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*Full Length Research Paper*

# A fuzzy group decision making model for multiple criteria based on Borda count

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**Most important decisions in organizations are made by groups of managers or experts. Methods for aggregating preferences and reconciling differences are needed in case decision makers have different viewpoints. Human judgments, including preferences are often vague and cannot be estimated in exact numerical values. This paper proposes a new method under the linguistic framework for heterogeneous group decision making. To accomplish this, an integrated fuzzy group decision making method based on Borda count is proposed, which allocates different weights for decision maker group members to use linguistic terms in order to express their fuzzy preferences for alternative solutions and for individual judgments. The proposed method is then supplied with a numerical example to illustrate the procedure and compare the results with other extant methods.**

**Key words:** Borda count, fuzzy numbers, group decision making, MCGDM.

## INTRODUCTION

Decision making is a usual human activity. It basically involves selecting the most preferred alternative(s) from a finite set of alternatives in order to achieve certain predefined objectives (Chuu, 2009a). Group decision making process can be defined as a decision situation where (1) there are two or more individuals different preferences but the same access to information, each characterized by his/her own perceptions, attitudes, motivations, and personalities; (2) all recognize the existence of a common problem; and (3) all attempt to reach a collective decision (Bui, 1987). There are two types of group decision making: (1) heterogeneous and (2) homogeneous. The heterogeneous group decision making environment allows the opinions of individuals to have different weights, which is contrary to the homogeneous group decision making environment (Chen and Chen, 2005). It is useful to compose a heterogeneous group with dissimilar individuals. When a group is diverse in terms of personalities, gender, age, education, functional specialization and expertise there is an increased possibility that the group will perform its task more effectively (Chakraborty and Chakraborty, 2007). Conflict always occurs in group decision making since members in a group generally do not reach the same decision. Resolving conflicts becomes an important issue in group decision making. For group decision making, the main approach is collective individual decision making (French, 1986) cited in Cheng and Lin (2002). If group members have different viewpoints, some method of aggregating preferences and reconciling differences are needed. Multi Criteria Decision Making (MCDM) methods have been developed to solve conflicting preferences among criteria for single decision makers (Corner and Kirkwood, 1991; Korhonen et al., 1984; Saaty, 1980; Keeney and Raiffa, 1976). MCDM has proven to be an effective methodology for solving a large variety of multi criteria evaluation and ranking problems (Hwang and Yoon, 1981) cited in Yeh and Chang (2008). A MCDM problem can be concisely expressed in matrix format as:

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \end{array} \quad (1)$$

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$W = [w_1\ w_2 \dots\ w_n]$; where $A_1, A_2, \dots, A_m$ are possible alternatives among which decision makers have to choose and $C_1, C_2, \dots, C_n$ are criteria with which alternative performance are measured, $x_{ij}$ is the rating of alternative $A_i$ with respect to criterion $C_j$ while $w_j$ is the weight of criterion $C_j$ (Chen, 2000). Using the opinions of several people that take decision instead of one person, of course, causes many intricacies in analyzing the decision that not only is because of access to collective agreement in ranking of alternatives, but also because of another act like possible differences between members who take group decisions' and possible different objectives and criteria that they have (Fletcher, 2001). In many situations decision makers may provide imprecise information which comes from a variety of sources such as unquantifiable information about alternatives with respect to attributes (Li et al., 2009). Decision Makers (DMs) judgments are uncertain and cannot be estimated by exact numerical values. Under many conditions, crisp data are inadequate to model real-life situations; human judgments, including preferences, are often vague and preferences cannot be estimated in exact numerical values (Zhang et al., 2008). Clearly, the Classical Multi Criteria Decision Making (MCDM) methods, both deterministic and random processes, cannot effectively handle Group Decision-Making problems with imprecise and linguistic information, therefore, fuzzy MCDM methods were developed (Chuu, 2009a).

