International Journal of the Physical Sciences Vol. 6(36), pp. 8183 - 8187, 30 December, 2011
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS11.1445
ISSN 1992 - 1950 ©2011 Academic Journals

Full Length Research Paper

Modified quadratic shepard and radial basis functions interpolations for determination of orthometric heights from global positioning system (GPS) heights

Leyla Cakir

Department of Geomatics, Engineering Faculty, Karadeniz Technical University, 61080 Trabzon, Turkey. E-mail: lcakir@ktu.edu.tr. Tel: +90 462 377 2765. Fax: +90 462 328 0918.

Accepted 12 October, 2011

We can obtain the position of any point on earth easily, accurately and quickly as geocentric coordinates (X, Y, Z) or geodetic coordinates (Ø, λ, h) in the World Geodetic System (WGS84) coordinate system with global positioning system (GPS) technique widely used in engineering studies today. However, we need to convert the ellipsoid height (h) to orthometric height to be able to use it in the applications. In this study, radial basis functions (RBF) and modified quadratic shepard (MQS) interpolation methods were used to determine orthometric heights from GPS measurements in two different study fields. It has been observed that multiquadric interpolation method gave better results when statistical values of test points in the study fields were considered.

Key words: Global positioning system (GPS) /leveling, geoid height, interpolation.

INTRODUCTION

Any height difference between any two points on earth can be obtained by classical spirit levelling (or trigonometric, barometric levelling etc). However spirit levelling is a method that requires time and workforce, having high costs. Other than this method, height information are obtained accurately and quickly after the progresses in global navigation satellite system (GNSS) positioning systems (GPS, GLONASS, GALILEO), space-borne/ airborne radar systems (satellite altimetry, LIDAR, SAR, SRTM) based on satellites (Fotopoulos, 2005).

We can obtain the position of any point on earth in three dimensions in WGS84 with global positioning system (GPS). From these measured values, the ellipsoidal height which does not have physical meaning is not used in practical surveying, engineering and geophysical applications. For example, when using ellipsoidal heights, there is the possibility of not taking into account the physical force of gravity, and water will appear to flow up-hill. By this reason GPS derived heights need to be converted to orthometric heights (Featherstone et al., 1998). To make such conversion geoid height needs to be known. If geoid height (N) of a point measured with a GPS receiver on earth is known, then the ellipsoidal height of that point can be easily converted to orthometric height by using the following formula (Heiskanen and Moritz, 1967; Lin, 2005).

$$H = h - N \tag{1}$$

h is the ellipsoidal height obtained from GPS measurements, which is based on WGS84 reference ellipsoid. H is the orthometric height based on geoid surface (mean sea level) obtained with geometric levelling. N is the geoid height value between reference ellipsoid and geoid across the normal of the reference ellipsoid.

There are basically two approaches for conversion of ellipsoidal heights to orthometric heights. First of them is geoid model determined with gravimetric measurements, and the second one is the geoid model determined with the combination of GPS-levelling measurements and the last is also called the geometric method (Featherstone et al., 1998). There are many different interpolation methods used for local geoid determination with geometric method (Zhong, 1997; Yanalak and Baykal, 2001; Erol and Celik, 2004). In this study, the GPS/levelling measurements were used. Local geoid surface was formed with radial basis functions and modified quadratic shepard interpolation methods for the selected application areas by using reference points. Then interpolation methods were evaluated according to obtained statistical values for test points.

MODIFIED QUADRATIC SHEPARD INTERPOLATION

This is a method developed by Franke and Nielson (1980) in order to eliminate the disadvantages of the interpolation method which is weighted with the inverse of the distance. If it is considered that there are n reference points around an interpolation point according to the method, the geoid height of the interpolation point is calculated with the following,

$$N_0 = \frac{\sum_{k=1}^{n} W_k Q_k}{\sum_{k=1}^{n} W_k} \quad (2)$$

Q_k in the formula, called nodal function, is the quadratic function value calculated with the least squares method weighted for the reference points. W_k weight value and nodal function Q_k are defined by

$$W_k = \left[\frac{(R_w - d_k)}{R_w \, d_k} \right]^2 \quad (3)$$

$$Q_k(x,y) = c_{k2}(x-x_i)^2 + c_{k3}(x-x_i)(y-y_i) + c_{k4}(y-y_i)^2 + c_{k5}(x-x_i) + c_{k6}(y-y_i) + f_k \quad (4)$$

