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Markov chain model for the dynamics of cooking fuel usage: Transition matrix estimation and forecasting

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Markov chain models are valuable tools for modeling data that vary over time. They are suitable models to use when modeling the transitions of variables between discrete states over time. In this paper, Markov chain model was applied to the data obtained from 300 households living in the headquarters of three Local Governments (Argungu, Arewa, and Augie) of Kebbi State, Nigeria. The data was information concerning the main source of fuel for cooking used by each of the households. The types of fuels were; fuel-wood, gas, kerosene and electricity. The initial distribution of the households was obtained based on the information in December, 2010 which was used as a baseline. Thereafter, the subsequent data for 2011, 2012 and 2013 were labeled as Periods 1, 2 and 3, respectively. Using this procedure, a transition matrix for the households was obtained and analyzed using Markov chain model. The model was implemented using R-statistical software version 3.0.2. The results obtained indicate a high probability of increase in the use of wood as fuel for cooking by the households. The probability of using other alternative fuels diminishes over time.

Key words: Households, cooking fuel, Markov chains, transition probability.

INTRODUCTION

According to Thierauf and Klekamp (1975), Markov chains originated with the studies of Markov (1906 - 1907) on the sequence of experiments connected in a chain, and with attempts to describe mathematically the physical phenomenon known as Brownian motion. In many real world problems, it is convenient to classify individuals or items into distinct categories or states. We can then analyze the transitions of these individuals or items from one state to another over time (Elwood and James, 1978). Markov chains is a method of studying changes in state of variables with respect to changes in time, in an effort to predict the future state of those variables (Richard and Charles, 1975). According to Sung et al. (2004) and Welton and Ades (2005), Markov chain models are useful tools for modeling data that vary over time. Markov models are appropriate for the analysis of problems in marketing, income tax auditing, car rental services, inventory, machine maintenance and replacement, stock market analysis and hospital administration (Kannan and Lakshmikanthan, 2002). Markov chain model is a suitable model to use when modeling the transitions of patients between discrete health states over time especially the progression over stages of a disease (McDonnel et al., 2002;

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Meredith, 1976). Sonnenberg and Beck (1993) applied Markov model in the prognosis of clinical problems. They modeled the events of interest as transition from one state to another.

Fuel is indispensable to human existence for warmth and food preparation. There are different ways a man obtains his required fuel; among these is the fuel-wood. It was approximated that 2.5 to 3.0 billion people rely on wood for fuel. Wood accounts for up to 58% of all energy requirements in African savanna areas (Williams, 2003). Personal interviews and observations indicate that majority of households in Argungu, Arewa and Augie Local Government headquarters are dependent on fuel-wood for cooking. This may not be unconnected to lack of inter-fuel substitution for household choice and the use of a given source of fuel, depend on socio-economic (e.g. family income), demographic (e.g. family size, household composition, life style and culture) and location attributes (e.g. proximity to sources of modern and traditional fuels) (Ayotebi, 2000; Adebaw, 2007).

The use of fuel-wood in Nigeria greatly contributes to desert encroachment and consequently has implications with regard to climate change. Upon this, a little comes to light about the drives and dynamics of fuel-wood consumption in Nigeria (Abebaw, 2007). For the purpose of this research, a household is defined as group of persons living together and maintaining unique eating arrangement. The head of a household is responsible for proving the necessities in the household (NBS, 2012).

MATERIALS AND METHODS

Data collection

In this research, the population was the entire households in the three Local Government headquarters (Argungu, Arewa, and Augie) of Kebbi State, Nigeria. A convenience sampling technique was used to select a sample of 300 households that use one of the common types of fuel as their major source of energy for cooking were interviewed for the purpose of determining and predicting the dynamics of fuel use for the period of 3 years (2010 to 2013). However, the selection of the sample was guided by National Bureau of Statistics Report (NBS, 2012). The detail of the sample was Argungu 185, Augie 50, and Arewa 65 households. The respondents were asked questions pertaining their main source of fuel for cooking among the following categories; A (fuel-wood), B (cooking gas), C (kerosene) and D (electricity).