In order to deal with vagueness of human thought, Zadeh (1965) first introduced the fuzzy set theory. A fuzzy set is an extension of a crisp set. Crisp sets only allow full membership or no membership at all, whereas fuzzy sets allow partial membership. In other words, an element may partially belong to a fuzzy set. The classical set theory is built on the fundamental concept of set of which is either a member or not a member. A sharp, crisp and unambiguous distinction exists between a member and non-member for any well-defined set of entities in this theory and there is a very precise and clear boundary to indicate if an entity belongs to the set. But many real world applications cannot be described and handled by classical set theory (Ertugrul and Karakasoglu, 2009). With different daily decision making problems of diverse intensity, the results can be misleading if the fuzziness of human decision making is not taken into account (Tsaur et al., 2002). The concept of fuzzy sets is one of the most fundamental and influential tools in the development of the computational intelligence (Herrera et al., 2006). Fuzzy sets and fuzzy logic are powerful mathematical tools for modeling: uncertain systems in industry, nature and humanity; and facilitators for common-sense reasoning in decision making in the absence of complete and precise information. Their role is significant when applied to complex phenomena not easily described by traditional mathematical methods, especially when the goal is to find a good approximate solution. A rational approach toward decision making should take into account human subjectivity, rather than employing only objective probability measures. This attitude towards imprecision of human behavior led to the study of a new decision analysis filed fuzzy decision making (Lai and Hwang, 1996).

The "method of marks" voting procedure proposed by the French scientist Jean-Charles de Borda (1733–1799) in Paris in 1781 represents an important step in the development of modern electoral systems, and indeed in the theory of voting more generally (Reilly, 2002). The Borda rule is an appropriate procedure in multi-person decision making when several alternatives are considered. The discrete Borda count allows the DMs only to show which alternatives are preferred in pair wise comparisons (García-Lapresta et al., 2009).

Following this, a lot of research has been conducted in the area of group decision making under the application of fuzzy set theory (Sadi-Nezhad and Akhtari, 2008; Li, 2007). Some of them have also applied the concept of linguistic variables proposed by Zadeh (1975) to handle linguistic terms and approximate reasoning in a group decision-making problem. Some researches (Fan et al., 2010; Ashtiani et al., 2009; Chuu, 2009b; Lu et al., 2008; Mahdavi et al., 2008; Yang and Hsieh, 2008; Shih et al., 2007; Wu and Chen, 2007; Chen and Chen, 2005; Kahraman et al., 2003; Cheng and Lin, 2002; Chang et al., 2000) have been carried out in describing the uncertainty of individual preferences for alternatives and aggregating these fuzzy individual preferences into a group decision making. This paper proposes a new fuzzy group decision-making method based on Borda count. The paper gives all preliminaries used, and also demonstrates a framework of fuzzy group decision making method. An example for using the method is shown and finally conclusions are discussed.

## PRELIMINARIES

Here, some basic definition of fuzzy sets, triangular fuzzy number and linguistic variables are reviewed.

### Definition 1

*A* fuzzy set presents a boundary with a gradual contour, by contrast with classical sets, which present a discrete border. Let U be the universe of discourse and *u* a generic element of U, then U $=\{u\}$. A fuzzy subset $\tilde{A}$, defined in U, is: $\tilde{A} = \{(u, \mu_{\tilde{A}}(u)) \mid u \in U\}$, Where $\mu_{\tilde{A}}(u)$ is designated as membership function or membership grade (also designated as degree of compatibility or degree of truth) of *u* in $\tilde{A}$. The membership function associates with each element *u*, of U, a real number $\mu_{\tilde{A}}(u)$, in the interval [0, 1] (Mario, 2000).

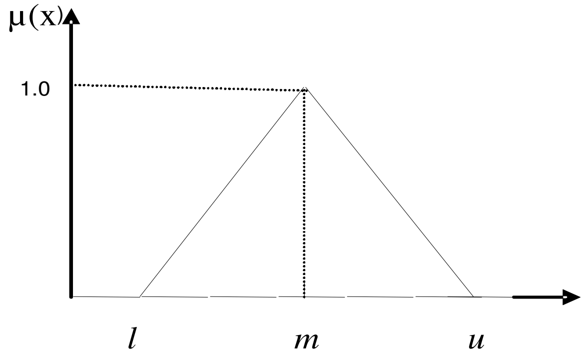


**Figure 1.** The triangular fuzzy membership function.

**Definition 2**

*A* is a fuzzy number if *A* is normal and convex fuzzy set of U. Triangular type membership function of *M* fuzzy number can be described as in Equation 2 and Figure 1. When $l=m=u$, it is a nonfuzzy number by convention (Onut et al., 2008).

