International Journal of Physical Sciences Vol. 7(8), pp. 1166 - 1174, 16 February, 2012
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS11.1753
ISSN 1992 - 1950

Full Length Research Paper

Reliable treatments of differential transform method for two-dimensional incompressible viscous flow through slowly expanding or contracting porous walls with small-to-moderate permeability

Saeed Dinarvand

Young Researchers Club, Central Tehran Branch, Islamic Azad University, Tehran, Iran.
E-mail: saeed_dinarvand@yahoo.com. Tel: +98 912 4063341.

Accepted 26 January, 2012

In this article, the problem of laminar, isothermal, incompressible and viscous flow in a rectangular domain bounded by two moving porous walls which enable the fluid to enter or exit during successive expansions or contractions is solved analytically by using the differential transform method (DTM). Graphical results are presented to investigate the influence of the non-dimensional wall dilation rate and seepage Reynolds number on the velocity, normal pressure distribution and wall shear stress. The validity of our solutions is verified by the numerical results obtained by shooting method coupled with Runge–Kutta scheme. Since the transport of biological fluids through contracting or expanding vessels is characterized by low seepage Reynolds numbers, the current study focuses on the viscous flow driven by small wall contractions and expansions of two weakly permeable walls.

Key words: Expanding or contracting porous walls, seepage Reynolds number, non-dimensional wall dilation rate, differential transform method.

INTRODUCTION

Most of the scientific problems and phenomena are modeled by non-linear ordinary or partial differential equations. In recent years, many powerful methods have been developed to construct explicit analytical solution of non-linear differential equations (He, 2006; Liao, 1992; Dinarvand, 2009, 2011a, 2010, 2011b). Zhou was the first one who introduced the differential transform method (DTM) as an efficient method to apply for electrical circuits in his paper entitled “differential transformation and its application for electrical circuits” (Zhou, 1986). It was used to solve both linear and nonlinear initial value problems in electric circuit analysis. The differential transform method (DTM) is an analytical method for solving ordinary differential equations, partial differential and integral equations. The method provides us with easily computable components and the solution is obtained in terms of convergent series. The main advantages of this method, compared to other analytic methods are controllable accuracy, and high efficiency which is exhibited by the rapid convergence of the solution. The DTM gives exact values of the nth derivative of an analytic function at a point in terms of known and unknown boundary conditions. This method constructs, for differential equations, an analytical solution in the form of a polynomial. It is different from the traditional high order Taylor series method which requires symbolic computations of the necessary derivatives of the data functions. The disadvantage of Taylor series method is that this method computationally has taken long time for large orders.

The DTM is an iterative procedure for obtaining analytic Taylor series solutions of differential equations in a fast manner. The DTM methodology introduces a promising approach for many applications in various domains of nonlinear problems. Various applications of DTM can be found in Zhou (1986), Biazar and Eslami (2010), Subhas and Mahesha (2009), Ayaz (2004), Arikoglu and Ozkol (2005), Liu and Song (2007), Kanth and Aruna (2009), Odibat et al. (2010), Chen and Chen (2009), Hosseini et al. (2011) and Chang et al. (1989). Studies of fluid transport in biological organisms often concern the flow of a particular fluid inside an expanding or contracting vessel with permeable walls. For a valved vessel exhibiting deformable boundaries, alternating wall contractions produce the effect of a physiological pump. The flow behavior inside the lymphatics exhibits a similar character. In such models, circulation is induced by successive contractions of two thin sheets that cause the downstream convection of the sandwiched fluid. Seepage across permeable walls is clearly important to the mass transfer between blood, air and tissue Chang et al. (1989). Therefore, a substantial amount of research work has been invested in the study of the flow in a rectangular domain bounded by two moving porous walls which enable the fluid to enter or exit during successive expansions or contractions. Dauenhauer and Majdalani (1999) studied the unsteady flow in semi-infinite expanding channels with wall injection. They presented a procedure that leads to an exact similarity solution of the Navier–Stokes equations in semi-infinite rectangular channels with porous and uniformly expanding walls. They considered the case of expanding wall combined with injection through two opposing porous walls.

Shooting method, coupled with a Runge–Kutta integration scheme was utilized to numerically solve the resulting fourth-order differential equation. Majdalani and Zhou (2003) studied moderate to large injection and suction driven channel flows with expanding or contracting walls. They considered the incompressible laminar flow in a porous channel with expanding or contracting walls. They assumed the head-end to be closed by a compliant membrane and downstream end is left unobstructed. Along the uniformly expanding porous walls, the Navier–Stokes equations are reduced to a single, non-linear ordinary differential equation for symmetric injection or suction. Using perturbations in cross-flow Reynolds number Re, the resulting equation is solved both numerically and analytically. Boutros et al. (2011) studied the solution of the Navier–Stokes equations which described the unsteady incompressible laminar flow in a semi-infinite porous circular pipe with injection or suction through the pipe wall whose radius varies with time. Their analysis simulates the flow field by the burning of inner surface of cylindrical grain in a solid rocket motor in which the burning surface regresses with time. They applied Lie-group method to the equations of motion for determining symmetry reductions of partial differential equations. The resulting fourth-order non-linear differential equation is then solved using small-parameter perturbations (Terrill and Thomas, 1969; Terrill, 1973) and the results are compared with numerical solutions using shooting method coupled with Runge–Kutta scheme. A homotopy based solution of the Navier–Stokes equations for a porous channel with orthogonally moving walls has been presented by Xu et al. (2010). Si et al. (2011a) have recently examined the existence of multiple solutions for the laminar flow in a porous channel with suction at both slowly expanding and contracting walls. Besides, the flow of a micropolar fluid through a porous channel with expanding or contracting walls has been investigated by them in other study (Si et al., 2011b). Our motivation in the present study is to investigate the problem of incompressible viscous flow through slowly expanding or contracting permeable walls using the DTM. We also intend to compare the results of simulation using the DTM with the results of simulation using the numerical method (shooting method coupled with fourth-order Runge-Kutta). Therefore, the paper is organized as follow. Subsequently, the mathematical formulation is presented. After which we extend the application of the DTM to construct the approximate solutions of the governing equation. Describing of the numerical scheme is covered thereafter. Then, it contains the results and discussion. The conclusions are summarized lastly.

