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Full Length Research Paper

A note on squeezing flow between two infinite parallel plates with slip boundary conditions

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The aim of this letter is to investigate an axisymmetric squeezing flow of an incompressible fluid generated by two large parallel plates, including fluid inertial effects. The governing equations have been transformed into a nonlinear ordinary differential equation using integribility condition. Solution to the problem is obtained by using an optimal homotopy asymptotic method (OHAM). The results reveal that the new method is very effective and simple.

Key words: Axisymmetric squeezing flow, slip boundary conditions, optimal homotopy asymptotic method (OHAM).

INTRODUCTION

The study of squeezing flows has been published in a wide variety of journals spanning a century or more due to its practical applications in chemical engineering and food industry. The basic research in this field was carried out by Stefan (1874). Of more recent origin is the interest in squeezing flows spurred by problems encountered in lubrications and dusty fluids (Chatraei et al., 1981; Thien and Tanner, 1984; Debnath and Ghosh, 1988; Hamdan and Baron, 1992; Kompani and Venerus, 2000). Also, the analytical and experimental study of these flows together with the inertial term between the rotating cylinders and parallel plates has resulted in increasing interest due to its importance in thin film of lubricants (Debbaut, 2001; Wang and Watson, 1979; Denn and Marrucci, 1999; Hoffner et al., 2001; Lee et al., 1984). The mathematical studies of these flows are concerned primarily with the non linear partial differential equations which arise from the Navier-Stokes equations. These equations have no general solutions and only a few of number of exact solutions have been attained (He, 2006). To solve practical problems, different perturbation and analytical techniques have been widely used in fluid mechanics and engineering (Ali et al., 2010).

Our purpose in this contribution is to study axisymmetric fluid flow between two large parallel plates with slip boundary conditions, taking into account the inertia effects. The optimal homotopy asymptotic method is applied to solve the title problem (Herisanu et al., 2008; Idrees et al., 2010a, b; Islam et al., 2010; Marinca and Herisanu, 2010a,b; Marinca et al., 2008, 2009; Hesameddini and Latifzadeh, 2009; Shah et al., 2010). Navier assumed that the velocity u_x, at a solid surface is pro-portional to the shear rate at the surface $u_x = \beta \partial u_x / \partial y$, where β is the slip length Navier (1823). If $\beta = 0$, we will get the general no-slip boundary condition. If β is finite, fluid slip occurs at the wall, but its effect depends upon the length scale of the flow. Estelle and Lanos (2007), Tretheway and Meinhart (2002), Laun et al. (1999) and Zhu and Granick (2001) have studied this fact in more detail.

BASIC EQUATION

Here, we determine the basic equations which are used in the rest of this paper. In the absence of body forces, the basic equations governing the flow in vorticity form are given by:

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} = 0 \tag{1}$$

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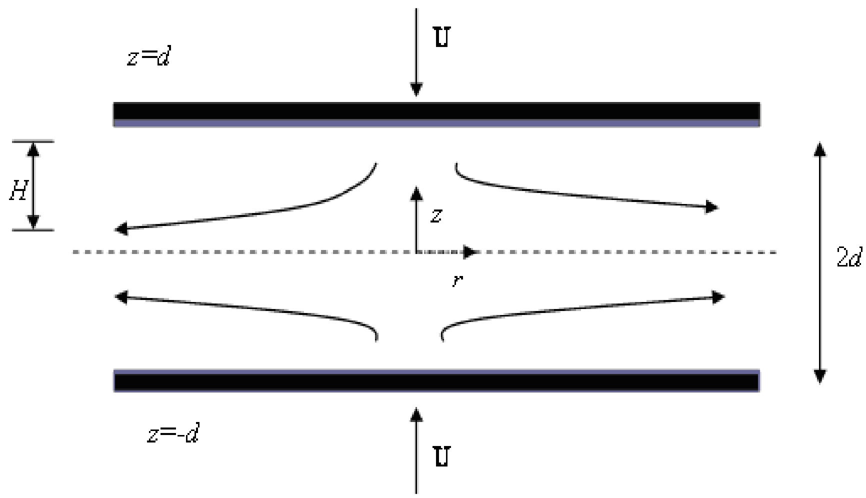

Figure 1. A steady squeezing axisymmetric fluid flow between two large parallel plates.

$$-\rho\left(\tilde{u}\times\tilde{\omega}\right)+\nabla\left(\frac{\rho}{2}\left|\tilde{u}\right|^{2}+p\right)=-\mu\nabla\times\tilde{u} \qquad (2)$$

where $\tilde{u}=\left(u_r\left(r, z, t\right),0,u_z\left(r, z, t\right)\right)$ is the velocity vector, ρ is the density of the fluid, p is the pressure, $\tilde{\omega}=\nabla\times\tilde{u}$ is the vorticity vector and μ is the dynamic viscosity of the fluid.

