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Short Communication

Multisoliton solutions to Hirota-Satsuma KdV system

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In this paper, we obtain one and two soliton solutions to Hirota-Satsuma KdV system with the aid of tanh method and the multiple exp method. We also obtain some rational solutions by using a rational ansatz. The system Hirota-Satsuma KdV system is reduced to a single nonlinear partial differential equation which is solved by the described methods.

Key words: Hirota-Satsuma KdV system, tanh method, multiple exp method one soliton solution, two soliton solution, HS coupled system.

INTRODUCTION

Hirota and Satsuma (1981) proposed the very first coupled KdV system, which describes interactions of two long waves with different dispersion relations. They then constructed the three-soliton solutions and five conserved quantities for this system. Subsequently, Hirota and Satsuma (1982), showed that the coupled KdV system is the four-reduction of the celebrated KP hierarchy and its soliton solutions can be derived from those of the KP equation.

The Hirota-Satsuma KdV system (HS-KdV system) reads

$$\begin{cases} u_t = \dfrac{1}{2}u_{xxx} + 3uu_x - 6vv_x. \\ v_t = -v_{xxx} - 3uv_x. \end{cases} \tag{1}$$

Our aim is to find one or two soliton solutions to system (1). To this end, we solve the second equation in (1) for u to obtain

$$u = -\frac{v_t + v_{xxx}}{3v_x} \tag{2}$$

We now substitute (2) into (1) and we get following pde:

$$-\frac{\partial}{\partial t}\frac{v_t+v_{xxx}}{3v_x} = -\frac{1}{2}\frac{\partial^3}{\partial x^3}\left(\frac{v_t+v_{xxx}}{3v_x}\right) - 3\frac{v_t+v_{xxx}}{3v_x}\frac{\partial}{\partial x}\left(\frac{v_t+v_{xxx}}{3v_x}\right) - 6vv_x = 0. \tag{3}$$

Thus, it is enough to find exact solutions to Equation (3). We will make use of two methods: the tanh method for one soliton solutions and and the multiple exp method for two-soliton solutions.

ONE SOLITON SOLUTIONS

To find one soliton solutions, we apply the tanh method. In view of this method, we seek solutions to Equation (3) in the form

$$v(x,t) = p + q\tanh^2(kx + \lambda t). \tag{4}$$

Substitution of Equation (4) into (3) gives a polynomial equation in the variable $z = \exp(kx + \omega t)$. Equating to zero the coefficients of the different powers of z gives following algebraic system:

$$\begin{cases} -244k^4 - 4k\lambda + 3pq + 63q^2 = 0. \\ -20k^4 - 4k\lambda + 3pq + 7q^2 = 0. \\ -4k^4 - 4k\lambda + 3pq + 3q^2 = 0. \\ 92k^4 - 4k\lambda + 3pq - 21q^2 = 0. \end{cases} \tag{5}$$

Solutions to system (5) are

$$p = \pm\frac{2(2k^3 - \lambda)}{3k} \text{ and } q = \mp 2k^2 \tag{6}$$

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Thus, the following are solutions to Equation (3)

$$v(x,t) = \pm\frac{2(2k^3-\lambda)}{3k} \mp 2k^2\tanh^2(kx+\lambda t) \tag{7}$$

From Equations (2) and (7) we obtain following solutions to HS-KdV system (1):

$$u(x,t) = \frac{8k^3-\lambda}{3k} - 4k\tanh^2(kx+\lambda t) \text{ and } v(x,t) = -\frac{2(2k^3-\lambda)}{3k} + 2k^2\tanh^2(kx+\lambda t)$$

And

$$u(x,t) = \frac{8k^3-\lambda}{3k} - 4k\tanh^2(kx+\lambda t) \text{ and } v(x,t) = \frac{2(2k^3-\lambda)}{3k} - 2k^2\tanh^2(kx+\lambda t)$$

TWO SOLITON SOLUTIONS

In order to find two soliton solutions we apply the multiple exp method. In view of this method, we seek solutions to Equation (3) in the form

$$v(x,t) = \frac{a_{22}\psi^2\phi^2 + a_{21}\psi^2\phi + a_{20}\psi^2 + a_{12}\psi\phi^2 + a_{11}\psi\phi + a_{10}\psi + a_{02}\phi^2 + a_{01}\phi + a_{00}}{b_{22}\psi^2\phi^2 + b_{21}\psi^2\phi + b_{20}\psi^2 + b_{12}\psi\phi^2 + b_{11}\psi\phi + b_{10}\psi + b_{02}\phi^2 + b_{01}\phi + 1}, \tag{8}$$

where

$$\phi = \phi(x,t) = \exp(k_1x+\lambda_1 t) \text{ and } \psi = \psi(x,t) = \exp(k_2x+\lambda_2 t)$$

