spin component s and four-momentum $p \equiv (p^-, \tilde{p})$, with $p^- = (m^2 + p^2)/2p^+$. For an antiquark the definition is analogous, except that, in the Fourier expansion (B. 4), we have to substitute the destruction operators c_s with the creation operators d_s^\dagger and p with $-p$ in the exponential.

Choosing the quantization axis along the hadron momentum P in the frame defined at the beginning of sect. 4, and substituting Equation (B. 4) into Equation (B. 1), we get

$$(\Gamma_0)_{ij}(p) = \frac{N}{2P} \sum_{s,s'} \int \frac{d^3\tilde{p}'}{(2\pi)^3} \langle P, S | c_s^\dagger(p) c_{s'}(p') | P, S \rangle \times [u_{s'}(p')]_i [\bar{u}_s(p)]_j \delta\left(p^- - \frac{m^2 + \mathbf{p}_\perp^2}{2p^+}\right). \tag{B. 8}$$

But owing to the second Equation (B. 6) we have

$$\Gamma_0(p) = [\Gamma_0^a(p) + \Gamma_0^b(p)] \delta\left(p^- - \frac{m^2 + \mathbf{p}_\perp^2}{2p^+}\right), \tag{B. 9}$$

where

$$\Gamma_0^a(p) = \frac{N}{2P} \sum_s \langle P, S | c_s^\dagger(p) c_s(p) | P, S \rangle u_s(p) \bar{u}_s(p), \tag{B. 10}$$

$$\Gamma_0^b(p) = \frac{N}{2P} \sum_s \langle P, S | c_{-s}^\dagger(p) c_s(p) | P, S \rangle u_{-s}(p) \bar{u}_s(p). \tag{B. 11}$$

Firstly we elaborate Γ_0^a . We have

$$u_s(p) \bar{u}_s(p) = \frac{1}{2} (\not{p} + m) (1 + 2s\gamma_5 S_\parallel^a). \tag{B. 12}$$

Here S_\parallel^a is a four-vector such that, in the quark rest frame, $S_\parallel^a \equiv (0, \lambda/|\lambda|\hat{P})$, $\lambda = \mathbf{S} \cdot \hat{P}$, $\hat{P} = \mathbf{P}/|\mathbf{P}|$ and \mathbf{S} is the unit spin vector of the hadron in its rest frame. Therefore

$$\Gamma_0^a(p) = \frac{N}{2P} \frac{1}{2} (\not{p} + m) [f_1(p) + \Delta' q(p) \gamma_5 S_\parallel^a], \tag{B. 13}$$

where

$$f_1(p) = \sum_s \langle P, S | c_s^\dagger(p) c_s(p) | P, S \rangle \tag{B. 14}$$

is the unpolarized transverse momentum distribution of the quark, while

$$\Delta' q(p) = \sum_s 2s \langle P, S | c_s^\dagger(p) c_s(p) | P, S \rangle. \tag{B. 15}$$

According to transformation properties of one-particle states under rotations, one has

$$|P, S\rangle = \cos\frac{\theta}{2}|P, +\rangle + i|P, -\rangle \sin\frac{\theta}{2}, \quad (\text{B. 16})$$

where \pm denotes the (positive or negative) helicity of the hadron and θ the angle between P and S. Substituting Equation(B. 16) into Equation(B. 15), and taking into account parity conservation, we get

$$\Delta'q(p) = \cos\theta g_{1L}(p). \quad (\text{B. 17})$$

Here

$$g_{1L}(p) = \sum_s 2s \langle P, + | c_s^\dagger(p) c_s(p) | P, + \rangle = - \sum_s 2s \langle P, - | c_s^\dagger(p) c_s(p) | P, - \rangle. \quad (\text{B. 18})$$

is the longitudinally polarized TMD distribution of the quark, the last equality following from parity conservation.

