International Journal of Physical Sciences Vol. 7(21), pp. 2860-2866, 30 May, 2012
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS12.071
ISSN 1992 - 1950 © 2012 Academic Journals

Full Length Research Paper

A new approach to Kudryashov's method for solving some nonlinear physical models

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Accepted 03 May, 2012

In this paper we give a new version of the Kudryashov's method for solving non-integrable partial differential equations in mathematical physics. Some exact solutions including 1-soliton and singular soliton solutions of the $K(m,n)$ equation with generalized evolution and time dependent damping and dispersion are obtained by using this new approach.

Key words: $K(m,n)$ equation with generalized evolution, Kudryashov's method, symmetric Fibonacci functions, soliton solution.

INTRODUCTION

The study of nonlinear evolution equations has become very important in the recent years. There are a lot of nonlinear evolution equations that are solved using different mathematical methods. For these physical problems, soliton solutions, compactons, cnoidal waves, singular solitons and the other solutions have been found. These types of solutions appear in various areas of applied sciences and engineering. In this paper, we consider the $K(m,n)$ equation with generalized evolution along with time-dependent damping and time-dependent dispersion as follows (Biswas, 2010):

$$(q^l)_t + a(t)q + dq^m q_x + b(t)(q^n)_{xxx} = 0. \tag{1}$$

Here $a(t)$ and $b(t)$ are real-valued functions while l, m and n are positive integers. In this paper we assume $l = n$ and Equation 1 gets

$$(q^n)_t + a(t)q + dq^m q_x + b(t)(q^n)_{xxx} = 0. \tag{2}$$

There are many methods that are used to obtain the integration of nonlinear partial differential equations.

Some of them are the exp-function method (He and Wu, 2006; Misirli and Gurefe, 2010a, b, 2011; Ebaid, 2012; Gurefe and Misirli, 2011a), the trial equation method (Gurefe et al., 2011, 2012), the (G'/G)-expansion method (Gurefe and Misirli, 2011a, b), the Hirota's method (Salas et al., 2011; Gurefe et al., 2012), the auxiliary equation method (Zhang et al., 2009), and many more. In this research, we modify Kudryashov's (2012) method to raise the effectiveness of this method. Our key idea is that traditional base e of the exponential function is replaced by an arbitrary base $a \neq 1$. So, new exact solutions of nonlinear evolution equations may be obtained by this simple modification.

THE MODIFIED KUDRYASHOV METHOD

We consider the following nonlinear partial differential equation for a function q of two real variables, space x and time t:

$$P(t, x, q, q_t, q_x, q_{tt}, q_{tx}, q_{xx}, ...) = 0. \tag{3}$$

The main steps of the modified Kudryashov method is summarize as follows:

Step 1

Our first step is to obtained the travelling wave solution of

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Equation 3 of the form

$$q(x,t) = u(\xi), \quad \xi = kx + \int^{t} w(t')dt', \tag{4}$$

where k is a free constant. Equatıon 3 was reduced to a nonlinear ordinary differential equation of the form:

$$N(t, x, u, u', u'', ...) = 0, \tag{5}$$

where the prime denotes differentiation with respect to ξ. Suppose that the highest order nonlinear terms in Equation 5 are $u^{l}(\xi)u^{(s)}(\xi)$ and $(u^{(p)})^{k}$.

Step 2

Suppose that the exact solutions of Equation 5 can be obtained in the following form:

$$u(\xi) = y(\xi) = \sum_{j=0}^{N} a_j Q^j, \tag{6}$$

where $Q = \dfrac{1}{1 \pm a^{\xi}}$. Where the function Q is solution of equation

$$Q_{\xi} = lna(Q^2 - Q). \tag{7}$$

Step 3

According to the proposed method, we assume that the solution of Equation 5 can be expressed in the form

$$u(\xi) = a_N Q^N + \ldots \tag{8}$$

To calculate the value N in Equation 8 that is the pole order for the general solution of Equation 5, we proceed analogously as in the classical Kudryashov method on balancing the highest order nonlinear terms in Equation (5). More precisely, by straightforward calculations, we have

$$u'(\xi) = a_N N Q^{N+1} + \ldots, \tag{9}$$

$$u''(\xi) = a_N N(N+1) Q^{N+2} + \ldots, \tag{10}$$

$$u^{(s)}(\xi) = a_N N(N+1)\ldots(N+s-1) Q^{N+s} + \ldots, \tag{11}$$

$$u^{l} u^{(s)}(\xi) = \overline{a_N} N(N+1)\ldots(N+s-1) Q^{(l+1)N+s} + \ldots, \tag{12}$$