PROBLEM STATEMENT AND MATHEMATICAL FORMULATION

Consider the laminar, isothermal, and incompressible flow in a rectangular domain bounded by two permeable surfaces that enable the fluid to enter or exit during successive expansions or contractions. A schematic diagram of the problem is shown in Figure 1. The walls expand or contract uniformly at a time-dependent rate $\dot{a}$. At the wall, it is assumed that the fluid inflow velocity V_w is independent of position. The equations of continuity and motion for the unsteady flow are given as follows:

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0, \tag{1}$$

$$\frac{\partial \tilde{u}}{\partial t} + \tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial \tilde{x}} + \nu\left[\frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2}\right], \tag{2}$$

$$\frac{\partial \tilde{v}}{\partial t} + \tilde{u}\frac{\partial \tilde{v}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{v}}{\partial \tilde{y}} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial \tilde{y}} + \nu\left[\frac{\partial^2 \tilde{v}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{v}}{\partial \tilde{y}^2}\right]. \tag{3}$$

In the aforementioned equations, $\tilde{u}$ and $\tilde{v}$ indicate the velocity components in the x and y directions, $\tilde{p}$ denotes the dimensional pressure, ρ, ν and t are the density, kinematic viscosity and time, respectively. The boundary conditions will be:

$$\begin{aligned}
&\tilde{u} = 0, \quad \tilde{v} = -V_w = -\dot{a}/c \quad at \quad \tilde{y} = a(t),\\
&\frac{\partial \tilde{u}}{\partial \tilde{y}} = 0, \quad \tilde{v} = 0, \quad at \quad \tilde{y} = 0,\\
&\tilde{u} = 0 \quad at \quad \tilde{x} = 0,
\end{aligned} \tag{4}$$

Where *c* ($c \equiv \dot{a}/V_w$) is the wall permeance or injection/suction coefficient, that is a measure of wall permeability. The stream function and mean flow vorticity can be introduced by putting:\

$$\tilde{u} = \frac{\partial \tilde{\psi}}{\partial \tilde{y}}, \qquad \tilde{v} = \frac{\partial \tilde{\psi}}{\partial \tilde{x}}, \qquad \tilde{\xi} = \frac{\partial \tilde{v}}{\partial \tilde{x}} - \frac{\partial \tilde{u}}{\partial \tilde{y}},$$
$$\frac{\partial \tilde{\xi}}{\partial t} + \tilde{u}\frac{\partial \tilde{\xi}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{\xi}}{\partial \tilde{y}} = \nu\left[\frac{\partial^2 \tilde{\xi}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{\xi}}{\partial \tilde{y}^2}\right]. \tag{5}$$

Due to mass conservation, a similar solution can be developed with respect to $\tilde{x}$ Majdalani et al. (2002). Starting with:

$$\tilde{\psi} = \frac{\nu \tilde{x}\tilde{F}(y,t)}{a}, \qquad \tilde{u} = \frac{\nu \tilde{x}\tilde{F}_y}{a^2}, \qquad \tilde{v} = \frac{-\nu \tilde{F}(y,t)}{a},$$
$$y = \frac{\tilde{y}}{a}, \qquad \tilde{F}_y \equiv \frac{\partial \tilde{F}}{\partial y}. \tag{6}$$

Substitution Equation 6 into Equation 5 yields:

$$\tilde{u}_{\tilde{y}t} + \tilde{u}\tilde{u}_{\tilde{y}\tilde{x}} + \tilde{v}\tilde{u}_{\tilde{y}\tilde{y}} = \nu \tilde{u}_{\tilde{y}\tilde{y}\tilde{y}}. \tag{7}$$

In order to solve Equation 7, one uses the chain rule to obtain:

$$\tilde{F}_{yyyy} + \alpha(y\tilde{F}_{yyy} + 3\tilde{F}_{yy}) + \tilde{F}\tilde{F}_{yyy} - \tilde{F}_y\tilde{F}_{yy} - a^2\nu^{-1}\tilde{F}_{yyt} = 0, \tag{8}$$

With the following boundary conditions:

$$\tilde{F} = 0, \quad \tilde{F}_{yy} = 0 \quad at \quad y = 0,$$
$$\tilde{F} = \mathrm{Re}, \quad \tilde{F}_y = 0 \quad at \quad y = 1, \tag{9}$$

Where, $\alpha(t) \equiv \dot{a}a/\nu$ is the non-dimensional wall dilation rate defined positive for expansion and negative for contraction. Furthermore, $\mathrm{Re} = aV_w/\nu$ is the seepage Reynolds number defined positive for injection and negative for suction through the walls. Equations 6, 8 and 9 can be normalized by putting:

$$\psi = \frac{\tilde{\psi}}{a\dot{a}}, \quad u = \frac{\tilde{u}}{\dot{a}}, \quad v = \frac{\tilde{v}}{\dot{a}}, \quad x = \frac{\tilde{x}}{a}, \quad F = \frac{\tilde{F}}{\mathrm{Re}}, \tag{10}$$

and so

$$\psi = \frac{xF}{c}, \quad u = \frac{xF'}{c}, \quad v = \frac{-F}{c}, \quad c = \frac{\alpha}{\mathrm{Re}},$$
$$F^{IV} + \alpha(yF''' + 3F'') + \mathrm{Re}\,FF''' - \mathrm{Re}\,F'F'' = 0, \tag{11}$$

$$\psi = \frac{xF}{c}, \quad u = \frac{xF'}{c}, \quad v = \frac{-F}{c}, \quad c = \frac{\alpha}{\mathrm{Re}},$$
$$F^{IV} + \alpha(yF''' + 3F'') + \mathrm{Re}\,FF''' - \mathrm{Re}\,F'F'' = 0, \tag{12}$$

The boundary conditions (Equation 9) will be:

$$F = 0, \quad F'' = 0 \quad at \quad y = 0,$$
$$F = 1, \quad F' = 0 \quad at \quad y = 1. \tag{13}$$

The resulting Equation 12 is the classic Berman's formula (Berman, 1953), with $\alpha = 0$ (channel with stationary walls).

