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Full Length Research Paper

The approximate analysis of nonlinear behavior of structure under harmonic loading

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In this work, attempt has been made to analyze the nonlinear behavior of structures analytically. Despite the increasing expenses of building structures, to maintain their linear behavior, nonlinearity has been inevitable and therefore, nonlinear analysis has been of great importance to the scientists in the field. Studying on nonlinear dynamics highlights the fact that essentially all dynamic systems encountered in the real world are nonlinear, meaning that their description as differential equations contains nonlinear terms. Such nonlinearities appear in different ways, such as through frictional terms, coriolis and centrifugal terms, large amplitude effects, or other structural nonlinearities. The nonlinearities make that standard linear dynamics not sufficient for the analysis and understanding of nonlinear mechanical systems. As structures confront lateral forces and intense earthquakes especially near fault regions, a part of the structure remains linear, but some part of it behaves nonlinearly; this is simulated by a damped nonlinear oscillator. In this paper, the nonlinear equation of oscillator with damping which is representative of the dynamic behavior of a structure has been solved analytically. In the end, the obtained results are compared with numerical ones and shown in graphs and in tables; analytical solutions are in good agreement with those of the numerical method.

Key words: Nonlinear oscillator equation, damping, analytical approximate method.

INTRODUCTION

Until recently, finding an exact analytical solution for non-linear equations is extremely difficult. Therefore, many analytical and numerical approaches have been investigated. The most useful methods for solving nonlinear equations are perturbation methods. They are not valid for strongly nonlinear equations and they have many shortcomings. Many new techniques have been shown in the open literature to overcome the short-comings, such as variational iteration (Barari et al., 2008a; Fouladi et al., 2010; Barari et al., 2008b), parameter-expansion (Kimiaeifar et al., 2010), energy balance (Momeni et al., 2010; Ganji et al., 2009), variational approach (Kachapi et al., 2009), max-min (Babazadeh et al., 2010; Ibsen et al., 2010; Sfahani et al., 2010), Adomian decomposition method (Mirgolbabaei et al., 2010), differential transform method (Omidvar et al., 2010), and etc. Nonlinear oscillations systems can be presented as nonlinear behavior of a structure under dynamics loads. In current research, attempt has been made to propose an analytical solution for such problem, which is much simpler for engineers to interpret and to use in their designs. This is because an equation is obtained rather than only some data.

In the dynamic model of this problem, the earthquake force has been modeled with a harmonic force and the columns with nonlinear behavior are modeled with the spring k_2 and the columns with linear behavior are modeled with the spring k_1. The coefficient c represents the damping which is based on the joints, materials and other parameters (Mohammad et al., 1992; Yashuda et al., 1988; Kerschen et al., 2003; Feeny et al., 2001; Liang and Feeny, 2006).

To fully demonstrate the problem, let us consider a structure which its columns are under the harmonic load (e.g. earthquake). This load results in a nonlinear behavior in a part of the structure, while another part still

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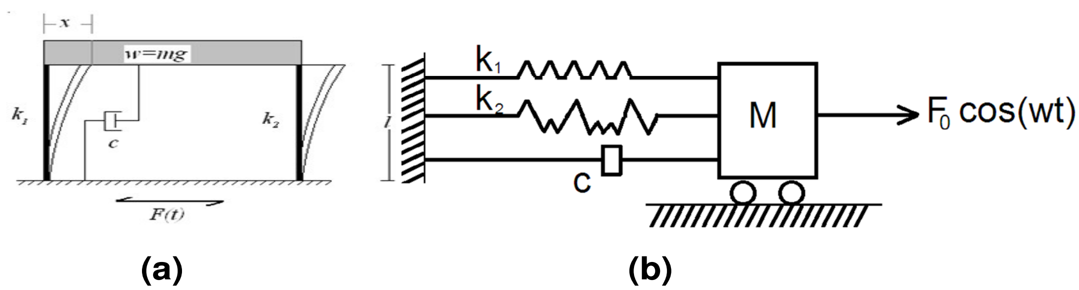


Figure 1. (a) Schematic view of a structure under harmonic load. (b) The dynamic model of a structure under harmonic load.

