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Full Length Research Paper

Rule based fuzzy logic (FL) time series prediction

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The singleton and non-singleton type-1 back propagation (BP) designed sixteen rule fuzzy logic system (FLS) on hourly averaged wind data for the years 1985 to 2004 are studied. The BP designed 16 rule non-singleton-type-1 FLS was found relatively a better forecaster than singleton-type-1. There are too many hidden or unraveled uncertainties, such as non-stationarity and stable attractors. These uncertainties make the data chaotic. Non-stationarity in the data can be properly handled with non-singleton type-1 FLS, therefore, there appears no reason to use a type-2 FLS. The stable attractors and non-stationarity in our data do not affect the predicted values as confirmed by Mackey Glass simulation. Parallel structure fuzzy systems and genetic logic may be one of the options to resolve sub crisps and chaos in time series data.

Key words: Back propagation, fuzzy logic system, singleton and non-singleton type1-FLS, cascade correlation algorithm, hybridization of intelligent systems with fuzzy logic, stable attractors.

INTRODUCTION

Rule based fuzzy logic systems (FLS), a powerful design methodology, minimize the effect of uncertainty (Mendel, 2001). The two most popular FLSs used by engineers today are the Mamdani and Takagi-Sugano-Kang (TSK) systems. Both are characterized by IF-Then rules and have the same antecedent structures. They differ in the structure of the consequents. The consequent of a Mamdani rule is a fuzzy set, whereas the consequent of a TSK rule is a function. The type-1 TSK FLSs have been widely used in control and other applications (Terano et al., 1994). The output of type-1 TSK forecaster occurs without a defuzzification step (Liang and Mendel, 1999, 2000) developed type-2 TSK FLSs. The FLS forecasters comprise of singleton type-1 (with virtually no uncertainties), non-singleton type-1 (with uncertainties), singleton type-2, type-1 non-singleton type-2, type-2 non-singleton type-2, type-1 TSK and type-2 TSK (Mendel, 2001). The rule based FLSs, both type-1 and type-2, handle uncertainties because modeling and minimization of uncertainties can be accomplished. If all uncertainties disappear, type-2 FL reduces to type-1 FL. In the same vein, if randomness disappears, probability reduces to determinism.

For basic singleton type-1 FLSs, we assume that there are no uncertainties; all fuzzy sets are of type-1, measurements are perfect and treated as crisp values, that is, as singletons. Thus, the non- singleton FLS do not yield crisp values, that is, uncertainties are inherently present. A FLS that is described completely in terms of type-1 fuzzy sets is called a type-1 FLS. Type-1 FLSs are unable to directly handle rule uncertainties, because they use type-1 fuzzy sets that are certain. Therefore, a better way to handle uncertainties is to use a type-2 FLS. But, a non-singleton type-1 FLS is a type-1 FLS whose inputs are modeled as type-1 fuzzy numbers; hence, it can be used to handle uncertainties. Moreover, the type-1 FL, in its applications, deciphers rule based systems as a powerful design methodology.

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The rules of a non singleton-type-1 FLS are the same as those for a singleton type-1 FLS (Mendel, 2001). The difference is of the fuzzifier, which treats the inputs, as type-1 fuzzy sets, and the effect of this on the inference block. The output of the inference block will again be a type-1 fuzzy set. So, the defuzzifiers that are described for a singleton type-1 FLS apply as well to a non-singleton type-1 FLS (Mendel, 2001).

We know that non-stationarity (randomness) in our wind data inherently exists (Jafri, 2008; Kamal and Jafri, 1996); therefore, uncertainties or randomness cannot be reduced. It can be handled properly with non-singleton type-1 FLS, therefore, there appears no reason to use a type-2 FLS. We recently performed fuzzy logic (FL) time series prediction modeling on hourly averaged wind speed (HAWS) data of 1985 to 2004 and used Mackey-Glass simulation, for Quetta, Pakistan (Jafri and Kamal, 2010). Quetta (30°11'/N, longitude 66° 57'/E), the capital of Balochistan is elevated at 1799 m above sea level. We shall use the results of wind data with the applications of rule based type-1 FLS. We used the MATLAB M-files which are available as freeware on the internet at the following: URL:http://sipi.usc.edu/~mendle/software. The M-files are available in three folders: type-1 FLS, general type-2 FLSs and interval type-2 FLSs. We used in this study, the following type-1FLSs:

1) Singleton Mamdani type-1 FLS sfls_type1.m: Compute the output(s) of a singleton type-1 FLS when the antecedent membership functions are Gaussian train_fls_type.1.m: tune the parameters of a singleton type-1 FLS when the antecedent membership functions are Gaussian using some input-output training data.
2) Non-singleton Mamdani type-1 FLS nsfls_type1.m: Compute the output(s) of a non-singleton type-1 FLS when the antecedent membership functions are Gaussian and the input sets are Gaussian train_nsfls_type1.m: tune the parameters of a non-singleton type-1 FLS when the antecedent membership functions are Gaussian, using some input-output training data.

The extraneous matter is avoided on the development and historical background of rule-based FLSs because we are concerned only with the use of FLSs in time series. The exhaustive literature and indeed critical review on rule-based FLSs are available in the form of a book (Mendel, 2001) However, we shall deliberate on fundamental rules extracted from the data under consideration (Jafri, 2008; Jafri et al., 2012).

The rules in fuzzy logic time-series are usually extracted from designing the FLSs. Prior to 1992, all FLSs reported in the open literature fixed the parameters, such as the type of fuzzification, composition, implication, t-norm (operators for fuzzy intersection), defuzzification (produces crisp output) and membership functions, arbitrarily, e.g. the locations and spreads of the membership functions were chosen by the designer independent of the numerical training data. Then, at the first IEEE conference in fuzzy systems, held in San Diago in 1992, three different groups of researchers (Horikawa et al., 1992; Jang, 1992; Wang and Mendel, 1992) presented the same idea: tune the parameters of a FLS using the numerical training data. Since that time, quite a few adaptive training procedures have been published. Because tuning of free parameters had been in feed forward neural network (FFNN) long before it was done in a FLS, a tuned FLS has also come to be known as a neural fuzzy system.

Designing a FLS (Mendel and Mouzouris, 1997) can be viewed as approximating a function or fitting a complex surface in a multidimensional space. Given a set of input-output pairs, tuning is essentially equivalent to determining a system that provides an optimal fit to input-output pairs, with respect to a cost function (tuning algorithm). Utilizing concepts from real analysis (Monzouris and Mendel, 1997) have proven that a non-singleton FLS can uniformly approximate any continuous function on a compact set. Although, the proof of approximation (Monzouris and Mendel, 1997) provides some insight, it does not tell us how to choose the parameters of the non-singleton FLS, nor does it tell us how many basis functions will be needed to achieve such performance. The latter are accomplished through design. The designing of FLSs require one-pass (OP), least square, back-propagation (BP, steepest descent), SVD-QR (SVD-QR is a matrix tool in numerical linear algebra used in signal processing, extracting fuzzy rules, reducing fuzzy rules and modeling the fuzzy rules) and iterative design methods. More and Deo (2003) employ the technique of neural networks to forecast daily, weekly and monthly wind speed. Both feed forward (FF) as well as recurrent networks (RN) are used and trained on past data in the autoregressive (AR) manner using BP and cascade correlation (CC) algorithm. They conclude that the CC algorithm yields better forecasts as compared to that of BP.