$$(u^{(p)})^{k}(\xi) = \left(a_N N(N+1)\ldots(N+s-1)\right)^{k} Q^{k(N+p)} + \ldots, \tag{13}$$

where a_N and $\overline{a_N}$ are constant coefficients. Balancing the highest order nonlinear terms of Equations 12 and (13), we have

$$(l+1)N + s = k(N+p), \tag{14}$$

so

$$N = \frac{s - kp}{k - l - 1}. \tag{15}$$

Step 4

Substituting Equation 6 into Equation 5 yields a polynomial $R(Q)$ of Q. Setting the coefficients of $R(Q)$ to zero, we get a system of algebraic equations. Solving this system, we shall determine $w(t)$ and the variable coefficients of $a_0(t), a_1(t), ..., a_N(t)$. Thus, we obtain the exact solutions to Equation 3.

APPLICATION OF KUDRYASHOV METHOD TO THE $K(m,n)$ EQUATİON

In this study, the modified Kudryashov method is applied to handle the $K(m,n)$ equation where Equation 4 is reduce to the ordinary differential equation by substituting it into Equation 2 which can be written as

$$(u^n)_{\xi}\left(x\frac{dk}{dt} + w(t)\right) + a(t)u + dk(t)u^m u_{\xi} + b(t)k^3(t)(u^n)_{\xi\xi\xi} = 0. \tag{16}$$

Upon integration and if $a(t) = 0$, then Equation 16 becomes

$$(u^n)\left(x\frac{dk}{dt} + w(t)\right) + \frac{dk(t)u^{m+1}}{m+1} + b(t)k^3(t)(u^n)_{\xi\xi} = C, \tag{17}$$

where C is the integration constant. For simplicity we take $C = 0$.
Using the transformation

$$u(\xi) = V^{\frac{1}{m-n+1}}(\xi) \tag{18}$$

Equation 17 becomes

$$V^2\left(x\frac{dk}{dt}+w(t)\right)+\frac{dk(t)}{m+1}V^3+\frac{b(t)k^3(t)n(2n-m-1)}{(m-n+1)^2}(V')^2+\frac{b(t)k^3(t)}{m-n+1}VV''=0. \quad (19)$$

From the first term in Equation 19, we take

$$\frac{dk}{dt}=0, \quad (20)$$

so, $k(t)=constant$. Also we take

$$V(\xi)=y(\xi)=\sum_{j=0}^{N}a_jQ^j, \quad (21)$$

where $Q=\frac{1}{1\pm a^{\xi}}$. We note that the function Q is solution of equation

$$Q_{\xi}=lna(Q^2-Q). \quad (22)$$

Using the balance formula (Equation 15) for the nonlinear terms VV'' and V^3 in Equation 19, we compute

$$N=2. \quad (23)$$

Therefore we have

$$V(\xi)=y(\xi)=\sum_{j=0}^{2}a_jQ^j=a_0+a_1Q+a_2Q^2, \quad (24)$$

and we substitute derivatives of the function $y(\xi)$ with respect to ξ. The required derivatives in Equation 19 are obtained

$$y_{\xi}=\ln a\left(-a_1Q+a_1Q^2-2a_2Q^2+2a_2Q^3\right), \quad (25)$$

$$y_{\xi\xi}=\ln^2 a\left(a_1Q-3a_1Q^2+2a_1Q^3+4a_2Q^2-10a_2Q^3+6a_2Q^4\right). \quad (26)$$

As a result of this, we have the system of algebraic equations that can be solved with Mathematica. Solving the systems, we obtain the coefficients $a_0(t), a_1(t)$ and $a_2(t)$ as follows:

$$a_0(t)=0, \quad a_1(t)=-\frac{2b(t)k^2(\ln a)^2(1+m)(m(2n-3)+n(-4n+5)-3)}{d(m-n+1)^2}, \quad (27)$$

$$a_2(t)=\frac{2b(t)k^2(\ln a)^2(1+m)(m(2n-3)+n(-4n+5)-3)}{d(m-n+1)^2},$$

$$w(t)=\frac{b(t)k^3(\ln a)^2(m(n-1)-2n(n-1)-1)}{(m-n+1)^2}, \quad (28)$$

where $b(t)$ is arbitrary function, k is free constant and m,n are positive integers. Substituting Equations 27 and 28 into 24, we have