behaves linearly. The important point is the analysis of such system to obtain the displacement equation, which is extremely useful to study the structure (Figure 1). The analytical method which is used in this article is Homotopy Perturbation Method (HPM) (He, 2005). This method has been implemented successfully to many engineering problems by many scientists in different fields (Beléndez, 2009; Yusufoğlu, 2009; Shaban et al., 2010; Ganji et al., 2010; Barari et al., 2008c; Miansari et al.,2010; Abdoul et al.,2008; Barari et al., 2009; Farrokhzad et al., 2009; Choobbasti et al.,2008). This method is capable of solving highly nonlinear problems while the constant coefficients are parametrically inserted into the equation. Therefore, the obtained results can be graphically shown and analyzed for different cases and by inserting different values for these parameters regarding each single case of study. Finally, a comparative study is conducted to verify the accuracy of the analytical method with the numerical one.

MATHEMATICAL MODELING

The general equation of an oscillator with a nonlinear spring, a linear spring and a damper under a harmonic load is as follows (Mann and Khasawneh, 2009; Rao, 1995; Rao et al., 2004):

$$m\ddot{x} + c\dot{x} + k_1 x + k_2 x^3 = F_0 \cos(\omega t) \tag{1}$$

Subject to the following initial conditions

$$x(0) = A, \qquad \dot{x}(0) = 0 \tag{2}$$

Where m is the mass, c is a viscous damping coefficient, k_1 is a linear stiffness coefficient, and k_2 is a nonlinear stiffness coefficient. The harmonic excitation force is characterized by the force amplitude, F_0, with excitation frequency of ω. A is the initial amplitude of displacement.

As in Rao (1995), ω can be found easily by having the parameters, A, c, m, k_1 and k_2:

$$((k_1 - m\omega^2)A + \frac{3}{4}k_2 A^3)^2 + (c\omega A)^2 = F_0^2 \tag{3}$$

Figure 2 shows how the stiffness coefficients of nonlinear and linear springs behave, where $f(x)$ is the spring force and x is the displacement.

In the following sections the basic concepts of the analytical and numerical methods as well as their applications to the discussed problem above were investigated.

THE BASIC CONCEPT OF THE SOLUTIONS

In this section, the basic of the utilized methods are explained for the better understanding of the reader.

HPM

To illustrate the basic ideas of this method, we consider the following equation:

$$A(x) - f(r) = 0 \qquad r \in \Omega \tag{4}$$

With the boundary condition of:

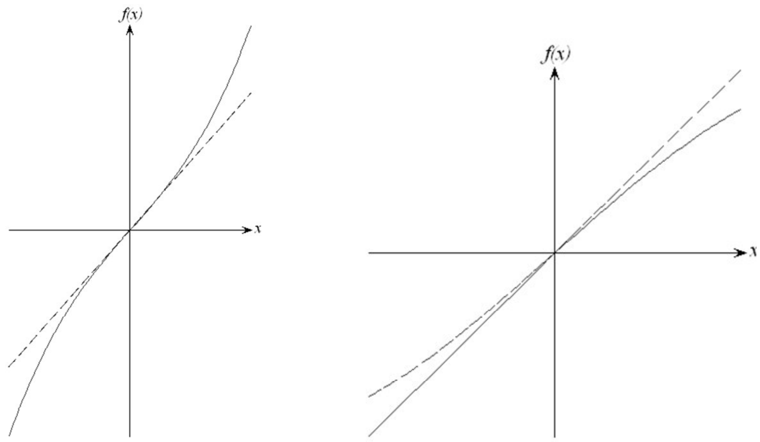
$$B\left(x, \frac{\partial x}{\partial t}\right) = 0 \qquad r \in \Gamma \tag{5}$$

Where A is a general differential operator, B a boundary operator, $f(r)$ a known analytical function and Γ is the boundary of the domain Ω.

A Can be divided into two parts of L and N, where L is linear and N is nonlinear. Equation (4) can therefore be rewritten as follows:

$$L(x) + N(x) - f(r) = 0 \qquad r \in \Omega \tag{6}$$

Homotopy perturbation structure is shown as follows:



(a) (b)

Figure 2. (a) Hard spring stiffness nonlinear behavior. (b) Soft spring stiffness nonlinear behavior.