Analytical technique

For the purpose of this research, Markov chain is used in the prediction of households' choice of means of fuel for cooking in the three Local Government headquarters of Kebbi State, Nigeria. A stochastic process is regarded as a sequence of random variables over time. A random variable taking one of the values 1, 2, 3 … *k* is associated with each point and the sequence is determined by Markov chain with transition matrix *P* (Tijms, 2003; Hsu, 1997; Schuss, 2010; Cox and Isham, 1980; Norris, 1997; Bailey, 1964). The sequence of number of household that use a particular fuel type is considered to be a realization of a stochastic process. If X_t denotes the number of households that maintain the use of a particular fuel for a given period, X_t is a random variable describing the outcome of the fuel usage on the t^{th} period and is termed as "the state" of the process. In Markov process, the probability of moving from one state to another depends only on the present state and not the history.

According to Sung et al. (2006), they defined $\{s_{m0}, s_{m1}, \ldots, \}$ as a sequence of random variables indexed by time, taking finite values in $\varepsilon = \{1, \ldots, J\}$. Assume that the sequence $\{s_{m0}, s_{m1}, \ldots, \}$ forms a first order Markov chains as the conditional probability distribution of s_{mt} given $s_{m,t-1}, \ldots, s_{m,0}$ depend only on the value of $s_{m,t-1}$. Let $X_{ij}(t)$ represents the transition from state *i* at time ($t-1$) to state *j* at time *t*. Let a matrix of state transition probabilities be defined where each row entry represents an initial state and each column entry represents a destination state. That is,

$$X(t) = \begin{bmatrix} x_{11}(t) & \cdots & x_{1M}(t) \\ \vdots & \ddots & \vdots \\ x_{M1}(t) & \cdots & x_{MM}(t) \end{bmatrix} \quad (1)$$

Where

$$\sum x_{ij} = 1. \quad (2)$$

And x_{ij} is defined as

$$x_{ij} = \Pr(X_n = j \mid X_{n-1} = i) \quad (3)$$

More generally, let n_{ij} denote the number of individuals who were in state *i* in period *t*-1 and are in state *j* in period *t*. The probability of an individual being in state *j* in period *t* given that they were in state *i* in period *t*-1, denoted by x_{ij}, can be estimated using the following formula:

$$x_{ij} = \frac{n_{ij}}{\sum_j n_{ij}} \quad (4)$$

Thus, the probability of transition from any given state *i* is equal to the proportion of individuals that started in state *i* and ended in state *j* as a proportion of all individuals in that started in state *i*. According to Hsu (1997), Bailey (1964) and Ross (2007), the Markov chain described above has an initial probability vector

$$X_0 = (i_1, i_2, i_3, \ldots i_n) \quad (5)$$

i's are the states and transition matrix $X_{ij} = P$, the probability vector after *n* repetitions of the experiment is

$$V = X_0 P^n \quad (6)$$

That is, for any regular transition matrix *P*, there is a unique vector *V* such that for any probability vector X_0 and for large value of *n*,

Table 1. Initial distribution of households over the states (t = 0).

State	Number of households	Percentage
A	278	92.7
B	11	3.6
C	9	3.0
D	2	0.7

$V = X_0 P^n$. Vector V is called the equilibrium vector of the Markov chain. From the above fact, and for large value of n,

$$X_0 \cdot P^n \cdot P = V \cdot P \qquad (7)$$
$$X_0 \cdot P^{n+1} = VP$$

as $n \to \infty, X_0 \cdot P^n \to V$ so that $X_0 \cdot P^{n+1} \to VP \to V$, (Danbaba and Isah, 2002). Moreover, at equilibrium, Equation 8 represents the proportion of the households in each state.

$$\lim_{n\to\infty} P^n = W\left(\lim_{n\to\infty} \Lambda^n\right) W^{-1} = L \qquad (8)$$

Where P is the matrix of transition probabilities, W and Λ are matrices of the eigenvectors and eigenvalues of P, respectively and the rows of L are all the same. (Burley and O'sullian, 1986; Ross, 2007).

Basic assumptions

It is assumed in this paper that;

a) Markov process is homogeneous and finite,
b) The number of fuel types (states) remain constant, that is, no new type of fuel used by the selected households,
c) Households used only one of the fuels at a regular interval, that is, yearly in this case.
d) No household leave the system throughout the periods of this research.

