International Journal of the Physical Sciences Vol. 6(15), pp. 3808-3811, 4 August, 2011
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS10.280
ISSN 1992-1950 ©2011 Academic Journals

Full Length Research Paper

Certain studies of the Liu-Srivastava linear operator on meromorphic functions

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Accepted 24 June, 2011

Motivated by the Liu-Srivastava linear operator, we introduce here a modification of the operator of multivalent meromorphic functions in the punctured unit disk. A new subclass of analytic functions involving this operator is given. Some sufficient conditions for star-likeness, which generalize and refine some previous results were determined.

Key words: Hypergeometric function, Liu-Srivastava linear operator, convolution, meromorphic function, univalent functions, starlike functions, convex functions, γ-convex function, Carlson-Shaffer linear operator, Ruscheweyh derivative operator.

INTRODUCTION

Consider the multivalent meromorphic functions of fractional power, which are analytic in the punctured unit disk $U^* := \{z \in \mathbb{C}, 0 < |z| < 1\}$, take the structure

$$F(z) = \frac{1}{z^{p+\alpha}(1-z)^{\alpha}}, \quad (z \in U^*)$$

where $\alpha \geq 0$. Then, we obtain

$$\begin{aligned} F(z) &= \frac{1}{z^{p+\alpha}(1-z)^{\alpha}} = \sum_{n=0}^{\infty} \frac{(\alpha)_n}{n!} z^{n-p-\alpha} \\ &= \frac{1}{z^{p+\alpha}} + \sum_{n=1}^{\infty} \frac{(\alpha)_n}{n!} z^{n-p-\alpha} \\ &= \frac{1}{z^{p+\alpha}} + \sum_{n=1-p}^{\infty} \frac{(\alpha)_{n+p}}{(n+p)!} z^{n-\alpha}. \end{aligned}$$

Let $\Sigma_{p,\alpha}$ denote the class of functions of the form

$$f(z) = \frac{1}{z^{p+\alpha}} + \sum_{n=1-p}^{\infty} a_n z^{n-\alpha}, \quad \Big(\alpha \geq 0, \, p \in \mathbb{N}\Big), \qquad (1)$$

which are analytic in the punctured unit disk U^*. The convolution of two power series f, given by (1) and $g(z) = \frac{1}{z^{p+\alpha}} + \sum_{n=1-p}^{\infty} b_n z^{n-\alpha}$ is defined as the following power series:

$$f(z) * g(z) = \frac{1}{z^{p+\alpha}} + \sum_{n=1-p}^{\infty} a_n b_n z^{n-\alpha}.$$

Remark 1: In view of the Uniformization Theorem – for every simply connected Riemann surface X, there exists a conformal homeomorphism $\phi : X_0 \to X$, where X_0 is one of the three standard regions, the Riemann sphere $\overline{C}$, the complex plane $\mathbb{C}$ or the unit disk $U := \{z \in \mathbb{C}, 0 < |z| < 1\}$. The conformal type of X is elliptic, parabolic or hyperbolic, respectively. The map ϕ is called the uniformizing map. The case which was studied most is that $X \subset \mathbb{C}$ is a simply connected region, $X \neq \mathbb{C}$. Then, X is of hyperbolic type and ϕ is a univalent function in U (Nevanlinna, 1953). A function $f \in \Sigma_{p,\alpha}$ belongs to the class $\mathcal{S}_{p,\alpha}(\mu)$, the class of meromorphically multivalent starlike functions of order μ where $0 \leq \mu < p + \alpha$, if and only if $f \neq 0$, and

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AMS mathematics subject classification: 30C45, 30C50, 33C20.

$-\Re\{\frac{zf'(z)}{f(z)}\} > \mu, \ \ (z \in U^*)$. A function $f \in \Sigma_{p,\alpha}$ belongs to the class $\mathcal{C}_{p,\alpha}(\mu)$, the class of meromorphically multivalent convex functions of order μ where $0 \leq \mu < p+\alpha$, if and only if $f' \neq 0$, and $-\Re\{1+\frac{zf''(z)}{f'(z)}\} > \mu, \ \ (z \in U^*)$. A function $f \in \Sigma_{p,\alpha}$ belongs to the class $\Sigma_{p,\alpha}^{\gamma}(\mu)$, the class of meromorphic multivalent γ-convex function, where $0 \leq \mu < p+\alpha$, if and only if $f(z)f'(z) \neq 0$, and $-\Re\Big\{(1-\gamma)[\frac{zf'(z)}{f(z)}] + \gamma[1+\frac{zf''(z)}{f'(z)}]\Big\} > \mu, \ \ (z \in U^*)$.