$$\mu m(x)= \begin{cases} 0, & x < l \\ (x-l)/(m-l) & l \le x \le m \\ (u-x)/(u-m) & m \le x \le u \\ 0, & x > u \end{cases} \tag{2}$$

**Definition 3**

The main operational laws for two triangular fuzzy numbers *M1* and *M2* are as follows (Kaufmann and Gupta, 1991):

$$\begin{aligned} &M1 + M2 = (l1 + l2,\ m1 + m2,\ u1 + u2), \\ &M1 \times M2 = (l1 \times l2,\ m1 \times m2,\ u1 \times u2), \\ &\omega \times M1 = (\omega l1,\ \omega m1,\ \omega u1),\ \ \omega>0,\ \omega \in R, \\ &M_1^{-1} = (1/u1,\ 1/m1,\ 1/l1). \end{aligned} \tag{3}$$

**Definition 4**

A linguistic variable is "a variable whose values are words or sentences in a natural or artificial language" (Wang, et al., 2009). These linguistic variables can be expressed in positive triangular fuzzy numbers as Tables 1 and 2 (Mahdavi et al., 2008). The triangular membership function is shown in Figure 2.

**Table 1.** Linguistic variables for the ratings.

| | | |
|---|---|---|
| Very Poor | VP | (0, 0, 1) |
| Poor | P | (0, 1, 3) |
| Medium Poor | MP | (1, 3, 5) |
| Fair | F | (3, 5, 7) |
| Medium Good | MG | (5, 7, 9) |
| Good | G | (7, 9, 10) |
| Very Good | VG | (9, 10, 10) |

**Table 2.** Linguistic variables for the importance weight of each criterion.

| | | |
|---|---|---|
| Very Low | VL | (0, 0, 0.1) |
| Low | L | (0, 0.1, 0.3) |
| Medium Low | ML | (0.1, 0.3, 0.5) |
| Medium | M | (0.3, 0.5, 0.7) |
| Medium High | MH | (0.5, 0.7, 0.9) |
| High | H | (0.7, 0.9, 1.0) |
| Very High | VH | (0.9, 1.0, 1.0) |

## THE PROPOSED METHODS

The purpose of this method is to enhance group agreement on the Group Decision Making outcome based on Borda count.

Let $A=\{A_1, A_2, \ldots, A_m\}$ be a discrete set of alternatives, $P=\{P_1, P_2, \ldots, P_k\}$ be the set of decision makers, and $\lambda=(\lambda_1, \lambda_2, \ldots, \lambda_p)$ be the weight vector of decision makers, where $\lambda_p \ge 0$, $P=1,2,\ldots,k$, and $\sum_{p=1}^{k} \lambda_p = 1$. Let $C=\{C_1, C_2, \ldots, C_n\}$ be the set of attributes, and $w=(w_1, w_2, \ldots, w_n)$ be the weight vector of attributes, where $w_n \ge 0$, $n=1,2,\ldots,j$, $\sum_{n=1}^{j} w_n = 1$. The fuzzy group decision problem can be concisely expressed as matrix format (Mahdavi et al., 2008):

$$\tilde{P}_t = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix} \end{array} \tag{4}$$

$\tilde{W}=[\tilde{w}_1, \tilde{w}_2, \cdots, \tilde{w}_n]$ Where $\tilde{x}_{ij}^k$ and $\tilde{w}_j^k$ are linguistic variables that can be shown by fuzzy numbers shown in