RESULTS AND DISCUSSION

At the beginning of data collection, at t = 0, there were a total of 300 households, out of which 278 or 92.7% were in Atate A (fuel-wood), 11 household or 3.6% were in State B (gas), 9 households or 3.0% were in State C (kerosene) and only 2 households or 0.7% use electricity as main source of fuel for cooking, that is State D. (Table 1).
Table 2 summarizes the flow of fuel users from one type of cooking fuel to another from December, 2010 to December, 2013.

Transition matrix

$$P = \begin{array}{c|cccc} & A & B & C & D \\ A & 0.9882 & 0.0083 & 0.0035 & 0 \\ B & 0.4643 & 0.3214 & 0.1786 & 0.0357 \\ C & 0.3636 & 0.1818 & 0.4546 & 0 \\ D & 0.2500 & 0.5000 & 0 & 0.2500 \end{array} \qquad (9)$$

The information in Table 2 is more useful when transformed into a transition probability matrix (9). To calculate the entries in the matrix, we sum up (say A to A for example) values and divide it by the row total ($\frac{836}{846} = 0.9882$). Continuing in this manner for other transition routes, we obtained a one-step transition matrix (9). The probabilities of the household moving from state A to States B, C and D are 0.0083, 0.0035 and 0, respectively. In other words, after one-step, the chance of making transition from fuel-wood to gas and kerosene is low. There is no chance of moving from fuel-wood to electricity. The probability of the household remaining in State A (continue using wood for cooking) is as high as 0.9882. The probability of making a forward transition $(\text{i.e., Wood} \to \text{gas} \to \text{kerosene} \to \text{electricity})$ is low while the probability of making backward transition (that is, $\text{Wood} \leftarrow \text{gas} \leftarrow \text{kerosene} \leftarrow \text{electricity})$ is high. The chance of abandoning the use of cooking gas for wood is relatively high (0.5). The chances of abandoning kerosene and electricity for wood are 0.4 and 0.3, respectively. Also, the chance of leaving electricity for gas is good (0.5). The tendency of the household to continue with the use of cooking gas, kerosene and electricity is 0.3, 0.5 and 0.3, respectively. Most importantly, the probability of leaving the use of wood for cooking for its alternative is approximately 0.

Transition diagram

Figure 1 shows a one step transition diagram. It shows the movement of households from one type of cooking fuel to another.

In 2011, 4 households migrate from wood to gas, one from wood to kerosene and none from wood to electricity. Five (5) moved from gas to wood, 1 each from gas to kerosene and electricity. Four households move from using kerosene to fuel-wood, 1 kerosene to gas, and none move from kerosene to electricity. One household moves from electricity to gas, and no other movement from electricity to other fuel types. The same explanation follows in 2012 and 2013. At the end of the data collection, the distribution of the households over the states changed as follows; 290 or 96.6% wood, 5 or 1.7% gas, and 5 or 1.7% kerosene.

To compute fuel shares of the households for a

Table 2. Flow of households from one type of cooking fuel to another (2010 - 2013)

State	December 2010	2011				2012				2013				December 2013
		A	B	C	D	A	B	C	D	A	B	C	D	
A	278	273	4	1	0	278	2	2	0	285	1	0	0	290
B	11	5	4	1	1	5	2	3	0	3	3	1	0	5
C	9	4	1	4	0	2	2	2	0	2	1	4	0	5
D	2	0	1	0	1	1	1	0	0	0	0	0	0	0

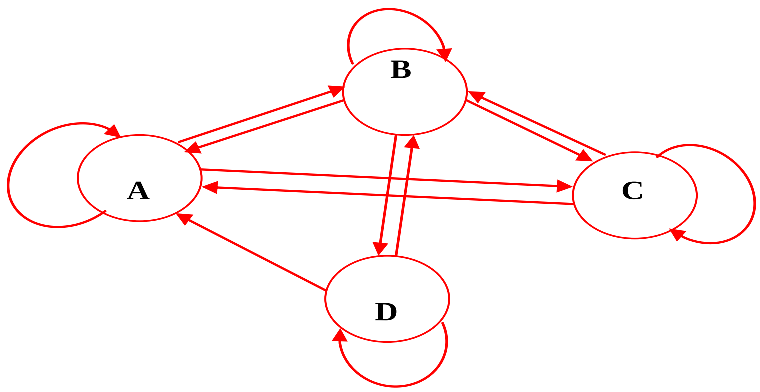

Figure 1. Transition diagram showing the possible transitions.