After the flow field is found, the normal pressure gradient can be obtained by substituting the velocity components into Equation 1 to 3. Hence, it is:

$$p_y = -[\mathrm{Re}^{-1}F'' + FF' + \alpha\,\mathrm{Re}^{-1}(F + yF')],$$
$$p = \frac{\tilde{p}}{\rho V_w^2}. \tag{14}$$

We can determine the normal pressure distribution, if we integrate Equation 14. Let p_c be the centreline pressure, hence:

$$\int_{p_c}^{p(y)} \mathrm{d}p = \int_0^y -[\mathrm{Re}^{-1}F'' + FF' + \alpha\,\mathrm{Re}^{-1}(F + yF')]\mathrm{d}y. \tag{15}$$

Then, using $FF' = (F^2)'/2$ and $(F + yF') = (yF)'$, the resulting normal pressure drop will be:

$$\Delta p_n = \mathrm{Re}^{-1}F'(0) - [\mathrm{Re}^{-1}F' + \frac{1}{2}F^2 + \alpha\,\mathrm{Re}^{-1}yF]. \tag{16}$$

Another important quantity is the shear stress. The shear stress can be determined from Newton's law for viscosity:

$$\tilde{\tau} = \mu(\tilde{v}_{\tilde{x}} + \tilde{u}_{\tilde{y}}) = \frac{\rho\nu^2\tilde{x}\tilde{F}''}{a^3}. \tag{17}$$

Introducing the non-dimensional shear stress $\tau = \tilde{\tau}/\rho V_w^2$, we have:

$$\tau = \frac{xF''}{\mathrm{Re}}. \tag{18}$$

ANALYTICAL APPROXIMATIONS BY MEANS OF DTM

The differential transform method is an analytical method for a vast variety of differential equations including ODEs and PDEs (Zhou, 1986). This method uses polynomials form to approximate the exact solutions. We now take a brief review to the DTM. The differential transform of the kth derivative of function $F(t)$ is defined as follows:

$$f(k) = \frac{1}{k!}\left[\frac{d^kF(t)}{dt^k}\right]_{t=t_0}, \tag{19}$$

Where $F(t)$ is the base function and $f(k)$ is the transformed function. The differential inverse transform of $f(k)$ is defined as:

$$F(t) = \sum_{k=0}^{\infty} f(k)(t - t_0)^k. \tag{20}$$

Equations 19 and 20 give the following:

$$F(t)=\sum_{k=0}^{\infty}\frac{(t-t_0)^k}{k!}\left[\frac{d^kF(t)}{dt^k}\right]_{t=t_0} \tag{21}$$

This shows that differential transform is derived from Taylor series expansion, but the method does not evaluate the derivatives symbolically. However, relative derivatives are calculated by an iterative way which is described by the transformed equations of the base function. We approximate $F(t)$ by a finite series and Equation 20 can be written as:

$$F(t)\approx\sum_{k=0}^{N}f(k)(t-t_0)^k. \tag{22}$$

The main steps of the DTM are the following. First, we apply the differential transform (Equation 19) to the given differential equation or a system of differential equations to obtain a recursive relation. Secondly, solving the recursive relation and then using the differential inverse transform (Equation 20), we obtain the solution of the problem. Using Equations 19 and 20, the following theorems can be deduced as follows:

Theorem 1

If $U(t)=X(t)\pm Y(t),$ then $u(k)=x(k)\pm y(k).$

Theorem 2

If $U(t)=\alpha X(t),$ then $u(k)=\alpha x(k),$ where α is a constant.

Theorem 3

If $U(t)=\left(\dfrac{d^mX(t)}{dt^m}\right),$ then $u(k)=\dfrac{(m+k)!}{k!}x(k+m).$

Theorem 4

If $U(t)=X(t)Y(t),$ then $u(k)=\sum_{r}^{k}x(r)y(k-r).$

Theorem 5

If $U(t)=t^n,$ then $u(k)=\delta(k-n);$ $\delta(k-n)=\begin{cases}1, & k=n,\\ 0, & k\neq n.\end{cases}$

Taking differential transform from Equation 12, one can obtain:

$$\begin{aligned}&(k+1)(k+2)(k+3)(k+4)f(k+4)+\\&\alpha\left(\sum_{r=0}^{\infty}\left[(k-r+1)(k-r+2)(k-r+3)f(k-r+3)\delta(r-1)\right]+3(k+1)(k+2)f(k+2)\right)+\\&\mathrm{Re}\left(\sum_{r=0}^{\infty}\left[(k-r+1)(k-r+2)(k-r+3)f(r)f(k-r+3)\right]-\sum_{r=0}^{\infty}\left[(r+1)(k-r+1)(k-r+2)f(r+1)f(k-r+2)\right]\right)=0.\end{aligned} \tag{23}$$

The boundary conditions (Equation 13) are transformed into:

$$f(0)=0 \quad ; \quad f(2)=0,$$
$$\sum_{k=0}^{\infty}f(k)=1 \quad ; \quad \sum_{k=0}^{\infty}kf(k)=0. \tag{24}$$

We approximate $F(t)$ using 25 terms and ignore the rest.