$$H(\nu,p)=(1-p)\left[L(\nu)-L(x_0)\right]+p\left[A(\nu)-f(r)\right]=0 \tag{7}$$

Where,

$$\nu(r,p):\ \Omega\times[0,1]\rightarrow R \tag{8}$$

In Equation (7), $p\in[0,1]$ is an embedding parameter and x_0 is the first approximation that satisfies the boundary condition. We can assume that the solution of Equation (4) can be written as a power series in p, as following:

$$\nu=\nu_0+p\nu_1+p^2\nu_2+\cdots=\sum_{i=0}^{n}\nu_i p^i \tag{9}$$

And the best approximation for the solution is:

$$x=\lim_{p\to 1}\nu=\nu_0+\nu_1+\nu_2+\cdots \tag{10}$$

Runge-Kutta

For the numerical approach to verify the analytic solution, the fourth RK (Runge-Kutta) method has been used. This iterative algorithm is written in the form of the following formulae for the second-order differential equation:

$$\dot{x}_{i+1}=\dot{x}_i+\frac{\Delta t}{6}(h_1+2h_2+2h_3+k_4)$$
$$x_{i+1}=x_i+\Delta t\,(\dot{x}_i+\frac{\Delta t}{6}(h_1+h_2+k_3)) \tag{11}$$

Where, Δt is the increment of the time and h_1, h_2, h_3, and h_4 are determined from the following formulae:

$$h_1=f\ (\dot{x},x_i,\dot{x}_i)k\ ,$$
$$h_2=f\ (t_i+\frac{\Delta t}{2},x_i+\frac{\Delta t}{2}\dot{x}_i,\dot{x}_i+\frac{\Delta t}{2}h_1),$$
$$h_3=f\ (t_i+\frac{\Delta t}{2},x_i+\frac{\Delta t}{2}\dot{x}_i,\frac{1}{4}\Delta t^2h_1,\dot{x}_i+\frac{\Delta t}{2}h_2),$$
$$h_4=f\ (t_i+\Delta t,x_i+\Delta t\dot{x}_i,\frac{1}{2}\Delta t^2h_2,\dot{x}_i+\Delta th_3). \tag{12}$$

The numerical solution starts from the boundary at the initial time, where the first value of the displacement function and its first-order derivative are determined from initial condition (Section 2). Then, with a small time increment Δt, the displacement function and its first-order derivative at the new position can be obtained using Equation (11). This process continues to the end of the time limit.

THE SOLUTIONS

In this section the applications of the two methods to the nonlinear equation of oscillator are discussed.

HPM (Analytic)

As the HPM was applied to the nonlinear equation of (1), we have:

$$(1-p)(m\ddot{x}+c\dot{x}+k_1x)+p(m\ddot{x}+c\dot{x}+k_1x+k_2x^3-F_0\cos(\omega t))=0 \tag{13}$$

After expanding the equation and collecting it based on the coefficients of p -terms, we have:

$$p^0: m\ddot{x}_0+c\dot{x}_0+k_1x_0=0$$
$$p^1: m\ddot{x}_1+c\dot{x}_1+k_1x_1+k_2x_0^{\ 3}-F_0\cos(\omega t)=0$$
$$P^2: m\ddot{x}_2+c\dot{x}_2+k_1x_2+3k_2x_0^{\ 2}x_1=0$$
$$P^3: m\ddot{x}_3+c\dot{x}_3+k_1x_3+3k_2x_0^{\ 2}x_2+3k_2x_0x_1^{\ 2}=0 \tag{14}$$

Table 1. The numerical values for x and $\dot{x}$ for eleven different points of time (Analytic), for $f=0.5,\ A=0.06,\ \omega=4.163379415$.

t	x	$\dot{x}$
0	0.06	0
1	-0.005350926	-0.078888606
2	0.019216633	-0.013109041
3	-0.007335627	0.072786956
4	-0.011085293	-0.064177133
5	0.018934134	-0.005887804
6	-0.008674275	0.070317298
7	-0.009880823	-0.067501783
8	0.018986752	0.000133187
9	-0.009935365	0.067363257
10	-0.00861736	-0.070438539

