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Full Length Research Paper

Bandelet based vector quantization coder design for gray scale image compression

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Providing efficient transform-based representations of images is an important problem in the area of image compression. In this paper, the geometrical flow of an image is analyzed by using the Bandelet transform and a new Bandelet based image coding scheme is proposed. First, the bandelet transform of the image is computed. The motivation behind the usage of the bandelet transform is that the geometry of the image is summarized with local clustering of similar geometric vectors, the homogeneous areas being taken from the quad tree structure. This will allow us to search for areas in the image that are geometrically similar to each other. Then the spatial and geometric interpixel redundancies present in the bandelet transformed coefficients are removed. The psycho-visual redundancies are removed using simple vector quantization (VQ) process. Finally, the consequential coefficients are encoded using Huffman encoder. Our experiments demonstrate that the proposed scheme achieves near-optimal rate-distortion performance for natural images. It is reported that a gain in the bit-rate of about 0.83 bpp over the wavelet based algorithms is achieved yielding similar quality factor.

Key words: N relations join, query optimization, distributed database, rank aware query, top K.

INTRODUCTION

Natural images possess edges, textured regions and geometric properties. The 2-D discrete wavelet transform (DWT) is the most important image compression technique of the last decade (Le Pennec and Mallat, 2003; Falzon and Mallat, 1998) because it provides a sparse multi-resolution representation of natural images due to the presence of vanishing moments(ω) in the High Pass (HP) filters (enforced by imposing zeros at ω = 0) (Mallat, 1997). This method is conceptually simple and has a low computational complexity because of the simple separable one-dimensional (1-D) filtering and sub-sampling operations. For these reasons, the 2-D WT has been adopted in the image compression standard JPEG-2000. However, the performance of the 2-D WT is limited by the spatial isotropy of the basis functions and the construction only along the horizontal and vertical directions, which does not provide enough directionality.

Therefore, the standard 2-D WT fails to provide a sparse representation of oriented 1-D discontinuities (edges or contours) in images (Vetterli and Kovacevic, 1995). These features are characterized by a geometrical coherence that is not properly captured by the isotropic wavelet basis functions. In the case of an edge, however, where a singularity extends along a contour, the number of 2-d wavelet basis functions overlapping the singularity grows exponentially at finer scales; many wavelet coefficients are required to reconstruct even a simple straight edge (Dahmen and Schneider, 2000). The abundance of significant coefficients describing geometry is not an immediate barrier to effective wavelet-domain image compression. There is, in fact, a strong coherency among the coefficients which is imposed by the structure of the geometry.

Several recently proposed directional approaches use the lifting scheme in image compression algorithms. This scheme has been exploited by Gerek and Cetin (2006) where transform directions are adapted pixel-wise throughout images. A similar adaptation is used by Chang

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and Girod (2006) but with eleven directions. However, even though these methods are computationally efficient and provide good compression results, they show a weaker performance when combined with zero-tree based compression algorithms. To enhance wavelets representations, Ding et al. (2007) have proposed to approximate the wavelet coefficients using adaptive vector quantization techniques. Following the work of Sweldens (Sweldens, 1996; Pennec and Mallat, 2000) on adaptive lifting schemes, new lifting algorithms have also been proposed to predict wavelet coefficients from their neighbors. These works are mostly algorithmic and do not provide mathematical bounds. They use the fact that wavelet coefficients inherit some regularity from the image geometric regularity.

Filter bank techniques uses windowing of the sub-band coefficients. It may lead to blocking effects. To overcome this problem, Do and Vetterli (2001) proposed the Pyramidal Directional Filter Bank (PDFB). This approach overcomes the block based approach of the curvelet by using a directional filter bank (Bamberger and Smith, 1992). Many adaptive geometric representations have also been proposed recently with good results in image processing. Instead of decomposing an image in a fixed *a priori* basis, an adaptive algorithm shall be used to modify the image representation. Adaptive techniques are techniques where the directional component of an image is adaptively estimated and as such, the transform is steered based on the estimate. For example the Bandelet transform of Pennec and Mallat (2000) links the significant wavelet coefficients along a discontinuity and represents it as a smooth 1-D curve geometry computed from the image.