NUMERICAL SIMULATION

The shooting method works by considering the boundary conditions as a multivariate function of initial conditions at some point, reducing the boundary value problem to finding the initial conditions that give a root. The advantage of the shooting method is that it takes advantage of the speed and adaptivity of methods for initial value problems. The basic concept of the shooting method can be obtained from Roberts and Shipman (1972). In this paper, the shooting method coupled with the fourth-order Runge-Kutta scheme is used for solving the flow of a viscous incompressible fluid through slowly expanding or contracting permeable walls.

RESULTS AND DISCUSSION

In Table 1, the function $F(y)$ obtained by the differential transform method is compared with the numerical results for the different values of seepage Reynolds number and wall dilation rate. We can see a very good agreement between the purely analytic results of the DTM and numerical results. Figures 1 to 4 illustrate the behaviour of $F'(\text{or } uc/x)$ for seepage Reynolds number $\mathrm{Re}=-4, \mathrm{Re}=0$ and $\mathrm{Re}=4,$ respectively, over a range of non-dimensional wall dilation rate α. For every level of injection or suction, in case of expanding wall ($\alpha>0$), increasing α leads to higher axial velocity near the centre and the lower near the wall. That is because the flow toward the centre becomes greater to make up for the space caused by the expansion of the wall and as a result the axial velocity also becomes greater near the centre. Similarly, for every level of injection or suction, in case of contracting wall ($\alpha<0$), increasing $|\alpha|$ leads to

Table 1. The analytic results of $F(y)$ compared with the numerical results for the different values of seepage Reynolds number and wall dilation rate.

y									
0.8		**0.6**		**0.4**		**0.2**		**Re**	**α**
Num	**DTM**	**Num**	**DTM**	**Num**	**DTM**	**Num**	**DTM**		
0.93161	0.93162	0.76449	0.76448	0.53779	0.53778	0.27678	0.27677	–1	
0.93279	0.93278	0.76679	0.76679	0.54007	0.54007	0.27815	0.27814	–0.5	
0.93388	0.93388	0.76895	0.76895	0.54224	0.54224	0.27944	0.27944	0	–1
0.93490	0.93490	0.77097	0.77097	0.54428	0.54428	0.28068	0.28067	0.5	
0.93583	0.93584	0.77287	0.77286	0.54621	0.54620	0.28184	0.28183	1	
0.94292	0.94292	0.78973	0.78973	0.56563	0.56563	0.29454	0.29454	–1	
0.94348	0.94348	0.79091	0.79091	0.56686	0.56686	0.29530	0.29530	–0.5	
0.94400	0.94400	0.79200	0.79200	0.56800	0.56800	0.29600	0.29600	0	0
0.94446	0.94446	0.79299	0.79299	0.56904	0.56904	0.29664	0.29664	0.5	
0.94488	0.94488	0.79390	0.79390	0.57001	0.57001	0.29724	0.29724	1	
0.95526	0.95525	0.81968	0.81967	0.60063	0.60062	0.31765	0.31764	–1	
0.95490	0.95489	0.81886	0.81885	0.59972	0.59972	0.31707	0.31707	–0.5	
0.95458	0.95458	0.81812	0.81812	0.59891	0.59891	0.31655	0.31655	0	1
0.95430	0.95430	0.81747	0.81747	0.59818	0.59818	0.31609	0.31609	0.5	
0.95405	0.95406	0.81689	0.81689	0.59753	0.59754	0.31567	0.31568	1	

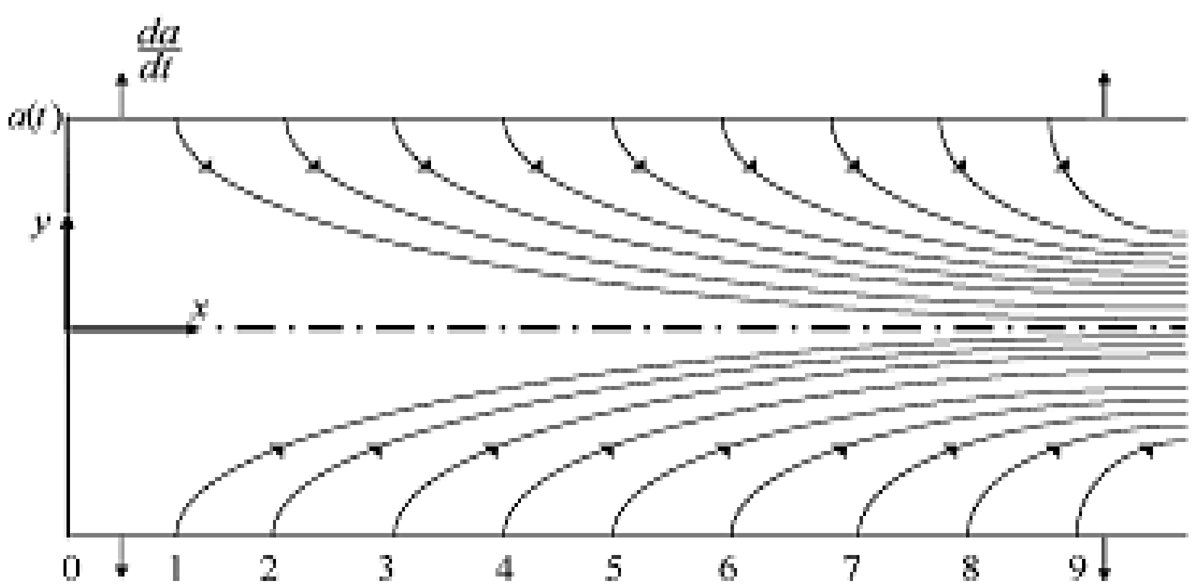

Figure 1. Two-dimensional domain with expanding or contracting porous walls.

lower axial velocity near the centre and the higher near the wall because the flow toward the wall becomes greater and as a result the axial velocity near the wall becomes greater. Furthermore, Figures 2 to 4 indicate the greater sensitivity to wall expansion in comparison with wall contraction. This behavior is greater sensible for suction case ($\mathrm{Re} = -4$).