Now we consider Γ_0^b . Equation (B. 16) yields, for $\theta = \pi/2$,

$$|\uparrow(\downarrow)\rangle = \frac{1}{\sqrt{2}}(|+\rangle \pm i|-\rangle), \quad (\text{B. 19})$$

where $|\pm\rangle$ and $|\uparrow(\downarrow)\rangle$ denote quark states with spin components, respectively, along \hat{P} and along

$$S_\perp = S - \lambda \hat{P}. \quad (\text{B. 20})$$

Substituting Equations (B. 16) and (B. 19) into Equation (B. 11), and taking into account again parity conservation, we get

$$\Gamma_0^b(p) = \frac{N}{2P} \frac{1}{2} \sin\theta h_{1T}(p) (|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|), \quad (\text{B. 21})$$

where

$$h_{1T}(p) = \langle P, - | c_+^\dagger(p) c_-(p) | P, + \rangle = \langle P, + | c_-^\dagger(p) c_+(p) | P, - \rangle \quad (\text{B. 22})$$

is the TMD transversity of the quark. Returning to the Dirac notation, we have

$$|\uparrow\rangle\langle\uparrow| = \frac{1}{2}(\not{p} + m)(1 + \gamma_5 \not{S}_\perp^b), \quad |\downarrow\rangle\langle\downarrow| = \frac{1}{2}(\not{p} + m)(1 - \gamma_5 \not{S}_\perp^b), \quad (\text{B. 23})$$

where S_\perp^b is such that $S_\perp^b \equiv (0, \hat{n})$ in the quark rest frame and

$$\hat{n} = \frac{S_\perp}{|S_\perp|}. \quad (\text{B. 24})$$

Then Equation (B. 21) goes over into

$$\Gamma_0^b(p; P, S) = \frac{N}{2P} \frac{1}{2} \sin\theta \Delta_{1T} q(p) (\not{p} + m) \gamma_5 \not{S}_\perp^b. \quad (\text{B. 25})$$

Substituting Equations (B. 13), (B. 17) and (B. 25) into Equation (B. 9) yields

$$\Gamma_0 = \frac{N}{2P} \frac{1}{2} (\not{p} + m) [f_1 + g_{1L} \gamma_5 \not{S}_\parallel^q + h_{1T} \gamma_5 \not{S}_\perp^q] \delta\left(p^- - \frac{m^2 + p_\perp^2}{2p^+}\right), \quad (\text{B. 26})$$

having set $S_\parallel^q = S_\parallel^a \cos\theta$ and $S_\perp^q = S_\perp^b \sin\theta$. Equation (B. 26) is a solution to Equation (B. 2), which is a consequence of the Politzer theorem at zero order in g . Since this equation survives renormalization - which generally implies only a weak Q-dependence (Sternan, 2005; Dokshitzer et al., 1980) - the structure of Γ_0 is not changed by QCD evolution.

Lastly we deduce the expressions of the four-vectors S_\parallel^q and S_\perp^q in the frame where the quark momentum is p. In the quark rest frame we have

$$S_\parallel^q \equiv (0, \lambda \hat{P}), \quad S_\perp^q \equiv (0, S_\perp). \quad (\text{B. 27})$$

In view of the Lorentz boost, it is convenient to further decompose $\lambda \hat{P}$ and S_\perp into components parallel and perpendicular to the quark momentum. We have

$$\lambda \hat{P} = \lambda \cos\alpha \hat{p} + \Sigma_\parallel, \quad \Sigma_\parallel = -\cos\alpha \frac{p_\perp}{|p|} + \sin^2\alpha \hat{P}, \quad (\text{B. 28})$$

$$S_\perp = \lambda \hat{p} + \Sigma_\perp, \quad \Sigma_\perp = |S_\perp| \cos\beta (\cos\beta \hat{n} - \sin\beta \hat{k}), \quad (\text{B. 29})$$

where

$$\hat{p} = \frac{p}{|p|}, \quad \hat{k} = \hat{n} \times \frac{\hat{p} \times \hat{n}}{|\hat{p} \times \hat{n}|}, \quad (\text{B. 30})$$

$$\alpha = \arccos(\hat{P} \cdot \hat{p}) \quad \text{and} \quad \beta = \arcsin(\hat{n} \cdot \hat{p}). \quad (\text{B. 31})$$