In the present paper, we consider another new class of meromorphic multivalent function $\Sigma_{p,\alpha}^{\gamma,\epsilon}(\mu), \ \epsilon \geq 0$ for functions, $f \in \Sigma_{p,\alpha}$, which is defined by

$$-\Re\Big\{\frac{zf'(z)}{f(z)}\Big((1-\gamma)[\frac{zf'(z)}{f(z)}] + \gamma[1+\frac{\epsilon zf''(z)}{f'(z)}]\Big)\Big\} > \mu, \ \ (z \in U^*)$$

for $f(z)f'(z) \neq 0$.

For $\alpha_j \in \mathbb{C}, \ j = 1, ..., l$ and $\beta_j \in \mathbb{C}\backslash\{0,-1,-2,...\}, \ j=1,...,m,$ the generalized hypergeometric function ${}_lF_m(\alpha_1,...,\alpha_l;\beta_1,...,\beta_m;z)$ is defined by the infinite series

$${}_lF_m(\alpha_1,...,\alpha_l;\beta_1,...,\beta_m;z) := \sum_{n=0}^{\infty} \frac{(\alpha_1)_n...(\alpha_l)_n}{(\beta_1)_n...(\beta_m)_n}\frac{z^n}{n!},$$
$$(l \leq m+1; \ l,m \in \mathbb{N}_0 := \{0,1,2,..\})$$

where $(a)_n$ is the Pochhammer symbol defined by

$$(a)_n := \frac{\Gamma(a+n)}{\Gamma(a)} = \begin{cases} 1, & (n=0); \\ a(a+1)(a+2)...(a+n-1), & (n \in \mathbb{N}). \end{cases}$$

Corresponding to the function h_p^{α} given by

$$h_p^{\alpha}(\alpha_1,...,\alpha_l;\beta_1,...,\beta_m;z) := z^{-p-\alpha} \ {}_lF_m(\alpha_1,...,\alpha_l;\beta_1,...,\beta_m;z)$$

We define the following linear operator

$$H_{m,p}^{l,\alpha}(\alpha_1,...,\alpha_l;\beta_1,...,\beta_m)f(z) := h_p^{\alpha}(\alpha_1,...,\alpha_l;\beta_1,...,\beta_m;z) * f(z)$$
$$= \frac{1}{z^{p+\alpha}} + \sum_{n=1-p}^{\infty} \frac{(\alpha_1)_{n+p}...(\alpha_l)_{n+p}}{(\beta_1)_{n+p}...(\beta_m)_{n+p}}\frac{a_n z^{n+\alpha}}{(n-p)!}. \qquad (2)$$

For convenience, we write

$$H_{m,p}^{l,\alpha}[\alpha_1]f(z) := H_{m,p}^{l,\alpha}(\alpha_1,...,\alpha_l;\beta_1,...,\beta_m)f(z).$$

Clearly, when $\alpha = 0$, the linear operator defined by Equation 2 would reduce immediately to the familiar Liu-Srivastava linear operator (Liu and Srivastava 2004a, b) which was studied by Ali et al. (2008). Special cases of the Liu-Srivastava linear operator include the meromorphic analogue of the Carlson-Shaffer linear operator (Liu and Srivastava, 2001; Liu, 2003; Yang, 2001). Also, the operator in Equation 2 reduces to the operator which is analogous to the Ruscheweyh derivative operator (Yang, 1996). Recently, Srivastava et al. (2011) defined and established a linear operator corresponding to the Dziok-Srivastava linear operator by using another class of analytic functions of fractional power. Next, by applying the operator in Equation 2 on the class $\Sigma_{p,\alpha}^{\gamma,\epsilon}(\mu)$ to obtain the subclass $\Sigma_{p,\alpha}^{\gamma,\epsilon}(\mu,\alpha_1)$ as follows:

$$-\Re\Big\{\frac{zH_{m,p}^{l,\alpha}[\alpha_1]f'(z)}{H_{m,p}^{l,\alpha}[\alpha_1]f(z)}\Big((1-\gamma)[\frac{zH_{m,p}^{l,\alpha}[\alpha_1]f'(z)}{H_{m,p}^{l,\alpha}[\alpha_1]f(z)}] + \gamma[1+\frac{\epsilon zH_{m,p}^{l,\alpha}[\alpha_1]f''(z)}{H_{m,p}^{l,\alpha}[\alpha_1]f'(z)}]\Big)\Big\} > \mu, \ \ (z \in U^*)$$

for $H_{m,p}^{l,\alpha}[\alpha_1]f(z)H_{m,p}^{l,\alpha}[\alpha_1]f'(z) \neq 0$.