One can now try to obtain the solution of different iterations (14), in the form of:

$$\begin{aligned}
x_0(t)=&\frac{1}{2}\frac{1}{c^2-4k_1m}((c\sqrt{c^2-4k_1m}+c^2-4k_1m)e^{\frac{1}{2}\frac{(-c+\sqrt{c^2-4k_1m})t}{m}}\\
&+\frac{1}{2}\frac{1}{c^2-4k_1m}(c^2-c\sqrt{c^2-4k_1m}-4k_1m)e^{\frac{-(c+\sqrt{c^2-4k_1m})t}{2m}}
\end{aligned} \tag{15}$$

$$\begin{aligned}
x_1(t)=&\frac{1}{2}\frac{1}{(c^2-4k_1m)^{\frac{5}{2}}}(((\int_0^t(-2e^{-\frac{5}{2}\frac{(c+\sqrt{c^2-4k_1m})_zl}{m}}(-\frac{1}{2}((c^3-3mck_1)\sqrt{c^2-4k_1m}\\
&+c^4-5c^2k_1m+4k_1^2m^2)\times k_2e^{\frac{1}{2}\frac{_zl(3c+7\sqrt{c^2-4k_1m})}{m}}+\frac{3}{2}mk_2k_1\frac{(c\sqrt{c^2-4k_1m})}{m}\\
&+c^2-4k_1m)e^{\frac{1}{2}\frac{_zl(3c+5\sqrt{c^2-4k_1m})}{m}}+F_0\cos(\omega t_zl)(c^2-4k_1m)^2e^{\frac{_zl(3c+2\sqrt{c^2-4k_1m})}{m}}\\
&-\frac{1}{2}(((-c^3+3mck_1)^2e^{\frac{_zl(3c+2\sqrt{c^2-4k_1m})}{m}}-\frac{1}{2}(((-c^3+3mck_1)\sqrt{c^2-4k_1m}+c4\\
&-5c^2k_1m+4k_1^2m^2)e^{\frac{1}{2}\frac{_zl(3c+\sqrt{c^2-4k_1m})}{m}}-3mk_1e^{\frac{3}{2}\frac{(c+\sqrt{c^2-4k_1m})_zl}{m}}\times\\
&(c^2-c\sqrt{c^2-4k_1m}-4k_1m)k_2))d_zl)e^{\frac{t\sqrt{c^2-4k_1m}}{m}}\\
&-(\int_0^t(-2e^{\frac{-1}{2}\frac{_zl(5c+3\sqrt{c^2-4k_1m})}{m}}(-\frac{1}{2}((c^3-3mck_1)\sqrt{c^2-4k_1m}\\
&+c^4-5c^2k_1m+4k_1^2m^2)k_2e^{\frac{1}{2}\frac{_zl(3c+7\sqrt{c^2-4k_1m})}{m}}\\
&+\frac{3}{2}mk_2k_1(c\sqrt{c^2-4k_1m}+c^2-4k_1m)e^{\frac{1}{2}\frac{_zl(3c+5\sqrt{c^2-4k_1m})}{m}}\\
&+F_0\cos(\omega t_zl)(c^2-4k_1m)^2e^{\frac{_zl(3c+2\sqrt{c^2-4k_1m})}{m}}\\
&-\frac{1}{2}(((-c^3+3mck_1)\sqrt{c^2-4k_1m}+c^4-5c^2k_1m+4k_1^2m^2)\times e^{\frac{1}{2}\frac{_zl(3c+\sqrt{c^2-4k_1m})}{m}}\\
&-3mk_1e^{\frac{3}{2}\frac{(3c+\sqrt{c^2-4k_1m})_zl}{m}}(c^2-c\sqrt{c^2-4k_1m}-4k_1m))k_2))d_zl))e^{\frac{-1}{2}\frac{(c+\sqrt{c^2-4k_1m})t}{m}})
\end{aligned} \tag{16}$$

And from Equation (10), $x(t)$ can be obtained:

$$\begin{aligned}
x(t)=&\frac{1}{2}\frac{1}{c^2-4k_1m}((c\sqrt{c^2-4k_1m}+c^2-4k_1m)e^{\frac{1}{2}\frac{(-c+\sqrt{c^2-4k_1m})t}{m}}\\
&+\frac{1}{2}\frac{1}{c^2-4k_1m}(c^2-c\sqrt{c^2-4k_1m}-4k_1m)e^{\frac{-(c+\sqrt{c^2-4k_1m})t}{2m}}\\
&\frac{1}{2}\frac{1}{(c^2-4k_1m)^{\frac{5}{2}}}(((\int_0^t(-2e^{-\frac{5}{2}\frac{(c+\sqrt{c^2-4k_1m})_zl}{m}}(-\frac{1}{2}((c^3-3mck_1)\sqrt{c^2-4k_1m}\\
&+c^4-5c^2k_1m+4k_1^2m^2)\times k_2e^{\frac{1}{2}\frac{_zl(3c+7\sqrt{c^2-4k_1m})}{m}}+\frac{3}{2}mk_2k_1\frac{(c\sqrt{c^2-4k_1m})}{m}\\
&+c^2-4k_1m)e^{\frac{1}{2}\frac{_zl(3c+5\sqrt{c^2-4k_1m})}{m}}+F_0\cos(\omega t_zl)(c^2-4k_1m)^2e^{\frac{_zl(3c+2\sqrt{c^2-4k_1m})}{m}}\\
&-\frac{1}{2}(((-c^3+3mck_1)^2e^{\frac{_zl(3c+2\sqrt{c^2-4k_1m})}{m}}-\frac{1}{2}(((-c^3+3mck_1)\sqrt{c^2-4k_1m}+c4\\
&-5c^2k_1m+4k_1^2m^2)e^{\frac{1}{2}\frac{_zl(3c+\sqrt{c^2-4k_1m})}{m}}-3mk_1e^{\frac{3}{2}\frac{(c+\sqrt{c^2-4k_1m})_zl}{m}}\times\\
&(c^2-c\sqrt{c^2-4k_1m}-4k_1m)k_2))d_zl)e^{\frac{t\sqrt{c^2-4k_1m}}{m}}\\
&-(\int_0^t(-2e^{\frac{-1}{2}\frac{_zl(5c+3\sqrt{c^2-4k_1m})}{m}}(-\frac{1}{2}((c^3-3mck_1)\sqrt{c^2-4k_1m}\\
&+c^4-5c^2k_1m+4k_1^2m^2)k_2e^{\frac{1}{2}\frac{_zl(3c+7\sqrt{c^2-4k_1m})}{m}}\\
&+\frac{3}{2}mk_2k_1(c\sqrt{c^2-4k_1m}+c^2-4k_1m)e^{\frac{1}{2}\frac{_zl(3c+5\sqrt{c^2-4k_1m})}{m}}\\
&+F_0\cos(\omega t_zl)(c^2-4k_1m)^2e^{\frac{_zl(3c+2\sqrt{c^2-4k_1m})}{m}}\\
&-\frac{1}{2}(((-c^3+3mck_1)\sqrt{c^2-4k_1m}+c^4-5c^2k_1m+4k_1^2m^2)\times e^{\frac{1}{2}\frac{_zl(3c+\sqrt{c^2-4k_1m})}{m}}\\
&-3mk_1e^{\frac{3}{2}\frac{(3c+\sqrt{c^2-4k_1m})_zl}{m}}(c^2-c\sqrt{c^2-4k_1m}-4k_1m))k_2))d_zl))e^{\frac{-1}{2}\frac{(c+\sqrt{c^2-4k_1m})t}{m}})
\end{aligned} \tag{17}$$

The obtained iteration is used to generate the equation for the next iteration, and therefore the second and third iterations are obtained. Since the two other ones and therefore the general solution are too long to be written in this article, we have shown them in graphs (see section 5). In Table 1, the numerical values for x and $\dot{x}$ for different points of time and for $f=0.5,\ A=0.06,\ \omega=4.163379415$ have been tabulated.

Runge-Kutta (Numerical)

In this section, the Maple Package has been utilized for the numerical analysis of the problem, in which the rkf45 is used to solve ODEs. The solution for the displacement and the velocity for eleven different points of time are shown in Table 2.

RESULTS AND DISCUSSION

In this section, the results for displacement and the velocity for different times are shown in Tables 3 and 4, for different f 's and A 's, in order to evaluate the

Table 2. The numerical values for x and $\dot{x}$ for eleven different points of time (Numerical), for $f = 0.5,\ A = 0.06,\ \omega = 4.163379415$.