Table 3. Fuel shares from December, 2010 to December, 2022.

Year	Period	Fuel shares of the households (%)			
		A	B	C	D
2010	1	98.82	0.83	0.35	0.00
2011	2	98.17	1.15	0.65	0.03
2012	3	97.79	1.31	0.85	0.05
2013	4	97.57	1.41	0.96	0.06
2014	5	97.43	1.47	1.03	0.07
2015	6	97.36	1.50	1.07	0.07
2016	7	97.31	1.52	1.10	0.07
2017	8	97.29	1.53	1.11	0.07
2018	9	97.27	1.54	1.12	0.07
2019	10	97.26	1.54	1.12	0.07
2020	11	97.25	1.54	1.13	0.07
2021	12	97.25	1.54	1.13	0.07
2022	13	97.25	1.55	1.13	0.07

Particular year, matrix P and the fuel shares of the preceding year are required. The household-shares of the four competing fuel types for the periods of December, 2010 to December, 2022 have been summarized in Table 3. The table indicates that if the present trends continue for instance, fuel-wood will have 97.25% of the households in the year 2020, while gas, kerosene and electricity will have 1.54, 1.13 and 0.07%, respectively.

Prediction of fuel usage by the households

Applying equation 6 to the initial probability vector

Table 4. Projection of transition probabilities for 13 years (2010 to 2022).

State	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
AA	0.988	0.982	0.978	0.975	0.974	0.973	0.973	0.973	0.973	0.973	0.973	0.973	0.973
AB	0.008	0.012	0.013	0.014	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015
AC	0.004	0.007	0.008	0.010	0.010	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011
AD	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
BA	0.464	0.682	0.803	0.873	0.914	0.938	0.952	0.961	0.966	0.968	0.970	0.971	0.971
BB	0.321	0.157	0.092	0.059	0.040	0.031	0.024	0.020	0.018	0.017	0.016	0.016	0.016
BC	0.179	0.140	0.094	0.062	0.042	0.029	0.022	0.018	0.015	0.014	0.013	0.012	0.012
BD	0.036	0.020	0.011	0.006	0.004	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001
CA	0.363	0.609	0.758	0.846	0.898	0.929	0.947	0.957	0.964	0.967	0.969	0.971	0.971
CB	0.182	0.144	0.098	0.066	0.046	0.033	0.026	0.022	0.019	0.018	0.017	0.016	0.016
CC	0.455	0.240	0.137	0.083	0.052	0.035	0.025	0.020	0.016	0.014	0.013	0.012	0.012
CD	0.000	0.007	0.007	0.005	0.004	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001
DA	0.250	0.542	0.722	0.826	0.887	0.923	0.944	0.955	0.962	0.967	0.969	0.971	0.971
DB	0.500	0.288	0.154	0.088	0.055	0.038	0.028	0.023	0.020	0.018	0.017	0.016	0.016
DC	0.000	0.090	0.094	0.073	0.052	0.036	0.026	0.020	0.017	0.014	0.013	0.012	0.012
DD	0.250	0.080	0.030	0.013	0.006	0.003	0.002	0.002	0.001	0.001	0.001	0.001	0.001