The forecasting of time-series following the rule-based FLSs designing employ only two methods, that is, one pass (OP) and BP methods, respectively. The OP design constructs 500 rules for each antecedent consequent membership functions. We set the value of the standard deviation equal to 0.1 for all Gaussian in a pre-defined OP design. But, the OP is exhaustive as compared to BP designing in FLSs. On the contrary, the BP constructs only 16 rules for each antecedent and consequent membership functions. The initial values of the standard deviation of Gaussian membership function are all set equal to 0.5240 in a pre-defined BP design. The BP designing, in many respects, is better than OP (Mendel, 2001). The predefined values of all four antecedent membership functions and for the centers of the consequent membership functions ($\bar{y}^l$-height defuzzifier) for each corresponding 16 rules in a BP design for FLSs are used in the form of a matrix as an input. We use the height defuzzifier ($\bar{y}^l$ or centers of the consequent membership functions) to be a random number from the interval (0, 1). After training and using BP design, the FLS forecaster was fixed.

MATERIALS AND METHODS

We use the learning parameter α = 0.2 in BP design. With tractable learning laws, we set the learning parameters. Alpha stable statistics model the impulsiveness as a parameterized family of probability density functions. Additive fuzzy systems can filter impulsive noise from signals. We used artificial neural fuzzy information system (ANFIS) to filter out infinite variances of noise in time series data. With α < 2 one gets impulsive noise and noise has infinite variance. The alpha in statistics is an exponent parameter. With α=2, we get the classical Gaussian case, that is, exponential tail and finite variance.

The predefined initial mean (center) values of antecedent membership functions along with height defuzzifiers (mean values of consequent membership functions) and the standard deviations of the Gaussian antecedent, in the form of matrix membership functions, as shown in Tables 1 and 2, are used for determining the values of singleton consequent membership functions, that is, $f_s(s^k)$ for hourly 600 trainee wind data and 120 or 144 (1/6 to 1/5th of the hourly averaged data of the month) testing wind data, respectively. The predefined final mean (center) values of antecedent membership functions along with height defuzzifiers (mean values of the consequent membership functions) and the standard deviations of the Gaussian antecedent membership functions, in the form of a matrix, after six epochs of training, as shown in Tables 2 and 3, are used for determining the values of non-singleton consequent member functions, that is, $f_{ns}(s^k)$, for hourly 600 trainee data and 120 or 144 testing data, respectively. In both cases, 600 trainee wind data and 120 or 144 testing data for all four antecedent membership functions are used as an input matrix, X, in sfls_type1.m and nsfls_type1.m, respectively. For trainee as well as for testing data, we calculated the predicted values (Jafri, 2008; Jafri et al., 2012). It is difficult to reproduce all predicted values and the values of consequent membership functions for singleton and non-singleton type-1 FLSs in this manuscript. Therefore, we compared root mean square error, that is, $RSME_s$ (BP) with $RSME_{ns}$ (BP) only for testing data (Jafri, 2008) and found for the non singleton type 1 back propagation designed sixteen rule FLS, better than singleton-type 1 (BP).

It is worth mentioning that trainee pairs are obtained with testing data, therefore, the analysis of testing data will be the same for trainee data. We input predefined initial mean values of all antecedent membership functions (Table 1) in case of a singleton type-1 FLS, because we assume that there are no uncertainties in the data. But, we cannot totally ignore the noisy measurement environment; therefore, we tested our final FLS forecasters on noisy testing data, that is,

$$x(k) = s(k) + n(k) \qquad (1)$$

where n (K) is O dB (decibel) uniformly distributed noise.

We accomplished this task for a Monte Carlo set of 60 realizations. This entire process was repeated 60 times using 60 independent sets of mean and standard deviation of 720 or 744 hourly wind data. The predefined BP $RMSE_s$ (BP) (Chu and Mendel, 1994) for each of the six epochs of tuning is:

$$RMSE_s\ (BP) = \{0.0548, 0.0431, 0.0322, 0.0261, 0.0237, 0.0232\} \qquad (2)$$

The non-singleton FLS shares most of the same parameters as the singleton FLS. So, we shall use the partially dependent BP design approach. In BP design, we use only two fuzzy sets for each of the four antecedents such that there are only 16 rules. Each rule is characterized by eight antecedent membership function parameters (the mean and standard deviation for each of the four Gaussian membership functions) and one consequent parameter, $\bar{y}$. More specifically, we initially chose the mean of each and every antecedents, two Gaussian membership functions as $m_x - 2\sigma_x$ or $m_x + 2\sigma_x$, respectively, and the standard deviations of these membership functions as $2\sigma_x$.

For the non-singleton type-1 FLS, we modeled each of the four noisy input measurements using a Gaussian membership function. Two choices are possible: (1) use a different standard deviation for each of the four input measurement membership functions, or (2) use the same standard deviation for each of the four input measurement membership functions. We tried both approaches and got similar results because the additive noise n(k) is stationary. The predefined average values and standard deviations of $RMSE_s$ (BP) and $RMSE_{ns}$ (BP), for each of the 6 epoch, mentioned by Mendel 2001) are used.

Theory

We consider a type-1 FLS having p inputs:

$x_1 \in X_1, \ldots\ldots\ldots\ldots x_p \in X_p$ and one output $y \in Y$.

Let suppose that it has M rules, where the lth rule has the form:

R^l: IF x_1 is F_1^l and……and x_p is F_p^l,

Then y is G^l; l=1,……M (3)

Equation 3 represents a type-1 fuzzy relation between the input space $X_1, X_2, \ldots, X_p$ and the output space, Y, of the FLS. F stands for fuzzy sets of antecedents and G for fuzzy sets of the consequents. A multiple-antecedent multiple consequent rule can always be considered as a group of multi-input single-output rules. Equation 3 describes the generic rule structure which comprises of six rules. The first five rules, such as, incomplete IF rules, mixed rules, fuzzy statement rules, comparative rules and unless rules were deciphered (Wang, 1994). The sixth is the quantifier rule. We are not concerned with the details of the rules. The main objective is to rephrase the BP design analysis in its theoretical form.