The normal pressure distribution for seepage Reynolds

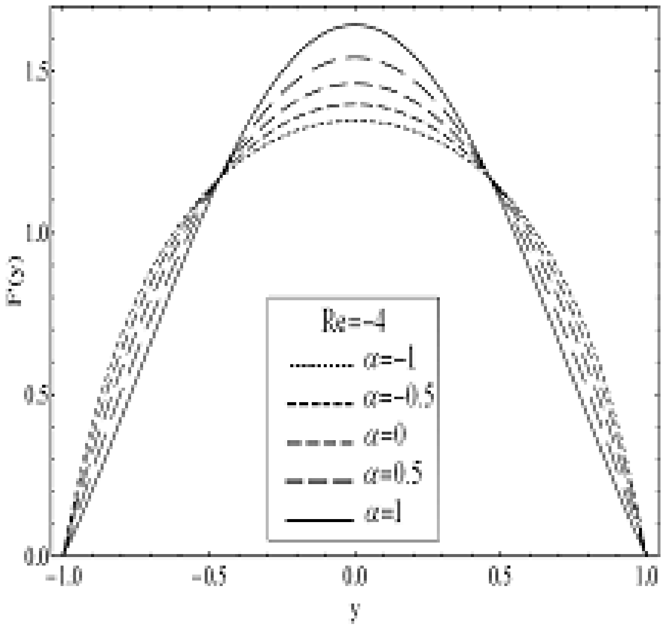

Figure 2. The effect of wall dilation rate α on axial velocity profiles ($F'(y)$) for $\mathrm{Re} = -4$.

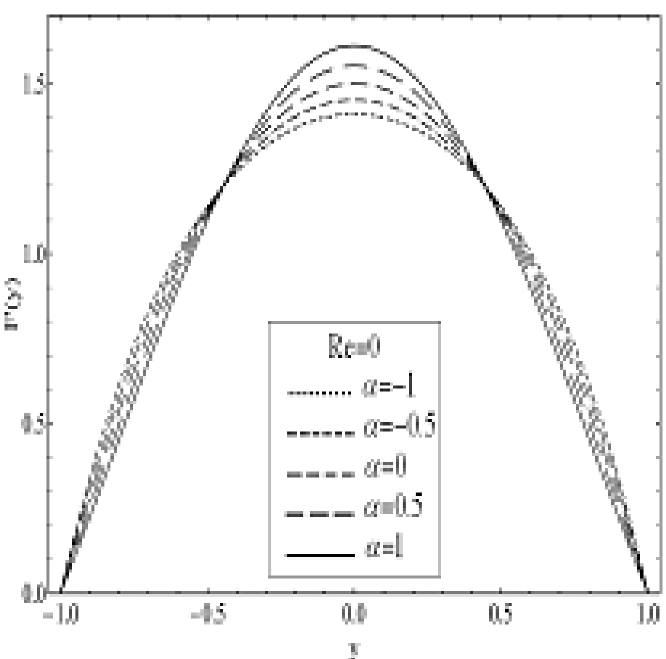

Figure 3. The effect of wall dilation rate α on axial velocity profiles ($F'(y)$) for $\mathrm{Re} = 0$.

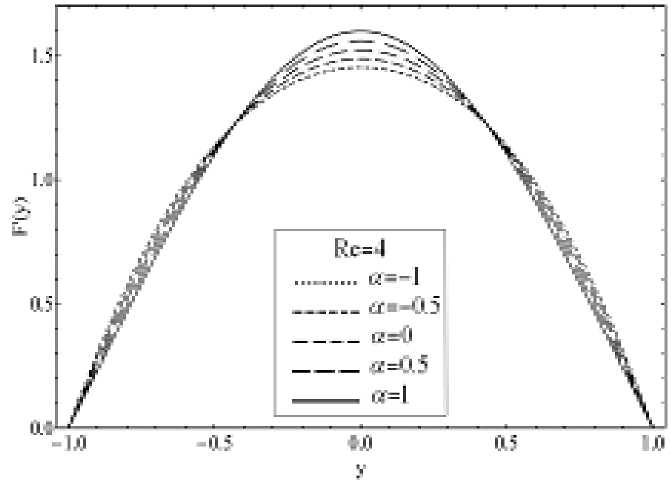

Figure 4. The effect of wall dilation rate α on axial velocity profiles ($F'(y)$) for $\mathrm{Re} = 4$.

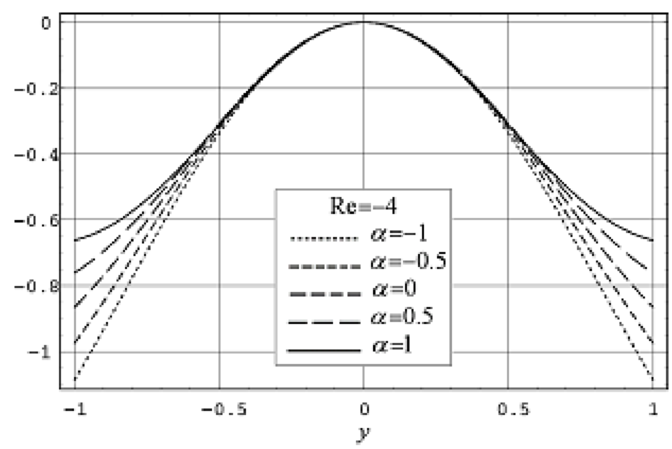

Figure 5. The effect of wall dilation rate α on the normal pressure drop (Δp_n) for $\mathrm{Re} = -4$.

