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Full Length Research Paper

Integrating a cost reduction delivery policy into an imperfect production system with repairable items

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This paper integrates a cost reduction delivery policy into an imperfect production system with repairable items for the purpose of lowering producer's stock holding cost. The present paper reexamines a manufacturing lot-size problem studied by a prior paper, and improves its production batch size solution in terms of lowering its inventory holding cost. An *n*+1 delivery policy is proposed here in lieu of *n* multi-delivery plan used by the prior paper. Under such a policy, an initial (additional) installment of finished products is shipped to customer within the production uptime for satisfying product demand during producer's regular production and rework times. Fixed quantity *n* installments of finished items are then distributed to customer at a fixed interval of time, at the end of rework. Mathematical modeling and analyses are employed in this study. As a result, the optimal replenishment lot size solution is derived. A numerical example is provided to show its practical usage as well as its significant savings in stock holding cost.

Key words: Industrial engineering, production lot size, multiple deliveries, random defective rate, rework, production control

INTRODUCTION

This paper integrates a cost reduction delivery policy into an imperfect economic production quantity (EPQ) model with repairable items (Chiu et al., 2009a) for the purpose of lowering producer's stock holding cost. The EPQ model employs mathematical modeling to balance production setup cost and holding cost, to assist producers in determining economic production lot size that minimizes the overall production- inventory costs (Hadley and Whitin, 1963; Hillier and Lieberman, 2001). Classic EPQ model implicitly assumes that all items produced are of perfect quality. But in real-life production systems, due to many controllable and/or uncontrollable factors, generation of defective items is inevitable. Therefore, many studies have been carried out to enhance EPQ model by addressing the imperfect production related issues (Nahmias, 2009). Examples of articles are surveyed as follows. Shih (1980) examined two inventory models to the case where the proportion of defective units in the accepted lot is a random variable with known probability distributions. Optimal solutions to the modified system were developed and comparisons with the traditional models were also presented via numerical examples. Rosenblatt and Lee (1986) studied an EPQ model that deals with imperfect quality. They assumed that at some random point in time, the process might shift from an in-control to an out-of-control state, and a fixed percentage of defective items are produced. Approximate solutions for obtaining an optimal lot size were developed in their paper. Zhang and Gerchak (1990) considered joint lot sizing and inspection policy in an EOQ model with random yield. Kim et al. (2001) studied the optimal production run length and inspection

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schedules in a deteriorating production process. They assumed that a production process is subject to a random deterioration from the in-control state to the out-of-control state and thus produces some proportion of defective items. By minimizing the overall production-inventory costs, an optimal production run length and an optimal number of inspections are derived, and unique properties of the proposed model are discussed (for more literature, see also Cheung and Hausman (1997), Chiu et al. (2005), Wazed et al. (2009), Koçyiğit et al. (2009), Chiu et al. (2006) and Baten and Kamil (2009).

Non conforming items sometimes can be reworked and repaired, so the overall production costs can be significantly reduced. For instance, the production processes in plastic injection molding, or printed circuit board assembly, etc., sometimes employs rework as an acceptable process in terms of level of quality. Yum and McDowell (1987) treated the allocation of inspection effort problem for serial systems as a 0-1 mixed integer linear programming problem (MILP). Their formulation permitted any different combination of scrap, rework, or repair at each station, and allowed the problem to be solved using standard MILP software packages. Grosfeld-Nir and Gerchak (2002) studied multistage production systems where defective units can be reworked repeatedly at every stage. They showed that a multistage system where only one of the stages requires a set-up can be reduced to a single-stage system. They proved that it is best to make the "bottle-neck" the first stage of the system and they also developed recursive algorithms for solving two- and three-stage systems. Chiu et al. (2007) studied the optimal lot-sizing decision for a production system with rework, a random scrap rate, and a service level constraint. They derived an optimal operating policy and prove that the expected overall costs of such a production system with backlogging permitted is less than or equal to that of the same model without backlogging. Then the relationship between the "imputed backorder cost" and maximal permitted shortage level was derived for decision-making on whether the required service level is achievable. In the case that the required service level is not attainable, an equation for calculating the intangible backorder cost was proposed, so that a new optimal lot-size policy that minimizes expected overall costs as well as satisfies the service level constraint can be derived accordingly (for more studies, see also Makis, 1998; Chiu and Chiu, 2006; Chiu et al., 2009b; Wazed et al., 2010; Chiu, 2010).

Continuous inventory issuing policy for satisfying product demand is another implicitly assumption of classic EPQ model. However, in real-life vendor- buyer integrated production-inventory-delivery system, multiple or periodic deliveries of finished products are commonly used. Schwarz (1973) studied a one-warehouse N-retailer deterministic inventory system. The objective was to determine the stocking policy which minimizes average system cost per unit time. Necessary properties of an optimal policy were derived and the optimal solutions for the one-retailer and N identical retailer problems were given. Heuristic solutions for the general problem were also suggested, tested against analytical lower bounds and on the basis of these tests, found to be near optimal. Goyal (1977) studied the integrated inventory model for the single supplier-single customer problem. He proposed a method that is typically applicable to those inventory problems where a product is procured by a single customer from a single supplier. An example was provided to illustrate his proposed method.