The coefficients of the function are determined with compensation with the least squares method by using the following condition,

$$\sum_{\substack{i=1 \\ k\neq i}}^{n} W_i \left[f_k + c_{k2}(x_i - x_k)^2 + \ldots + c_{k6}(y_i - y_k) - f_i \right]^2 \quad (5)$$

The weight value in the Formula (5) is given by

$$W_i = \left[\frac{(R_q - d_i)}{R_q \, d_i} \right]^2 \quad (6)$$

Before making these calculations R_w and R_q critical circle radiuses need to be determined.

$$R_q = \frac{D}{2}\sqrt{\frac{n_q}{n}} \quad , \quad R_w = \frac{D}{2}\sqrt{\frac{n_w}{n}} \quad (7)$$

n_q in the formulas indicates the number of reference points used for calculating the quadratic function; n_w indicates the number of reference points used for calculating the weight values; D indicates the maximum distance between two points in the data set, n indicates the total point number in the data set. n_q and n_w are the parameters determined by the user (Franke and Nielson, 1980; Nielson, 1993). According to Renka (1988), the results yielded better if n_q = 19 and n_w = 13 for two dimensional data (Renka, 1988). In this study, Renka's values are used.

RADIAL BASIS FUNCTIONS INTERPOLATION

The method of radial basis functions is a global interpolation method used for interpolation of scattered data. It is the generalized version of the multiquadric method developed by Hardy for making topographical maps in the cartography field (Hardy, 1971; Franke et al., 1993). The interpolation function in this method is defined as follows:

$$\mathrm{N}(x,y) = \sum_{i=1}^{n} c_i \left[Q(x,y,x_i,y_i) \right] + \sum_{j=1}^{m} b_j B_j(x,y) \qquad m < n \quad (8)$$

where Q is radial basis function and B is the polynomial basis function. The matrix notation of the (8) equation

$$\underline{N} = \underline{Q} \, . \, \underline{c} + \underline{B} \, \underline{b} \quad (9)$$

and $\underline{b}$ and $\underline{c}$ indicate the unknown coefficient vector of the polynom and RBF respectively. The polynomial term has been added for polynomial sensitivity of the method in the Equation (8) and guarantees unique approximation. For example, in case a linear polynomial is used as polynomial, and by adding the conditions following (Franke et al., 1993; Buhmann, 2000; Micchelli, 1986),

$$\underline{B}\,\underline{c} = 0, \quad \sum_{i=1}^{n} c_i = \sum_{i=1}^{n} c_i\, x_i = \sum_{i=1}^{n} c_i\, y_i = 0 \tag{10}$$

unknown c_i and b_i coefficients can be obtained separately from the matrix equations (Eberly, 2009)

$$\underline{c} = \underline{Q}^{-1}(\underline{\Delta N} - \underline{B}\underline{b}) \tag{11}$$

$$\underline{b} = (\underline{B}^{T}\underline{Q}^{-1}\underline{B})^{-1}\underline{B}^{T}\underline{Q}^{-1}\underline{N} \tag{12}$$

or can be calculated as well with a combined solution algorithm like:

$$\begin{bmatrix} Q & B \\ B^{T} & 0 \end{bmatrix}\begin{bmatrix} c_i \\ b \end{bmatrix} = \begin{bmatrix} N \\ 0 \end{bmatrix} \tag{13}$$