Suppose we are given a collection of N input-output numerical data training pairs $(x^{(1)}:y^{(1)}), (x^{(2)}:y^{(2)}) \ldots (x^{(N)},y^{(N)})$, where x is the vector input and y is the scalar output of a FLS. To begin, we must know how the training data can be interpreted as a collection of IF-Then rules. Each rule is governed by Equation 3, where F_1^l are fuzzy sets described by Gaussian membership functions, that is,

$$\mu_{F_i^l}(x_i) = \exp\left\{-\frac{1}{2}\left[\frac{(x_i - m_{F_i^l})}{\sigma_{F_i^l}}\right]^2\right\} \qquad (4)$$

where i=1,…..p and l=1,…..M. Each design method establishes how to specify the parameters $m_{F_i^l}$ and $\sigma_{F_i^l}$ of these membership

Table 1. Initial values for the centers of the Gaussian antecedent membership functions and the centroid of the consequent set.

Rule number	Initial value for centers of the four antecedent membership functions				Initial value for y^{-l}
1	0.3793	0.3793	0.3793	0.3793	0.5314
2	0.3793	0.3793	0.3793	1.4272	0.3831
3	0.3793	0.3793	1.4272	0.3793	0.0159
4	0.3793	0.3793	1.4272	1.4272	0.8181
5	0.3793	1.4272	0.3793	0.3793	0.6931
6	0.3793	1.4272	1.4272	0.3793	0.1209
7	0.3793	1.4272	1.4272	0.3793	0.4647
8	0.3793	1.4272	1.4272	1.4272	0.9975
9	1.4272	0.3793	0.3793	0.3793	0.9522
10	1.4272	0.3793	0.3793	1.4272	0.6991
11	1.4272	0.3793	1.4272	0.3793	0.2673
12	1.4272	0.3793	1.4272	1.4272	0.7625
13	1.4272	1.4272	0.3793	0.3793	0.6460
14	1.4272	1.4272	0.3793	1.4272	0.6483
15	1.4272	1.4272	1.4272	0.3793	0.3793
16	1.4272	1.4272	1.4272	1.4272	0.8687

$m_2 - 2\sigma_s = 0.3793$ and $m_s + 2\ \sigma_s = 1.4272$.

Table 2. Final values for the centers of the Gaussian antecedent membership functions and the centroid of the consequent set, after six epoch of tuning.

Rule number	Final value for centers of the four antecedent membership functions				Initial value for y^{-l}
1	0.4001	0.3613	0.3076	0.1694	0.4986
2	0.3075	0.2707	0.1988	1.5524	0.3860
3	0.4273	0.3821	1.3487	0.2121	0.0035
4	0.2586	0.3205	1.3205	1.3434	0.8631
5	0.3451	1.5229	0.3352	0.3297	0.6929
6	0.2942	1.5375	0.2316	1.4938	0.1211
7	0.3473	1.4700	1.4523	0.2704	0.4622
8	0.5727	1.1876	1.2624	1.3675	1.2695
9	1.5721	0.3604	0.3790	0.3960	0.9506
10	1.4782	0.2994	0.2817	1.4598	0.7021
11	1.4265	0.4093	1.3689	0.3367	0.2556
12	1.4560	0.2404	1.4518	1.4497	0.7644
13	1.4648	1.4641	0.2593	1.4445	0.6486
14	1.4748	1.4641	0.2593	1.4445	0.6486
15	1.4555	1.4210	1.3917	0.3730	0.3868
16	1.3964	1.4352	1.4933	1.6955	0.8715

functions, as well as the centers of the consequent membership functions:

$$y(x) = f_s(x) = \sum_{l=1}^{M} \bar{y}\phi_l(x) \tag{5}$$

where $\phi_l(x)$ is known as fuzzy basis function (FBF) (Wang, 1994).

$$\phi_l(x) = \frac{\prod_{i=1}^{p} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{M} \prod_{i=1}^{p} \mu_{F_i^l}(x_i)}, \quad i=1,\ldots..M \tag{6}$$

Using the training pairs and with tuning, we abide by the commonly used design principle, that is, the number of rules M<N, that is, the number of input-output numerical data training pairs. In BP (steepest descent) design, none of the antecedent or consequent Parameters are fixed ahead of time. They are all tuned using a

Table 3. Final values for the standard deviations of the Gaussian antecedent membership functions after six epoch of tuning.

Rule number	Final values of the standard deviations			
1	0.5649	0.5224	0.4531	0.2268
2	0.4646	0.4094	0.2630	0.3728
3	0.5403	0.5075	0.7142	0.1399
4	0.3043	0.4044	0.6618	0.5931
5	0.5109	0.3561	0.4589	0.4267
6	0.4224	0.3640	0.2616	0.4736
7	0.4987	0.4718	0.5109	0.2772
8	0.6775	0.7240	0.6280	0.3527
9	0.2497	0.5214	0.5362	0.5489
10	0.4766	0.4106	0.3519	0.4872
11	0.4682	0.5493	0.6171	0.2966
12	0.5154	0.2404	0.5106	0.5082
13	0.4408	0.4884	0.5000	0.4111
14	0.4754	0.4926	0.2648	0.5216
15	0.4618	0.5299	0.5777	0.3553
16	0.5861	0.5408	0.4678	0.1198

steepest descent method. Using Equation 6 in Equation 5, we have:

$$y(x^{(i)}) = f_s(x^{(i)}) = \sum_{l=1}^{M} \bar{y}\phi_l(x^{(i)})$$

$$\frac{\sum_{l=1}^{M} \bar{y}^l \prod_{k=1}^{p} \exp\left\{-\frac{(x_k^{(i)} - m_{F_k^l})^2}{2\sigma_{F_k^l}^{\ 2}}\right\}}{\sum_{l=1}^{M} \prod_{k=1}^{p} \exp\left\{-\frac{(x_k^{(i)} - m_{F_k^l})^2}{2\sigma_{F_k^l}^{\ 2}}\right\}}$$

$l = 1, \ldots\ldots N$ (7)

Given an input-output training pair $(x^{(i)}:y^{(i)})$, we design the FLS in Equation 7 such that the following error function is minimized.

$$e^i = \frac{1}{2}[f_s(x^i) - y^{(i)}]^2,\ i = 1, \ldots N \qquad (8)$$

It is evident from Equation 7 that f_s is completely characterized by

$\bar{y}^l$, $m_{F_k^l}$ and $\sigma_{F_k^l}$ (l=1,….M and k=1,……p).