$$V(\xi)=-\frac{2b(t)k^2(\ln a)^2(1+m)(m(2n-3)+n(-4n+5)-3)}{d(m-n+1)^2}\left(\frac{1}{1\pm a^{\xi}}-\frac{1}{(1\pm a^{\xi})^2}\right), \quad (29)$$

where

$$\xi=kx+\int^{t}\frac{b(t')k^3(\ln a)^2(m(n-1)-2n(n-1)-1)}{(m-n+1)^2}dt'.$$

Substituting Equation 29 into 18, we can write the solutions

$$u(x,t)=\left(-\frac{2b(t)k^2(\ln a)^2(1+m)(m(2n-3)+n(-4n+5)-3)}{d(m-n+1)^2}\frac{a^{kx+\int^{t}\frac{b(t')k^3(\ln a)^2(m(n-1)-2n(n-1)-1)}{(m-n+1)^2}dt'}}{\left(1+a^{kx+\int^{t}\frac{b(t')k^3(\ln a)^2(m(n-1)-2n(n-1)-1)}{(m-n+1)^2}dt'}\right)^2}\right)^{\frac{1}{m-n+1}}, \quad (30)$$

$$u(x,t)=\left(\frac{2b(t)k^2(\ln a)^2(1+m)(m(2n-3)+n(-4n+5)-3)}{d(m-n+1)^2}\,\frac{a^{kx+\int^t\frac{b(t')k^3(\ln a)^2(m(n-1)-2n(n-1)-1)}{(m-n+1)^2}dt'}}{\left(1-a^{kx+\int^t\frac{b(t')k^3(\ln a)^2(m(n-1)-2n(n-1)-1)}{(m-n+1)^2}dt'}\right)^2}\right)^{\frac{1}{m-n+1}}. \quad (31)$$

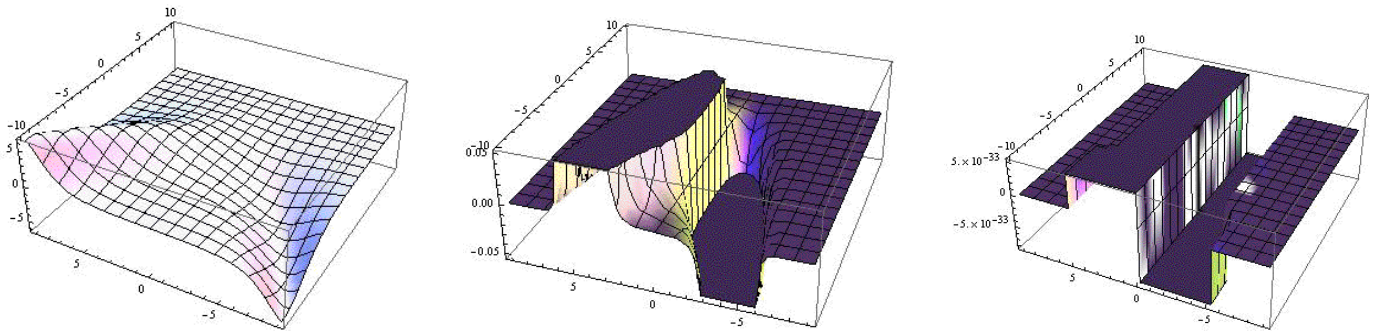

Figure 1. Solution of $u_1(x,t)$ is shown at $k=d=m=n=1$, $b(t)=t$.

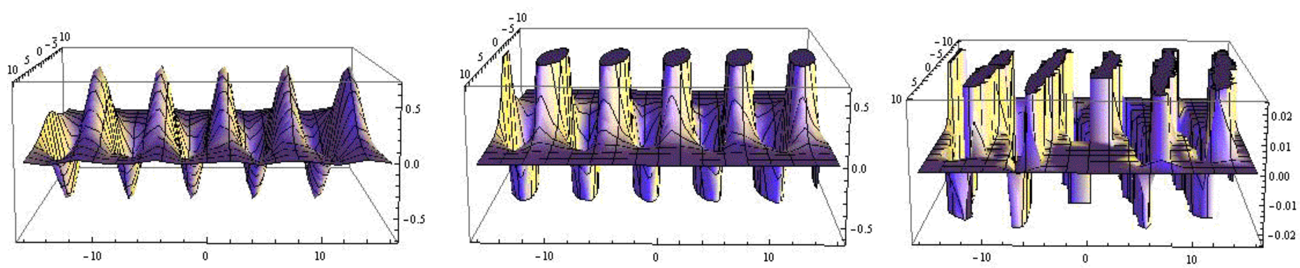

Figure 2. Solution of $u_1(x,t)$ is shown at $k=d=m=n=1$, $b(t)=sin(t)$.