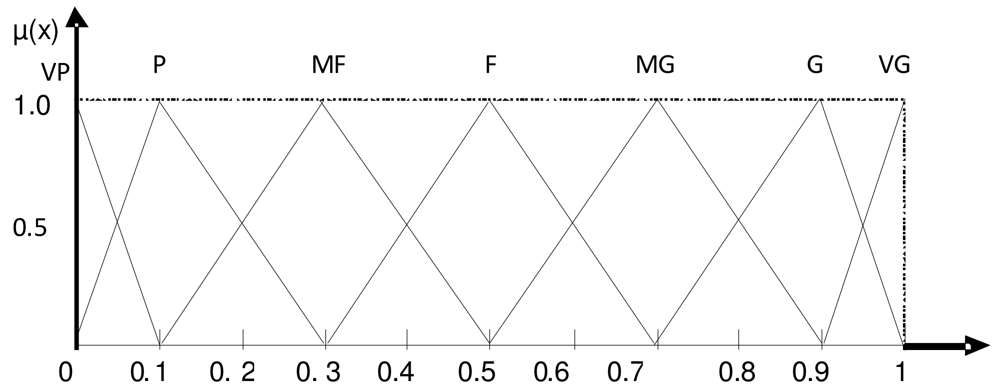


**Figure 2.** The triangular fuzzy membership function.

Tables 1 and 2. The proposed models are linearly described in the following 11 steps:

1. Identifying evaluation criteria.
2. Generating alternatives.
3. Identifying weights of criteria and weights of decision makers.
4. Presenting preferences on the part of each decision maker (every decision maker gives preferences to per alternative based on every attribute according to linguistic terms such as: Very Poor, Poor, Medium Poor, Fair, Medium Good, Good, and Very Good).
5. Construction of fuzzy decision matrix. In fuzzy decision matrix, we suppose that, each $\tilde{x}_{ij}^{k}$ is fuzzy number.
6. Construct the normalized fuzzy decision matrix that can be found in (Kahraman, et al., 2007; Tsao, 2006). Normalized fuzzy decision matrix for triangular fuzzy numbers is:

$$\tilde{R} = \left[\tilde{r}_{ij}\right]_{m \times n} \tag{5}$$

If $(\tilde{x}_{ij}, i = 1,2,\ldots,m,\ j = 1,2,\ldots,n)$ are triangular fuzzy numbers, then the normalization process can be performed by (Wang et al., 2009):

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}\right) \quad i = 1,2,\ldots,m, \quad j \in B$$

$$\tilde{r}_{ij} = \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right) \quad i = 1,2,\ldots,m, \quad j \in C \tag{6}$$

Where *B* and *C* are the set of benefit criteria and cost criteria, respectively, and

$$c_j^* = \max_i \quad c_{ij} \quad j \in B ,$$
$$a_j^- = \min_i \quad a_{ij} \quad j \in C$$

The normalization method mentioned above is to preserve the property that the ranges of normalized fuzzy numbers belong to [0, 1]. In order to avoid these computations and make an easier and practical procedure, we can define all the fuzzy numbers in this interval to avoid normalization method (Mahdavi et al., 2008; Wang and Lee, 2007; Saghafian and Hejazi, 2005).

7. Construction of defuzzification decision matrix: Defuzzification is a technique to convert the fuzzy number into crisp real numbers; the procedure of defuzzification is to locate the Best Nonfuzzy Performance (BNP) value. There are several available methods to serve this purpose: Mean-of-Maximum, Center-of-Area, and α-cut Method (Tsaur et al., 2002). This study utilizes the Center-of-Area method due to its simplicity and because it does not require analyst's personal judgment. The defuzzified value of fuzzy number based on Equation 2 can be obtained from Equation 7.

$$BNP_i = [(U_i - L_i) + (M_i - L_i)]/3 + L_i ,\ \forall i \tag{7}$$

8. Considering proper value (DM weights) of every decision making group member idea

$$N_{ij\lambda} = N_{ij} \times \lambda_p \tag{8}$$

$N_{ij}$ is an element of defuzzification decision matrix for every DM, and $\lambda_p$ is the weight of per DM idea.