Using the steepest descent algorithm to minimize $e^{(i)}$, it is straight forward to obtain the following recursions to update all the design parameters of this FLS (k=1…..,p, l=1, ….M and i=0,1,….):

$$m_{F_k^l}(i+1) = m_{F_k^l}(i) - \alpha_m [f_s(x^{(i)}) - y^{(i)}][\bar{y}^l(i) - f_s(x^{(i)})]\left[\frac{x_k^i - m_{F_k^{l(i)}}}{\sigma_{F_k^l}^{\ 2}(i)}\right]\phi_l(x^{(i)}) \qquad (9)$$

$$\bar{y}^l(i+1) = \bar{y}^l(i) - \alpha_{\bar{y}}[f_s(x^{(i)}) - y^{(i)}]\phi_l(x^{(i)}) \qquad (10)$$

and

$$\sigma_{F_k^l}(i+1) = \sigma_{F_k^l}(i) - \alpha_\sigma [f_s(x^{(i)}) - y^{(i)}][\bar{y}^l(i) - f_s(x^{(i)})]\left[\frac{(x_k^i - m_{F_k^{l(i)}})^2}{\sigma_{F_k^l}^{\ 3}(i)}\right]\phi_l(x^{(i)}) \qquad (11)$$

In Equation 11, we update $\sigma_{F_k^l}$ instead of $\sigma_{F_k^l}^{\ 2}$, because $\sigma_{F_k^l}^{\ 2}$ must be positive (that is, $\sigma_{F_k^l}^{\ 2}$ is constrained) whereas $\sigma_{F_k^l}$ can be positive or negative (that is, $\sigma_{F_k^l}$ is unconstrained). We then square up $\sigma_{F_k^l}$ to obtain $\sigma_{F_k^l}^{\ 2}$. Values for $m_{F_k^l}(0)$, $\bar{y}^l(0)$ and $\sigma_{F_k^l}(0)$ must be provided to initialize Equations 9 to 11. To make these equations application dependent, we used predefined initial untrained values of mean, final trained values of mean for six epochs, final trained values of standard deviation for six epochs and the centers of the values of the consequents, that is, $\bar{y}^l$ or height defuzzifier (Tables 1, 2 and 3). The learning parameters α_m, α_y and α_σ must be chosen with care. Frequently, they are chosen to be the same, say α. The choosing of learning parameters for an algorithm to converge much faster was studied by Chu and Mendel (1994). Wang (1992) was the first to show that the FLS described by Equations 4 and 5 could also be viewed as a layered architecture, one with three layers. Equations 9 to 11 are therefore referred to as a back-propagation algorithm, because of their dependence on error $f_s(x)^{(i)}-y^{(i)}$, which propagates from the output layer of the FLS down into lower layers. A drawback in BP design is the choice for selecting the number of feedback fuzzies (FBF_s) and M. The SVD-QR method can resolve this drawback (Mendel, 2001). With comparison of a non-singleton type-1 FLS with a singleton type-1 FLS using BP design, we found out that the non-singleton FLS shares most of the parameters same as the singleton FLS. Therefore, we shall use the partially dependent design approach. We assume that all the antecedent, consequent, or input measurement membership function's parameters are to be tuned. Many of the results described in the aforementioned paragraph are similar but the associated equations in non-singleton type-1 FLS for BP design are somewhat different. Hence, we briefly describe the tuning method for a non-singleton type-1 FLS for BP design because the training pairs are noisy. The FLS, for a non-singleton type-1 is represented as:

$$y(x) = f_{ns}(x) = \sum_{k=1}^{M} \phi_l(x) \quad (12)$$

where $\phi_l(x)$ is FBF, given as:

$$\phi_l(x) = \frac{\prod_{k=1}^{p} \mu_{Q_k^l}(x_{k,\max}^l)}{\sum_{l=1}^{M} \prod_{k=1}^{p} \mu_{Q_k^l}(x_{k,\max}^l)}, \quad l = 1,\ldots,M \quad (13)$$

$$\mu_{Q_k^l}(x_{k,\max}^l) = \exp\left\{-\frac{1}{2}\frac{(x_k' - m_{F_s^l})^2}{\sigma_x^2 + \sigma_{F_k^l}^{\ 2}}\right\} \quad (14)$$

where k = 1,…p and l =1,….M.

Equation 14 shows that, in the special case of Gaussian membership functions and product t-norm, it is possible to interpret the non-singleton FLS as a singleton FLS. Thus,

$$y(x^{(i)}) = f_{ns}(x^{(i)}) = \frac{\sum_{l=1}^{M} y^{-l} \prod_{k=1}^{p} \mu_{Q_k^l}(x_{k,\max}^{l,(i)})}{\sum_{l=1}^{M} \prod_{k=1}^{p} \mu_{Q_k^l}(x_{k,\max}^{l,(i)})}$$

$$= \frac{\sum_{l=1}^{M} y^{-l} \prod_{k=1}^{p} \exp\left\{-\frac{1}{2}\frac{(x_k^{(i)} - m_{F_s^l})^2}{\sigma_x^2 + \sigma_{F_k^l}^{\ 2}}\right\}}{\sum_{l=1}^{M} \prod_{k=1}^{p} \exp\left\{-\frac{1}{2}\frac{(x_k^{(i)} - m_{F_s^l})^2}{\sigma_x^2 + \sigma_{F_k^l}^{\ 2}}\right\}} \quad (15)$$

where i=1,…N. We wish to design the FLS in Equation 15 such that the following error funcion is minimized:

$$e^{(i)} = \frac{1}{2}[f_{ns}(x^{(i)}) - y^{(i)}]^2, \quad i=1,\ldots\ldots,N \quad (16)$$

It is evident from Equation 17 that f_{ns} is completely characterized by $\bar{y}^l$, $m_{F_k^{l(i)}}$, $\sigma_{F_k^l}$ and σx (l=1,…M and k=1,….,p). Using a steepest descent algorithm to minimize $e^{(i)}$, it is straightforward to obtain the following recursions to update all the design parameters of this FLS (k=1,….,p, l=1,2,…M and i=0,1,…).

$$m_{F_k^l}(i+1) = m_{F_k^l}(i) - \alpha\,[f_{ns}(x^{(i)}) - y^{(i)}][\bar{y}^l(i) - f_{ns}(x^{(i)})]\left[\frac{x_k^i - m_{F_k^{l(i)}}}{\sigma_x^2(i) + \sigma_{F_k^l}^{\ 2}(i)}\right]\phi_l(x^{(i)}) \quad (17)$$

$$\bar{y}^l(i+1) = \bar{y}^l(i) - \alpha_{\bar{y}}\,[f_{ns}(x^{(i)} - y^{(i)}]\,\phi_l\,(x^{(i)}) \quad (18)$$

$$\sigma_{F_k^l}(i+1) = \sigma_{F_k^l}(i) - \alpha_{\sigma}[f_{ns}(x^{(i)}) - y^{(i)}][\bar{y}^l(i) - f_{ns}(x^{(i)})]\,\sigma_{F_k^l}^{(i)}\left[\frac{(x_k^i - m_{F_k^{l(i)}})}{\sigma_x^2(i) + \sigma_{F_k^l}^{\ 2}(i)}\right]^2\phi_l(x^{(i)}) \quad (19)$$

and

$$\sigma_x(i+1) = \sigma_x(i) - \alpha_x[f_{ns}(x^{(i)}) - y^{(i)}][\bar{y}^l(i) - f_{ns}(x^{(i)})]\,\sigma_x^{(i)}\left[\frac{x_k^i - m_{F_k^{l(i)}}}{\sigma_x^2(i) + \sigma_{F_k^l}^{\ 2}(i)}\right]^2\phi_l(x^{(i)}) \quad (20)$$