Many studies have since been carried out to address various aspects of supply chain optimization. For instance, Schwarz et al. (1985) examined the system fill-rate of a one-warehouse N-identical retailer distribution system as a function of warehouse and retailer safety stock. Using approximation model of a prior study, they examined the problem of maximizing system fill-rate subject to a constraint on system safety stock. Optimal safety stock policy is characterized to be the intersection of a fill-rate policy line and the safety stock budget line. Properties of fill-rate policy lines are given. These properties may be used to provide managerial insight into system optimization and as the basis for heuristics. Sarker and Parija (1994) considered a manufacturing system which procures raw materials from suppliers and processes them to convert to finished products. They proposed a model that was used to determine an optimal ordering policy for procurement of raw materials, and the manufacturing batch size to minimize the total cost for meeting equal shipments of the finished products, at fixed intervals, to the buyers. Sarker and Khan (2001) addressed the problem of a manufacturing system that procures raw materials from suppliers in a lot and processes them to convert to finished products. They proposed an ordering policy for raw materials to meet the requirements of a production facility. In turn, this facility must deliver finished products demanded by outside buyers at fixed interval points in time.

A general cost model was developed first considering both raw materials and finished products. Then this model was used to develop a simple procedure to determine an optimal ordering policy for procurement of raw materials as well as the manufacturing batch size to minimize the total cost for meeting the customer demand on time. Diponegoro and Sarker (2006) studied an ordering policy for raw materials as well as an economic batch size for finished products that are delivered to customers frequently at a fixed interval of time for a finite planning horizon. The problem was also extended to compensate for the lost sales of finished products. A closed-form solution to the problem was obtained for the minimal total cost. A lower bound on the optimal solution was also developed for problem with lost sale. It was shown that the solution and the lower bound were consistently tight. Kim et al. (2008) examined benefits of buyer-supplier partnerships over lot-for-lot (that is single setup single delivery (SSSD)) systems and suggested two policies that the supplier can pursue in order to meet customers' needs: (1) Single setup multiple delivery (SSMD), and (2) Multiple setup multiple delivery (MSMD). In order to provide guidelines for the policy selection, they examined the interactions among variables such as production capacity, learning rate, and holding costs for both parties. They also discussed the benefit sharing plan, which is according to the contribution (or sacrifice) each party made to partnership efforts. Chiu et al. (2009a) derived the production lot size with the reworking of random defective items and fixed quantity multiple deliveries. They assumed that fixed quantity multiple installments of the finished batch can only be delivered to customers if the whole lot is quality assured at the end of rework. A closed-form optimal lot size solution to the problem was obtained (for more literature, see also Viswanathan, 1998; Goyal and Nebebe, 2000; Buscher and Lindner, 2005; Tang et al., 2008; Sarker; Diponegoro, 2009).

This paper improves the lot size solution derived by Chiu et al. (2009a) by introducing a cost reduction delivery policy to their model, with the purpose of lowering producer's stock holding cost. An n+1 delivery policy is proposed here in lieu of their n multi-delivery plan for this specific EPQ model with repairable items. The joint effect of the n+1 multi-delivery policy and the reworking of defective items on the optimal replenishment batch size for this integrated EPQ model are investigated.

METHODS

Mathematical modeling and derivations

This research reexamines the specific EPQ model studied by Chiu et al. (2009a). Description of the proposed model is as follows: Consider a production system may produce x portion of random defective items at a production rate d. All nonconforming items are assumed to be repairable and are reworked at a rate P_1, within the same cycle when regular production ends. Under the regular operating schedule, the constant production rate P is larger than the sum of demand rate λ and production rate of defective items d. That is: $(P-d-\lambda)>0$; where d can be expressed as $d=Px$. Other cost related parameters include the setup cost K, unit production cost C, unit holding cost h, unit rework cost C_R, unit holding cost h_1 for reworked item, fixed delivery cost K_1 per shipment, and delivery cost C_T per item shipped to customers. Additional notation is listed in nomenclature (see Appendix A).

Under the proposed n+1 delivery policy, an initial installment of finished (perfect quality) products is delivered to customer for satisfying the demand during production uptime and rework time. At the end of rework, when the rest of the production lot is quality assured, fixed quantity n installments of finished items are distributed to customer at a fixed interval of time. Such an n+1 delivery policy is intended to reduce supplier's stock holding cost.