9. Formation of $R_j$ matrixes; while the rows of the matrix are alternatives and its columns are DMs opinions based on *j* criterion. So *n* matrixes in lieu of *j* attributes were established ($R_j$):

$$R_j = \begin{matrix} A_1 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{matrix} \begin{bmatrix} r_{1,j}^{1} & \cdots & r_{1,j}^{p} & \cdots & r_{1,j}^{k} \\ \vdots & & \vdots & & \vdots \\ r_{i,j}^{1} & \cdots & r_{i,j}^{p} & \cdots & r_{i,j}^{k} \\ \vdots & & \vdots & & \vdots \\ r_{m,j}^{1} & \cdots & r_{m,j}^{p} & \cdots & r_{m,j}^{k} \end{bmatrix} \quad \begin{matrix} , i = 1,2,...,m \\ , j = 1,2,...,n \\ , p = 1,2,...,k \end{matrix} \tag{9}$$

10. Computing linear sum in lieu of *P* decision makers

$$\left( \sum_{p=1}^{k} r_{i,j}^{p} \right)$$

and final grade of every alternative in lieu of *j* attributes would be calculated. In this matrix the line with the highest mark is the first rank and the line with the lowest mark is *m* rank.

$$R_G = \begin{matrix} \\ A_1 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{matrix} \begin{matrix} \begin{matrix} C_1 & \cdots & C_j & \cdots & C_n \end{matrix} \\ \begin{bmatrix} r'_{1,1} & \cdots & r'_{1,j} & \cdots & r'_{1,n} \\ \vdots & & \vdots & & \vdots \\ r'_{i,1} & \cdots & r'_{i,j} & \cdots & r'_{i,n} \\ \vdots & & \vdots & & \vdots \\ r'_{m,1} & \cdots & r'_{m,j} & \cdots & r'_{m,n} \end{bmatrix} \end{matrix} \tag{10}$$

11. Changing $R_G$ matrix into Borda count, i.e. alternative with first rank based on per criterion would have *m-1* relative value on the basis of *m* alternatives. The same goes for, alternative with second rank (*m-2* relative value). Alternatives with *m* rank would receive zero relative values. We multiply the Borda count matrix with the corresponding weight vector of attributes.

$$\begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = \begin{bmatrix} b_{11}.w_1 + & b_{12}.w_2 + & \cdots & b_{1n}.w_n \\ b_{21}.w_1 + & b_{22}.w_2 + & \cdots & b_{2n}.w_n \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1}.w_1 + & b_{m2}.w_2 & \cdots & b_{mn}.w_n \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_m \end{bmatrix} \tag{11}$$

The alternative sum with the highest value would be considered as the first rank and the lowest represents the last rank.





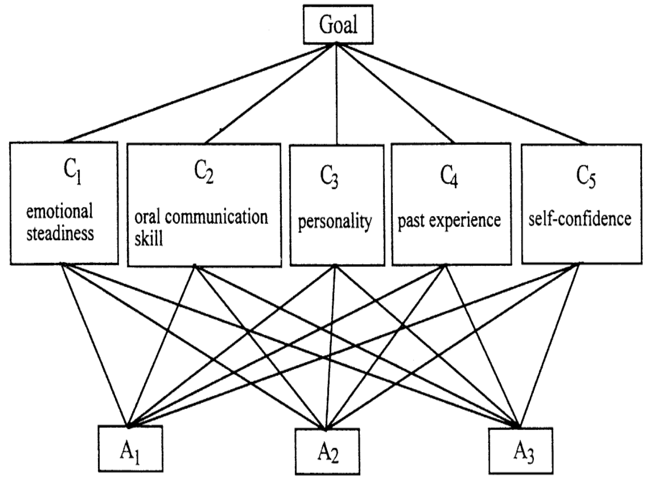


**Figure 3.** The hierarchical structure.

## NUMERICAL EXAMPLE

Here, we work out a numerical example, taken from (Chen, 2000), to illustrate the proposed methods for decision making problems with fuzzy data. Suppose that a software company desires to hire a system analysis engineer. After preliminary screening, three candidates (alternatives) $A = (A_1, A_2, A_3)$ remain for further evaluation. A committee of three decision makers $P = (P_1, P_2, P_3)$ has been formed with weight vector of $\lambda = \left( 1/3, 1/3, 1/3 \right)$ to conduct interview and select the most suitable candidate.

Five benefit criteria are considered: $C_1$: Emotional steadiness, $C_2$: Oral communication skill, $C_3$: Personality, $C_4$: Past experience, and $C_5$: Self-confidence.