In this work, the problem of image compression is addressed from a new angle. Instead of using the multiresolution theory, it is projected as a problem of a geometrical similarity optimization. This work introduces a new class of bases, called bandelet bases, which decompose the image along multi-scale vectors that are elongated in the direction of a geometric flow. This geometric flow indicates the direction in which the image grey levels have regular variations. The image decomposition in a bandelet basis is implemented with a fast sub-band filtering algorithm. Bandelet bases lead to optimal approximation rates for geometrically regular images. Comparisons are made for image compression with wavelet bases.

BANDELET BASICS

The Bandelet transform (Mallat and Peyre, 2007) exploits geometric regularity that is found in images by constructing orthogonal vectors that are elongated in the direction where the function has a maximum of regularity. Bandelet bases, which decompose the image along multi-scale vectors that are elongated in the direction of a geometric flow. This geometric flow indicates directions in which the image grey levels have regular variations. Bandelets in a region Ω are computed by applying a bandeletization to warped wavelets, which are separable along a fixed direction (horizontal or vertical) and along the flow lines. The geometric flow in a region Ω_i is a vector field $\vec{V}_i[m,n]$ defined over the image sampling grid G. If the flow is parallel vertically then $\vec{V}_i[m,n]=\vec{V}_i[m]=(1,C_i^{'}[m])$ where $C_i^{'}[m]$ measures an average relative displacement of the image gray levels Ω_i in along the line m with respect to the line n □1. If the geometric flow is parallel horizontally in□ Ω_i□□, then□ $\vec{V}_i[m,n]=(C_i^{'}[m],1)$. Given the original image sample values $f[m,n]$, at each grid point $G_i[k_1,k_2]$ the re-sampling computes an interpolated image value that is written $V_i[k_1,k_2]$. For a flow parallel vertically, the grid points $(k_1,k_2+C_i[k_1])\in\Omega_i$ are obtained with one-dimensional translations along the y direction of the integer sampling grid $(m,n)\in\Omega_i$. If the flow is parallel horizontally then the one-dimensional translation is along the x direction. For image compression and noise removal applications, the geometric flow is optimized with fast algorithms, so that the resulting bandelet basis produces a minimum distortion with $O(n^2(\log_2(n^2))$ operations for an image of n^2 pixels, because the geometry is structured by aggregating nearly independent building blocks. This optimization requires establishing the link between the image geometry and the distortion-rate of the image coder. A full detailed description of the bandelet basics is discussed by some studies (Le Pennec and Mallat, 2005; Peyre and Mallat, 2005).

THE BANDELET TRANSFORM

The input image is decomposed using the Warped Haar transform based on an orthogonal basis formed by the translation and dilation of three mother wavelets for the horizontal, vertical and diagonal directions. Once the transform is applied, the quad-tree is computed by dividing the image into dyadic squares. For each square in the quad-tree the optimal geometrical direction is computed by the minimization of a Lagrangian. The Lagrangian approach proposed by Ramachandran and Vetterli (1993) is used which finds the best basis that minimizes dR □λR, where λ □is a Lagrange multiplier, d is the distortion and R is the number of bits. If dR is convex, which is usually the case; by letting λ vary dR shall be minimized. If dR is not convex, then this strategy leads to a dR that is at most a factor 2 larger than the minimum. Even in squares with no geometric features (on which the function is constant), the algorithm chooses some arbitrary orientation. This is because in these squares the function does not have zero mean, so a bandelet transform (with any direction) is better than leaving the data untransformed. This situation does not appear in the wavelet-bandelet algorithm, since in flat areas, a wavelet transform has zero mean. Then a projection of the transform coefficients along the optimal direction is performed. Finally a 1D haar transform is carried on the projected coefficients. Particularly, the size and the optimal geometrical direction of each square will be used as criteria to study the similarity. The inverse discrete bandelet transform computes the image values on the original integer sampling grid □m,n□□from the sample values $V_i[k_1,k_2]$ along the flow lines in each Ω_i where $(k_1,k_2)\in z^2$.