We consider viscous incompressible fluid, squeezed between two large planar and parallel plates, separated by a distance 2*d*. The plates are moving towards each other with velocity *U*, as shown in the Figure 1. The surfaces of both plates are covered by special material with slip length (slip coefficient) β. For small values of *U,* the gap distance 2*d* between the plates varies slowly with the time *t*, so that it can be taken as constant and the flow as quasi-steady (Zhu and Granick, 2001; Papanastasiou, 2000; Siddiqui et al., 2007; Idrees et al., 2010; Ran et al., 2009). The velocity field $\tilde{u}$ is given as follows.

$$\tilde{u}=\left(u_r\left(r, z, t\right),0,u_z\left(r, z, t\right)\right) \qquad (3)$$

It is easily shown that the stream function $\psi\left(r, z\right)$, defined by $u_r=-\frac{1}{r}\frac{\partial\psi}{\partial z}, u_z=\frac{1}{r}\frac{\partial\psi}{\partial z}$ satisfies the continuity equation identically. Substituting u_r and u_z into the *z*- and *r*-components of the Navier-Stokes equation, and eliminating the pressure lead to the following equation

$$-\rho\left(\frac{\partial\psi}{\partial r}\frac{\partial\left(\frac{E^2\psi}{r^2}\right)}{\partial z}-\frac{\partial\left(\frac{E^2\psi}{r^2}\right)}{\partial r}\frac{\partial\psi}{\partial z}\right)=\frac{\mu}{r}\left(E^2\right)^2\psi \qquad (4)$$

with the slip boundary conditions

$$z=d, \quad \text{then} \quad u_r=\beta\frac{\partial u_r}{\partial z}, \quad u_z=-U, z=0, \quad \text{then} \quad u_z=0, \quad \frac{\partial u_r}{\partial z}=0, \qquad (5)$$

where the differential operator E^2 is defined by

$$E^2=\frac{\partial^2}{\partial r^2}-\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}.$$

Equation (4) admits a solution of the form (Stefan, 1874):

$$\psi\left(r, z\right)=r^2F\left(z\right). \qquad (6)$$

By virtue of Equation (6), the compatibility Equation (4) and boundary conditions (5) becomes

$$\frac{d^4F}{dz^4}+2\frac{\rho}{\mu}F\frac{d^3F}{dz^3}=0, \qquad (7)$$

subjecting to the boundary conditions

$$F\left(0\right)=0, \; F''\left(0\right)=0,$$
$$F\left(d\right)=-\frac{U}{2}, \; F'\left(d\right)=\gamma F''\left(d\right). \qquad (8)$$

To deal with the problem, we introduce dimensionless parameters given by

$$F^* = \frac{F}{U/2}, \quad z^* = \frac{z}{d}, \quad \gamma = \frac{\beta}{d}, \; R = \frac{\rho d U}{\mu},$$

and dropping '*' for simplicity, the boundary value problem (7) becomes,

$$\frac{d^4F}{dz^4} + 2\frac{\rho}{\mu}F\frac{d^3F}{dz^3} = 0, \tag{9}$$

which satisfies the boundary conditions

$$F(0) = 0, \; F''(0) = 0, \; F(1) = 1, \; F'(1) = \gamma F''(1). \tag{10}$$

SOLUTION BY OPTIMAL HOMOTOPY ASYMPTOTIC METHOD

Here we apply the basic idea of OHAM (Idrees et al., 2010) to Equations (9) and 10). Defining linear and non linear operators respectively as:

$$L(\phi(z,p)) = \frac{\partial^4\phi(z,p)}{\partial z^4}, \tag{11}$$

$$N(\phi(z,p)) = R\phi(z,p)\frac{\partial^2\phi(z,p)}{\partial z^2}, \tag{12}$$

$$g(z) = 0. \tag{13}$$

Equating the coefficients of like powers of *p*, we get the following problems of different orders.

Zeroth order problem

$$F_0^{(4)} = 0,$$
$$F_0(0) = 0, F_0'(0) = 0, F_0(1) = 1, F_0'(1) = \gamma F_0''(1) \tag{14}$$

First order problem

$$F_1^{(4)}(z, C_1) = (1 + C_1)F_1^{(4)}(z) + RC_1F_0(z)F_0''(z), \tag{15}$$

$$F_1(0) = 0, \; F_1'(0) = 0, \; F_1(1) = 0, \; F_1'(1) = \gamma F_1''(1),$$

Second order problem

$$F_2^{(4)}(z, C_1, C_2) = \begin{pmatrix} (1+C_1)F_1^{(4)}(z,C_1) + C_2F_0^{(4)}(z) + RC_1F_0(z)F_1'''(z,C_1) \\ +R\;C_1\;F_1(z)\;F_1'''(z,C_1) + R\;C_2\;F_0(z)\;F_1'''(z,C_1) \end{pmatrix}, \tag{16}$$

$$F_2(0) = 0, \; F_2'(0) = 0, \; F_2(1) = 0, \; F_2'(1) = \gamma F_2''(1),$$

Now we solve the problems (14) to (16) in succession and obtain the series solutions with unknown constants.