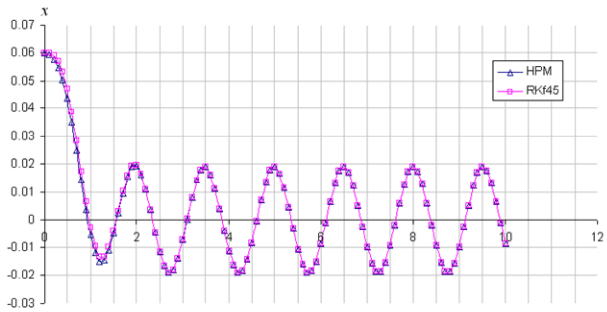
t	x	$\dot{x}$
0	0.06	0.00
1	-0.0028167883	-0.0825804788
2	0.0195449692	-0.0138897877
3	-0.0073091136	0.0727177020
4	-0.0110834704	-0.0641821083
5	0.0189342438	-0.0058881623
6	-0.0086742697	0.0703173716
7	-0.0098808263	-0.0675016034
8	0.0189867450	0.0001330720
9	-0.0099353577	0.0673626924
10	-0.0086173519	-0.0704385763

Table 3. A comparative table for error detection of the analytic method, for $f = 0.5,\ A = 0.06,\ \omega = 4.163379415$.

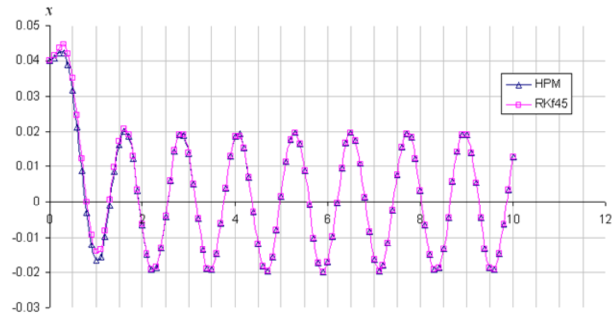
	x		$\dot{x}$	
t	**HPM**	**RKf45**	**HPM**	**RKf45**
0	0.06	0.06	0.00	0.00
1	-0.0053509257	-0.0028167883	-0.0788886055	-0.0825804788
2	0.0192166329	0.0195449692	-0.0131090406	-0.0138897877
3	-0.0073356269	-0.0073091136	0.0727869563	0.0727177020
4	-0.0110852934	-0.0110834704	-0.0641771328	-0.0641821083
5	0.0189341339	0.0189342438	-0.0058878042	-0.0058881623
6	-0.0086742754	-0.0086742697	0.0703172984	0.0703173716
7	-0.0098808233	-0.0098808263	-0.0675017833	-0.0675016034
8	0.0189867516	0.0189867450	0.0001331867	0.0001330720
9	-0.0099353650	-0.0099353577	0.0673632572	0.0673626924
10	-0.0086173596	-0.0086173519	-0.0704385389	-0.0704385763

Table 4. A comparative table for error detection of the analytic method, for $f = 0.7,\ A = 0.04,\ \omega = 5.147879675$.

	x		$\dot{x}$	
t	**HPM**	**RKf45**	**HPM**	**RKf45**
0	0.04	0.04	0.00	0.00
1	-0.0166844795	-0.0141498204	-0.0179037352	-0.0215962732
2	-0.00676624361	-0.0064378310	-0.0955008558	-0.0962817733
3	0.0138665780	0.01389313562	-0.0721831694	-0.0722528614
4	0.0185603233	0.0185621543	0.0342636907	0.0342587020
5	0.00179512000	0.0017952350	0.101082818	0.1010825368
6	-0.0170457813	-0.0170457707	0.0510209037	0.0510209094
7	-0.0161768383	-0.0161768394	-0.0580359551	-0.0580359227
8	0.00339720702	0.0033972117	-0.0999860509	-0.0999864615
9	0.0190431014	0.0190430958	-0.0263242834	-0.0263237149
10	0.0126696636	0.0126696551	0.0777768330	0.0777768691

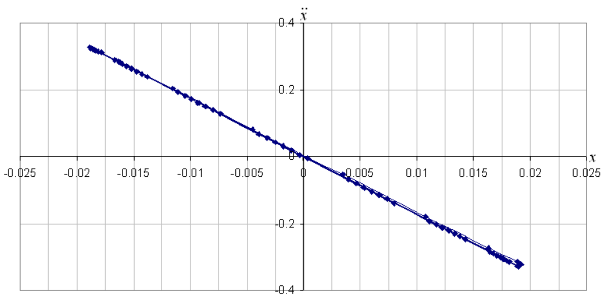


(a)