number $\mathrm{Re} = -4$ and $\mathrm{Re} = 4$, over a range of non-dimensional wall dilation rate α are plotted in Figures 5 and 6, respectively. From Figures 5 and 6, for every level of injection or suction, the absolute pressure change in the normal direction ($|\Delta p_n|$) is lowest near the central portion. For every level of suction, in case of expanding wall ($\alpha > 0$), increasing α leads to lower $|\Delta p_n|$ and in case of contracting wall ($\alpha < 0$), increasing $|\alpha|$ leads to higher $|\Delta p_n|$. The effect of seepage Reynolds number on the normal pressure distribution for wall dilation rate $\alpha = -1$ and $\alpha = 1$ is illustrated in Figures 7 and 8, respectively. The wall shear stress ($\tau_w = xF''(1)/\mathrm{Re}$) for seepage Reynolds number $\mathrm{Re} = -4$ and $\mathrm{Re} = 4$ over a range of non-dimensional wall dilation rate α are plotted in Figures 9 to 10, respectively. Figures 9 to 10 illustrate

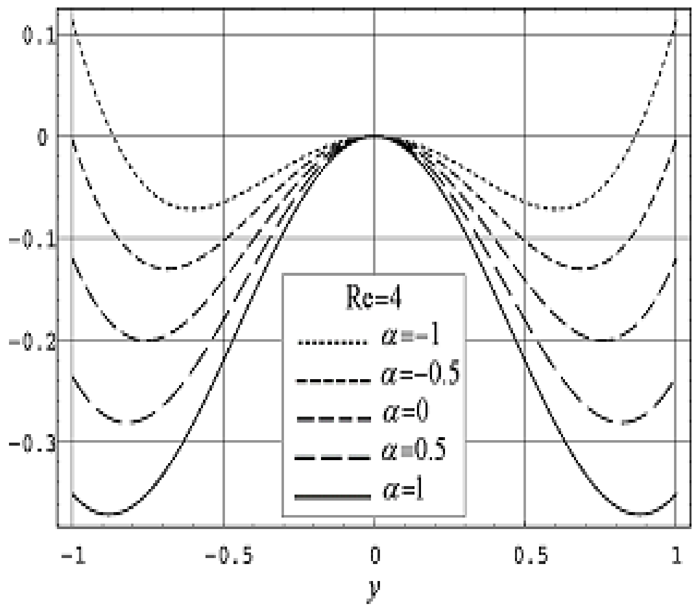

Figure 6. The effect of wall dilation rate α on the normal pressure drop (Δp_n) for $\mathrm{Re} = 4$.

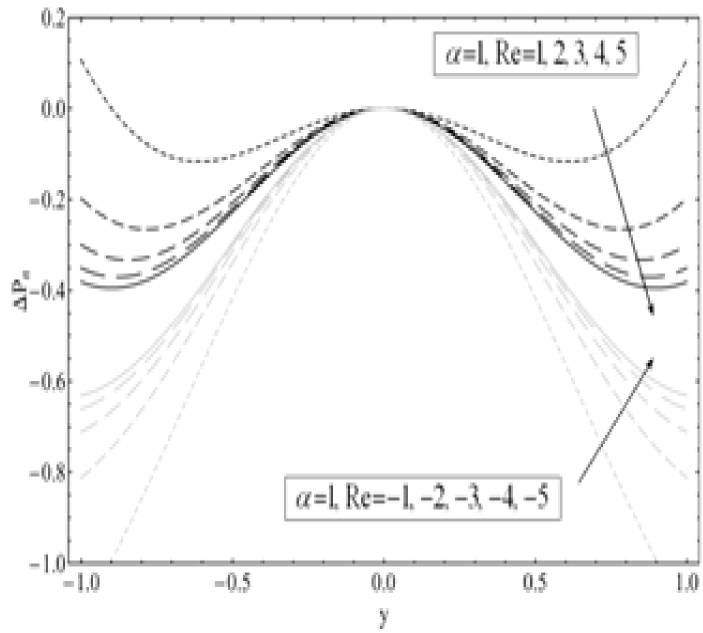

Figure 8. The effect of seepage Reynolds number on the normal pressure drop (Δp_n) for $\alpha = 1$.

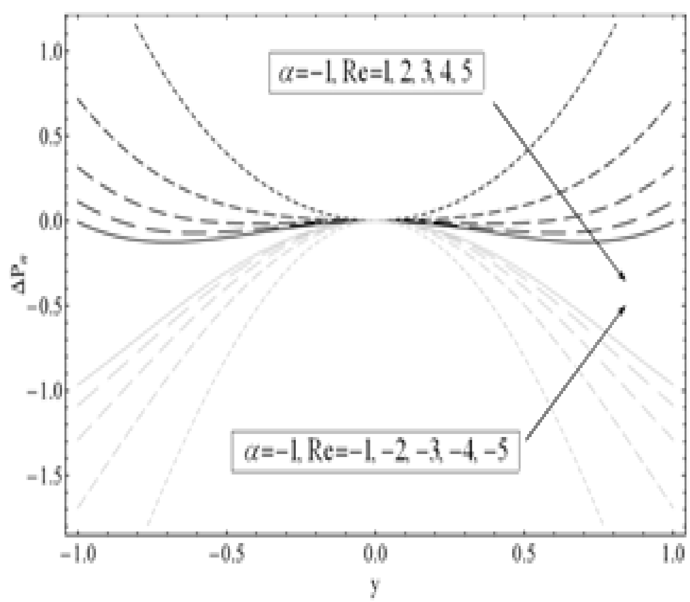

Figure 7. The effect of seepage Reynolds number on the normal pressure drop (Δp_n) for $\alpha = -1$.