The boost which transforms the four-momentum of the quark from $(m, 0)$ to (E, p) , with $E = \sqrt{m^2 + p^2}$ changes only the components along \hat{p} of $\lambda \hat{P}$ and of S_\perp . In particular, the boost transforms the four-vector $(0, p)$ to p/m , with $p \equiv (|p|, E\hat{p})$. Therefore, since α and β are $O(|p_\perp|/|p|)$ and $|p|/P = O(1)$, Equations (B. 27) go over into

$$S_{\parallel}^q = \lambda \left(\frac{\bar{p}}{m} - \bar{\eta}_{\perp} \right) + O(\bar{\eta}_{\perp}^2), \quad S_{\perp}^q = S_{\perp} + \bar{\lambda}_{\perp} \frac{\bar{p}}{m_q} + O(\bar{\eta}_{\perp}^2), \quad (\text{B. 32})$$

where

$$\bar{\eta}_{\perp} = p_{\perp}/P \text{ and } \bar{\lambda}_{\perp} = -S \cdot \bar{\eta}_{\perp}.$$

B.2. The Quark-Gluon-Quark Correlator

Now we deduce a parameterization for the quark-gluon-quark correlator, defined by

$$[\Phi_{\mu}^{(1)}(p, k)]_{ij} = N \int \frac{d^4x}{(2\pi)^4} e^{i(p-k)x} \langle P, S | \bar{\psi}_j(0) [\hat{A}_{\mu}(k) + \tilde{A}_{\mu}(k)] \psi_i(x) | P, S \rangle. \quad (\text{B. 33})$$

As shown in Appendix A, the Politzer theorem implies, at order 1 in the coupling,

$$(\not{p} - \not{k} - m) \Phi_{\mu}^{(1)}(p, k) = 0, \quad (\text{B. 34})$$

which holds also after renormalization. Therefore our line of reasoning is the same as for Γ_0 , that is, we start from unrenormalized fields and we take on-shell quarks, whose field satisfies expansion (B. 4). Substituting this expansion into Equation (B. 33), we get

$$\Phi_{\mu}^{(1)}(p, k) = \Psi_{\mu}(p, k) \delta \left(p_1^- - \frac{m^2 + P_{1\perp}^2}{2p_1^+} \right), \quad (\text{B. 35})$$

$$\Psi_{\mu}(p, k) = N \int \frac{d^3\tilde{p}'}{(2\pi)^3} \frac{1}{2\sqrt{\mathcal{P}_1\mathcal{P}'}} \sum_{s,s'} \mathcal{A}_{s,s',\mu}(p', k) u_s(p_1) \bar{u}_{s'}(p'). \quad (\text{B. 36})$$

Here $d^3\tilde{p}'$ and \mathcal{P}' are defined analogously to Equations (B. 5),

$$p_1 = p - k, \quad \mathcal{P}_1 = p_1^+ / \sqrt{2} \quad (\text{B. 37})$$

and

$$\mathcal{A}_{s,s',\mu}(p', k) = \langle P, S | c_s^{\dagger}(p') [\hat{A}_{\mu}(k) + \tilde{A}_{\mu}(k)] c_{s'}(p_1) | P, S \rangle. \quad (\text{B. 38})$$