The on-hand inventory of perfect quality items of the proposed model is illustrated in Figure 1 (in blue). The expected reduction in stock holding costs (in yellow/shade) for the proposed model in comparison with Chiu et al.'s model (2009a) (in red), is also displayed in Figure 1. From Figure 1 and the assumption of the proposed model, the following expressions can be derived accordingly (as in Chiu et al., 2009a):

$$t=\frac{\lambda(t_1+t_2)}{P-d} \tag{1}$$

$$H=(P-d)t=\lambda(t_1+t_2) \tag{2}$$

$$H_1=Q(1-x)-\lambda(t_1+t_2) \tag{3}$$

$$H_2=H_1+P_1t_2 \tag{4}$$

$$t_1=\frac{Q}{P}=\frac{H_1+H_2}{P-d} \tag{5}$$

$$t_3=nt_n=T-(t_1+t_2) \tag{6}$$

$$T=t_1+t_2+t_3=\frac{Q}{\lambda} \tag{7}$$

It is noted that the maximum level of defective items is dt_1. These nonconforming items are reworked and repaired during t_2, which is the time needed for rework, as shown in Equation 8.

$$t_2=\frac{xQ}{P_1} \tag{8}$$

Total production-inventory-delivery costs per cycle $TC(Q)$ consists of the variable production cost, the setup cost, variable rework cost, (n+1) fixed distribution costs and variable delivery cost, holding cost for perfect quality items during production uptime t_1 and reworking time t_2, holding cost for defective items during t_1, variable holding cost for items reworked during t_2, and holding cost for finished goods during the delivery time t_3 where n fixed-quantity installments of the finished batch are delivered to customers at a fixed interval of time (for computation of the last term (refer to Appendix-2 of Chiu et al. 2009c).

$$\begin{aligned}TC(Q)&=CQ+K+C_R\left[xQ(1-\theta)\right]+(n+1)K_1+C_TQ\\&+h\left[\frac{H}{2}(t)+\frac{H_1}{2}(t_1-t)+\frac{H_2+H_1}{2}(t_2)+\frac{dt_1}{2}(t_1)\right]\\&+h_1\left[\frac{dt_1}{2}(t_2)\right]+h\left[\left(\frac{n-1}{2n}\right)H_2t_3\right]\end{aligned} \tag{9}$$

Taking the randomness of defective rate x into account (where x is assumed to be a random variable with a known probability density function), one uses the expected values of x in the related cost analysis. Substituting all related parameters from Equation 1 to 8 in $TC(Q)$, the expected production-inventory- delivery cost per unit time $E[TCU(Q)]$ can be obtained as follows (see appendix B for details).

$$\begin{aligned}E\left[TCU(Q)\right]&=\frac{E\left[TC(Q)\right]}{T}\\&=\lambda\left[C+\frac{\left[(n+1)K_1+K\right]}{Q}+C_RE(x)+C_T\right]\\&+\frac{hQ}{2}\left\{\begin{array}{l}\frac{2\lambda^3}{P^3}E\left(\frac{1}{1-x}\right)+\frac{4\lambda^3}{P^2P_1}E\left(\frac{x}{1-x}\right)-\frac{\lambda^2}{P^2}+\frac{2\lambda^3}{PP_1^2}E\left(\frac{x^2}{1-x}\right)\\-\frac{2\lambda^2E(x)}{PP_1}-\frac{\lambda\left[E(x)\right]^2}{P_1}-\frac{\lambda^2\left[E(x)\right]^2}{P_1^2}+\left(1-\frac{\lambda}{P}\right)\\-\left(\frac{1}{n}\right)\left[1-\frac{2\lambda}{P}-\frac{2\lambda E(x)}{P_1}+\frac{2\lambda^2E(x)}{PP_1}+\frac{\lambda^2}{P^2}+\frac{\lambda^2\left[E(x)\right]^2}{P_1^2}\right]\end{array}\right\}\\&+\frac{h_1Q\lambda\left[E(x)\right]^2}{2P_1}\end{aligned} \tag{10}$$