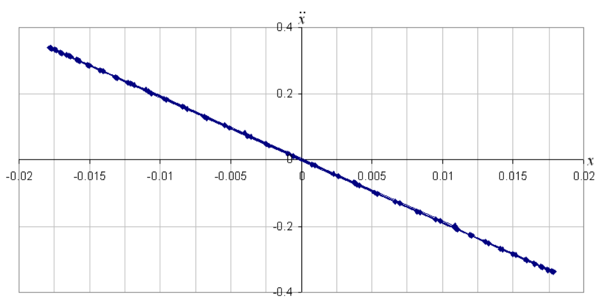


(b)

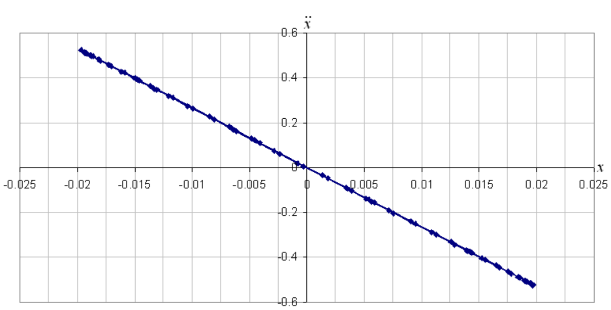
Figure 3. Displacement x based on time t for (a) $f = 0.5,\ A = 0.06,\ \omega = 4.163379415$), and (b) $f = 0.7,\ A = 0.04,\ \omega = 5.147879675$).



(a)



(b)



(c)

Figure 5. Acceleration $\ddot{x}$ based on displacement x for (a) $f = 0.5,\ A = 0.06,\ \omega = 4.163379415$), (b) $f = 0.6,\ A = 0.05,\ \omega = 4.582639115$ (c) $f = 0.7,\ A = 0.04,\ \omega = 5.147879675$).

accuracy of the analytic solution.

As it is obviously seen, the results of the analytic and numerical approaches have shown excellent compatibility. In order to have a better scheme of the problem, displacement x is shown in Figure 3 based on time, for ten seconds (different f 's and A 's are assumed).

In the Figure 4, the velocity of each position is drawn versus its position; therefore, the velocity of any specific point x can be easily read. This procedure can only be performed using the analytic method; since the equation of displacement is readily given by this method, the first and second differentiations can be simply done by differentiating with respect to t .

Also using Figure 5, the acceleration of any specific point x can be easily read. As mentioned earlier, this can only be done using the analytic approach.

The important point which cannot be seen on the figures of $(\dot{x} - x)$ and $(\ddot{x} - x)$ is that the starting part of these diagrams refereeing to the times between t 's from 0 to 1, is not drawn. The reason is that in this period of time, the behavior of the displacement equation has not yet become harmonic and therefore, the velocity and acceleration is not in the rage of the above diagrams.

Conclusions

As structures are exposed to lateral harmonic forces and intense earthquakes, parts of the structure remains linear, but some parts of it inevitably behave nonlinearly; this is simulated by a damped nonlinear oscillator.

In this work, HPM which is a new analytical method has been applied to the nonlinear equation of an oscillator with damping and the results have been compared with those of the numerical solution. The results, as in section5, have shown good agreement with the numerical ones. By obtaining the displacement equation, one is able to determine the velocity and acceleration equations.

The target of the present work was to determine the displacement, velocity and acceleration equations of the structure under the specified harmonic load, which give a better viewpoint for engineering design to scientist in the field. The obtained displacement equation can be used by designers to minimize displacements.

The main advantage of applying HPM is that the results are readily obtained and a few iterations are used. The significant merit of the analytic approach is to provide scientists with the general parametric relation between the dependent and independent variables, namely, displacement and time, respectively. Therefore, the related equations can be simply obtained, giving one the opportunity for further studies, for different cases and thereby different parameters.

REFERENCES

Abdoul RG, Barari A, Ganji DD (2008). Solving ratio-dependent predator–prey system with constant effort harvesting using homotopy perturbation method. J. Math. Prob. Eng., Article ID 945420, 1-8.

Babazadeh H, Domairry G, Barari A, Azami R, Davodi AG (2010). Numerical analysis of strongly nonlinear oscillation systems using He's max-min method, Front. Mech. Eng. Chin., in press.

Barari A, Omidvar M, Abdoul RG, Ganji DD (2008a), Application of Homotopy Perturbation Method and Variational Iteration Method to Nonlinear Oscillator Differential Equations, Acta Appl. Math., 104: 161-171.