We update $\sigma_{F_k^l}$ and σ_x instead of $\sigma_{F_k^l}^{\ 2}$ and $\sigma_x^{\ 2}$, because $\sigma_{F_k^l}^{\ 2}$ and $\sigma_x^{\ 2}$ must be positive values whereas $\sigma_{F_k^l}$ and σ_x can be positive or negative. We then square up $\sigma_{F_k^l}$ and σ_x to obtain $\sigma_{F_k^l}^{\ 2}$. Equations 17 to 20 need to be initiated by $m_{F_k^{l(i)}}(0)$, $\bar{y}^l(0)$, $\sigma_{F_k^l}(0)$ and $\sigma_x(0)$. The choosing of α in a non-singleton type-1 FLS is the same as in a singleton type-1 FLS. If $x_{k,\max}^l$ cannot be computed due to non-availability of specific choice for membership functions, than a different kind of optimization algorithm, that is, random search algorithm must be used to minimize $e^{(i)}$ (Mendel, 2001).

RESULTS AND DISCUSSION

We considered the month of March (Spring Season; February to April) of Quetta (Pakistan), while the results of the remaining months of the years 1985 to 2004 are mentioned elsewhere (Jafri, 2008).

Table 4 shows the unpredicted six hundred hourly averaged values of wind data of March, 1985 to 2004 (data read column-wise). Table 5 shows the predicted

Table 4. Unpredicted hourly averaged wind speed (m/s) values of March, 1985 to 2004.

3.0864	1.0288	0.0000	4.1152	9.2592	8.2304	3.0864	7.2016	17.4800	0.0000	2.0576	0.0000
3.0864	1.0288	0.0000	4.1152	9.2592	7.2016	2.0576	7.2016	17.4800	1.0288	1.0288	0.0000
3.0864	1.0288	1.0288	3.0864	8.2304	6.1728	3.0864	6.1728	16.4600	1.0288	1.0288	0.0000
3.0864	3.0864	5.1440	4.1152	5.1440	3.0864	4.1152	5.1440	2.0576	0.0000	2.0576	0.0000
1.0288	3.0864	2.0576	3.0864	5.1440	0.0000	2.0576	5.1440	15.4300	1.0288	2.0576	0.0000
0.0000	1.0288	4.1152	3.0864	4.1152	0.0000	2.0576	4.1152	15.4300	0.0000	2.0576	0.0000
0.0000	0.0000	7.2016	3.0864	3.0864	0.0000	3.0864	9.2592	9.2592	0.0000	0.0000	0.0000
4.1152	0.0000	6.1728	3.0864	4.1152	0.0000	2.0576	9.2592	5.1440	0.0000	2.0576	0.0000
5.1440	4.1152	5.1440	2.0576	2.0576	0.0000	2.0576	8.2304	2.0576	0.0000	0.0000	4.1152
7.2016	5.1440	3.0864	6.1728	3.0864	2.0576	2.0576	7.2016	2.0576	0.0000	8.2304	0.0000
8.2304	7.2016	2.0576	7.2016	4.1152	2.0576	0.0000	5.1440	0.0000	0.0000	7.2016	2.0576
5.1440	8.2304	2.0576	7.2016	2.0576	0.0000	0.0000	5.1440	1.0288	0.0000	6.1728	2.0576
7.7160	5.1440	5.1440	7.2016	2.0576	2.0576	0.0000	7.2016	2.0576	0.0000	5.1440	2.0576
5.1440	7.7160	4.1152	6.1728	0.0000	0.0000	0.0000	7.2016	2.0576	2.0576	5.1440	5.1440
2.0576	5.1440	3.0864	5.1440	2.0576	0.0000	0.0000	8.2304	1.0288	1.0288	10.2880	5.1440
1.0288	2.0576	4.1152	5.1440	1.0288	0.0000	1.0288	8.2304	4.1152	2.0576	11.3100	5.1440
1.0288	1.0288	5.1440	4.1152	0.0000	0.0000	2.0576	9.2592	6.1728	8.2304	10.2880	1.5432
2.0576	1.0288	0.0000	2.0576	0.0000	0.0000	3.0864	10.2880	8.2304	8.2304	10.2880	5.1440
1.0288	2.0576	4.1152	2.0576	1.0288	3.0864	1.0288	13.3700	7.2016	8.2304	10.2880	4.1152
0.0000	1.0288	3.0864	2.0576	2.0576	4.1152	5.1440	13.3700	5.1440	10.2880	8.2304	2.0576
0.0000	0.0000	1.0288	1.0288	2.0576	1.0288	9.2592	13.3700	7.2016	8.2304	10.2880	3.0864
0.0000	0.0000	1.0288	2.0576	5.1440	2.0576	9.2592	13.3700	7.2016	4.1152	9.2592	3.0864
0.0000	0.0000	3.0864	5.1440	4.1152	8.2304	9.2592	8.2304	7.2016	3.0864	9.2592	4.1152
0.0000	0.0000	5.1440	3.0864	5.1440	7.2016	7.2016	4.1152	6.1728	0.0000	8.2304	2.0576
0.0000	0.0000	6.1728	10.2880	1.0288	7.2016	4.1152	8.2304	5.1440	1.0288	10.2880	4.1152
0.0000	0.0000	7.2016	3.0864	6.1728	7.2016	3.0864	9.2592	5.1440	0.0000	8.2304	2.0576
0.0000	0.0000	9.2592	8.2304	6.1728	6.1728	1.0288	10.2880	4.1152	4.1152	8.2304	3.0864
0.0000	0.0000	8.2304	8.2304	4.1152	5.1440	1.0288	9.2592	3.0864	1.0288	7.2016	4.1152
0.0000	0.0000	9.2592	8.2304	4.1152	1.0288	0.0000	7.2016	3.0864	0.0000	7.2016	2.0576
0.0000	0.0000	10.2880	7.2016	3.0864	1.0288	0.0000	4.1152	1.0288	1.0288	5.1440	6.1728
0.0000	0.0000	9.2592	8.2304	4.1152	0.0000	0.0000	4.1152	0.0000	0.0000	4.1152	5.1440
0.0000	0.0000	8.2304	5.1440	3.0864	0.0000	1.0288	3.0864	0.0000	0.0000	5.1440	4.1152
2.0576	0.0000	7.2016	4.1152	3.0864	1.0288	0.0000	8.2304	0.0000	0.0000	4.1152	5.1440
6.1728	5.1440	6.1728	4.1152	2.0576	2.0576	1.0288	7.2016	0.0000	1.0288	5.1440	5.1440
6.1728	6.1728	6.1728	3.0864	2.0576	2.0576	0.0000	8.2304	0.0000	1.0288	3.0864	10.2880
5.1440	6.1728	6.1728	5.1440	2.0576	0.0000	0.0000	2.0576	0.0000	2.0576	3.0864	9.2592
2.0576	5.1440	5.1440	4.1152	0.0000	0.0000	0.0000	7.2016	0.0000	4.1152	1.0288	9.2592
2.0576	0.0000	5.1440	2.0576	0.0000	0.0000	0.0000	11.3168	0.0000	7.2016	2.0576	10.2880
2.0576	2.0576	4.1152	1.0288	2.0576	0.0000	0.0000	10.2880	0.0000	9.2592	2.0576	9.2592
1.0288	2.0576	3.0864	0.0000	2.0576	0.0000	1.0288	1.0288	4.1152	10.2880	2.0576	10.2880
1.0288	2.0576	2.0576	0.0000	0.0000	0.0000	1.0288	12.3400	6.1728	10.2880	5.1440	10.2880
2.0576	1.0288	2.0576	1.0288	2.0576	3.0864	4.1152	12.8600	0.0000	9.2592	2.0576	11.3100
2.0576	1.0288	1.0288	0.0000	0.0000	5.1440	5.1440	15.4300	7.2016	9.2592	4.1152	10.2880
1.0288	2.0576	3.0864	0.0000	1.0288	5.1440	9.2592	23.1500	9.2592	6.1728	5.1440	11.3100
0.0000	2.0576	0.0000	3.0864	2.0576	5.1440	9.2592	23.1400	8.2304	5.1440	2.0576	10.2880
1.0288	2.0576	1.0288	1.0288	2.0576	4.1152	10.2880	18.0040	9.2592	0.0000	1.0288	10.2880
0.0000	0.0000	0.0000	0.0000	3.0864	5.1440	9.2592	18.0040	7.2016	1.0288	2.0576	10.2880
0.0000	0.0000	2.0576	3.0864	9.2592	7.2016	5.1440	15.4320	5.1440	1.0288	2.0576	10.2880
0.0000	0.0000	2.0576	5.1440	7.2016	8.2304	5.1440	17.4800	1.0288	2.0576	0.0000	9.2592
0.0000	0.0000	2.0576	8.2304	8.2304	7.2016	5.1440	16.4600	0.0000	2.0576	0.0000	8.2304