Moreover the matrix element (B. 38) fulfils a relation of the type

$$\mathcal{A}_{s,s',\mu}(p', k) = (2\pi)^3 \mathcal{C}_{s,s',\mu}(p', k) \delta^3(\tilde{\mathbf{p}}' - \tilde{\mathbf{p}}_1 - \tilde{\mathbf{k}}), \quad (\text{B. 39})$$

where $\mathcal{C}_{s,s',\mu}(p', k)$ is a quark-gluon correlator and $\tilde{\mathbf{p}}', \tilde{\mathbf{p}}_1$ and $\tilde{\mathbf{k}}$ are defined by Equation (B. 7). Then Equation (B. 36) yields

$$\Psi_{\mu}(p, k) = \frac{N}{2\sqrt{\mathcal{P}_1\mathcal{P}}} \sum_{s,s'} \mathcal{C}_{s,s',\mu}(p, k) u_s(p_1) \bar{u}_{s'}(p_0) \quad (\text{B. 40})$$

and

$$p_0 \equiv (p_0^-, \tilde{\mathbf{p}}), \quad p_0^- = \frac{P_{\perp}^2 + m^2}{2p^+}. \quad (\text{B. 41})$$

We rewrite Equation (B. 40) as

$$\Psi_{\mu}(p, k) = \frac{N}{2\sqrt{\mathcal{P}_1\mathcal{P}}} (\Psi_{\mu}^a + \Psi_{\mu}^b), \quad (\text{B. 42})$$

where

$$\Psi_{\mu}^a = \sum_s \mathcal{C}_{s,s,\mu}(p_1, k) u_s(p_1) \bar{u}_s(p_0), \quad (\text{B. 43})$$

$$\Psi_{\mu}^b = \sum_s \mathcal{C}_{s,-s,\mu}(p_1, k) u_s(p_1) \bar{u}_{-s}(p_0). \quad (\text{B. 44})$$

Taking into account the appropriate Lorentz transformations for the spinors involved, we have

$$u_s(p_1) \bar{u}_s(p_0) = \frac{1}{2} (\not{p}_1 + m) U(p_1, p_0) (1 + 2s\gamma_5 S_{0\parallel}^q), \quad (\text{B. 45})$$

$$u_s(p_1) \bar{u}_{-s}(p_0) = \frac{1}{2} (\not{p}_1 + m) U(p_1, p_0) \gamma_5 (\cos\chi S_{0\perp}^q + \sin\chi \bar{S}_{\perp}). \quad (\text{B. 46})$$

$$U(p_1, p_0) = \exp \left[\frac{1}{2} (\phi_1 \hat{\mathbf{p}}_1 - \phi_0 \hat{\mathbf{p}}_0) \cdot \vec{\alpha} \right], \quad (\text{B. 47})$$

$$\phi_1 = \ln \frac{E_1 + |\mathbf{p}_1|}{m}, \quad \hat{\mathbf{p}}_1 = \frac{\mathbf{p}_1}{|\mathbf{p}_1|}, \quad (\text{B. 48})$$

$$p_1 \equiv (p_{1\perp}, \frac{1}{\sqrt{2}}(p_1^+ - p_1^-)), \quad E_1 = \sqrt{p_{1\perp}^2 + m^2}, \quad (\text{B. 49})$$

analogous definitions holding for ϕ_0 and $\hat{\mathbf{p}}_0$. Moreover $S_{0\parallel}^q$ and $S_{0\perp}^q$ refer to the PL vector of a quark with four-momentum \mathbf{p}_0 , directly connected with nucleon polarization; they can be related to the nucleon longitudinal and transverse PL vectors, using the formulae elaborated at the end of sect. B1. \bar{S}_{\perp} refers to the spin caused by spin-orbit coupling,

$$\sqrt{|p_{0\perp}^2|} \bar{S}_{\perp\alpha} = \epsilon_{\alpha\beta\gamma\rho} n_+^{\beta} n_-^{\gamma} p_{0\perp}^{\rho}. \quad (\text{B. 50})$$

Last, χ is a real, "soft" parameter, which in general will depend on \mathbf{p}_0 and \mathbf{p}_1 ; it will be included in the definitions of two of the "soft" correlation functions.