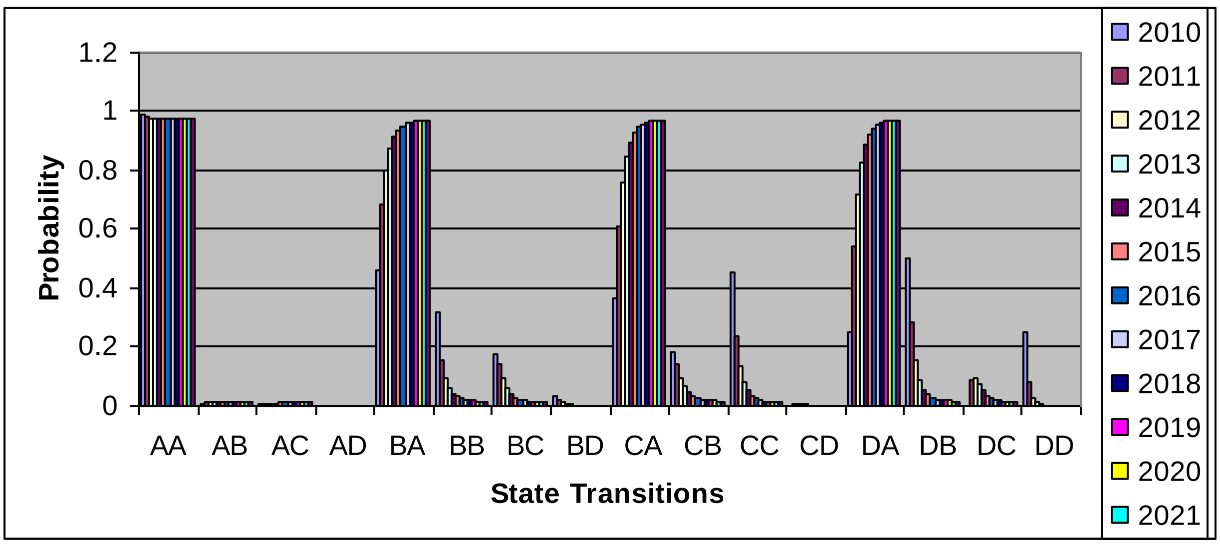

Figure 2. Projection of transition probabilities of moving to each state from 2010 to 2022.

described in Table 1, that is, $X_0 = (0.927 \quad 0.036 \quad 0.030 \quad 0.007)$ and Transition matrix (9), the projected transition probabilities in Table 4 were obtained. Table 4 gives the projection of transition probabilities for 13 years (that is, 2010 to 2022).

Figure 2 is a graphical representation of the four competitive fuel type's transition probabilities for the periods under study. The transition probability of the fuels decreases over the years in favor of the fuel-wood. That is, the probability of the households using gas, kerosene and electricity reduces steadily over the years. The probability of preference of the fuel-wood over other types of fuels for cooking increases across the years. In other words, the probability of the households moving from gas to wood, kerosene to wood and electricity to wood increases steadily over the years. The hope for the households changing cooking fuel from fuel-wood to its alternative is very little. Eventually, all the transition probabilities remain constant in 2021 that is after 12 years.

Conclusion

This paper used Markov chain model to analyze the behavior of the households in respect of using four types of fuel for cooking in three local government headquarters of Kebbi State, Nigeria. Three years observation periods were used and transition probabilities were calculated up to equilibrium stage. At that state, it is predicted that the probability of households using wood as fuel for cooking is 0.971. This means 97.1% of the household will use wood as their main source of fuel for cooking in the year 2022. This is a sharp increase of fuel-wood users on the initial figure of 92.7%. The probability of the households using gas for cooking is 0.016 or only 1.6% of the households will use gas for cooking,

indicating a sharp decline from the initial figure 3.6%. The same situation was observed for kerosene and electricity declining from 3.0 and 0.7% to 1.2 and 0.1%, respectively. These finding indicates that the high demand of wood for cooking will continue to linger among the households. In view of the above, fuel wood accounted for major part of the fuel sources for cooking in the three local governments. As more and more households depend on the use of fuel wood as a source of fuel, the demand for its exploitation has continued to increase. As a result, fuel-wood exploitation has thus gone beyond mere gathering of dead wood to deliberate and indiscriminate cutting of live trees. The disturbing aspect of fuel-wood extraction is that it can hardly be replaced. Therefore, utilization of fuel-wood in these local government areas will certainly contribute greatly to desert encroachment, and consequently has implications on the climate change and other ecological problems. Hence, the rate of rising exploitation of fuel-wood calls for serious and urgent concern at national and local levels.

Conflict of Interests

The author(s) have not declared any conflict of interests.

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