THE PROPOSED CODING SCHEME

To concentrate on the properties of the bandelet transform itself, a relatively simple transform coder with a quantization and entropy coding of all coefficients is used in this paper. The input image is decomposed into the bandelet basis associated to the optimized

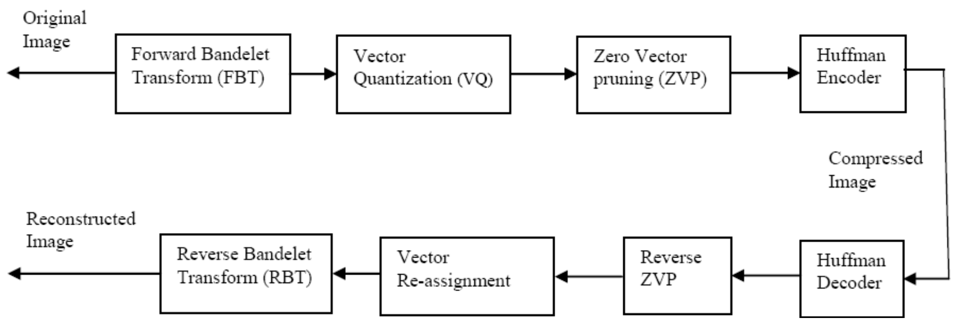


Figure 1. The proposed BVQC scheme.

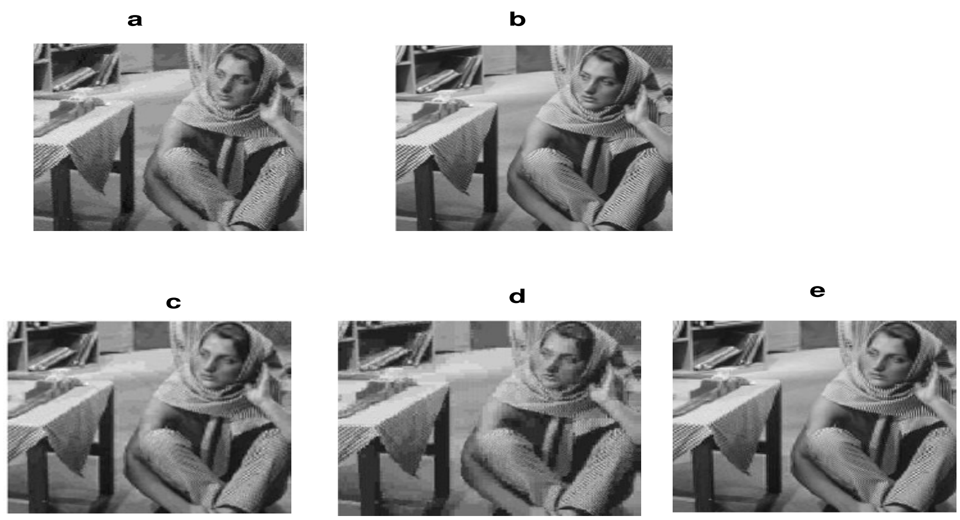


Figure 2. Original image and respective reconstructed images obtained using the proposed NSVQ, existing WVQC, WPVQC and WPLBGC methods. **a.** 256 x 256 Original Image **b:** Reconstructed image using the proposed BVQC. **c.** Reconstructed Image using WVQC. **d.** Reconstructed Image using WVQC **e.** Reconstructed Image using WPLBGC(PSNR: 24.21db).

partition and its geometric flow. This process of Bandeletization of an image results in psycho-visual and inter-pixel redundancies. Therefore to reduce the inter-pixel redundancy the bandeletized coefficients are subject to Zero Vector Pruning (ZVP) process. ZVP identifies all non-zero vectors along with their row indices thereby eliminating redundant zero vectors. Then the psycho-visual redundancy is reduced using a simple vector quantization process. The quantizer identifies the correlation among either along the row or the column vector. The correlation coefficient r is given in Equation 1.

$$r = \frac{\sum_{m}\sum_{n}(A_{mn}-\bar{A})(B_{mn}-\bar{B})}{\sqrt{\left(\sum_{m}\sum_{n}(A_{mn}-\bar{A})^2\sum_{m}\sum_{n}(B_{mn}-\bar{B})^2\right)}} \quad (1)$$

Where A,B are the row vectors in the transformed input matrix; $\bar{A}$, $\bar{B}$ are the respective mean values of the row vectors A, B. For all test images used in this work the correlation among the row vectors is high. Hence, the near similar row vectors are clustered together and code book consisting of a representative code vector for each cluster is generated. The quantized coefficients are coded with Huffman encoder. The compressed image is decompressed using Huffman decoder, vector reassignment procedure followed by the reverse Bandelet transform to reconstruct the image. The flow of the proposed work is depicted in Figure 1.

