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Modification of the coupling constant in the gravitational field equation and the growing celestial bodies

Jian Liang Yang

College of Physics, Zhengzhou University, Henan, China.

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The coupling constant $-8\pi G$ in the gravitational field equation $R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=-8\pi GT_{\mu\nu}$ is calculated based on a prior assumption that the pressure is zero under weak field approximation which has logical defect. It is precisely this deficiency that leads to infinite pressure inside celestial bodies and some singularities that should not have appeared. This article first proves that under the weak field approximation, the pressure acting as a gravitational source is negative and has the following relation with density namely $p=-\rho$. Then based on this premise the coupling constant in the gravitational field equation is modified from the original $-8\pi G$ to the present $4\pi G$, improved and perfected the theoretical foundation of gravitation. Turn out a new cosmic evolution model of the continuous generation of matter, cyclic expansions and eliminate the based difficulties in existing gravitational theory and cosmology. Derive a new relation between distance and red-shift consistent with observation and show that the universe is alive and galaxies or celestial bodies are continuously growing with cosmic expansion, provide formulas for the evolution of their mass and radius over time, and demonstrate that dark energy is the binding energy within matter rather than the independent entities.

Key words: Field equation coupling constant, weak field approximation, negative pressure

INTRODUCTION

The coupling constant $-8\pi G$ in the gravitational field equation of general relativity is obtained together with the weak field approximation metric under the assumption of zero pressure. This author believes that this prior assumption has logical flaws, and the discussion of its rationality has not been seen in any literature. In fact, it is precisely this defect that leads to this inappropriate coupling constant, manifested as the pressure solution of

E-mail: bps267890@163.com.

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the field equation appearing infinitely large in celestial bodies, as well as some other singularities that should not occur (Weinberg, 1972; Norbert, 2017). In cosmology, the need to introduce strange phenomena such as dark matter, dark energy, and inflation to solve theoretical problems indicates serious problems with this coupling constant, which has become an obstacle to the further development of gravity theory and modifications are imperative. The reason why this problem was not discovered in the early days of general relativity was because the external solution of the field equation was used without involving this coupling constant. For example, the external solution was used to describe the precession of Mercury, the delay of radar waves, and the bending of light rays. The problem of the internal solution of the equation when applied to the universe is fully exposed. Firstly, the distance redshift relationship derived from it does not match the observation seriously. Secondly, there are insurmountable difficulties in the horizon and time starting point. Although people constantly propose various repair plans, they cannot fundamentally eliminate these difficulties such as proposing inflation models, which seem to bring more problems. Starting from the field equation itself, this article first determines the form of the pressure that should be taken as the gravitational source and then re determines this coupling constant based on this premise, so as to easily solve various difficult problems. It cannot be said that the reliability of supplementary or research foundations is not at the forefront, but rather a more important frontier. The author attempts to take a step forward in this regard.

Note that the discussion in this article is in the natural unit system, namely speed of light takes 1, and agree on that the metric $\eta_{\mu\nu}$ of a flat spacetime only has diagonal elements -1, +1, +1, +1. Ricci tensor $R_{\mu\nu} \equiv R^{\sigma}_{\mu\nu\sigma} = \Gamma^{\sigma}_{\mu\sigma},_{\nu} - \Gamma^{\sigma}_{\mu\nu,\sigma} + \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\lambda\nu} - \Gamma^{\lambda}_{\mu\nu}\Gamma^{\sigma}_{\lambda\sigma}$

THE RELATION BETWEEN PRESSURE AND DENSITY AS A GRAVITATIONAL SOURCE

Using field equations to solve the pressure p under weak field approximation

The pressure p here refers to the pressure acting as a gravitational source which is the component of the energy-momentum tensor of the material acting as a gravitational source. The expression for the energy-momentum tensor of the gravitational source is (Weinberg, 1972) $T_{\mu\nu} = (\rho + p)U_{\mu}U_{\nu} + pg_{\mu\nu}$, where ρ is the density of the matter, p is the pressure, $U_{\mu} = g_{\mu\nu}U^{\nu}$ is the covariant velocity of the medium, and the corresponding inversion velocity is $U^{\mu} = \frac{dx^{\mu}}{d\tau}$, τ is the proper time, and $g^{\mu\nu}$ is the inverse matrix of $g_{\mu\nu}$. $ds^2 = -d\tau^2 = g_{\mu\nu}dx^{\mu}dx^{\nu}$, $U_{\mu}U^{\mu} = -1$, $T = g_{\mu\nu}T^{\mu\nu} = 3p - \rho$. Note that the repeated superscripts and subscripts represent the summation over 0, 1, 2, 3. In the article, $x^0 = t$, Greek alphabets $\mu, \nu, \lambda, \sigma = 0, 1, 2, 3$ and Latin alphabets $i, j, k = 1, 2, 3$ are only for spatial indicators. Einstein gravitational field equation is (Weinberg, 1972; Norbert, 2017):