We assume $\theta_0, \theta_1 \ll 1$, where θ_0 and θ_1 are, respectively, the angle between \mathbf{p}_0 and \mathbf{P} and the one between \mathbf{p}_1 and \mathbf{P} . Then

$$U(p_1, p_0) \approx \cosh\varphi + \frac{1}{2\varphi}\gamma_0(\gamma_3 a + \gamma_i r_{\perp}^i) \sinh\varphi, \quad (\text{B. 51})$$

with

$$\varphi = \frac{1}{2}\sqrt{(\phi_0 - \phi_1)^2 + \theta^2 \phi_0 \phi_1}, \quad \theta = \theta_1 - \theta_0, \quad (\text{B. 52})$$

Then Ψ_{μ} results in

$$a = \phi_1 - \phi_0 - \frac{1}{2}(\phi_1 \theta_1^2 - \phi_0 \theta_0^2), \quad \mathbf{r}_{\perp} = \frac{\phi_1}{|\mathbf{p}_1|} \mathbf{p}_{1\perp} - \frac{\phi_0}{|\mathbf{p}_0|} \mathbf{p}_{0\perp}, \quad (\text{B. 53})$$

$$\Psi_{\mu}(p_1, k) \approx \frac{1}{2}(\not{p}_1 + m) \mathcal{L} [C_{\mu} + \Delta C_{\mu} \gamma_5 \not{S}_{\parallel}^a + \Delta_T C_{\mu} \gamma_5 \not{S}_{\perp}^a + \Delta_T C_{\mu}' \gamma_5 \not{S}_{\perp}^b], \quad (\text{B. 54})$$

where

$$\mathcal{L} = \frac{N}{2\sqrt{\mathcal{P}_1 \mathcal{P}}} [\cosh\varphi + \frac{1}{2\varphi}\gamma_0(\gamma_3 a + \gamma_i r_{\perp}^i) \sinh\varphi] \quad (\text{B. 55})$$

$$C_{\mu}(p_1, k) = \sum_s C_{s, s, \mu}(p_1, k), \quad (\text{B. 56})$$

$$\Delta C_{\mu}(p_1, k) = \sum_s 2s C_{s, s, \mu}(p_1, k), \quad (\text{B. 57})$$

$$\Delta_T C_{\mu}(p_1, k) = \sum_s \cos\chi C_{s, -s, \mu}(p_1, k), \quad (\text{B. 58})$$

$$\Delta_T C_{\mu}'(p_1, k) = \sum_s \sin\chi C_{s, -s, \mu}(p_1, k) \quad (\text{B. 59})$$

are correlation functions. In order to parameterize these functions, we recall the definition (B. 33) of quark-gluon-quark correlator and Equation (9), concerning the gauge used. Therefore we have to take into account the available transverse four-vectors, whence it follows that

$$C_{\mu} = C_1 p_{1\perp\mu} + \epsilon_{\mu\nu\rho\sigma} n_{\perp}^{\nu} (C_2 \lambda n_{\perp}^{\rho} p_{1\perp}^{\sigma} + C_3 M S_{\perp}^{\rho} n_{\perp}^{\sigma}), \quad (\text{B. 60})$$

$$\Delta C_{\mu} = \Delta C p_{1\perp\mu}, \quad (\text{B. 61})$$

$$\Delta_T C_{\mu} = \Delta_T C p_{1\perp\mu}, \quad (\text{B. 62})$$

$$\Delta_T C_{\mu}' = \Delta_T C' p_{1\perp\mu}. \quad (\text{B. 63})$$

Here $C_1, C_2, C_3, \Delta C, \Delta_T C$ and $\Delta_T C'$ are "soft" functions of \mathbf{p} and \mathbf{k} . The parameterization of $\Phi_{\mu}^{(1)}$ is obtained by inserting Equations (B. 54) and (B. 60) to (B. 63) into Equation (B. 35). Again, as in the case of Γ_0 , the Politzer theorem, of which Equation (B. 4) is a consequence, implies that renormalization effects preserve the structure of that parameterization.