EXPERIMENTAL RESULTS

To evaluate the performance of this bandelet based compression algorithm a comparison is made with the same coder applied to a wavelet and wavelet packet-based compression. Figure 2a shows 256 x 256 Barbara input image. Figure 2b shows the respective recon-structed image of the Barbara image in Figure 2a using the proposed Bandelet based VQ Coder [BVQC] while Figures 2c, d and e depict the same using wavelet

Table 1. Performance of the proposed work at various window sizes for thecameraman image.

S/No. window	Size (w)	MSE	PSNR (db)	Ratio	Space (%)	Rate(bpp)	Computation time (sec)
1	4	252.12	24.11	11.16	91.04	1.43	8.79
2	8	183.56	25.49	11.14	91.02	1.44	33.23

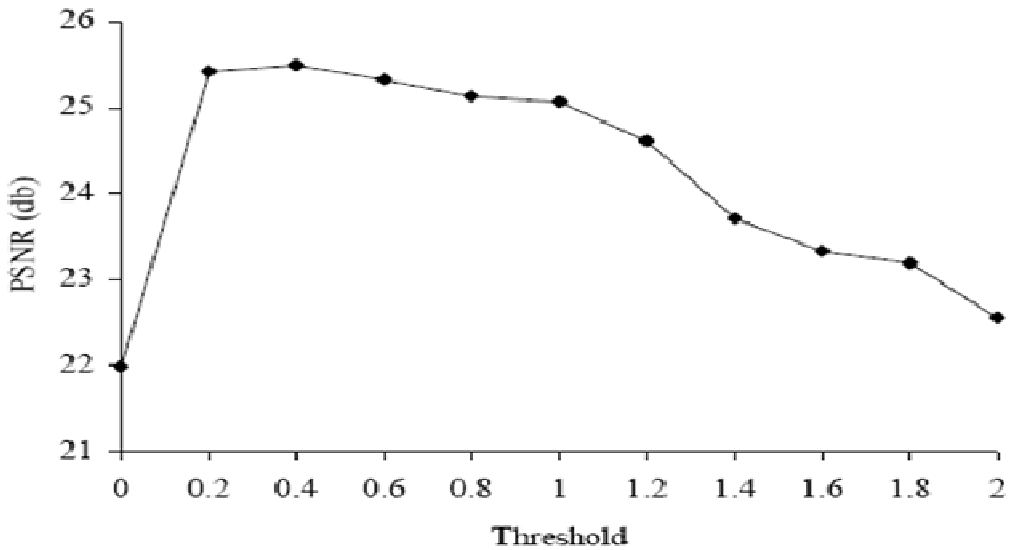


Figure 3. Threshold selection for Lagrangian computation.

based VQ Coder [WVQC], Wavelet-Packet based VQ Coder [WPVQC] and the Wavelet based Linde - Buzo - Gray Coder [WPLBGC] respectively.

It is observed from the figure (Figure 2) that WVQC (Figure 2c) suffers from more pronounced blocking artifacts. And WPVQC (Figure 2d) eliminates blocking artifacts but detail information is not preserved substantially. Though the effect of blocking artifacts is reduced using WPLBGC (Figure 2e), it is observed that this method suffers from smoothening effect and hence ignoring the detail information. The proposed BVQC (Figure 2b) because of its geometry preserving nature preserves details and reduces blocking artifacts seen in the reconstructed image thereby improving the subjective psycho-visual quality remarkably. Barbara image is purposely chosen as the test image since it contains more detail information which helps in the measure of the subjective evaluation of the quality of the reconstructed images using the proposed and other existing methods, quality of reconstructed image using WPLBGC is low, we increase quality of compressing Image is shown in Figure 2.