$$R_{\mu\nu} = \gamma(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T) \tag{1}$$

where Ricci tensor $R_{\mu\nu} = \Gamma^{\sigma}_{\mu\sigma},_{\nu} - \Gamma^{\sigma}_{\mu\nu,\sigma} + \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\lambda\nu} - \Gamma^{\lambda}_{\mu\nu}\Gamma^{\sigma}_{\lambda\sigma}$, and $\Gamma^{\sigma}_{\mu\nu} = \frac{g^{\sigma\rho}}{2}(\frac{\partial g_{\mu\rho}}{\partial x^{\nu}} + \frac{\partial g_{\rho\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\rho}})$.

Note that the subscript commas in the formula represent taking the derivative of the corresponding indicator, such as $B_{,\nu} = \frac{\partial B}{\partial x^{\nu}}$. Another form of field Equation 1 is $R^{\nu}_{\mu} - \frac{1}{2}\delta^{\nu}_{\mu}R = \gamma T^{\nu}_{\mu}$. Here γ is the coupling constant to be determined in this paper, which is treated as a universal constant. The constant previously determined under the assumption of zero pressure under weak field approximation is $-8\pi G$. The author believes that this prior assumption may lead to incorrect results. In fact, some singularities that should not appear in general relativity are directly or indirectly related to this constant such as causing infinite internal pressure in celestial bodies and introducing strange phenomena such as dark matter dark energy inflation in cosmology to solve the problem, all of which indicate serious problems with this coupling constant (Weinberg, 1972). Starting from the field equation itself, this article first determines the form of the pressure that should be taken as the gravitational source, and then redetermines this coupling constant based on this premise. In order to compare with Newtonian gravity, it can still be discussed that the weak field approximation of the static spherically symmetric gravitational field in the Cartesian coordinates $(x^0, x^1, x^2, x^3) = (t, x, y, z)$. As a weak gravitational field, it can order $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $|h_{\mu\nu}| << 1$, while $\eta_{\mu\nu} = \eta^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ is the Minkowski metric.

And it is worth emphasizing in particular that zero covariant divergence of Einstein tensor transforms into zero ordinary divergence under the weak field approximation, that is from $(R^{\mu}_{\nu}-\frac{1}{2}R\delta^{\mu}_{\nu})_{;\ \mu}=0$ to $(R^{\nu}_{\mu}-\frac{1}{2}\delta^{\nu}_{\mu})_{,\nu}=0$, where subscript semicolon represents covariant divergence. Therefore, $T^{\nu}_{\mu,\ \nu}=0$ can be directly obtained from the field equation (Weinberg, 1972; Norbert, 2017). Here, is the proof of the process $R^{\mu}_{\nu;\ \mu}=R^{\mu}_{\nu,\ \mu}+\Gamma^{\mu}_{\lambda\mu}R^{\lambda}_{\nu}-\Gamma^{\mu}_{\lambda\nu}R^{\lambda}_{\mu}=R^{\mu}_{\nu,\ \mu}+O(h^2)=R^{\mu}_{\nu,\ \mu}$, then $0=(R^{\mu}_{\nu}-\frac{1}{2}R\delta^{\mu}_{\nu})_{;\ \mu}=R^{\mu}_{\nu,\mu}-\frac{1}{2}R_{,\nu}$, and on the other hand, field equation gives $R=-\gamma T$, so $R^{\mu}_{\nu,\mu}=\gamma(T^{\mu}_{\nu}-\frac{1}{2}T\delta^{\mu}_{\nu})_{;\ \mu}=\gamma T^{\mu}_{\nu\ ,\mu}+\frac{1}{2}R_{,\nu}$, that is $T^{\nu}_{\mu,\ \nu}=0$. And applying the result to the static gravitational source, noting that $T_{\mu\nu}=(\rho+p)U_{\mu}U_{\nu}+pg_{\mu\nu}$, it has

$$0=T^{\nu}_{\mu,\nu}=\left[(\rho+p)U_{\mu}U^{\nu}\right]_{,\nu}+\frac{\partial p}{\partial x^{\nu}}\delta^{\mu}_{\nu}=\frac{\partial p}{\partial x^{\mu}} \tag{2}$$

Note that for the static source $\left[(\rho+p)U_{\mu}U^{\nu}\right]_{,\nu}=0$. Equation 2 tells that the pressure p is uniform in the weak gravitational source. On the other hand, for the static source the strict zero covariant divergence $T^{\nu}_{\mu;\ \nu}=0$ surely provides the following equation

$$\frac{\partial p}{\partial x^{\mu}}=-\frac{\rho+p}{2g_{00}}\cdot\frac{\partial g_{00}}{\partial x^{\mu}} \tag{3}$$