If we express Equation (13) more explicitly, it is shown as

$$\underbrace{\left[\begin{array}{ccccc|cc} 0 & q_{12} & \cdots & q_{1n} & 1 & x_1 & y_1 \\ q_{21} & 0 & \cdots & q_{21} & 1 & x_2 & y_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ q_{n1} & q_{n1} & \cdots & 0 & 1 & x_n & y_n \\ \hline 1 & 1 & \cdots & 1 & 0 & 0 & 0 \\ x_1 & x_2 & \cdots & x_n & 0 & 0 & 0 \\ y_1 & y_2 & \cdots & y_n & 0 & 0 & 0 \end{array}\right]}_{A} \underbrace{\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \\ b_0 \\ b_1 \\ b_2 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ N_n \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{L} \tag{14}$$

here, $\underline{A}$ is a diagonal matrix and its diagonal members are null. $\underline{X}$, is consisted of c_i and b_i unknown coefficients, and $\underline{L}$ is consisted of geoid height values of the reference points. From here, unknown coefficients can be calculated from the matrix equation

$$\underline{X} = \underline{A}^{-1}\,\underline{L} \tag{15}$$

Finally, geoid height value of the interpolation point is obtained by placing the (x_0,y_0) position values into the Equation (8) (URL-1, 2011)

Many RBF types are defined and in this study we used multiquadric (MQ) ($Q(d) = \sqrt{d^2 + \delta^2}$) and thin plate spline function (TPS) ($Q(d) = d^2 \log d$) from radial basis functions. d in the formulas is the distance between the test points and the reference point and it is calculated as:

$$d_i = \sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2} \qquad i = 1, 2, \ldots, n \tag{16}$$

Moreover, δ shape parameter in the MQ function from the previous defined RBF equations is a fixed number determined by the user and affects the accuracy of the method substantially. The issue of which value the shape parameter δ indicating the smoothness or the sharpness of the surface will take poses an essential problem. According to Rippa (1999), δ parameter depends on many factors like the number of support points, the distribution of support points, RBF type, condition of the matrix and sensitivity of the computer calculation etc.

In the literature, various approaches are recommended for δ shape parameter in multiquadric radial basis functions (Hardy, 1971; Fogel and Tinney, 1996; Fasshauer, 2002). In this study, δ shape parameter value was calculated with the method recommended by Rippa (1999).

Rippa (1999) suggests finding an optimal value for the δ parameter as to minimize the cost function value defined by taking some norm of the error vector $E = (E_1, E_2, ..., E_n)^T$, for the reference points with leave-one-out-cross validation technique. Here the cost function is more expressly as follows:

$$E_i = N_i - \hat{N}_i \tag{17}$$

N_i is the real value of ith reference point, $\hat{N}_i$ is the RBF estimation value formed by using all reference points except ith reference point.

NUMERICAL APPLICATION

The data used in this study are the geodesic network points, ellipsoidal heights of which are determined with GPS and orthometric heights of which are determined with geometric levelling, within the boundaries of provinces Rize and Karaman located in the Northeast and South of Turkey respectively (Figure 1). Rize study field is consisted of a total of 41 GPS/levelling points, 38 of them being reference points and 13 of them being test points and encompasses a territory of 6*13 km^2. Karaman test field is consisted of a total of 191 GPS/levelling points, 142 of them being reference points and 49 of them being test points and encompasses a territory of 15*11 km^2. The topography in the study field in Rize has rough structure and its orthometric height alteration is about 570 m. The study field in Karaman does not have much rough surface and its orthometric height alteration is about 146 m. The topographical structure of the test fields and the distribution of the points are shown in Figure 1.

For both study fields in the application, local geoid surfaces were formed with MQ, TPS and MQS interpolation methods based on reference points. Then the geoid heights of the test points were estimated for formed surfaces and minimum, average and maximum difference values for test points were calculated. In addition, RMS values used as the conformity measure between real height values and estimation values of the m test points have been calculated with Formula (18) (El-Shafie and Seyed, 2011). These statistical values with which we assessed the sensitivities of the interpolation methods are shown in Table 1.