The hierarchical structure of this decision problem is shown as Figure 3. The proposed method is currently applied to solve this problem and the computational procedure is summarized as follows:

Step 3: Decision-makers use the linguistic weighting variables shown in Table 2 to assess the importance of the criteria presented in Table 3.
Step 4: Decision makers use the linguistic rating variables shown in Table 1 to evaluate the rating of alternatives with respect to each criterion presented in Table 4.
Step 5: Converting linguistic evaluation (Tables 3 and 4) into triangular fuzzy numbers to construct the fuzzy decision matrix and determining the weight of each criterion based on (Equation 7) as shown in Table 5.
Step 6: Construction of normalized fuzzy decision matrix as shown in Table 6.

**Table 3.** Weight importance of the criteria.

| | $P_1$ | $P_2$ | $P_3$ |
|---|---|---|---|
| $C_1$ | H | VH | MH |
| $C_2$ | VH | VH | VH |
| $C_3$ | VH | H | H |
| $C_4$ | VH | VH | VH |
| $C_5$ | M | MH | MH |

**Table 4.** Ratings of three candidates by decision makers under all criteria.

| | $C_1$ | | | $C_2$ | | | $C_3$ | | | $C_4$ | | | $C_5$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $A_1$ | $A_2$ | $A_3$ | $A_1$ | $A_2$ | $A_3$ | $A_1$ | $A_2$ | $A_3$ | $A_1$ | $A_2$ | $A_3$ | $A_1$ | $A_2$ | $A_3$ |
| $P_1$ | MG | G | VG | G | VG | MG | F | VG | G | VG | VG | G | F | VG | G |
| $P_2$ | G | G | G | MG | VG | G | G | VG | MG | G | VG | VG | F | MG | G |
| $P_3$ | MG | MG | F | F | VG | VG | G | G | VG | VG | VG | MG | F | G | MG |

**Table 5.** The fuzzy decision matrix and criteria weights.

| | | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---|---|---|---|---|---|---|
| $P_1$ | $A_1$ | (5, 7, 9) | (7, 9, 10) | (3, 5, 7) | (9, 10, 10) | (3, 5, 7) |
| | $A_2$ | (7, 9, 10) | (9, 10, 10) | (9, 10, 10) | (9, 10, 10) | (9, 10, 10) |
| | $A_3$ | (9, 10, 10) | (5, 7, 9) | (7, 9, 10) | (7, 9, 10) | (7, 9, 10) |
| $P_2$ | $A_1$ | (7, 9, 10) | (5, 7, 9) | (7, 9, 10) | (7, 9, 10) | (3, 5, 7) |
| | $A_2$ | (7, 9, 10) | (9, 10, 10) | (9, 10, 10) | (9, 10, 10) | (5, 7, 9) |
| | $A_3$ | (7, 9, 10) | (7, 9, 10) | (5, 7, 9) | (9, 10, 10) | (7, 9, 10) |
| $P_3$ | $A_1$ | (5, 7, 9) | (3, 5, 7) | (7, 9, 10) | (9, 10, 10) | (3, 5, 7) |
| | $A_2$ | (5, 7, 9) | (9, 10, 10) | (7, 9, 10) | (9, 10, 10) | (7, 9, 10) |
| | $A_3$ | (3, 5, 7) | (9, 10, 10) | (9, 10, 10) | (5, 7, 9) | (5, 7, 9) |
| **Weights** | | (0.7, 0.87, 0.97) | (0.9, 1, 1) | (0.77, 0.93, 1) | (0.9, 1, 1) | (0.43, 0.63, 0.83) |
| **Weights** | | 0.2 | 0.22 | 0.21 | 0.22 | 0.15 |