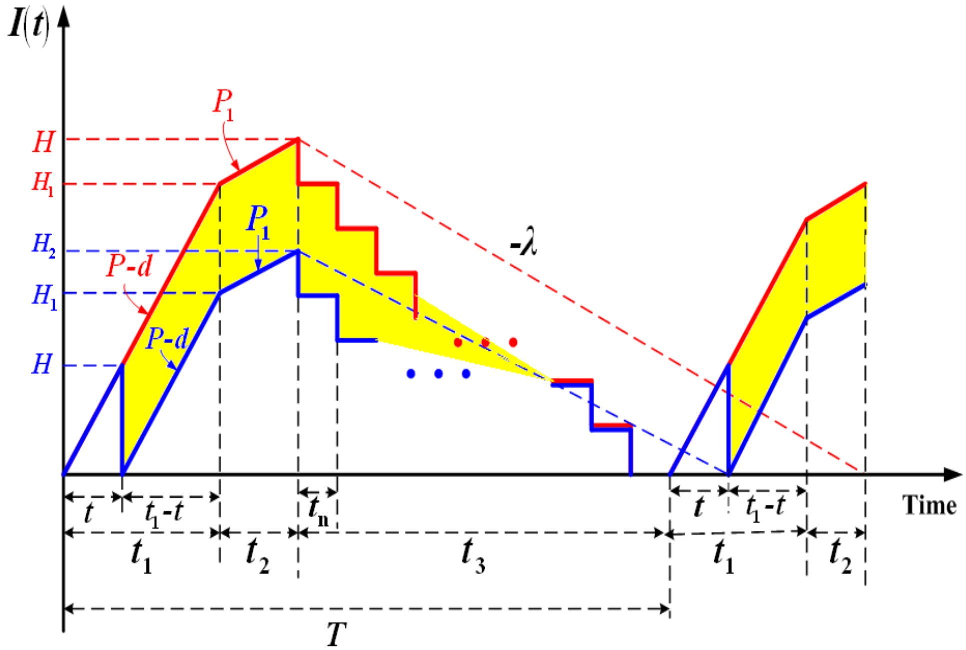

Figure 1. Expected reduction in stock holding costs (in yellow) of the proposed model in comparison with Chiu et al.'s model (2009a).

Derivations of the optimal replenishment lot size

The optimal replenishment lot size can be obtained by minimizing the expected cost function *E[TCU(Q)]*. Differentiating *E[TCU(Q)]* with respect to *Q*, the first and the second derivatives of *E[TCU(Q)]* are shown in Equations 11 and 12.

$$\frac{dE\left[TCU(Q)\right]}{dQ}=\frac{-\left[(n+1)K_1+K\right]\lambda}{Q^2}+\frac{h}{2}\left\{\begin{array}{l}\frac{2\lambda^3}{P^3}E\left(\frac{1}{1-x}\right)+\frac{4\lambda^3}{P^2P_1}E\left(\frac{x}{1-x}\right)-\frac{\lambda^2}{P^2}+\frac{2\lambda^3}{PP_1^2}E\left(\frac{x^2}{1-x}\right)\\ -\frac{2\lambda^2E(x)}{PP_1}-\frac{\lambda\left[E(x)\right]^2}{P_1}-\frac{\lambda^2\left[E(x)\right]^2}{P_1^2}+\left(1-\frac{\lambda}{P}\right)\\ -\left(\frac{1}{n}\right)\left[1-\frac{2\lambda}{P}-\frac{2\lambda E(x)}{P_1}+\frac{2\lambda^2E(x)}{PP_1}+\frac{\lambda^2}{P^2}+\frac{\lambda^2\left[E(x)\right]^2}{P_1^2}\right]\end{array}\right\}+\frac{h_1\lambda\left[E(x)\right]^2}{2P_1} \quad (11)$$

$$\frac{d^2E\left[TCU(Q)\right]}{dQ^2}=\frac{2\left[(n+1)K_1+K\right]\lambda}{Q^3} \quad (12)$$

One notes that Equation 12 is resulting positive because *K, n,* K_1*, λ,* and *Q* are all positive. The second derivative of *E[TCU(Q)]* with respect to *Q* is greater than zero, and hence *E[TCU(Q)]* is a convex function for all *Q* different from zero.

It follows that the optimal production lot size *Q** can be derived by setting the first derivative of *E[TCU(Q)]* equal to zero.

$$\frac{dE\left[TCU(Q)\right]}{dQ}=\frac{-\left[(n+1)K_1+K\right]\lambda}{Q^2}+\frac{h}{2}\left\{\begin{array}{l}\frac{2\lambda^3}{P^3}E\left(\frac{1}{1-x}\right)+\frac{4\lambda^3}{P^2P_1}E\left(\frac{x}{1-x}\right)-\frac{\lambda^2}{P^2}+\frac{2\lambda^3}{PP_1^2}E\left(\frac{x^2}{1-x}\right)\\ -\frac{2\lambda^2E(x)}{PP_1}-\frac{\lambda\left[E(x)\right]^2}{P_1}-\frac{\lambda^2\left[E(x)\right]^2}{P_1^2}+\left(1-\frac{\lambda}{P}\right)\\ -\left(\frac{1}{n}\right)\left[1-\frac{2\lambda}{P}-\frac{2\lambda E(x)}{P_1}+\frac{2\lambda^2E(x)}{PP_1}+\frac{\lambda^2}{P^2}+\frac{\lambda^2\left[E(x)\right]^2}{P_1^2}\right]\end{array}\right\}+\frac{h_1\lambda\left[E(x)\right]^2}{2P_1}=0 \quad (13)$$