Appendix C

Here, we consider the parameterization of the correlator in terms of the Dirac components, up to and including twist-3 terms. This parameterization is similar to the usual ones (Boer and Mulders, 1998; Goetze et al., 2005), also as regards notations, except for an energy scale μ_0 , which we leave undetermined here, and for the twist-2, T-odd sector, which we omit because it has no place in our procedure. The scale μ_0 , usually set equal to the rest mass of the hadron, is determined in sects. 4 and 5, with a different result.

The parameterization reads

$$\Phi = 2p^+ \left[(\Psi_E^f + \Psi_H^f) \delta(p^2 - m^2) + (\Psi_O^i + \Psi_H^i) \frac{1}{\pi(p^2 - m^2 + i\epsilon)} \right]. \quad (\text{C. 1})$$

Here

$$\begin{aligned} \Psi_E^f &= \frac{\mathcal{P}}{\sqrt{2}} \{ f_1 \not{p}_+ + (\lambda g_{1L} + \lambda_{\perp} g_{1T}) \gamma_5 \not{p}_+ + \frac{1}{2} h_{1T} \gamma_5 [\not{S}_{\perp}, \not{p}_+] \\ &+ \frac{1}{2} (\lambda h_{1L}^{\perp} + \lambda_{\perp} h_{1T}^{\perp}) \gamma_5 [\not{p}_{\perp}, \not{p}_+] \}, \end{aligned} \quad (\text{C. 2})$$

$$\begin{aligned} \Psi_H &= \frac{1}{2} \{ (f^{\perp} + \lambda g_L^{\perp} \gamma_5 + \lambda_{\perp} g_T^{\perp} \gamma_5) \not{p}_{\perp} + \frac{1}{4} \lambda_{\perp} h_T^{\perp} \gamma_5 [\not{S}_{\perp}, \not{p}_{\perp}] \\ &+ \frac{1}{2} x M \left(e + g_T' \gamma_5 \not{S}_{\perp} + \frac{1}{2} (\lambda h_L + \lambda_{\perp} h_T) \gamma_5 [\not{p}_{\perp}, \not{p}_+] \right) \}, \end{aligned} \quad (\text{C. 3})$$

$$\begin{aligned} \Psi_O^i &= \epsilon_{ij} S_{\perp}^i (p_{\perp}^j e_T^{\perp} + M \gamma^j f_T) + \epsilon_{ij} \bar{S}_{\perp}^i p_{\perp}^j e_T^{\perp} + \gamma_5 (x M e_L \lambda \\ &+ e_T p_{\perp} \cdot S_{\perp} + e_T' p_{\perp} \cdot \bar{S}_{\perp}) + \epsilon_{ij} \gamma_i p_{\perp}^j (f_L^{\perp} \lambda + f_T^{\perp} \lambda_{\perp} + \gamma_5 g^{\perp}) \\ &+ \gamma_5 \not{p}_{\perp} \not{S}_{\perp} h' + \frac{1}{2} \gamma_5 [\gamma_+, \gamma_-] p_{\perp} \cdot \bar{S}_{\perp} h'^{\perp}. \end{aligned} \quad (\text{C. 4})$$

Here, Ψ_H denotes the "hybrid" term, both interaction free (Ψ_H^f , T-even) and interaction dependent (Ψ_H^i , T-odd): the two terms have the same parameterization, but behave quite differently. For the "soft" functions, we have adopted notations similar to those employed by

Goeke et al. (2005). Note, however, that in the expression of Ψ_O^i the functions $e_T'^{\perp}$, e_T' and h'^{\perp} do not appear in the parameterization proposed by those authors; on the contrary, we have not taken into account the functions and $f_T'^{\perp}$, defined by them.