Applying several simple transformations to these solutions, we obtain new exact solutions to Equation 2, respectively:

$$u_1(x,t)=\frac{A_1(t)}{\mathrm{cFs}^{\frac{2}{m-n+1}}[Bx-v(t)]}, \quad (32)$$

$$u_2(x,t)=\frac{A_2(t)}{\mathrm{sFs}^{\frac{2}{m-n+1}}[Bx-v(t)]}, \quad (33)$$

where

$$A_\epsilon(t)=\left(\frac{2(-1)^\epsilon b(t)k^2(\ln a)^2(1+m)(m(2n-3)+n(-4n+5)-3)}{5d(m-n+1)^2}\right)^{\frac{1}{m-n+1}}$$

, $(\epsilon=1,2.)$, $B=\frac{k}{2}$ and

$$v(t)=\int^t\frac{b(t')k^3(\ln a)^2(2n(n-1)-m(n-1)+1)}{2(m-n+1)^2}dt'.$$

Here, $A_1(t), A_2(t)$ represent the amplitude of the solitons, while B is the inverse width of the solitons and $v=v(t)$ represent the velocity of the solitons. Also, Equation 33 represents a singular soliton solution for Equation 2. Figures 1 to 7. show the solutions

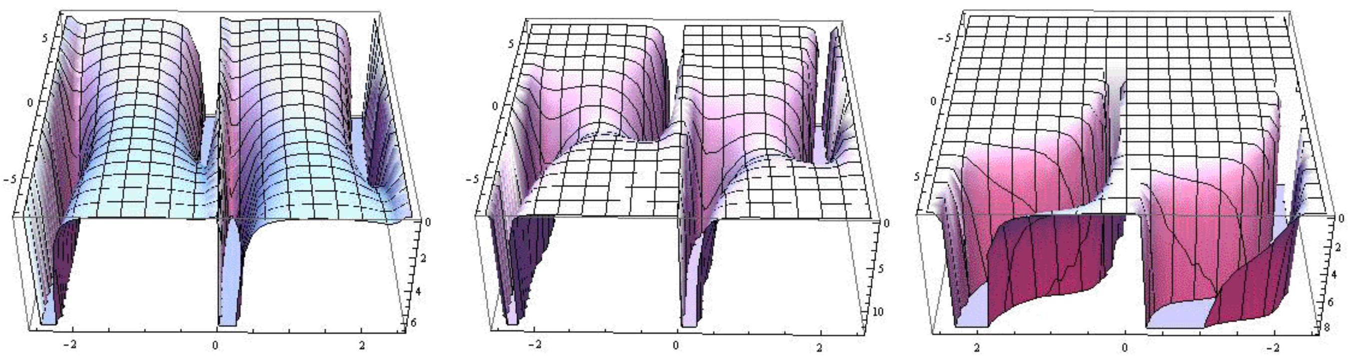

Figure 3. Solution of $u_1(x,t)$ is shown at $k = d = m = n = 1$, $b(t) = Weierstrass - elliptic - function\ [t,1/3]$.