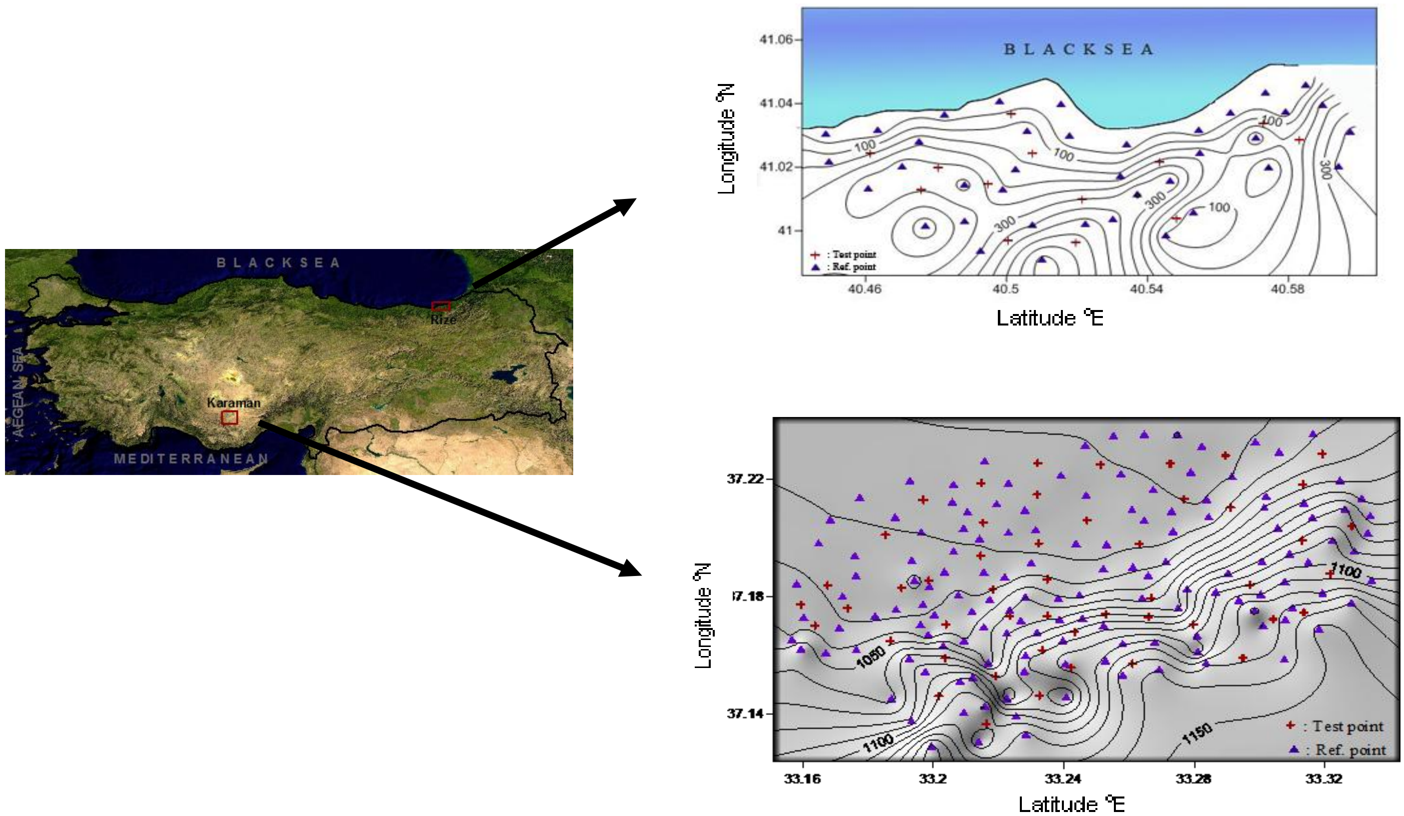

Figure 1. Locations of the study areas in Turkey and the distributions of the reference points and test points in the study areas.

Table 1. Statistical results related with the interpolation methods (cm).

Data	*Method*	*Min*	*ort*	*maks*	*RMS*
	MQ	-11.42	-7.41	-0.31	8.09
Rize	TPS	-14.26	-7.60	-0.95	8.55
	MQS	-15.16	-7.95	0.47	9.11
	MQ	-3.42	0.29	4.54	1.69
Karaman	TPS	-3.21	0.37	6.01	2.03
	MQS	-2.96	0.38	5.06	1.95

$$RMS = \sqrt{\frac{\sum_{i=1}^{m} (N_i^{estimated} - N_i^{real})^2}{m}} \qquad (18)$$

RESULTS

Conversion from ellipsoidal heights to orthometric heights has become an important issue as the geometric levelling is a high cost measurement method requiring time and workforce. As the GPS technique is used widely, it is practically used to give the position of values accurately.