**Table 6.** The fuzzy normalized decision matrix.

| | | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---|---|---|---|---|---|---|
| $P_1$ | $A_1$ | (0.5, 0.7, 0.9) | (0.7, 0.9, 1) | (0.3, 0.5, 0.7) | (0.9, 1, 1) | (0.3, 0.5, 0.7) |
| | $A_2$ | (0.7, 0.9, 1) | (0.9, 1, 1) | (0.9, 1, 1) | (0.9, 1, 1) | (0.9, 1, 1) |
| | $A_3$ | (0.9, 1, 1) | (0.5, 0.7, 0.9) | (0.7, 0.9, 1) | (0.7, 0.9, 1) | (0.7, 0.9, 1) |
| $P_2$ | $A_1$ | (0.7, 0.9, 1) | (0.5, 0.7, 0.9) | (0.7, 0.9, 1) | (0.7, 0.9, 1) | (0.3, 0.5, 0.7) |
| | $A_2$ | (0.7, 0.9, 1) | (0.9, 1, 1) | (0.9, 1, 1) | (0.9, 1, 1) | (0.5, 0.7, 0.9) |
| | $A_3$ | (0.7, 0.9, 1) | (0.7, 0.9, 1) | (0.5, 0.7, 0.9) | (0.9, 1, 1) | (0.7, 0.9, 1) |
| $P_3$ | $A_1$ | (0.56, 0.78, 1) | (0.3, 0.5, 0.7) | (0.7, 0.9, 1) | (0.9, 1, 1) | (0.3, 0.5, 0.7) |
| | $A_2$ | (0.56, 0.78, 1) | (0.9, 1, 1) | (0.7, 0.9, 1) | (0.9, 1, 1) | (0.7, 0.9, 1) |
| | $A_3$ | (0.33, 0.56, 0.78) | (0.9, 1, 1) | (0.9, 1, 1) | (0.5, 0.7, 0.9) | (0.5, 0.7, 0.9) |

**Table 7.** The defuzzification decision matrix.

| | | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---|---|---|---|---|---|---|
| | **A₁** | 0.7 | 0.87 | 0.5 | 0.97 | 0.5 |
| **$P_1$** | **$A_2$** | 0.87 | 0.97 | 0.97 | 0.97 | 0.97 |
| | **$A_3$** | 0.97 | 0.7 | 0.87 | 0.87 | 0.87 |
| | **$A_1$** | 0.87 | 0.7 | 0.87 | 0.87 | 0.5 |
| **$P_2$** | **$A_2$** | 0.87 | 0.97 | 0.97 | 0.97 | 0.7 |
| | **$A_3$** | 0.87 | 0.87 | 0.7 | 0.97 | 0.87 |
| | **$A_1$** | 0.78 | 0.5 | 0.87 | 0.97 | 0.5 |
| **$P_3$** | **$A_2$** | 0.78 | 0.97 | 0.87 | 0.97 | 0.87 |
| | **$A_3$** | 0.56 | 0.97 | 0.97 | 0.7 | 0.7 |

**Table 8.** $R_j$ matrix.

| $C_1$ | **$P_1$** | **$P_2$** | **$P_3$** | ∑ | **Rank** | $C_4$ | **$P_1$** | **$P_2$** | **$P_3$** | ∑ | **Rank** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **$A_1$** | 0.23 | 0.29 | 0.26 | 0.78 | 3 | **$A_1$** | 0.32 | 0.29 | 0.32 | 0.93 | 2 |
| **$A_2$** | 0.29 | 0.29 | 0.26 | 0.84 | 1 | **$A_2$** | 0.32 | 0.32 | 0.32 | 0.96 | 1 |
| **$A_3$** | 0.32 | 0.29 | 0.19 | 0.8 | 2 | **$A_3$** | 0.29 | 0.32 | 0.23 | 0.84 | 3 |
| $C_2$ | **$P_1$** | **$P_2$** | **$P_3$** | ∑ | **Rank** | $C_5$ | **$P_1$** | **$P_2$** | **$P_3$** | ∑ | **Rank** |
| **$A_1$** | 0.29 | 0.23 | 0.17 | 0.69 | 3 | **$A_1$** | 0.17 | 0.17 | 017 | 0.51 | 3 |
| **$A_2$** | 0.32 | 0.32 | 0.32 | 0.96 | 1 | **$A_2$** | 0.32 | 0.23 | 0.29 | 0.84 | 1 |
| **$A_3$** | 0.23 | 0.29 | 0.32 | 0.84 | 2 | **$A_3$** | 0.29 | 0.29 | 0.23 | 0.81 | 2 |
| $C_3$ | **$P_1$** | **$P_2$** | **$P_3$** | ∑ | **Rank** | | | | | | |
| **$A_1$** | 0.17 | 0.29 | 0.29 | 0.75 | 3 | | | | | | |
| **$A_2$** | 0.32 | 0.32 | 0.29 | 0.93 | 1 | | | | | | |
| **$A_3$** | 0.29 | 0.23 | 0.32 | 0.84 | 2 | | | | | | |