In order to obtain our results, we need the following lemmas:

Lemma 1: (Miller and Mocanu, 1978) Let $\phi(u,v)$ be a complex function, $\phi : D \to \mathbb{C}, \ D \in \mathbb{C} \times \mathbb{C}$ and let $u = u_1 + iu_2, v = v_1 + iv_2$.. Suppose that the function $\phi(u,v)$ satisfies the following conditions:

(i) $\phi(u,v)$ is continuous in D,
(ii) $(1,0) \in D$ and $\Re(\phi(1,0)) > 0$,
(iii)$\Re(\phi(iu_2,v_1)) \leq 0, \forall \ (iu_2,v_1)) \in D$ such that $v_1 \leq -\frac{1+u_2^2}{2}$. Let $p(z) = 1 + p_1(z) + p_2z^2 + ...$ be analytic in U such that $(p(z), zp'(z)) \in D$ for all $z \in U$. If $Re\{\phi(p(z), zp'(z)) > 0\}, \ z \in U,$ then $\Re\{p(z)\} > 0, \ z \in U$.

Lemma 2: (Oros, 2008) Let n be a positive integer. Suppose that functions $A, B, C, D : U \to \mathbb{C}$ satisfy $\Re\{B(z)\} \geq 0, \ \Re\{A(z)\} \geq -\Re\{\frac{n^2}{2}B(z) + \frac{n}{2}C(z)\}$ and $\Re\{D(z)\} \leq \Re\{\frac{n^2}{4}B(z) + \frac{n}{2}C(z)\}$. If $p \in \mathcal{H}[1,n] := 1 + a_nz^n + ...$ and $\Re\{A(z)p^2(z) - B(z)(zp'(z))^2 + C(z)(zp'(z)) + D(z)\} > 0$, then $\Re\{p(z)\} > 0$.

Lemma 3: (Miller and Mocanu, 1981) Let the function $\Psi : \mathbb{C}^2 \to \mathbb{C}$ satisfy $\Re\{\Psi(ix,y)\} \leq 0$ for all real x and for all real $y, \ y \leq -\frac{1+x^2}{2}$. If $p(z) = 1 + p_1z + ...$ is analytic in the unit disk U and $\Re\{\Psi(p(z), zp'(z))\} > 0$ then $\Re\{p(z)\} > 0$ for $z \in U$.

RESULTS

We begin with the following theorem:

Theorem 1: Let $f \in \Sigma_{p,\alpha}$,

$f(z)f'(z) \neq 0, \ z \in U^*$ and we let a function $\Phi(t), \ t > 0$ given by:

$$\Phi(t) := -[1-\gamma(1-\epsilon)]t^{2\theta}\cos\theta\pi - \gamma\epsilon\frac{(1+t^2)}{2} - \mu,$$

where $\gamma \geq 0, 0 \leq \epsilon \leq 1$ and satisfy $2\gamma(1-\epsilon) > \mu + 1$.

If for the aforestated real γ, ϵ, μ and θ satisfy that $f \in \Sigma_{p,\alpha}^{\gamma,\epsilon}(\mu, \alpha_1)$ and $\Phi(t) \leq 0,\ t > 0$, then

$$-\Re\{\frac{zH_{m,p}^{l,\alpha}[\alpha_1]f'(z)}{H_{m,p}^{l,\alpha}[\alpha_1]f(z)}\} > 0, \quad (z \in U^*).$$

Proof: Our aim is to satisfy Lemma 1. First we let $f \in \Sigma_{p,\alpha}$ and set $q(z) := -\frac{zH_{m,p}^{l,\alpha}[\alpha_1]f'(z)}{H_{m,p}^{l,\alpha}[\alpha_1]f(z)}, \quad (z \in U^*)$ and

$$H(z) = -\left\{\frac{zH_{m,p}^{l,\alpha}[\alpha_1]f'(z)}{H_{m,p}^{l,\alpha}[\alpha_1]f(z)}\left((1-\gamma)[\frac{zH_{m,p}^{l,\alpha}[\alpha_1]f'(z)}{H_{m,p}^{l,\alpha}[\alpha_1]f(z)}] + \gamma[1+\frac{\epsilon zH_{m,p}^{l,\alpha}[\alpha_1]f''(z)}{H_{m,p}^{l,\alpha}[\alpha_1]f'(z)}]\right)\right\} - \mu$$
$$= -[1-\gamma(1-\epsilon)]q^2(z) + [\gamma(1-\epsilon)]q(z) + \gamma\epsilon zq'(z) - \mu.$$