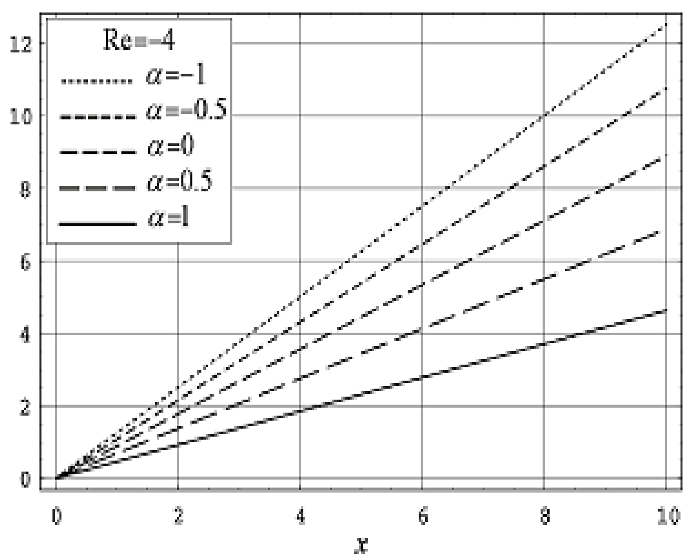

Figure 9. The effect of wall dilation rate α on the wall shear stress (τ_w) for $\mathrm{Re} = -4$.

that the absolute shear stress along the wall surface increases in proportion to *x*. Furthermore, in case of expanding wall ($\alpha > 0$), the wall shear stress (absolute value) increases as α decreases, while it increases as $|\alpha|$ increases in case of contracting wall ($\alpha < 0$). The effect of seepage Reynolds number on the wall shear stress for wall dilation rate $\alpha = -1$ and $\alpha = 1$ is illustrated in Figures 11 and 12, respectively.

Conclusions

In this article, the differential transform method (DTM) is employed to study the laminar, isothermal, incompressible

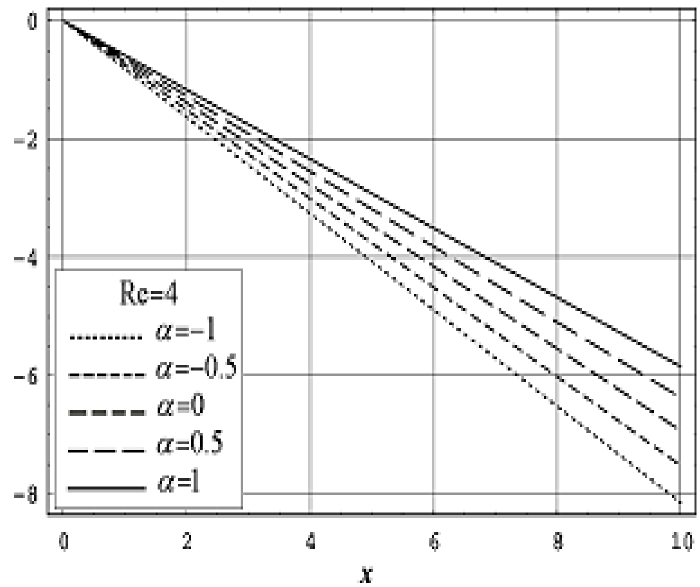

Figure 10. The effect of wall dilation rate α on the wall shear stress (τ_w) for $\mathrm{Re} = 4$.

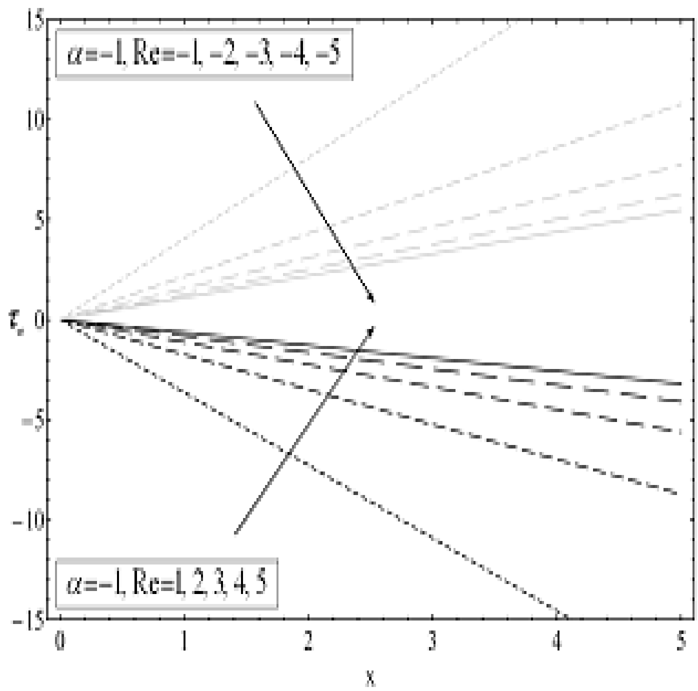

Figure 11. The effect of seepage Reynolds number on the wall shear stress (τ_w) for $\alpha = -1$.

and viscous flow in a rectangular domain bounded by two moving porous walls presented by Majdalani et al. (2002). Since the transport of biological fluids through contracting or expanding vessels is characterized by low seepage Reynolds numbers, the current study focuses

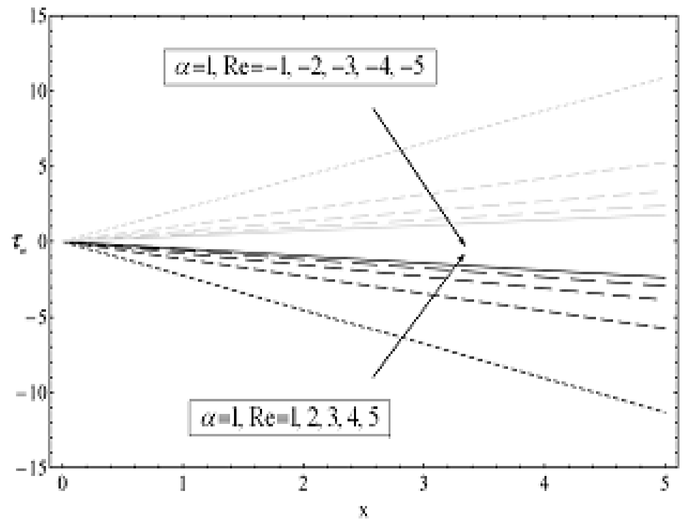

Figure 12. The effect of seepage Reynolds number on the wall shear stress (τ_w) for $\alpha = 1$.

on the viscous flow driven by small wall contractions and expansions of two weakly permeable walls. The DTM results are compared with the numerical solution obtained using shooting method, coupled with Runge–Kutta scheme. The obtained solutions, in comparison with the numerical solutions, demonstrate remarkable accuracy. The results show that the DTM does not require small parameters in the equations, so the limitations of the traditional perturbation methods can be eliminated. Besides, the reliability of the method and reduction in the size of computational domain give this method a wider applicability. Therefore, this method can be applied to many nonlinear integral and differential equations without linearization, discretization or perturbation.