Barari A, Omidvar M, Ganji DD, Tahmasebi AP (2008b). An approximate solution for boundary value problems in structural engineering and fluid mechanics. J. Math. Problems Eng., Article ID 394103, 1-13.

Barari A, Abdoul RG, Farrokhzad F, Ganji DD (2008c). Variational iteration method and Homotopy-perturbation method for solving different types of wave equations. J. Appl. Sci., 8(2008): 120-126.

Barari A, Janalizadeh A, Farrokhzad F, Ganji DD (2009). Application of homotopy perturbation method and variational iteration method to a nonlinear diffusion equation with a reaction term, J. Appl. Functional Anal., 4(1): 300-311.

Beléndez A (2009). Homotopy perturbation method for a conservative x1/3 force nonlinear oscillator, Comp. Math. Appl., 58: (11-12): 2267-2273.

Choobbasti AJ, Barari A, Farrokhzad F, Ganji DD (2008). Analytical investigation of a fourth order boundary value problem in deformation of beams and plate deflection theory, J. Appl. Sci., 8(11): 2148-2152.

Farrokhzad F, mosavi AK, Barari A, Choobasti AJ, Ganji DD (2009). Solution of Beam Deformation Equation Using Perturbation and Homotopy-Perturbation Methods, J. Appl. Functional Anal., 4(1): 392-400.

Feeny BF, Yuan CM, Cusmano JP (2001). Parametric identification of an experimental magneto-elastic oscillator, J. Sound Vib., 247(5): 785-806.

He JH (2005). Homotopy perturbation method for bifurcation of nonlinear problems, Int. J. Nonlinear. Sci. Numer. Simulat., 6(2): 207-8.

Ibsen LB, Barari A, Kimiaeifar A (2010), Analysis of Highly Nonlinear Oscillation Systems Using He's Max-Min Method and Comparison with Homotopy Analysis and Energy Balance Methods, Sadhana, in press.

Kerschen G, Lenaerts V, Golinval JC (2003). Identification of a continuous structure with a geometrical nonlinearity. Part1: conditioned reversed path method, J. Sound Vib., 262(4): 889-906.

Liang JW, Feeny BF (2006). Balancing energy to estimate damping parameters in forced oscillators, J. Sound Vib., 295: 988-998.

Mann BP, Khasawneh FA (2009). An Energy-Balance Approach for Oscillator Parameter Identification, J. Sound Vib., 321(1-2): 65-78.

Miansari MO, Miansari ME, Barari A, Domairry G (2010). Analysis of Blasius Equation for Flat-Plate Flow with Infinite Boundary Value, Intl. J. Comput. Meth. Eng. Sci. Mech., 11(2): 79-84.

Mirgolbabaei H, Barari A, Ibsen L.B, Sfahani MG (2010). Numerical Solution of Boundary Layer Flow and Convection Heat Transfer Over a Flat Plate, Arch. Civil Mech. Eng., in press.

Mohammad KS, Worden K, Tomlinson GR (1992). Direct parameter estimation for linear and nonlinear structures, J. Sound Vib., 152(3): 471-499.

Omidvar M, Barari A, Momeni M, Ganji DD (2010). New Class of Solutions for Water Infiltration Problems in Unsaturated Soils, Geomechanics and Geoen-gineering: Int. J., 5(2): 127-135.

Rao SS (1995). Mechanical Vibrations, 3rd Edition, Addison-Wesley.

Rao VD, Srinivas J. Textbook of Mechanical Vibrations, Prentice Hall, 2004.

Sfahani MG, Ganji SS, Barari A, Mirgolbabaei H, Domairry G (2010). Analytical Solutions to Nonlinear Conservative Oscillator with Fifth-Order Non-linearity, Earthquake Eng. Eng. Vib., in press.

Shaban M, Ganji DD, Alipour MM (2010). Nonlinear fluctuation, frequency and stability analyses in free vibration of circular sector oscillation systems, Curr. Appl. Phys., 10(5): 1267-1285.

Yashuda K, Kawamura S, Watanabe K (1988). Identification of nonlinear multi-degree-of-freedom systems (presentation of an identification technique), JSME Int. J., 31: 8-14.

Yusufoğlu E (2009). An improvement to homotopy perturbation method for solving system of linear equations, Comp. Math. Appl., 58: (11-12): 2231-2235.