As shown in the work of Weinberg (1972), $\frac{\partial p}{\partial x^{\mu}}=0$ requires $-\frac{\rho+p}{2g_{00}}\cdot\frac{\partial g_{00}}{\partial x^{\mu}}$ to degenerate to zero under weak field approximation, and since the gravitation inside the celestial body or source is not zero, that is $\frac{\partial g_{00}}{\partial x^{i}}\neq 0$, then there must be $p=-\rho=const$, which is the relation between the pressure and the density of matter under the weak field approximation that is been looked for. Note that outside source $p=-\rho=0$. Previously, it was not appropriate to assume that the pressure p inside the weak gravitational source was zero, and even the interior of the so-called loose medium cannot be arbitrarily assumed to be zero because $\frac{\partial p}{\partial x^{i}}+\frac{\rho+p}{2g_{00}}\cdot\frac{\partial g_{00}}{\partial x^{i}}=\frac{\rho}{2g_{00}}\cdot\frac{\partial g_{00}}{\partial x^{i}}\neq 0$ as $p=0$, that is $T^{\nu}_{\mu;\ \nu}\neq 0$, and on the other hand only $T^{\nu}_{\mu;\ \nu}=0$ the field equation can hold.

It should be emphasized that $p=-\rho$ is the result of solving the field equation rather than being deliberately introduced by the author, so this is not something we are willing to accept, but something we must accept. It is generally believed that p is equal to zero, but obviously Equation 3 cannot hold. In short, we must face the result of $p=-\rho$ squarely and cannot avoid it.

The physical meaning of negative pressure

Then, what is the physical meaning of this negative pressure? Taking a negative value of p indicates that it is not the usual pressure and it is well known that matter is ultimately composed of various basic particles such as electron, protons, neutrons, and quarks, there are four basic interactions between these basic particles, the negative pressure represents exactly the energy of the interaction between particles inside matter, which is the potential energy. This understanding is consistent with Einstein's explanation back then. At the beginning, Einstein did not deny the possibility of negative pressure (Einstein, 2004). Considering that $T_{\mu\nu}=\rho U_{\mu}U_{\nu}$ cannot reflect the true situation, Einstein said:" *But in addition, we shall add a pressure term that may be physically established as follows. Matter consists of electrically charged particles. On the basis of Maxwell's theory these cannot be conceived of as electromagnetic fields free from singularities. In order to be consistent with the facts, it is necessary to introduce energy terms, not contained in Maxwell's theory, so that the single electric particles may hold together in spite of the mutual repulsions between their elements, charged with electricity of one sign. For the sake of consistency with this fact, Poincaré has assumed a pressure to exist inside these particles which balances the electrostatic repulsion. It cannot, however, be asserted that this pressure vanishes outside the particles. We shall be consistent with this circumstance if, in our phenomenological presentation, we add a pressure term. This must not, however, be confused with a hydrodynamical pressure, as it serves only for the energetic presentation of the dynamical relations inside matter. Accordingly, we put* $T_{\mu\nu}=\rho U_{\mu}U_{\nu}+pg_{\mu\nu}$". Later this form is further modified to $T_{\mu\nu}=(\rho+p)U_{\mu}U_{\nu}+pg_{\mu\nu}$. Obviously, Einstein did not treat the pressure of the gravitational source as the usual fluid dynamic pressure, but rather as the binding energy of matter. The reason why it is still called as pressure today is because it has the same calculation effect as pressure in the equation, but fundamentally it is not the same thing. Therefore, it is normal to take a negative value, similar to absolute temperature, you can also call this negative pressure absolute pressure. Based on the above interpretation of p, $T_{\mu\nu}=(\rho+p)U_\mu U_\nu+pg_{\mu\nu}$ as a gravitational source, is no longer just an ideal fluid, which means that the form of its gravitational source is the same for any material system. The previous explanation of p as ordinary pressure actually negated the possibility of a solid as a source of gravity (because the ordinary pressure inside a solid is usually considered zero and not cumulative). It is obviously absurd that solids cannot serve as a source of gravity. Now a more precise definition of the ordinary pressure of a fluid should be $P=-\int_r^{r_0}\rho\Gamma^1_{00}dr$ corresponding to Newton's $P=\int_r^{r_0}\rho\frac{GM(r)}{r^2}dr$. Obviously, neither P nor p will exhibit singularities in celestial bodies. It is absurd for p to appear infinitely large in celestial bodies, as shown in the work of Weinberg (1972).

Under weak field approximation, celestial bodies can be treated as incompressible fluids

The conclusion that the density of matter in weak gravitational sources or not too large celestial bodies is uniform is also basically consistent with reality. For example, the density of rocks on the Earth's surface is about 4 g/cm^3, and the average density of the Earth is about 5 g/cm^3, indicating that a radius of 6400 km did not significantly increase the density of the Earth's center. In fact, the effect of gravity on the aggregation of matter is negligible, while the determining force is the strong electromagnetic and weak forces between particles. The conclusion of uniform density inside relatively small celestial bodies is also reasonable.