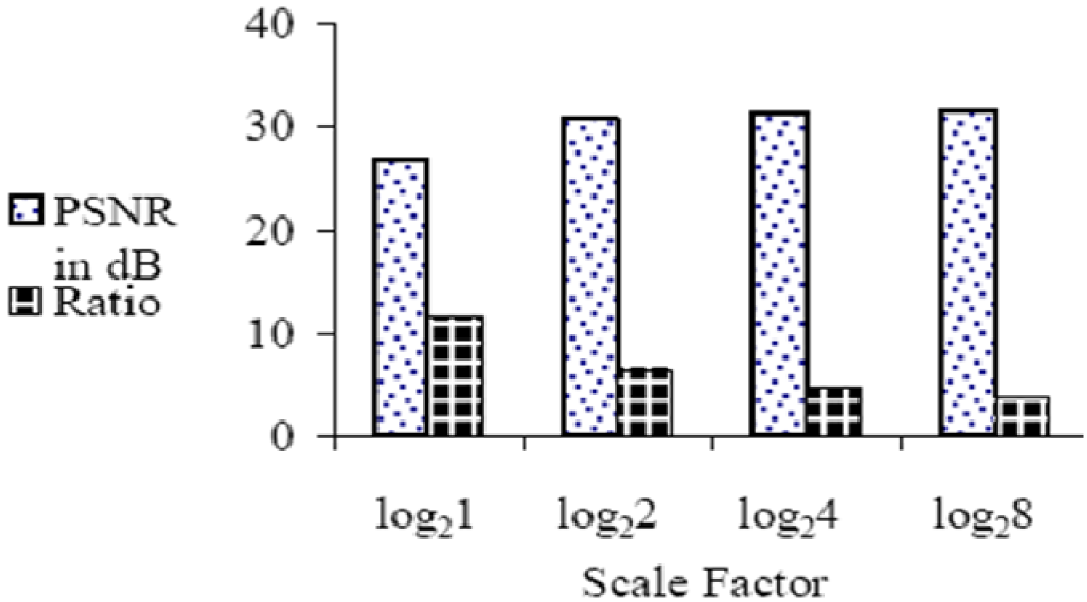
The implementation stage of the proposed work includes fixing the minimum window size for the dyadic squares, the threshold selection for Lagrangian computation and a scale factor for Bandeletization. The initialization process is as follows:

a) Window size (w) selection: The proposed work is tested with the two possible window sizes, w=4 and w=8. The results are recorded in Table 1. it is observed from this table that though the computation time increases with an increase in window size, the reconstructed image quality is improved significantly with a negligible change in the bit rate. Hence the window size w=8 is used in this work.

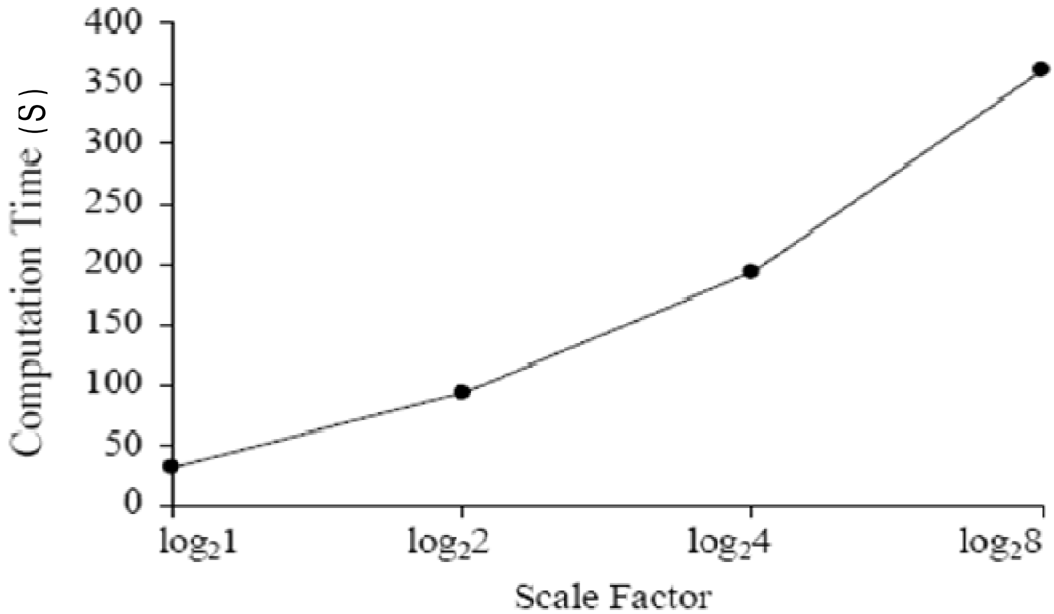
b)Threshold (T) selection: The graph shown in depicts the impact of the Threshold (T) value for Lagrangian computation. It is obvious from the graph that PSNR increases as T increases to a certain limit (0 to 0.4) and start decreasing with further increase in T.It is seen in Figure 3. That the gain in PSNR is maximum with T=0.4.