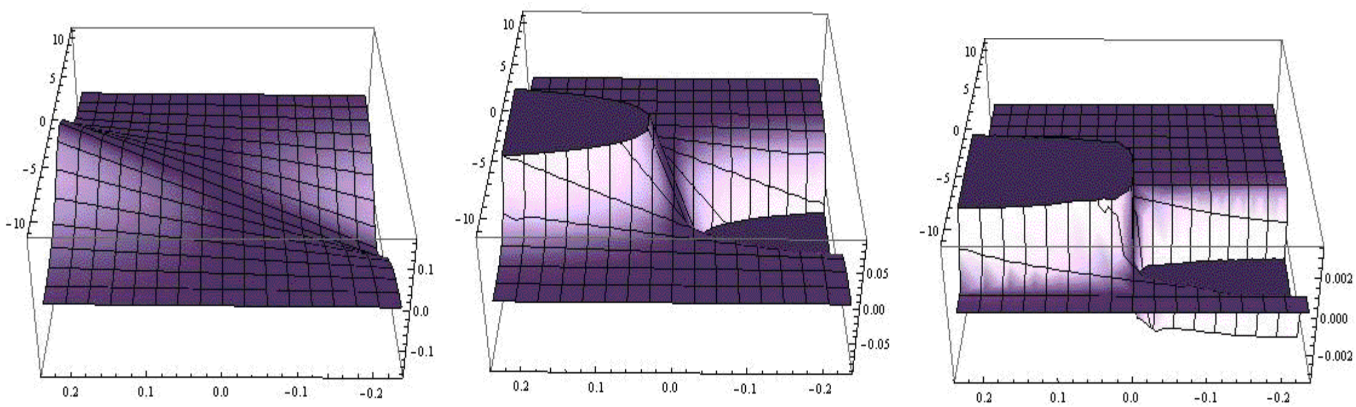

Figure 4. Solution of $u_1(x,t)$ is shown at $k = d = m = n = 1$, $b(t) = Jacobi - elliptic - functionsc\ [t,1/3]$.

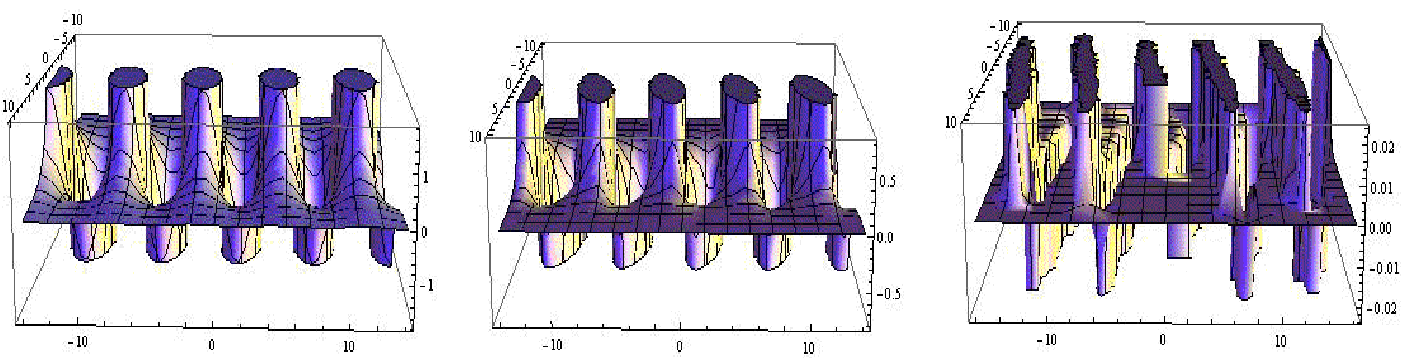

Figure 5. Solution of $u_2(x,t)$ is shown at $k = d = m = n = 1$, $b(t) = sin(t)$.

$u_1(x,t), u_2(x,t)$ for the values $k = d = m = n = 1$, the function $b(t)$ and a takes respectively, Golden Mean, e and 10. If we take $a = e$ in Equation 32, then we can find the solution obtained by using the Ansatz method in (Biswas, 2008).

REMARKS AND CONCLUSIONS

Our aim in this section is to show that general Exp $_a$ - function with Kudryashov method could be used in the solutions in the form of symmetrical hyperbolic Fibonacci