With rearrangement, one obtains the following:

$$\frac{\left[(n+1)K_1+K\right]\lambda}{Q^2}=\frac{h}{2}\left\{\begin{array}{l}\frac{2\lambda^3}{P^3}E\left(\frac{1}{1-x}\right)+\frac{4\lambda^3}{P^2P_1}E\left(\frac{x}{1-x}\right)-\frac{\lambda^2}{P^2}+\frac{2\lambda^3}{PP_1^2}E\left(\frac{x^2}{1-x}\right)\\ -\frac{2\lambda^2E(x)}{PP_1}-\frac{\lambda\left[E(x)\right]^2}{P_1}-\frac{\lambda^2\left[E(x)\right]^2}{P_1^2}+\left(1-\frac{\lambda}{P}\right)\\ -\left(\frac{1}{n}\right)\left[1-\frac{2\lambda}{P}-\frac{2\lambda E(x)}{P_1}+\frac{2\lambda^2E(x)}{PP_1}+\frac{\lambda^2}{P^2}+\frac{\lambda^2\left[E(x)\right]^2}{P_1^2}\right]\end{array}\right\}+\frac{h_1\lambda\left[E(x)\right]^2}{2P_1} \quad (14)$$

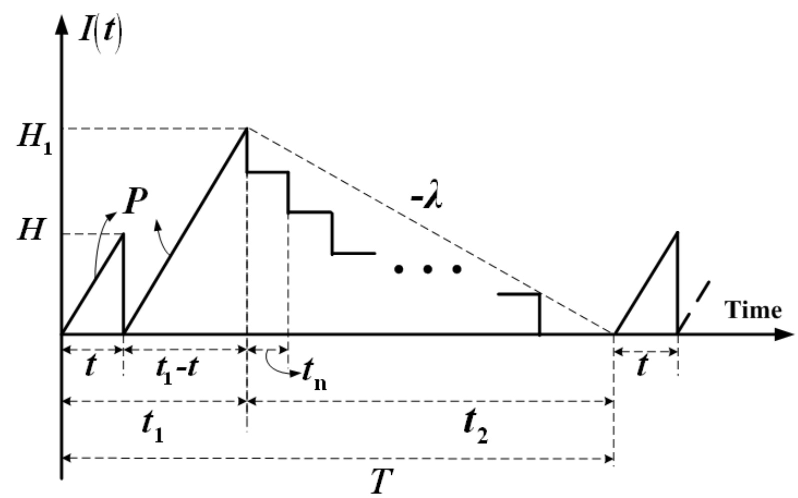

Figure 2. On-hand inventory of finished items in EPQ model with (n+1) delivery policy – the special case (x=0) model.

or

$$Q^2 = \sqrt{\frac{2\left[(n+1)K_1+K\right]\lambda}{h\left\{\begin{array}{l}\dfrac{2\lambda^3}{P^3}E\left(\dfrac{1}{1-x}\right)+\dfrac{4\lambda^3}{P^2P_1}E\left(\dfrac{x}{1-x}\right)-\dfrac{\lambda^2}{P^2}+\dfrac{2\lambda^3}{PP_1^2}E\left(\dfrac{x^2}{1-x}\right)\\ -\dfrac{2\lambda^2E(x)}{PP_1}-\dfrac{\lambda\left[E(x)\right]^2}{P_1}-\dfrac{\lambda^2\left[E(x)\right]^2}{P_1^2}+\left(1-\dfrac{\lambda}{P}\right)\\ -\left(\dfrac{1}{n}\right)\left[1-\dfrac{2\lambda}{P}-\dfrac{2\lambda E(x)}{P_1}+\dfrac{2\lambda^2E(x)}{PP_1}+\dfrac{\lambda^2}{P^2}+\dfrac{\lambda^2\left[E(x)\right]^2}{P_1^2}\right]\end{array}\right\}+\dfrac{h_1\lambda\left[E(x)\right]^2}{P_1}}} \quad (15)$$

Therefore, with further rearrangements, one obtains the optimal replenishment lot size as

$$Q^* = \sqrt{\frac{2\left[(n+1)K_1+K\right]\lambda}{h\left\{\begin{array}{l}\dfrac{\lambda^2}{P^2}\left[\dfrac{2\lambda}{P}E\left(\dfrac{1}{1-x}\right)+\dfrac{4\lambda}{P_1}E\left(\dfrac{x}{1-x}\right)-1\right]+\left(1-\dfrac{\lambda}{P}\right)\\ +\dfrac{\lambda^2}{P_1^2}\left[\dfrac{2\lambda}{P}E\left(\dfrac{x^2}{1-x}\right)-\left[E(x)\right]^2\right]-\dfrac{\lambda E(x)}{P_1}\left[\dfrac{2\lambda}{P}+\left[E(x)\right]\right]\\ -\left(\dfrac{1}{n}\right)\left[1+\dfrac{\lambda}{P}\left[-2+\dfrac{\lambda}{P}\right]+\dfrac{\lambda E(x)}{P_1}\left[-2+\dfrac{2\lambda}{P}+\dfrac{\lambda\left[E(x)\right]}{P_1}\right]\right]\end{array}\right\}+\dfrac{h_1\lambda\left[E(x)\right]^2}{P_1}}} \quad (16)$$