It should also be noted that $p=-\rho=const$ only holds true under weak field approximation, indicating that celestial bodies are solids or incompressible fluids under weak approximation. Strictly speaking, celestial bodies are not incompressible and when the gravitational field is quite strong, the density may be a function of r. In principle, this situation p must be solved from the Equation 3. Its general solution can be written as $p=p(r,C)$, the integral constant C is determined from $\int_0^{r_0}p(r,C)4\pi r^2dr=-M$, and the metric must also strictly solve the field equation in the quite strong gravitational field (Hamuy, 2003).

From another perspective, density is the statistical average of the mass per unit volume, and for relatively small celestial bodies, it can be considered a volume unit for statistical purposes. Density is naturally a constant. That is to say, the uniformity of density itself has a certain artificial degree, depending on the size of the selected unit volume for statistical purposes. For example, if the entire Earth is treated as a statistical volume unit, the density of each point within the Earth is the average density of the Earth. Therefore, for the convenience of calculation, treating the density of celestial bodies as a constant would not pose a principle problem (otherwise the concept of density would have to be removed from physics), that is, $p=-\rho=const$ is still approximately valid in general situation.

By the way, it should be pointed out that $p=-\rho$ is the equation of state for dark energy introduced in cosmology. Therefore, dark energy is the binding energy of matter rather than an independent existence, and it is a property of matter that does not need to be introduced separately.

Redetermining the coupling constant γ in field equation

It can be observed that when $p=-\rho$, the coupling constant of the field is 4πG instead of -8πG. As usual, in order to determine the constant γ we need to solve the weak field approximation metric of the spherically symmetric static gravitational field, which can be compared with Newtonian gravity. That is to say, the coupling constant is determined simultaneously with the static weak field approximation metric.

Under the weak field approximation, the rise and fall of the indexes are replaced by η instead of g, that is

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu},\qquad |h_{\mu\nu}|<<1,\qquad h^\mu_\beta=\eta^{\mu\rho}h_{\rho\beta},$$
$$h=h^\mu_\mu=\eta^{\mu\rho}h_{\mu\rho},\ h^i_j=h_{ij}=h_{ji}=h^j_i.$$

Equation 1 under weak field approximation linearizes to

$$R_{\mu\nu}=\gamma(T_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}T),$$

where Ricci tensor linearizes to

$$R_{\mu\nu}=\Gamma^\sigma_{\mu\sigma},_\nu-\Gamma^\sigma_{\mu\nu},_\sigma=\frac{1}{2}\eta^{\sigma\rho}(\frac{\partial h_{\rho\mu}}{\partial x^\sigma}+\frac{\partial h_{\rho\sigma}}{\partial x^\mu}-\frac{\partial h_{\mu\sigma}}{\partial x^\rho})_{,\nu}-\frac{1}{2}\eta^{\sigma\rho}(\frac{\partial h_{\rho\mu}}{\partial x^\nu}+\frac{\partial h_{\rho\nu}}{\partial x^\mu}-\frac{\partial h_{\mu\nu}}{\partial x^\rho})_{,\sigma}$$
$$=\frac{1}{2}(\frac{\partial^2h^\sigma_\mu}{\partial x^\sigma\partial x^\nu}+\frac{\partial^2h}{\partial x^\mu\partial x^\nu}-\frac{\partial^2h^\rho_\mu}{\partial x^\rho\partial x^\nu})-\frac{1}{2}\frac{\partial^2h^\sigma_\mu}{\partial x^\nu\partial x^\sigma}-\frac{1}{2}\frac{\partial^2h^\sigma_\nu}{\partial x^\mu\partial x^\sigma}+\frac{1}{2}\eta^{\sigma\rho}\frac{\partial h_{\mu\nu}}{\partial x^\rho\partial x^\sigma}$$

$$=\frac{1}{2}(\frac{\partial^2h}{\partial x^\mu\partial x^\nu}-\frac{\partial^2h^\rho_\mu}{\partial x^\rho\partial x^\nu}-\frac{\partial^2h^\sigma_\nu}{\partial x^\mu\partial x^\sigma})+\frac{1}{2}\eta^{\sigma\rho}\frac{\partial h_{\mu\nu}}{\partial x^\rho\partial x^\sigma}$$

Substitute it into the linearized field equation:

$R_{\mu\nu}=\gamma(T_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}T)$ obtains

$$\frac{1}{2}(\frac{\partial^2 h}{\partial x^\mu \partial x^\nu}-\frac{\partial^2 h^\rho_\mu}{\partial x^\rho \partial x^\nu}-\frac{\partial^2 h^\sigma_\nu}{\partial x^\mu \partial x^\sigma})+\frac{1}{2}\eta^{\sigma\rho}\frac{\partial h_{\mu\nu}}{\partial x^\rho \partial x^\sigma}=\gamma(T_{\mu\nu}-\frac{1}{2}T\eta_{\mu\nu}) \quad (4)$$

For static $U_0=\eta_{0\mu}U^\mu=-1$, $U_j=\eta_{j\lambda}U^\lambda=0$. Pay attention to $T=3p-\rho$ and outside the celestial body $p=-\rho=0$. And from Equation 4 we obtain

$$\nabla^2 h_{00}=\gamma(\rho+3p) \quad (5)$$

Substituting $p=-\rho$ into Equation 5 obtains $\nabla^2 h_{00}=-2\nabla^2\varphi=\gamma(\rho+3p)=-2\gamma\rho$, whose solution is $h_{00}=\frac{\gamma}{2\pi}\int\frac{\rho dV}{|\mathbf{r}-\mathbf{r}'|}$. Note the spherical symmetry, $h_{00}=\frac{\gamma M}{2\pi r}$ outside the source, $M=\int_0^{r_0}\rho dxdydz$ is its mass, $r=\sqrt{x^2+y^2+z^2}$, r_0 is the radius of the source. In order to make $h_{00}=\frac{2GM}{r}$ outside the celestial body, there must be $\gamma=4\pi G$, which is the coupling constant we have determined. In *SI* $\gamma=4\pi G/c^4$.

Note that, $h_{00}=\frac{2GM}{r}$ is the requirement for the geodesic of a low speed moving object to return to Newtonian gravity under weak field approximation. When $h_{00}=\frac{2GM}{r}$ the geodesic equation is $\frac{d^2x^i}{dt^2}=-\Gamma^i_{00}=-\frac{GM}{r^3}x^i$ which is exactly Newtonian gravity. It is precisely this geodesic equation that links the theory of curved spacetime with gravitational phenomena. It can be seen that $h_{00}=-2\varphi$, φ is Newtonian gravitational potential.

Obviously, when $p=0$, Equation 5 provides $\nabla^2 h_{00}=\gamma\rho$, since $h_{00}=-2\varphi$, $-2\nabla^2\varphi=\gamma\rho$, and φ satisfies Poisson's equation $\nabla^2\varphi=4\pi\rho$, by comparison we have $\gamma=-8\pi G$ (in SI $\gamma=-8\pi G/c^4$) which is the previous result and now must be discarded because p is not equal to zero. So far field Equation 1 can be written as:

$$R_{\mu\nu}=4\pi G(T_{\mu\nu}-\frac{1}{2}g_{\mu\nu}T) \quad (6)$$

or $R^\nu_\mu-\frac{1}{2}\delta^\nu_\mu R=4\pi G T^\nu_\mu$ or $R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=4\pi G T_{\mu\nu}$ and so on.

As long as you are familiar with the calculation details rather than just being literal, you will not think that such modification makes gravity become repulsive.

Equation 6 is only used to determine the metric, and what reflects attraction or repulsion is the geodesic equation:

$$\frac{d^2x^k}{dt^2}+\Gamma^k_{00}+\Gamma^k_{0i}\frac{dx^i}{dt}+\Gamma^k_{ij}\frac{dx^i}{dt}\frac{dx^j}{dt}-\Gamma^0_{\mu\nu}\frac{dx^k}{dt}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}=0.$$

Note that in natural unit speed is a small quantity. Obviously, when $x^k>0$, $\Gamma^k_{00}=\frac{GM}{r^3}x^k>0$, $\frac{d^2x^k}{dt^2}<0$, and when $x^k<0$, $\Gamma^k_{00}=\frac{GM}{r^3}x^k<0$, $\frac{d^2x^k}{dt^2}>0$, the acceleration always points towards the center, representing attraction. Note that for the static gravitational field $\Gamma^k_{0i}=0$.

For completeness all the components of $h_{\mu\nu}$ was listed without further deduction. Inside the source $h_{00}=\frac{3GM}{r_0}-\frac{GM}{r_0^3}r^2$ and outside the source $h_{00}=\frac{2GM}{r}$. Time inversion symmetry requires $h_{0i}=0$, and spherical symmetry and constant speed of light require $h_{ij}=\omega(r)\delta^{ij}-\frac{h_{00}+\omega(r)}{r^2}x^ix^j$, inside the source $\omega(r)=-\frac{3GM}{r_0}+\frac{2GM}{3r_0^3}r^2$ and outside the source $\omega(r)=\frac{4GM\ln r_0-7GM/3}{r}-\frac{4GM\ln r}{r}$. Note that, for $i=j$, $\delta^{ij}=1$, and for $i\neq j$, $\delta^{ij}=0$. Readers can verify that they satisfy Equation 4.