Data are read columnwise.

Table 5. Predicted values of averaged wind data for the month of March (1985-2004).

S/N	x(t-18)	x(t-12)	x(t-6)	x(t)	x(t+6)
1	1.0288	0	1.0288	4.1152	5.1440
2	2.0576	0	3.0864	5.1440	2.0576
3	2.0576	0	3.0864	7.2016	1.0288
4	1.0288	0	1.0288	8.2304	1.0288
5	0	1.0288	0	5.1440	2.0576
6	1.0288	1.0288	0	7.7160	1.0288
7	0	1.0288	4.1152	5.1440	0
8	0	3.0864	5.1440	2.0576	0
9	0	3.0864	7.2016	1.0288	0
10	0	1.0288	8.2304	1.0288	0
11	1.0288	0	5.144	2.0576	0
12	1.0288	0	7.7160	1.0288	0
13	1.0288	4.1152	5.144	0	0
14	3.0864	5.144	2.0576	0	0
15	3.0864	7.2016	1.0288	0	0
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591	5.1440	9.2592	8.2304	2.0576	0
592	5.1440	9.2592	6.1728	2.0576.	0
593	4.1152	10.288	5.1440	2.0576	1.0288
594	4.1152	1.0288	1.0288	2.0576	2.0576
595	10.288	2.0576	1.0288	2.0576	3.0864
596	9.2592	2.0576	1.0288	3.0864	2.0576
597	9.2592	1.0288	1.0288	0	3.0864
598	9.2592	2.0576	2.0576	0	7.2016
599	10.288	2.0576	3.0864	1.0288	2.0576
600	9.2092	1.0288	0	2.0576	8.2304

six hundred hourly values of wind data of March, 1985 to 2004 (x(t+6) is read column wise) (Jafri, 2008; Jafri et al., 2012). Tables 6 and 7 are the corresponding values of the singleton consequent membership functions, $f_s(x^k)$ (read column wise) and the non-singleton consequent membership functions, $f_{ns}(x^k)$ (read column wise) for BP designed type-1 FLS forecasters, respectively for the month of March, 1985 to 2004. Values for other months of the year 1985 to 2004 are not shown in this paper. NaN in singleton type-1 FLS for values of consequent membership functions, that is, Table 6 shows that fuzzifiers are not working. One may ascribe this 'anomaly' to non-existence of centroid of the consequent set or height defuzzifier for all the corresponding four Gaussian antecedent membership functions; and indeed for sixteen rules in a BP design. Defuzzification produces a crisp output for FLS, from the fuzzy sets that appear at the output. Therefore, one can conjecture that NaN is producing an empty or null set, as an output. An empty or null set is also a subset of a set; therefore, NaN is a sub-crisp output which needs further handling on a fuzzy rule.

We considered the 120 or 144 predicted values of testing data over the months of 1985 to 2004, obtained the corresponding values of both the singleton and non-singleton consequent membership functions, that is, $f_s(x^k)$ and $f_{ns}(x^k)$, respectively. The parameters of input-output training data were tuned both for singleton and non-singleton type-1 FLS for Gaussian antecedent membership functions, and for six epochs by using sixteen rules in a BP design (Jafri, 2008).

Stochastic simulation and time series models were studied and developed to forecast synthetic sequences of wind speed and global solar radiations, respectively (Kamal and Jafri, 1996, 1997).The fuzzy autoregressive (FAR) model can never be described by the stochastic model (Kezuhiro et al., 1997).

The fuzzy parameters for autoregression are determined by linear programming (operations research). FAR model represents a possibility of occurrence of a certain set of data in future when the present data are dependent to some degree on the past data (Ozawa and Niimura, 1999). We do not find in recent years, any significant analysis on fuzzy time series and its prediction modeling. A parallel structure fuzzy system (PSFS) for

Table 6. Values of the consequent membership functions (m/s), f(x), for BP designed singleton type-1 FLS of March 1985 to 2004.