A computation yields

$$H(z) = -\left\{\frac{zH_{m,p}^{l,\alpha}[\alpha_1]f'(z)}{H_{m,p}^{l,\alpha}[\alpha_1]f(z)}\left((1-\gamma)[\frac{zH_{m,p}^{l,\alpha}[\alpha_1]f'(z)}{H_{m,p}^{l,\alpha}[\alpha_1]f(z)}] + \gamma[1+\frac{\epsilon zH_{m,p}^{l,\alpha}[\alpha_1]f''(z)}{H_{m,p}^{l,\alpha}[\alpha_1]f'(z)}]\right)\right\} - \mu$$
$$= -[1-\gamma(1-\epsilon)]q^2(z) + [\gamma(1-\epsilon)]q(z) + \gamma\epsilon zq'(z) - \mu.$$

Next, if we put that $q(z) = u, zq'(z) = v$, and set

$$\phi(u,v) := -[1-\gamma(1-\epsilon)]u^{2\theta} + [\gamma(1-\epsilon)]u + \gamma\epsilon v - \mu,$$

we can observe that $\phi(u,v)$ is continuous in $D = (\mathbb{C}\backslash\{0\}, \mathbb{C})$ with $(1,0) \in D$ and $\Re\{\phi(1,0)\} = 2\gamma(1-\epsilon) - \mu - 1 > 0$, that is the conditions (i) and (ii) of Lemma 1 are satisfied. In addition, for $(iu_2, v_1) \in D$ such that $v_1 \leq -\frac{1+u_2^2}{2}$, we pose

$$\Re\{\phi(iu_2, v_1) = -[1-\gamma(1-\epsilon)]|u_2|^{2\theta}\cos\theta\pi + [\gamma(1-\epsilon)]|u_2|\cos\frac{\pi}{2} + \gamma\epsilon v_1 - \mu$$
$$\leq -[1-\gamma(1-\epsilon)]|u_2|^{2\theta}\cos\theta\pi - \gamma\epsilon[\frac{1+|u_2|^2}{2}] - \mu$$
$$\leq 0,$$

This shows that condition (iii) of Lemma 1 is satisfied. Then from Lemma 1, we readily arrive at the conclusion asserted by Theorem 1. In view of Theorem 1, if we put $\alpha = 0, l = 2, m = 1, \alpha_1 = a, \alpha_2 = 1$ and $\beta_1 = c$, then we obtain the next result for Carlson-Shaffer's linear operator; Liu and Srivastava (2001) and Liu (2003).

Corollary 2: Let the assumptions of Theorem 1 hold. Then

$$-\Re\{\frac{zL(a,c)f'(z)}{L(a,c)f(z)}\} > 0, \quad (z \in U^*).$$

Again in virtue of Theorem 1, if we set $\alpha = 0, l = m+1, \alpha_l = 1$ and $\frac{(\alpha_1)_{n-1}...(\alpha_{l-1})_{n-1}}{(\beta_1)_{n-1}...(\beta_m)_{n-1}} = 1$, then we obtain the next result (Goodman, 1983; Duren, 1983).

Corollary 3: Let the assumptions of Theorem 1 hold. Then

$$-\Re\{\frac{zf'(z)}{f(z)}\} > 0, \quad (z \in U^*).$$

In the next result we introduce another sufficient condition for star-likeness by using Lemma 2.

Theorem 4: Let $f \in \Sigma_{p,\alpha}, f(z)f'(z) \neq 0,\ z \in U^*$. If

$$\Re\left\{\frac{zH_{m,p}^{l,\alpha}[\alpha_1]f'(z)}{H_{m,p}^{l,\alpha}[\alpha_1]f(z)}[\frac{zH_{m,p}^{l,\alpha}[\alpha_1]f'(z)}{H_{m,p}^{l,\alpha}[\alpha_1]f(z)} - (1+\frac{zH_{m,p}^{l,\alpha}[\alpha_1]f''(z)}{H_{m,p}^{l,\alpha}[\alpha_1]f'(z)})]\right\} > 0,$$

Then

$$-\Re\{\frac{zH_{m,p}^{l,\alpha}[\alpha_1]f'(z)}{H_{m,p}^{l,\alpha}[\alpha_1]f(z)}\} > 0, \quad (z \in U^*).$$

Proof: Letting

$$p(z) = -\Re\{\frac{zH_{m,p}^{l,\alpha}[\alpha_1]f'(z)}{H_{m,p}^{l,\alpha}[\alpha_1]f(z)}\}.$$

Assume that $n = 1,\ A(z) = 0, B(z) = 0, C(z) = 1\ and\ D(z) = 0$, then the result follows from Lemma 2.