APPLICATION OF MODIFIED FIELD EQUATIONS IN COSMOLOGY

The application of field equations with coupling constant $4\pi G$ in cosmology

The correctness of a theory lies not only in its logical consistency but also in its consistency with reality. The distance redshift relationship derived from the field Equation 6 is consistent with the observation height, which strongly demonstrates that the modification is progressive. Now apply Equation 6 to cosmology. In the co-move coordinate system, the spacetime metric to describe the universe is the Robertson-Walker metric:

$$ds^2=g_{\mu\nu}dx^\mu dx^\nu=-dt^2+R^2(t)[\frac{1}{1-kr^2}dr^2+r^2d\theta^2+r^2\sin^2\theta d\varphi^2] \quad (7)$$

Equation 7 provides $g_{00}=-1$, $g_{11}=R^2(t)\frac{1}{1-kr^2}$, $g_{22}=R^2(t)r^2$, $g_{33}=R^2(t)r^2\sin^2\theta$, $g_{\mu\nu}=0$ ($\mu\neq\nu$, and $U^i=0$, $U^0=1$, $T_{00}=\rho$, $T_{ii}=pg_{ii}$, $T_{\mu\nu}=0(\mu\neq\nu)$ substituting these into field Equation 6 yields the following two equations: The first equation is similar to the previous Friedman equation, and the second equation is the same energy conservation equation as before

$$(\frac{dR(t)}{dt})^2+k=-\frac{4\pi G}{3}\rho R^2(t) \tag{8}$$

$$\frac{d\rho}{dt}=-3\frac{dR}{Rdt}(\rho+p) \tag{9}$$

Substituting $p=-\rho$ into Equations 9 yields $p=-\rho=const$, which means that density is independent of whether space expands. This is a universe where matter is continuously generated, which is more testable than the Big Bang model (where all matter is created at the moment of the Big Bang). Hoyle also strongly advocated for the continuous creation of matter and opposed the instantaneous generation of the Big Bang, but mistakenly assumed the number density of galaxies remains unchanged and is negated by observations.

The general solution of Equation 8 is $R(t)=C\sin(t\sqrt{\frac{4\pi G\rho}{3}}+\alpha)$, indicating that the expansion and contraction of the universe cycle infinitely, with no starting point in time. It is advisable to define the start time of the latest round of expansion as $t=0$, and take a negative value for the previous time. So, this solution can be written as:

$$R(t)=C\sin(t\sqrt{\frac{4\pi G\rho}{3}}),$$

where C is the integral constant.

Now lets take a look at the distance redshift relationship given by Equation 8. Assuming that the light is emitted from the celestial body at r_a at time t_a and reaches Earth at time t_0 with a redshift of z, satisfying $z=\frac{R(t_0)}{R(t)}-1$ which is derived in detail in reference [2]. Lets set today's $R(t_0)=1$, so $dz=-\frac{dR}{R^2(t)}=-\frac{dR}{R(t)dt}\frac{dt}{R(t)}=-H(t)\frac{dt}{R(t)}$, that is, $-\frac{dz}{H(t)}=\frac{dt}{R(t)}$, here $H(t)\equiv\frac{dR(t)}{R(t)dt}$, write $q\equiv\frac{4\pi G\rho}{3H^2(t)}$, today's $q_0\equiv\frac{4\pi G\rho}{3H_0{}^2}$, $H_0\equiv H(t_0)$. From Equation 8, we have $q=-R\frac{d^2R}{dt^2}/(\frac{dR}{dt})^2$, which means that the expansion process of the universe is decelerating while the contraction process is accelerating. Take care of $R(t_0)=1$ and $\frac{1}{R(t)}=z+1$, from Equation 8 we have $k=-H_0{}^2(1+q_0)$, $H(t)=H_0\sqrt{(1+q_0)(1+z)^2-q_0}$. The constant $k<0$ indicates that space is open or infinite. For rays moving radially $ds^2=0$, from Equation 7 we have $\frac{dt}{R(t)}=-\frac{dr}{\sqrt{1-kr^2}}$. So, we have

$$\int_0^z\frac{dz}{H}=\int_0^z\frac{dz}{H_0\sqrt{(1+q_0)(1+z)^2-q_0}}=\int_{t_a}^{t_0}\frac{dt}{R}=\int_0^{r_a}\frac{dr}{\sqrt{1-kr^2}} \tag{10}$$

by integrating z with Equation 10, we obtain the relationship between distance and redshift

$$H_0d_L=\frac{z+1}{\sqrt{1+q_0}}\ln\frac{(z+1)\sqrt{1+q_0}+\sqrt{(q_0+1)(1+z)^2-q_0}}{1+\sqrt{1+q_0}} \tag{11}$$

Here, the luminosity distance formula of Weinberg (1972), $d_L=(1+z)\int_0^{r_a}\frac{dr}{\sqrt{1-kr^2}}$ is used. In z=0 neighborhood, expanding the logarithm in Equation 11 into Taylor's series we can obtain $H_0d_L=z+\frac{1-q_0}{2}z^2+\frac{3q_0{}^2-2q_0-1}{6}z^3+\cdots$, to keep the first term on the right, it is the usual Hubble's law.