c) Scale factor: Selecting an optimal value for the scale factor is influenced by parameters like compression ratio and the computation time is shown in Figure 4. The impact of scale factor on the gain in quality (PSNR) and the compression ratio is shown in Figure 4a. It is evident that the quality of the reconstructed image is not affected radically. But the compression ratio varies from 1: 11.54 to 1: 3.79 as the scale factor is varied logarithmically as shown for the cameraman image. Also it shall be noted from Figure 4b.

That computation time increases as the scale factor increases logarithmically. The performance of the proposed work is analyzed with w = 8; T = 0.4 and a scale



A



B

Figure 4. The impact of Scale factor on various performance parameters. (A). Scale factor Vs PSNR and ratio. (B). scale factor Vs computation time

factor log21. The results are tabulated for various images in Table 2. Compression ratio is the ratio of the input image size to the compressed image size. Space saving gives the amount of memory space saved due to compression. It is given in Equation 2.

$$SpaceSaving = (1-(1/C_R))\times 100 \qquad (2)$$

where, C_R is the compression ratio.

It is perceived from this Table 2 that on an average the proposed work gives 1:10 compression ratio leading to 1.5 bits per pixel representation for the compressed file with 90% space saving. Also it maintains the PSNR value to about 28 dB. Table 3 illustrates the result of comparison of the proposed work with the existing methods for the Barbara image. It is inferred that as compared with the other existing methods the proposed work gives a high compression ratio of 1: 9.67 resulting in a gain in the bit

Table 2. Performance analysis of the proposed BVQC for various images.

S/No.	Image	MSE	PSNR (db)	Ratio	Space (%)	Rate (bpp)
1	Pepper	96.27	28.30	11.55	91.34	1.39
2	Baboon	78.50	29.18	10.25	90.24	1.56
3	Cameraman	183.56	25.49	11.14	91.02	1.44
4	Rice	70.27	29.66	10.76	90.70	1.49
5	Bird	56.07	30.64	10.14	90.14	1.58
6	Barbara	246.48	24.21	9.67	89.66	1.65
7	Zelta	82.59	28.96	9.78	89.78	1.64
8	Keyboard	56.55	30.61	9.92	89.92	1.61

Table 3. Performance comparison of the proposed BVQC with existing methods for the Barbara image for a quality factor of 24.21db.

S/No.	Algorithm	Ratio	Space (%)	Rate (bits/pixel)
1	Proposed	9.67	89.66	1.65
2	Wavelet Packet-Kmean	7.28	86.26	2.20
3	Wavelet-Kmean	7.70	87.01	2.08
4	Wavelet Packet-LBG	5.64	82.27	2.84

rate of 0.83 bpp with 24.21 db quality factor.

Conclusion

A novel anisotropic transform for images that use separable filtering in many directions (not only horizontally and vertically) is proposed in this work. The associated basis functions, called Bandelets, have geometric flow in the direction where the function has a maximum of regularity. These transforms retain critical sampling and the simplicity of the filter design from the standard wavelet transform (WT). Still, multi-directionality and anisotropy overcome the weakness of the standard WT in presence of edges or contours, that is, they allow for sparser representations of these directional anisotropic features. Because of the critical sampling and geometrical regularity bandelets are applied in the approximation and compression methods based on Lagrangian optimization. The new transform provides a convenient orthogonal basis with the functions spanning different scales analogously to those of the digital wavelet transform but aligned anisotropically along the dominant geometrical regularity. This new transform is reversible and introduces redundancies. At the same time, the compression algorithm obtained as a combination of Bandelets, quantization and coding outperforms the state-of-the-art methods in terms of both the numerical criterion and the subjective visual quality increase 7.5%

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