Theorem 5: Let $f \in \Sigma_{p,\alpha},\ f(z)f'(z) \neq 0,\ z \in U^*$. If $f \in \Sigma_{p,\alpha}^{\gamma,\epsilon}(\mu, \alpha_1)$

$$\Re\left\{\gamma[\frac{zH_{m,p}^{l,\alpha}[\alpha_1]f'(z)}{H_{m,p}^{l,\alpha}[\alpha_1]f(z)}]^2 - \gamma\frac{zH_{m,p}^{l,\alpha}[\alpha_1]f'(z)}{H_{m,p}^{l,\alpha}[\alpha_1]f(z)}(1+\frac{zH_{m,p}^{l,\alpha}[\alpha_1]f''(z)}{H_{m,p}^{l,\alpha}[\alpha_1]f'(z)})\right\} - \mu > 0,$$

Then

$$-\Re\{\frac{zH_{m,p}^{l,\alpha}[\alpha_1]f'(z)}{H_{m,p}^{l,\alpha}[\alpha_1]f(z)}\} > 0, \quad (z \in U^*).$$

Proof: Letting

$$p(z) = -\Re\{\frac{zH^{l,\alpha}_{m,p}[\alpha_1]f'(z)}{H^{l,\alpha}_{m,p}[\alpha_1]f(z)}\}.$$

Assume that $\epsilon = 1,\ n = 1,\ A(z) = \gamma, B(z) = 0, C(z) = \gamma,\ and\ D(z) = \mu,$ with $\mu \leq \frac{\gamma}{2}$ then the result follows from Lemma 2. Finally, by applying Lemma 3, we obtain the next result which describes new sufficient conditions for star-likeness subclass containing the operator (2).

Theorem 6: Let $f \in \Sigma_{p,\alpha},\ f(z)f'(z) \neq 0,\ z \in U^*$. If

$$-\Re\Big\{1 + \frac{zH^{l,\alpha}_{m,p}[\alpha_1]f''(z)}{H^{l,\alpha}_{m,p}[\alpha_1]f'(z)}\Big\} > 0,$$

Then

$$-\Re\{\frac{zH^{l,\alpha}_{m,p}[\alpha_1]f'(z)}{H^{l,\alpha}_{m,p}[\alpha_1]f(z)}\} > 0,\ \ (z \in U^*).$$

Proof. Assuming

$$p(z) = -\Re\{\frac{zH^{l,\alpha}_{m,p}[\alpha_1]f'(z)}{H^{l,\alpha}_{m,p}[\alpha_1]f(z)}\}.$$

Define a function $\Psi : \mathbb{C}^2 \to \mathbb{C}$ as follows

$$\Psi(p(z), zp'(z)) := p(z) - \frac{zp'(z)}{p(z)},$$

then by the hypotheses of the theorem, we obtain that

$$\Re\{\Psi(p(z), zp'(z))\} = -\Re\Big\{1 + \frac{zH^{l,\alpha}_{m,p}[\alpha_1]f''(z)}{H^{l,\alpha}_{m,p}[\alpha_1]f'(z)}\Big\} > 0.$$

In order to apply Lemma 3, we must verify that $\Re\{\Psi(ix; y)\} \leq 0$ whenever x and y are real numbers such that $y \leq -\frac{(1+x^2)}{2}$. We have $\Re\{\Psi(ix; y)\} = \Re\{ix)\} - y\Re\{\frac{1}{ix}\} \leq -[x^2 + \frac{(1 + x^2)}{2x^2}] < 0.$ Hence by Lemma 3, we conclude that $\Re\{p(z)\} > 0$.

CONCLUSIONS

From the aforementioned, we conclude that by using the same method of this work, we can find the sufficient conditions for convexity, close to convexity, $\gamma-$convexity, uniformly star-likeness and spiral-likeness (Darus and Ibrahim, 2008; Ibrahim and Darus, 2009a). Other studies related to subordination and superodinations can be read up in Ibrahim and Darus (2010a, b), and Ibrahim and Darus (2009b).

ACKNOWLEDGEMENTS

The work here was supported by UKM-ST-06-FRGS0107-2009, MOHE Malaysia. The first author has recently obtained her Phd degree under the supervision of Prof. Maslina Darus. We would like to thank the referee for giving us some valuable suggestions to improve the presentation of the article.

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