The following Figure 1 is a Hubble diagram of the data of distance and redshift observed from Ia supernova (Dai and Liang, 2004; Wang et al., 2009). The curved line in the figure is exactly the image of the function $d_L=d_L(z)$ defined by Equation 11, which fits well with the observed data, indicates that the universe is still decelerating, that is $\frac{d^2R}{dt^2}<0$. According to the observations of Weinberg (1972), the density of the universe $\rho=3.1\times10^{-28}kg/m^3$, so here we take $q_0=\frac{4\pi G\rho}{3H_0{}^2}=0.0147$, in which $H_0=75km.s^{-1}.Mpc^{-1}$ is the today's values of the Hubble parameter, and Distance-Modulus $=5\lg d_L+25$, and the unit of d_L is Mpc, $1Mpc=3.1\times10^{19}km$

By the way, it should be noted that the new distance redshift relationship fits well with the observed data and

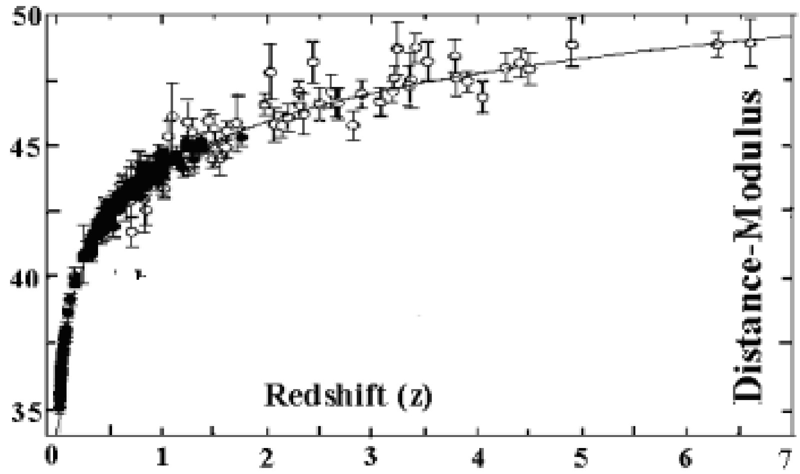

Figure 1. the recent Hubble diagram of 69 GRBs and 192 SNe Ia

does not introduce the so-called dark energy, indicating that dark energy is the binding energy of matter, which appears in the field equation in the form of negative pressure rather than independent existence. So, does dark matter necessarily exist? The author believes that it may not necessarily exist either. After considering the mass of the galactic halo and the mass of the galactic disk itself, the rotational velocity curve of the material on the spiral arm of a disc-shaped galaxy can be explained without introducing dark matter (Nan et al., 2008). The galactic core is treated as a rigid body, and its rotational speed is not controlled by gravity. The calculated velocity on the disk is $v=\sqrt{\frac{a}{r}+br^2}$, r is the distance from the material on the disk to the center, and a and b are two positive parameters, which fit well with observations and do not require the introduction of dark matter. In fact, the sun travels through space with the Earth, and if there are really so-called dark matter and dark energy sitting on Earth, it can be detected without launching any instruments to explore in space.

Returning to the topic, using $H=\frac{dR}{Rdt}=2\sqrt{\frac{\pi G\rho}{3}}ctg\left(2t\sqrt{\frac{\pi G\rho}{3}}\right)$, we can obtain the time from the beginning of the latest round of expansion to today, which is the age of our universe, $t_0=\frac{tg^{-1}\sqrt{q_0}}{H_0\sqrt{q_0}}$ = $1.37\times10^{10}\,year$, the same result as the Big Bang.

The proper distance of a celestial body is $d_p=R(t)\int_0^r\frac{dr}{\sqrt{1-kr^2}}$. When $R(t)=0$ the volume of the universe is zero and everything disappears. This is the end of the previous contraction and the beginning of the next expansion. The information from the previous cycle cannot be seen by the observer in the next cycle. By direct calculation, it can be inferred that the particle horizon at any moment after the start of the latest round of expansion is

$$l_H(t)\equiv R(t)\int_0^t\frac{dt}{R(t)}=\sin(t\sqrt{\frac{4\pi G\rho}{3}})\int_0^t\frac{dt}{\sin(t\sqrt{4\pi G\rho/3})}=\infty \quad (12)$$

Integral divergence indicates that our universe always appears infinite, and the difficulty of horizon or uniformity is automatically eliminated. Note that although the volume of the universe is zero at time $R(t)=0$, this state is unobservable because any observation is always completed within a certain time period. The so-called quasi flatness problem no longer exists. The time from expansion to contraction is a quarter of the period of the sine function of the scaling factor, that is $t_m=\frac{1}{4}\cdot\frac{2\pi}{\omega}=\frac{1}{4}\sqrt{\frac{3\pi}{G\rho}}=1.6\times10^{11}$ years and is greater than the age of the universe today. Therefore, the universe is currently in the expansion stage, so is the observation.