Two study fields, topographic structures of which are different from each other are considered in the study performed, and the behaviors of the interpolation methods used for these fields have been investigated. According to the numerical application results given in Table 1, RMS values of test points were calculated as 8.09, 8.55, 9.11 cm in Rize study field while RMS values in Karaman study field were revealed as 1.69, 2.03 and 1.95 cm. As can be observed, even if there are not many differences between the methods in estimation of geoid surfaces of the test points, MQ method gave better results than TPS and MQS methods in both study fields.

REFERENCES

Buhmann MD (2000). Radial Basis Functions, Acta Numerica (Cambridge University Press), pp. 1–38.

Eberly D (2009). Thin Plate Splines. Geometric Tools, http://www.geometrictools.com/Documentation/ThinPlateSplines.pdf

El-Shafie A OJ, Seyed A (2011), Adaptive neuro-fuzzy inference system based model for rainfall forecasting in Klang River, Malaysia, Int. J. Phys. Sci., 6(12): 2875-2888.

Erol B, Celik RN (2004). Modelling local GPS/Levelling geoid with the assessment of inverse distance weighting and geostatistical kriging methods. XXth ISPRS Congress, Istanbul, Turkey.

Fasshauer GE (2002). Newton iteration with multiquadrics for the solution of nonlinear PDEs, Comput. Math. Appl., 43(3-5): 423-438.

Featherstone WE, Denith MC, Kirby JF (1998). Strategies for the accuratedetermination of orthometric heights from GPS, *Survey Rev.,* 34(267): 278-296.

Fogel DN, Tinney LR (1996). Image registration using multiquadric functions the finite element method bivariate mapping polynomials and thin plate spline, National Center for Geographic Information and Analysis Teknik Raporu, Santa Barbara. p. 63.

Fotopoulos G (2005). Calibration of geoid error models via a combined adjustment of ellipsoidal, orthometric and gravimetric geoid height data. J. Geodesy, 79: 111-123.

Franke R, Hagen H, Nielson GM (1993). Repeated knots in least squares multiquadric functions, Technical Report NPS-MA-94-004, Naval Postgraduate School, Monterey CA. 18.

Franke R, Nielson G (1980). Smooth interpolation of large sets of scattered data, Int. J. Numerical Methods Eng., 15: 1691-1704.

Hardy RL (1971). Multiquadric equations of topography and other irregular surfaces, J. Geophys. Res., 76: 1905-1915.

Heiskanen WA, Moritz H (1967). Physical Geodesy. W.H. Freeman and Company, San Fransisco. p. 364.

Lin S (2005). Study on developing regional grid-based geoid model using GPS and Levelling data. Proceedings of ACRS 2005, Asian Association on Remote Sensing,Hanoi, Vietnam. (http://www.a-a-r-s.org/acrs/proceeding/ACRS2005/Papers/NAS4.pdf)

Micchelli CA (1986). Interpolation of Scattered Data: Distance Matrices and Conditionally Positive Definite Functions, Const. Approx., 2: 11-22.

Nielson GM (1993). Scattered data modeling, IEEE Comput. Graphics Appl., 13: 60-70.

Renka RJ (1988). Multivariate Interpolation of Large Sets of Scattered Data, ACM Trans. Math. Software, 14(2): 139-148.

Rippa S (1999). An algorithm for selecting a good parameter c in radial basis function interpolation. Adv. Comput. Math., 11: 193-210.

URL-1, (2011), Thin Plate Spline Interpolation on Large 2D Grids, Barrodale Computing Services Ltd, http://www.barrodale.com/bcs/docs/Thin%20Plate%20Spline%2 Evaluation%20on%20Large%202D%20Grids.pdf ,

Yanalak M, Baykal O (2001). Transformation of Elllipsoid Heights to Local Levelling Heights, ASCE, J. Surveying Eng., 127(3): 1-14.

Zhong D (1997). Robust estimation and optimal selection of polynomial parameters for the interpolation of GPS geoid heights. *J. Geodesy Berlin*, 71(9): 552–561.