1.1181	1.1181	11.547	4.0017	6.4113	8	8	8	8	8	13.999	13.999
1.1181	1.1181	4	8	8	8	8	8	8.0011	12	13.087	13.087
1.1181	1.1181	4	8	8	8	8	NaN	8	12	10.284	10.284
1.1181	1.1181	6.7195	8	8	8	8	NaN	8	12.533	9.1834	9.1834
1.1181	1.1181	4	8	8	8.3477	8	NaN	8	14	10.084	10.084
1.1181	1.1181	4	8	8	14.87	8	8	8	4	4.0414	4.0414
1.1181	1.1181	4	8	8.0003	8.5774	8	8	8.0582	4	9.7513	9.7513
1.1181	1.1181	7.9996	8.0001	8	8.0015	8	8	9.1834	4	11.55	11.55
1.1181	1.1181	7.9995	8	8.0025	8	4.0036	8	4	4	5.1938	5.1938
1.1181	8.214	8	8	8	8	4	8	4	4	4.0001	4.0001
1.1181	8	8	8.0002	8	12.3	7.9991	8	4	4	4.003	4.003
1.1181	8	8	8	11.656	14	7.9991	8	4	4	4.1005	4.1005
1.1181	8	8.3447	8	8.0794	14	4	8	4	4	8	8
1.1181	8.0005	8	14	13.875	14	4	4	4.003	4	4.0002	4.0002
1.1181	8	8	13.823	4	4	8	8	4	4	4.0026	4.0026
1.1181	14	8	14	4	4.0284	8	8	3.0006	4	7.9908	7.9908
1.1181	8	14	12.493	4	11.879	8	8	3.9192	4	4.0001	4.0001
1.1181	13.629	8	12	4	9.2626	8	8.0169	3	4	8	8
1.1181	14	14	4.0103	4	3.0056	8	4.0002	4	8	8	8
1.1181	14	14	11.982	11.984	3.0337	8	8	8.0007	8	8	8
1.1181	4	8	12	3.5532	3	8	8	8	8	8.0008	8.0008
1.1181	4	12	9.2626	4.0049	3.4133	8	8	8	8	14	14
1.1181	4	12	13.042	7.0023	3.5072	8	8	8	8	7.9999	7.9999
1.1181	4	8.2238	4.9683	7.99	8	8	8.0543	8	8	8.0004	8.0004
1.1181	4	8	7.9999	4.7289	8	8	8	8	8	8	8
1.1181	12	8	8	7.9908	8	8	8	8.0583	8	8	8
1.1181	8	8	8	8	8	8	8	8.0008	8	4	4
1.1181	4	4.0011	8	8	8	8	8	8	8	12	12
1.1181	8	8	10.272	8	13.187	8	8	8.0002	8	4	4
1.1181	8	8	8.0005	8	8.6149	8	8	12	8	4	4
1.1181	8	8	8	8.0001	8	8	8	14	8	4	4
1.1181	8	8	8	9.028	8.0015	8	8.0009	13.995	8	4	4
1.1181	8	8	11.177	8	8.0001	8	14	4.038	8	4	4
1.1181	8	8	8.0003	8.0001	11.656	14	8.0089	4	8	4	4
1.1181	8	8	13.187	12	11.726	12	12.404	4	8	4	4
1.1181	8	8.5998	8.0413	12	14	NaN	13.602	4	8	4	4
1.1181	8	8	8.0001	13.032	4	NaN	13.779	8.0006	8	4	4
1.1181	8	8	8.0021	13.999	4	14	4.215	3	8	4	4
1.1181	8	8	13.988	13.941	4	NaN	6.0662	3.0423	8	8	8
1.1181	8	8	4.1984	4	4	4	4.0414	3.5299	8.0003	8	8
1.1181	8	8.0001	13.365	4	4	NaN	4.0049	8	11.955	8	8
1.1181	8	14	12.234	4	4	NaN	3.0001	8	8.2304	8	8
1.1181	8	12	13.144	4.0009	4	NaN	3.0006	8	14.87	8	8
1.1181	8.203	12.235	4.0001	9.1971	4	NaN	8	8	10.838	8	8
1.1181	8.0129	13.944	7.8612	9.2626	4	NaN	8	8	8.5998	8	8
1.1181	8	12	7.9834	4.2819	4.902	8	3.1214	8	14.916	8	8
1.1181	10.678	12	3.005	8	8	NaN	8	8	13.187	8	8
1.1181	8	4	3.0005	8	8	NaN	8	8	8.8366	8	8
1.1181	11.863	7.9951	3.0033	8	8	NaN	8	8	13.087	8	8
1.1181	10.338	4	3.2795	8	8	8	8	8	12.591	8	8

Data are read column-wise.

Table 7. Values of the consequent membership functions (m/s), f(x), for BP designed non-singletion type- 1 FLS of March, 1985 to 2004.