The special case

If all items produced are of perfect quality (that is x=0). Figure 2 depicts the on-hand inventory of finished items for this special case model. Let $TC_1(Q)$ denote the total production-inventory-delivery cost per cycle for such a case, then

$$TC_1(Q)=K+CQ+(n+1)K_1+C_TQ + h\left[\frac{H}{2}(t)+\frac{H_1}{2}(t_1-t)\right]+h\left[\left(\frac{n-1}{2n}\right)H_1t_2\right] \quad (17)$$

By using the similar derivations, one obtains $E[TCU_1(Q)]$ as follows.

$$E\left[TCU_1(Q)\right]=C\lambda+\frac{\left[(n+1)K_1+K\right]\lambda}{Q}+C_T\lambda + \frac{hQ}{2}\left[\frac{\lambda^2}{P^2}\left(\frac{2\lambda}{P}-1\right)+\left(1-\frac{\lambda}{P}\right)\left[1-\left(\frac{1}{n}\right)\left(1-\frac{\lambda}{P}\right)\right]\right] \quad (18)$$

The second derivative of $E[TCU_1(Q)]$ is shown in Equation 19. One can verify that $E[TCU_1(Q)]$ is a convex function.

$$\frac{d^2E\left[TCU_1(Q)\right]}{dQ^2}=\frac{2\left[(n+1)K_1+K\right]\lambda}{Q^3} \quad (19)$$

By setting the first derivative of $E[TCU(Q)]$ equal to zero, the following optimal production lot size Q^* can be obtained.

$$Q^*=\sqrt{\frac{2\left[(n+1)K_1+K\right]\lambda}{h\left[\dfrac{\lambda^2}{P^2}\left(\dfrac{2\lambda}{P}-1\right)+\left(1-\dfrac{\lambda}{P}\right)\left[1-\left(\dfrac{1}{n}\right)\left(1-\dfrac{\lambda}{P}\right)\right]\right]}} \quad (20)$$

Numerical example and discussion

With the purpose of comparison of the proposed model and Chiu et al.'s model (2009a), here the same numerical example was adopted as in Chiu et al. (2009a). Consider that a product can be manufactured at a rate of 60,000 units per year and this item had experienced a flat demand rate of 3,400 units per year. During production

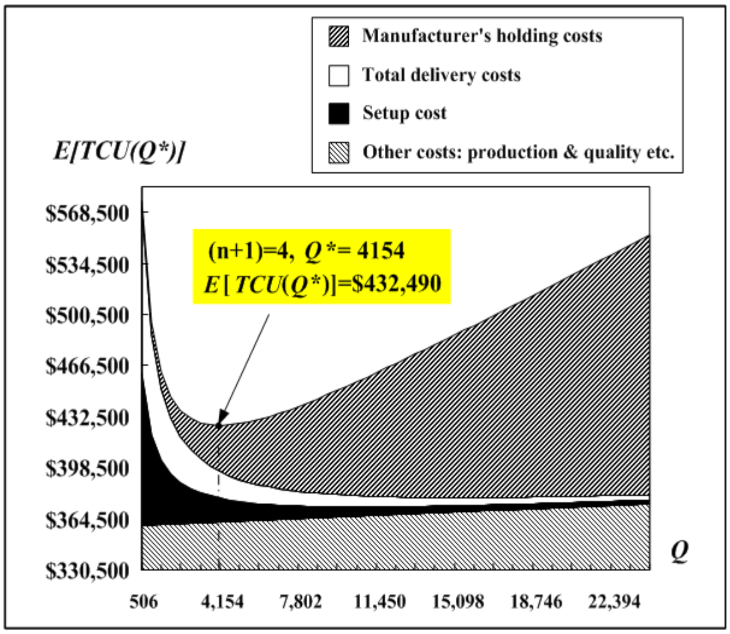

Figure 3. Variation of the replenishment lot size effects on E[*TCU*(*Q*)] and on different components of E[*TCU*(*Q*)]

process, a random defective rate x is assumed to be uniformly distributed over the interval [0, 0.3]. All defective items are assumed to be repairable and are reworked when regular production ends, at a rate of rework P_1 = 2,200 units per year. Additional values of parameters include: K=$20,000 per production run; C=$100 per item; C_R=$60 per item reworked; h=$20 per item per year; h_1=$40 per item reworked; a fixed cost K_1=$4,400 per shipment; and C_T=$0.1 per item delivered. In order to show practical usages of our research results, the following two different scenarios are demonstrated, respectively:

Scenario 1

Let total number of delivery remain 4 (that is n=4 as was used in Chiu et al., 2009a). For the proposed model, it is (n+1)=4. An initial installment of finished products is distributed to customer during t_1, for satisfying the product demand during producer's production uptime and rework time. Then, at the end of rework, fixed quantity 3 other installments of finished items are delivered to customer at a fixed interval of time. Also, for the purpose of comparison, we use the lot-size solution Q=3,427 (from Chiu et al., 2009a) in calculating the expected production-inventory-delivery cost (Equation 10 of the proposed model) and obtain E[*TCU*(3427)]=$433,633. One notes that there is a reduction in manufacturer holding costs amounts to $11,921, or 12.73% of total other related costs (that is, E[*TCU*(*Q*)]-(λC): total cost excludes the variable production cost).