At first glance, the continuous generation of matter violates the conservation of energy, but in fact, this is not the case because Equation 9 represents exactly the conservation of energy. By changing Equation 9 to the following equation $d(\rho R^3)=-pdR^3=\rho dR^3$, it can be seen that the continuous generation of matter is the work done by the expansion force in space, and the energy lost in space is this negative pressure. However, there is no corresponding mechanism for the instantaneous generation of matter at the Big Bang singularity. So, this theory of continuous creation is more scientific.

Continuous generation of matter and gradual growth of galaxies

The expansion and contraction of the universe are cyclical, and the contraction process is the reverse of expansion. During the expansion stage, matter gradually forms, while during the contraction stage, matter gradually disappear. Currently, it is the expansion stage, which is also the stage of gradual generation of matter. This means that matter and space are being generated simultaneously. This article only elaborates on the observable behavior during the expansion stage, while the behavior during the contraction stage is treated as the inverse process of the expansion process, which we cannot see.

Due to the zero pressure and density outside the celestial body, actual material generation can only occur within the interior of a celestial body or galaxy. This means that the celestial bodies or galaxies grow with the expansion of the universe, and the celestial bodies or galaxies undergo equal density expansion, the mass $M \propto R^3(t)$, that is $dM = 3MHdt$, and the radius $r \propto R(t)$, that is $dr = Hrdt$, for example, the current mass and radius of Earth increase by 1.2 trillion tons and 0.5 mm respectively a year, and the radius of the disk of the Milky Way increases by 0.00005 light years a year. In a word, universe is alive, and currently galaxies or celestial bodies are synchronously growing with the expansion of space, and stars become brighter instead of darker. Of course, contraction is the inverse process of expansion.

The reason why the spiral arms of a galaxy are not intertwined is that while they rotate, they also expand outward (Melissa and Dustin 2016; Cristina et al., 2017). The period of rotation or motion around the center remains unchanged, but only the rotational speed changes, as seen in a magnifying glass, where everything is magnified proportionally. Obviously, if the spiral arms did not expand outward, they would already be intertwined, and the shape of the galaxy would have been disrupted.

The conclusion is that there is formation of galaxies and celestial bodies from gradually growing, rather than the aggregation of existing matter, and new matter is gradually generated, and this process is still ongoing. As for the microscopic generation mechanism of matter, which can be imagined as the splitting of neutrons, it needs to be tested, and this is another issue that will not be discussed here.

In short, by modifying this coupling constant, existing cosmological problems no longer exist such as the difficulty of the event horizon, the singularity of the Big Bang, the asymmetry of positive and negative matter, the difficulty of dark matter and dark energy, etc (Melissa and Dustin, 2016; Yu and Shi, 2023). In fact, dark matter and dark energy are the negative pressure here, which is the interaction energy between fundamental particles inside matter and cannot exist independently of matter. This is consistent with the fact that celestial bodies composed of dark matter alone have not been observed in daily life. Readers can refer to Jian's (2019) work for the exact solution

CONCLUSIONS

The negative pressure as a gravitational source represents the total binding energy inside the material, and the coupling constant of the gravitational field equation should be modified from the original -8 π G to the current 4 π G The matter in the universe is continuously generated, and there is no Big Bang singularity with infinite density and temperature. Dark matter dark energy is the binding energy within the matter rather than an independent existence. The evolution of ancient and modern times follows the same law, no longer distinguishing between radiation dominated and matter dominated stages. The expansion and contraction of the universe cycle back and forth, and is currently in the stage of expansion, The celestial body gradually grows and becomes brighter, with higher temperatures. The radius changes over time to satisfy $r \propto R(t)$, while the mass changes over time to satisfy $M \propto R^3(t)$ Using the mass-light relationship $L \propto M^4$, the temperature T and apparent brightness l_p of the same celestial body satisfy the following relationship $L = 4\pi r_e^{\,2} \cdot \sigma T^4 = 4\pi r_p^{\,2} \cdot l_p \propto R^{12}(t)$, where r_e and r_p are respectively the radius of the celestial body and the distance to Earth, both proportional to $R(t)$, celestial body temperature $T \propto R^{5/2}(t)$, and apparent brightness $l_p \propto R^{10}(t)$.

CONFLICT OF INTERESTS

The author has not declared any conflict of interests

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