Step 7: Conversion of normalized fuzzy decision matrix to the defuzzification decision matrix by Equation 7 as shown in Table 7.

Steps 8, 9: Considering proper value (DM weights) of every decision making group member idea by Equation 8 and establishing *n* matrixes lieu of *j* attribute as shown in Table 8.

Step 10: In Table 8, linear sum would be reached in lieu of *P* decision makers and final rank of every alternative in lieu of *j* attribute would be calculated. In these matrixes, the line with the highest mark is the first rank and the line with the lowest mark is *m* rank.

Step 11: We change the $R_G$ matrix into Borda count; multiply the Borda count matrix with the corresponding weight vector of attributes by Equation 11. The alternative sum with the highest value would be considered as the first rank and the lowest represents the last rank.



$$\begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix}\begin{bmatrix} 3 & 3 & 3 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 3 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.2 \\ 0.22 \\ 0.21 \\ 0.22 \\ 0.15 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & + & 0 & + & 0 & + & 1\times0.22 & + & 0 \\ 2\times0.2 & + & 2\times0.22 & + & 2\times0.21 & + & 2\times0.22 & + & 2\times0.15 \\ 1\times0.2 & + & 1\times0.22 & + & 1\times0.21 & + & 0 & + & 1\times0.15 \end{bmatrix} = \begin{bmatrix} 0.22 \\ 2 \\ 0.78 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

we get the order of three alternatives as follows: A2 >> A3 >> A1. Therefore, A2 is the optimal candidate.

## Conclusion

In Multi Criteria Group Decision Making with linguistic variables, the DMs may have vague information, limited

**Table 9.** Comparison with other methods.

| Methods | Ordering | Proposed method ordering |
|---|---|---|
| (Chen, 2000) | $A_2 \rangle A_3 \rangle A_1$ | $A_2 \rangle A_3 \rangle A_1$ |
| (Mahdavi et al., 2008) | $A_2 \rangle A_3 \rangle A_1$ | $A_2 \rangle A_3 \rangle A_1$ |
| (Xu, 2004) | $A_3 \rangle A_1 \rangle A_2 \rangle A_4$ | $A_3 \rangle A_1 \rangle A_2 \rangle A_4$ |
| (Zhang et al., 2008) | $A_1 \rangle A_2 \rangle A_3 \rangle A_6 \rangle A_5 \rangle A_4$ | $A_1 \rangle A_2 \rangle A_5 \rangle A_3 \rangle A_6 \rangle A_4$ |
| (Wu and Chen, 2007) | $A_3 \rangle A_1 \rangle A_2 \rangle A_4$ | $A_3 \rangle A_1 \rangle A_2 \rangle A_4$ |

attention and different information processing capabilities. This paper proposes a new fuzzy group decision making method which allows group members to express their fuzzy preferences in linguistic terms for alternative selection and for individual judgments. The method then aggregates these elements into compromised group decisions which seem to be more acceptable. The proposed method is compared with other methods, with Table 9 listing the results of the comparison. The proposed method can solve problems in uncertain environments. The proposed method covered both homogeneous and heterogeneous group decision making by considering the decision makers' viewpoint weights. The results are very similar to other methods. Furthermore, in Table 9, the selection made by the proposed method approximately is identical with the five already established methods, which is expressive in itself and possibly approves of the reliability and validity of the proposed methods. The approach is computationally simple and its underlying concept is logical and comprehensible, thus facilitating its implementation in a computer-based system. Moreover, their ease of solution in various group decision making circumstances is considered as a strong merit.

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