Scenario 2

Let total number of deliveries remain 4 (that is (n+1)=4 in our model). By applying Equations 16 and 10, one obtains the optimal replenishment lot size Q^*=4,154 and the expected total costs E[*TCU*(4154)]=$432,490, respectively. It is noted that the overall reduction in production-inventory- delivery costs amounts to $13,064, or 14.12% of total other related costs.

Variation of the replenishment lot size effects on E[*TCU*(*Q*)] and on different components of E[*TCU*(*Q*)] are depicted in Figure 3.

Conclusions

This paper integrates a cost reduction delivery policy into an imperfect EPQ model with repairable items (Chiu et al., 2009a), for the purpose of cutting down manufacturer's inventory holding cost. Chiu et al. (2009a) derived the lot size solution for an EPQ model with the reworking of random defective items and fixed quantity multiple deliveries. In their model, it is assumed that fixed quantity multiple installments of the finished batch can only be distributed to customers if the whole lot is quality assured at the end of rework. For the purpose of lowering supplier's stock holding cost, this study extends Chiu et al.'s model (2009a) and proposes an n+1 delivery policy in lieu of their n multi-delivery plan.

Mathematical modeling and analyses are employed, and the expected integrated production-inventory-delivery cost per unit time is derived and proved to be a convex function. The closed-form optimal replenishment lot size solution to the problem is obtained. A numerical example is provided to show practical usage of our research result and demonstrate its significant savings in producer's stock holding cost. For future research, one may examine the effect of multiple customers on the lot size decision for the same model.

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APPENDIX A

Nomenclature: ***t*****,** the production time needed for producing enough perfect items for satisfying product demand during the production uptime t_1 and the rework time t_2**;** $\boldsymbol{t_1}$**,** the production uptime for the proposed EPQ model; $\boldsymbol{t_2}$**,** time required for reworking of defective items; $\boldsymbol{t_3}$**,** time required for delivering the remaining quality assured finished products; ***T*****,** cycle length; ***H*****,** the level of on-hand inventory in units for satisfying product demand during manufacturer's regular production time t_1 and rework time t_2; $\boldsymbol{H_1}$**,** maximum level of on-hand inventory in units when regular production ends; $\boldsymbol{H_2}$**,** the maximum level of on-hand inventory in units when rework process finishes; ***Q*****,** production lot size to be determined for each cycle; ***n*****,** number of fixed quantity installments of the rest of finished batch to be delivered to customer during t_3; t_n, a fixed interval of time between each installment of **products delivered during** $\boldsymbol{t_3}$**;** ***I*****(*****t*****),** on-hand inventory of perfect quality items at time *t*; $\boldsymbol{I_d(t)}$**,** on-hand inventory of defective items at time *t*; ***TC*****(*****Q*****),** total production-inventory-delivery costs per cycle for the proposed model; $\boldsymbol{TC_1(Q)}$**,** total production-inventory-delivery costs per cycle for the special case model; **E[*****TCU*****(*****Q*****)],** the long-run average costs per unit time for the proposed model; $\mathbf{E}[\boldsymbol{TCU_1(Q)}]$**,** the long-run average costs per unit time for the special case.

APPENDIX B

Computation of Equation 10

Recall Equation 9 as follows:

$$TC(Q)=CQ+K+C_{\mathrm{R}}\left[xQ(1-\theta)\right]+(n+1)K_1+C_{\mathrm{T}}Q+h\left[\frac{H}{2}(t)+\frac{H_1}{2}(t_1-t)+\frac{H_2+H_1}{2}(t_2)+\frac{dt_1}{2}(t_1)\right]+h_1\left[\frac{dt_1}{2}(t_2)\right]+h\left[\left(\frac{n-1}{2n}\right)H_2t_3\right] \quad (9)$$