REFERENCES

Arikoglu A, Ozkol I (2005). Solution of boundary value problems for integro-differential equations by using differential transform method. Appl. Math. Comput., 168: 1145–1158.

Ayaz F (2004). Solutions of the system of differential equations by differential transform method. Appl. Math. Comput., 147: 547–567.

Berman AS (1953). Laminar flow in channels with porous walls. J. Appl. Phys., 24: 1232–1235.

Biazar J, Eslami M (2010). Analytic solution for Telegraph equation by differential transform method. Phys. Lett. A., 374: 2904–2906.

Boutros YZ, Abd-el-Malek MB, Badran NA, Hassan HS (2011). Lie-group method for unsteady flows in a semi-infinite expanding or contracting pipe with injection or suction through a porous wall. J. Comput. Appl. Math, 197(2): 465-494.

Chang HN, Ha JS, Park JK, Kim IH, Shin HD (1989). Velocity field of pulsatile flow in a porous tube. J. Biomech., 22: 1257–1262.

Chen S, Chen C (2009). Application of the differential transformation method to the free vibrations of strongly non-linear oscillators. Nonlinear Anal. Real World Appl., 10(2): 881-888.

Dauenhauer EC, Majdalani J (1999). Unsteady flows in semi-infinite expanding channels with wall injection. AIAA, 5: 99–3523.

Dinarvand S (2009). A reliable treatment of the homotopy analysis method for viscous flow over a non-linearly stretching sheet in presence of a chemical reaction and under influence of a magnetic field. Cent. Eur. J. Phys., 7(1): 114–122.
Dinarvand S (2010). On explicit, purely analytic solutions of off-centered stagnation flow towards a rotating disc by means of HAM. Nonlinear Anal. Real World Appl., 11: 3389–3398.
Dinarvand S (2011a). The laminar free-convection boundary-layer flow about a heated and rotating down-pointing vertical cone in the presence of a transverse magnetic field. Int. J. Numer. Meth. Fluids., 67: 2141–2156.
Dinarvand S (2011b). Viscous flow through slowly expanding or contracting porous walls with low seepage Reynolds number: a
He JH (2006). Homotopy perturbation method for solving boundary value problems. Phys. Lett, A., 350: 87–88.
Hosseini R, Poozesh S, Dinarvand S (2011). MHD flow of an incompressible viscous fluid through convergent or divergent channels in presence of a high magnetic field. J. Appl. Math. In Press.
Kanth A, Aruna K (2009). Differential transform method for solving the linear and nonlinear Klein-Gordon equation. Comput. Phys. Commun. 180: 708–711.
Liao SJ (1992). The proposed homotopy analysis technique for the solution of non-linear problems,.Ph.D. Thesis. Shanghai Jiao Tong University.
Liu H, Song Y (2007). Differential transform method applied to high index differential-algebraic equation. Appl. Math. Comput., 184(2): 748–753.
Majdalani J, Zhou C (2003). Moderate-to-large injection and suction driven channel flows with expanding or contracting walls. ZAMM. Z. Angew. Math. Mech., 83: 181–196.
Majdalani J, Zhou C, Dawson CA (2002). Two-dimensional viscous flow between slowly expanding or contracting walls with weak permeability. J. Biomech., 35: 1399–1403.
model for transport of biological fluids through vessels. Comput. Meth. Biomech. Biomed. Eng., 14: 853–862.
Odibat Z, Bertelle C, Aziz-Alaoui MA, Duchamp GHE (2010). A multi-step differential transform method and application to non-chaotic or chaotic systems. Comput. Math. Appl. 59: 1462–1472.
Roberts SM, Shipman JS (1972). Two Point Boundary Value Problems: Shooting Methods. American Elsevier, New York, pp. 1–231.
Si X, Zheng L, Zhang X, Chao Y (2011a). Existence of multiple solutions for the laminar flow in a porous channel with suction at both slowly expanding and contracting walls. I. J. Min. Metal. Mater., 18: 494–501.
Si X, Zheng L, Zhang X, Chao Y (2011b). The flow of a micropolar fluid through a porous channel with expanding or contracting walls. Cent. Eur. J. phys., 9: 825–834.
Subhas Abel M, Mahesha N (2009). Heat transfer in a liquid film over an unsteady stretching surface with viscous dissipation in presence of external magnetic field. Appl. Math. Model., 33: 3430–3441.
Terrill RM (1973). On some exponentially small terms arising in flow through a porous pipe. Quart. J. Mech. Appl. Math., 26 (3): 347–354.
Terrill RM, Thomas PW (1969). On laminar flow through a uniformly porous pipe. Appl. Sci. Res., 21: 37–67.
Xu H, Lin ZL, Liao SJ, Wu JZ, Majdalani J (2010). Homotopy based solutions of the Navier–Stokes equations for a porous channel with orthogonally moving walls. Phys. Fluids., 22: 053601.
Zhou JK (1986). Differential Transformation and Its Applications for Electrical Circuits, Huazhong University Press. Wuhan. China. (in Chinese).