Substitution of the ansatz (8) into (3) gives a system of two polynomial equations in the variables $z = \exp(k_1x+\lambda_1 t)$ and $\zeta = \exp(k_2x+\lambda_2 t).$ Equating to zero the coefficients of the different powers of w and z gives a large algebraic system. Solving it gives

$$a_{00} = a_{01} = a_{02} = a_{11} = a_{12} = a_{20} = a_{21} = a_{22} = 0.$$

$$a_{10} = a_{10},\ a_{11} = a_{10}\frac{k_1^2 - 2k_1k_2 + 2k_2^2}{k_1^2 + 2k_1k_2 + 2k_2^2}$$

$$a_{02} = a_{10} = a_{11} = a_{12} = b_{22} = 0.$$

$$b_{01} = 1,\ b_{20} = \frac{a_{10}^2}{8k_2^4},\ b_{21} = \frac{a_{10}^2}{8k_2^4}\left(\frac{k_1^2 - 2k_1k_2 + 2k_2^2}{k_1^2 + 2k_1k_2 + 2k_2^2}\right)^2$$

$$\lambda_1 = \frac{k_1^3}{2},\ \lambda_2 = -k_2^3.$$

We see that a two soliton solution to Equation (3) is

$$v(x,t) = \frac{a_{10}\psi + a_{10}\dfrac{k_1^2 - 2k_1k_2 + 2k_2^2}{k_1^2 + 2k_1k_2 + 2k_2^2}\phi\psi}{1 + \phi + \dfrac{a_{10}^2}{8k_2^4}\psi^2 + \dfrac{a_{10}^2}{8k_2^4}\left(\dfrac{k_1^2 - 2k_1k_2 + 2k_2^2}{k_1^2 + 2k_1k_2 + 2k_2^2}\right)^2\phi\psi^2}. \tag{9}$$

and then a solution to HS-KdV system (1) is easily obtained from Equation (2) and (9) taking into account that

$$\phi = \exp(k_1x + \frac{k_1^3}{2}t) \text{ and } \psi = \exp(k_2x - k_2^3 t)$$

RATIONAL SOLUTIONS TO HS-KDV SYSTEM

We seek solutions to Equation (3) in the form

$$v(x,t) = \frac{f(x,t)}{g(x,t)},$$

where

$$\begin{cases} f(x,t) = f = \sum_{i,\,j=0}^{3} a_{ij}t^ix^j, \\ g(x,t) = g = \sum_{i,\,j=0}^{3} b_{ij}t^ix^j. \end{cases} \tag{10}$$

We substitute ansatz (10) into Equation (3) to get a system of two polynomial equations in the variables t and x. Equating to zero the coefficients of the different powers of t and x yields a large algebraic system in the unknowns a_{ij}, b_{ij}. Solving it with the aid of a computer, we obtain many solutions. We included here only two of them:

$$v(x,t) = \frac{2b_{03}\left(a_{10} + t^3\right)}{2a_{03}a_{10} + 2a_{03}t^3 - a_{10}b_{03}x^2 + 2a_{10}x + 2a_{21}tx^2 - b_{03}t^3x^2 + 2t^3x}. \tag{11}$$

$$v(x,t) = \frac{2a_{23}\left(a_{12}t^2 + 10a_{10} + t^3\right)}{a_{23}t^3x^2 + a_3t^3 + a_{12}a_{23}t^2x^2 + a_{12}t^2x + a_3a_{12}t^2 + a_{21}tx^2 + a_{10}a_{23}x^2 + a_{10}x + a_3a_{10} + t^3x}. \tag{12}$$

Finally, two rational solutions to HS-KdV system (1) may be obtained from Equation (2).

CONCLUSION

We successfully applied the tanh method and the multiple exp method to obtain one and two soliton solutions to Hirota-Satsuma KdV equations. We think that some of the results in the present work are new in the open literature. Other methods for solving nonlinear partial differential equations (pde's) may be found in the works by Salas and Castillo (2011) and Salas (2008, 2009, 2010, 2011).

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