7.611	7.611	9.232	8.282	9.032	9.443	10.419	11.788	12.267	12.029	10.927	13.995
7.611	7.611	9.959	9.722	11.075	11.445	11.656	14.053	8.907	12.211	9.213	14.212
7.611	7.611	9.317	9.696	9.473	12.156	12.181	13.257	11.240	12.498	9.826	13.573
7.611	7.611	10.369	10.032	10.044	12.014	13.407	14.469	11.393	11.627	10.265	14.229
7.611	7.611	9.296	11.567	10.684	11.298	13.632	14.926	11.130	10.968	8.829	14.074
7.611	7.611	9.851	10.558	10.388	10.478	13.891	14.537	11.830	8.457	8.480	13.994
7.611	7.611	8.978	9.863	10.695	11.173	13.527	14.612	11.185	8.746	9.458	13.541
7.611	7.611	10.547	10.938	12.451	12.228	11.989	13.508	10.265	8.548	9.012	13.567
7.611	7.611	10.293	11.866	11.112	11.834	11.920	14.554	8.780	9.426	8.001	11.867
7.611	8.937	10.229	12.433	11.130	11.606	11.582	14.419	8.016	9.520	8.201	12.909
7.611	10.024	11.243	12.543	11.662	10.065	12.397	14.822	8.016	9.109	8.164	13.327
7.611	9.793	11.240	12.589	11.138	9.529	12.397	14.416	8.588	9.315	8.332	13.297
7.611	9.901	10.382	12.115	10.574	10.049	11.817	13.447	8.622	7.960	9.443	12.071
7.611	11.488	11.833	11.354	9.877	10.603	11.259	9.911	8.164	10.134	8.394	12.116
7.611	9.993	11.509	11.430	8.508	9.181	11.808	13.310	8.857	10.244	8.624	8.623
7.611	10.528	11.310	10.528	8.009	9.347	11.511	13.652	8.148	10.283	8.871	12.153
7.611	12.847	11.156	9.942	8.796	9.615	13.009	12.895	8.217	8.763	8.903	12.580
7.611	12.184	11.147	10.282	8.353	9.459	12.878	11.503	8.517	10.391	9.322	11.878
7.611	11.287	10.355	9.783	8.103	9.191	12.329	9.758	8.630	9.546	11.544	10.115
7.611	10.644	12.255	10.285	9.262	9.475	12.074	10.521	8.726	13.108	9.543	10.489
7.611	10.416	12.598	10.454	9.287	8.635	12.266	9.879	9.848	12.884	9.732	10.981
7.611	9.109	12.057	9.459	8.273	9.036	12.530	10.586	9.583	12.758	10.502	12.806
7.611	11.703	11.928	9.358	10.102	8.379	12.719	10.900	9.634	12.188	10.475	12.348
7.611	11.080	11.739	8.958	9.300	9.372	12.700	10.534	10.835	12.435	11.198	12.333
7.611	9.957	11.220	10.021	8.558	9.783	12.385	10.310	10.452	14.040	11.594	13.507
7.611	10.696	11.482	10.070	8.871	10.353	12.672	11.703	9.944	14.297	11.687	13.102
7.611	11.596	11.700	9.315	9.757	10.712	13.710	12.497	12.010	14.083	9.809	12.770
7.611	9.358	10.281	9.443	9.460	10.889	13.301	12.710	11.448	14.007	11.234	13.138
7.611	11.752	10.998	8.913	10.889	9.200	14.466	12.094	11.697	13.943	11.249	12.276
7.611	11.456	10.199	10.561	11.353	11.038	14.185	11.380	11.959	13.497	9.607	10.507
7.611	10.618	10.502	10.696	9.792	11.660	14.362	11.870	12.442	13.899	10.507	11.968
7.611	10.629	10.968	11.580	10.234	11.891	14.509	11.918	10.313	13.654	10.600	10.863
7.611	11.667	12.257	10.987	11.675	11.481	12.769	11.973	9.597	13.611	10.079	10.310
7.611	12.553	11.455	11.337	11.341	11.138	10.850	10.756	8.370	12.932	9.866	10.731
7.611	12.894	13.762	9.200	11.183	10.010	11.754	10.496	8.715	13.061	10.952	10.792
7.611	13.244	10.742	11.370	11.183	9.804	9.598	10.484	8.393	12.950	9.222	11.028
7.611	13.441	12.538	11.728	11.099	8.531	10.910	10.058	10.377	12.604	10.418	12.201
7.611	13.034	12.252	10.639	10.927	8.786	12.860	9.780	8.746	12.444	10.777	12.933
7.611	13.082	11.922	10.429	8.997	8.076	10.139	9.963	8.273	12.379	10.102	13.484
7.611	13.462	12.459	10.181	8.814	8.397	8.096	8.480	9.224	11.337	11.840	13.123
7.611	12.938	12.487	10.057	8.301	8.449	7.788	8.273	9.308	10.441	11.733	12.510
7.611	12.932	10.987	9.666	8.381	8.753	7.627	8.055	9.997	11.224	11.123	12.560
7.611	12.159	10.485	10.240	8.923	8.109	8.124	8.148	10.155	10.478	11.876	12.128
7.611	11.713	10.124	9.763	9.332	8.704	7.728	9.262	10.371	10.777	11.626	11.950
7.611	11.262	9.577	9.372	9.459	8.456	9.400	9.801	10.455	10.742	13.667	13.793
7.611	11.592	11.086	9.851	9.238	9.480	9.997	8.182	10.911	10.000	13.215	13.668
7.611	10.884	10.536	8.201	9.619	9.699	8.974	9.529	11.950	9.200	13.755	13.361
7.611	11.404	9.421	8.428	9.654	10.168	8.441	9.679	12.707	9.971	13.937	13.082
7.611	10.406	9.593	9.788	9.502	9.787	8.571	9.518	12.898	9.213	13.720	13.206
7.611	10.179	8.524	9.827	9.088	10.191	8.459	11.757	11.937	8.945	13.982	12.374

Data are read column-wise.

Table 8. RMSE values.

Month	$RMSE_s$ (BP)	$RMSE_{ns}$ (BP)
January	4.1089	3.7623
February	4.2079	3.6529
March	3.9876	3.5976
April	3.8988	3.4271
May	3.4423	3.5613
June	3.3399	3.2376
July	3.4765	3.3219
August	3.4498	3.3754
September	3.4569	3.1226
October	3.5678	3.3578
November	3.7629	3.2866
December	3.7628	3.5674

prediction of time series, especially the chaotic time series data (like Lorenz time series and Laser time series) was developed (Kim and Kong, 2001). The PSFS consists of multiple numbers of fuzzy systems connected in parallel. Each component fuzzy system in the PSFS predicts the same future data independently based on its past time series data with different embedding dimension and time delay. We observed chaos in our wind data (Jafri et al., 2012); therefore, PSFS could be used more beneficially for prediction of short term time series data as compared to other models. Tsai and Wu (2001) developed a model to improve the performance of the root mean square error of the forecast. They followed fuzzy time series models of Song and Chissom (1993) which employ fuzzy relational equations, definition of various time series, properties of fuzzy time series and a step by step procedure for the implementation of the fuzzy time series with linguistic values. These methods are, however, based on linguistic values of fuzzy rules and are not suitable for chaotic heavy set of data, like wind. The second order modeling of fuzzy time series (Chao et al., 1999) can be extended and applied to the prediction of traditional (stationary) numerical time series.

We did not find any comparison with BP designed singleton type-1 with non-singleton type-1 FLS in any literature known to us. Although, we did not establish any comparison with OP designed 500 rule type-1 FLS, yet we conformed on the basis of RMSE values (Table 8) that the BP-designed 16 rules non-singleton type-1 for chaotic data seemed relatively better than singleton type-1 FLS (Jafri, 2008).

We assumed that there are no uncertainties in BP designed singleton type-1 FLS. But, our data is chaotic and have too many unraveled0/hidden uncertainties. The BP design is a solution to handle data where uncertainties exist. We identified two kinds of uncertainties, that is, stable attractor and the other non-stationarity in wind data (Jafri, 2008; Kamal and Jafri, 1996; Jafri et al., 2012). Valenzuela et al. (2008) exploited hybridization of intelligent techniques and ARIMA model for time series prediction. We conjecture that there is a dire need to include cascade correlation algorithm in FLSs to make the forecaster more efficient than BP designed FLS. Moreover, the hybridization of intelligent techniques with FLS could also be a promising solution for efficient and reliable forecasters.

We infer from the present study the following conclusions:

1) The BP designed non-singleton type-1 FLS is a better forecaster to handle data where uncertainties exist (such as in our data which is chaotic).
2) The wind data have too many unraveled/hidden uncertainties, such as, non-stationarity and stable attractors. These uncertainties do not influence the predicted values as confirmed by Mackey glass simulation (Jafri et al., 2008).
3) An anomaly (NaN) for some of the hourly wind data, especially for a singleton type-1 FLS forecaster exists. This confirms to the fact that fuzzifiers for consequents are not working.
4) The parallel structure fuzzy system (PSFS) if opted for our wind data would have produced better results as a short duration forecaster. The chaos can be effectively resolved through time- delays in a time series data.
5) The introduction of cascade correlation algorithm in fuzzy logic systems can make the forecaster more reliable, efficient and sustainable than BP designed FLS.
6) Hybridization of intelligent systems with fuzzy logic (FL) is an alternate option for developing new forecasters.

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