Substituting all related parameters from Equations 1 to 8 in Equation 9 one obtains

$$TC(Q)=CQ+K+(n+1)K_1+C_{\mathrm{R}}\left[xQ\right]+C_{\mathrm{T}}Q+\frac{hQ^2}{2}\left\{\begin{array}{l}\frac{2\lambda^2}{P^3(1-x)}+\frac{4\lambda^2x}{P^2P_1(1-x)}+\frac{2\lambda^2x^2}{PP_1^2(1-x)}-\frac{\lambda}{P^2}\\-\frac{2\lambda x}{PP_1}-\frac{x^2}{P_1}\left(1+\frac{\lambda}{P_1}\right)+\left(1-\frac{1}{n}\right)\left[\frac{1}{\lambda}-\frac{1}{P}\right]\\-\left(\frac{1}{n}\right)\left[\frac{1}{P}\left(\frac{\lambda}{P}-1\right)-\frac{2x}{P_1}+\frac{2\lambda x}{PP_1}+\frac{\lambda x^2}{P_1^2}\right]\end{array}\right\}+\frac{h_1x^2Q^2}{2P_1} \quad \text{(B-1)}$$

or

$$TC(Q)=CQ+K+(n+1)K_1+C_{\mathrm{R}}\left[xQ\right]+C_{\mathrm{T}}Q+\frac{hQ^2}{2}\left\{\begin{array}{l}\frac{2\lambda^2}{P^3(1-x)}+\frac{4\lambda^2x}{P^2P_1(1-x)}+\frac{2\lambda^2x^2}{PP_1^2(1-x)}\\-\frac{\lambda}{P^2}-\frac{2\lambda x}{PP_1}-\frac{x^2}{P_1}\left(1+\frac{\lambda}{P_1}\right)+\frac{1}{\lambda}-\frac{1}{P}\\-\left(\frac{1}{n}\right)\left[\frac{1}{\lambda}-\frac{2}{P}-\frac{2x}{P_1}+\frac{2\lambda x}{PP_1}+\frac{\lambda}{P^2}+\frac{\lambda x^2}{P_1^2}\right]\end{array}\right\}+\frac{h_1x^2Q^2}{2P_1} \quad \text{(B-2)}$$

Because

$$T=\frac{Q}{\lambda} \quad \text{(B-3)}$$

and

$$E\left[TCU(Q)\right]=\frac{E\left[TC(Q)\right]}{T} \quad \text{(B-4)}$$

Substituting Equation (B-2) and (B-3) in Equation (B-4) one obtains

$$\frac{E\left[TC(Q)\right]}{T}=C\lambda+\frac{\left[(n+1)K_1+K\right]\lambda}{Q}+C_{\mathrm{R}}E(x)\lambda+C_{\mathrm{T}}\lambda+\frac{hQ\lambda}{2}\left\{\begin{array}{l}\frac{2\lambda^2}{P^3}E\left(\frac{1}{1-x}\right)+\frac{4\lambda^2}{P^2P_1}E\left(\frac{x}{1-x}\right)+\frac{2\lambda^2}{PP_1^2}E\left(\frac{x^2}{1-x}\right)\\-\frac{\lambda}{P^2}-\frac{2\lambda E(x)}{PP_1}-\frac{\left[E(x)\right]^2}{P_1}-\frac{\lambda\left[E(x)\right]^2}{P_1^2}+\frac{1}{\lambda}-\frac{1}{P}\\-\left(\frac{1}{n}\right)\left[\frac{1}{\lambda}-\frac{2}{P}-\frac{2E(x)}{P_1}+\frac{2\lambda E(x)}{PP_1}+\frac{\lambda}{P^2}+\frac{\lambda\left[E(x)\right]^2}{P_1^2}\right]\end{array}\right\}+\frac{h_1Q\lambda\left[E(x)\right]^2}{2P_1} \quad \text{(B-5)}$$

With further rearrangements one has

$$E\left[TCU(Q)\right]=\frac{E\left[TC(Q)\right]}{T}=\lambda\left[C+\frac{\left[(n+1)K_1+K\right]}{Q}+C_{\mathrm{R}}E(x)+C_{\mathrm{T}}\right]+\frac{hQ}{2}\left\{\begin{array}{l}\frac{2\lambda^3}{P^3}E\left(\frac{1}{1-x}\right)+\frac{4\lambda^3}{P^2P_1}E\left(\frac{x}{1-x}\right)-\frac{\lambda^2}{P^2}+\frac{2\lambda^3}{PP_1^2}E\left(\frac{x^2}{1-x}\right)\\-\frac{2\lambda^2E(x)}{PP_1}-\frac{\lambda\left[E(x)\right]^2}{P_1}-\frac{\lambda^2\left[E(x)\right]^2}{P_1^2}+\left(1-\frac{\lambda}{P}\right)\\-\left(\frac{1}{n}\right)\left[1-\frac{2\lambda}{P}-\frac{2\lambda E(x)}{P_1}+\frac{2\lambda^2E(x)}{PP_1}+\frac{\lambda^2}{P^2}+\frac{\lambda^2\left[E(x)\right]^2}{P_1^2}\right]\end{array}\right\}+\frac{h_1Q\lambda\left[E(x)\right]^2}{2P_1} \quad (10)$$