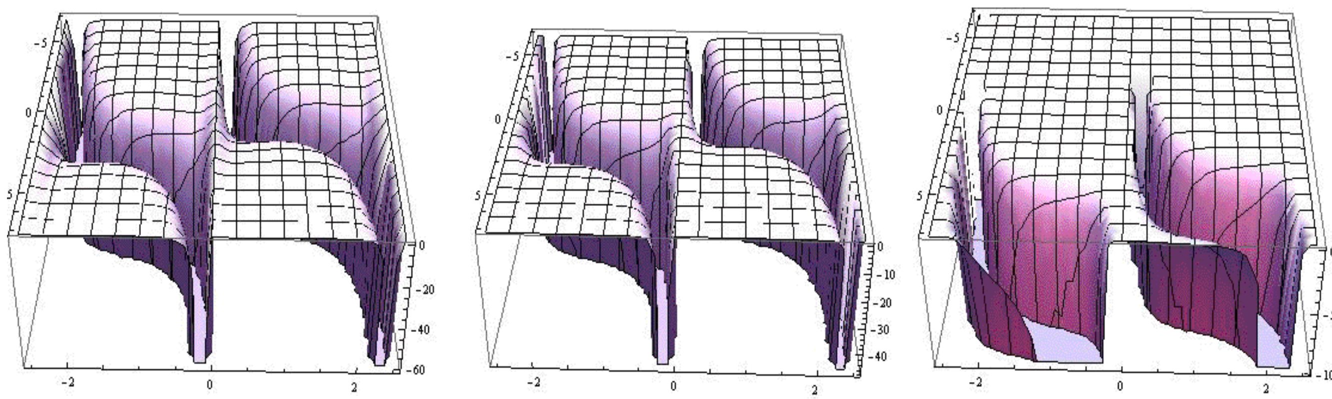

Figure 6. Solution of $u_2(x,t)$ is shown at $k=d=m=n=1$, $b(t)=Weierstrass-elliptic-function\ [t,1/3]$.

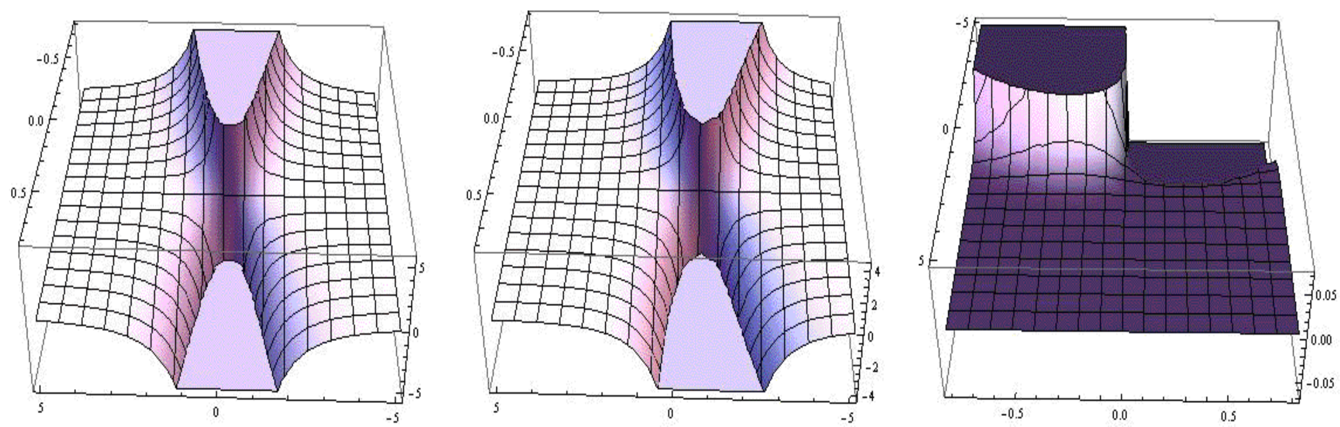

Figure 7. Solution of $u_2(x,t)$ is shown at $k=d=m=n=1$, $b(t)=Jacobi-elliptic-functionsc\ [t,1/3]$.

and Lucas functions. We highlight briefly the definitions of symmetrical hyperbolic Fibonacci and Lucas functions. Also Stakhov and Rozin (2005) defined all details of symmetrical hyperbolic Fibonacci and Lucas functions. We only give several formulas with respect to these functions here. Symmetrical Fibonacci sin, cosine, secant and cosecant functions are respectively defined as

$$sFs(x)=\frac{a^x-a^{-x}}{\sqrt{5}},\quad cFs(x)=\frac{a^x+a^{-x}}{\sqrt{5}},\quad scFs(x)=\frac{\sqrt{5}}{a^x+a^{-x}},\quad csFs(x)=\frac{\sqrt{5}}{a^x-a^{-x}}. \tag{34}$$

Analogously, symmetrical Lucas sine and cosine functions are respectively defined as

$$sLs(x)=a^x-a^{-x},\quad cLs(x)=a^x+a^{-x}, \tag{35}$$

where $a=\frac{1+\sqrt{5}}{2}$, which is known in literature as Golden Mean (Ahmad and Ezzat, 2010). From this study, it is therefore possible to find more general (or more larger classes of) solutions in applying the modified Kudryashov method with Symmetrical Fibonacci functions. However, If $a=e$, then the other solutions can be obtained.

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