Zeroth order solution

$$F_0(z) = \frac{1}{2(3\gamma - 1)}\left(z^3 + (6\gamma - 3)z\right). \tag{17}$$

First order solution

$$F_1 = \frac{1}{560\,(-1+3\gamma)^3}\begin{pmatrix} 19\,R\,z\,C_1 - 39\,R\,z^3\,C_1 + 21\,R\,z^5\,C_1 - R\,z^7\,C_1 - \\ 171\,R\,z\,\gamma\,C_1 + 273\,R\,z^3\,\gamma\,C_1 - 105\,R\,z^5\,\gamma\,C_1 + \\ 3Rz^7\gamma C_1 + 294Rz\gamma^2C_1 - 420Rz^3\gamma^2C_1 + 126Rz^5\gamma^2C_1 \end{pmatrix}. \tag{18}$$

Second order solution

$$F_2 = \frac{1}{2587200\,(-1+3\gamma)^5}\begin{pmatrix} R\,z\,(-1+z^2)\,(4620\,(1-3\gamma)^2\,(-19+3\,(57-98\gamma)\,\gamma \\ +z^4\,(-1+3\gamma)+2\,z^2\,(10-51\gamma+63\gamma^2))C_1+(4620 \\ (1-3\gamma)^2\,(-19+3\,(57-98\gamma)\,\gamma+z^4\,(-1+3\gamma)+2\,z^2 \\ (10-51\gamma+63\gamma^2))+R\,(63\,z^8(1-3\gamma)^2+7z^6(1-3\gamma)^2 \\ (-211+440\gamma)+z^4\,(-1+3\gamma)(-10205+7\gamma(10609+ \\ 660\gamma(-37+27\gamma)))-z^2(-1+3\gamma)(-5503+7\gamma(9587+ \\ 264\gamma(-107+90\gamma)))-12(274+\gamma(-4384+7\gamma(2405+ \\ 99\gamma(-38+25\gamma))))))+4620(1-3\gamma)^2(-19+3(57-98\gamma)\gamma \\ +z^4\,(-1+3\gamma)+2\,z^2\,(10-51\gamma+63\gamma^2))\;C_2) \end{pmatrix}. \tag{19}$$

Thus the optimal solution up to second order is given by:

$$F = \frac{1}{2587200\,(-1+3\gamma)^5}\begin{pmatrix} z(1293600\,(1-3\gamma)^4\,(-3+z^2+6\gamma)+R(-1+z^2)(9240 \\ (1-3\gamma)^2(-19+3\,(57-98\gamma)\,\gamma+z^4\,(-1+3\gamma)+2z^2(10 \\ -51\gamma+63\gamma^2)\,)C_1+(4620\,(1-3\gamma)^2(-19+3(57-98\gamma) \\ \gamma+z^4(-1+3\gamma)+2z^2(10-51\gamma+63\gamma^2))+R(63z^8(1-3\gamma)^2 \\ +7z^6(1-3\gamma)^2(-211+440\gamma)+z^4(-1+3\gamma)(-10205+7\gamma \\ (10609+660\gamma(-37+27\gamma)))-z^2(-1+3\gamma)(-5503+7\gamma \\ (9587+264\gamma(-107+90\gamma)))-12(274+\gamma(-4384+7\gamma \\ (2405+99\gamma(-38+25\gamma))))))C_1^2+4620(1-3\gamma)^2(-19+ \\ 3(57-98\gamma)\gamma+z^4(-1+3\gamma)+2z^2(10-51\gamma+63\gamma^2))C_2)) \end{pmatrix}. \tag{20}$$

Here we used the method of least squares for finding the values of C_1 and C_2. Thus for $\gamma = 1$ and $R = 1$, we have C_1=-0.9453415636834306, C_2=0.003973197981442457

In view of the values of C_1 and C_2, the simplified form of our solution is;

$$F(z) = \frac{z}{82790400}\begin{pmatrix} 20697600\,(3+z^2)+(-1+z^2)(-34866.4(-142+44z^2 \\ +2z^4)+0.893671(-44592-60380\,z^2+35716\,z^4 \\ +6412\,z^6+252z^8+18480\,(-142+44\,z^2+2z^4))) \end{pmatrix}. \tag{21}$$

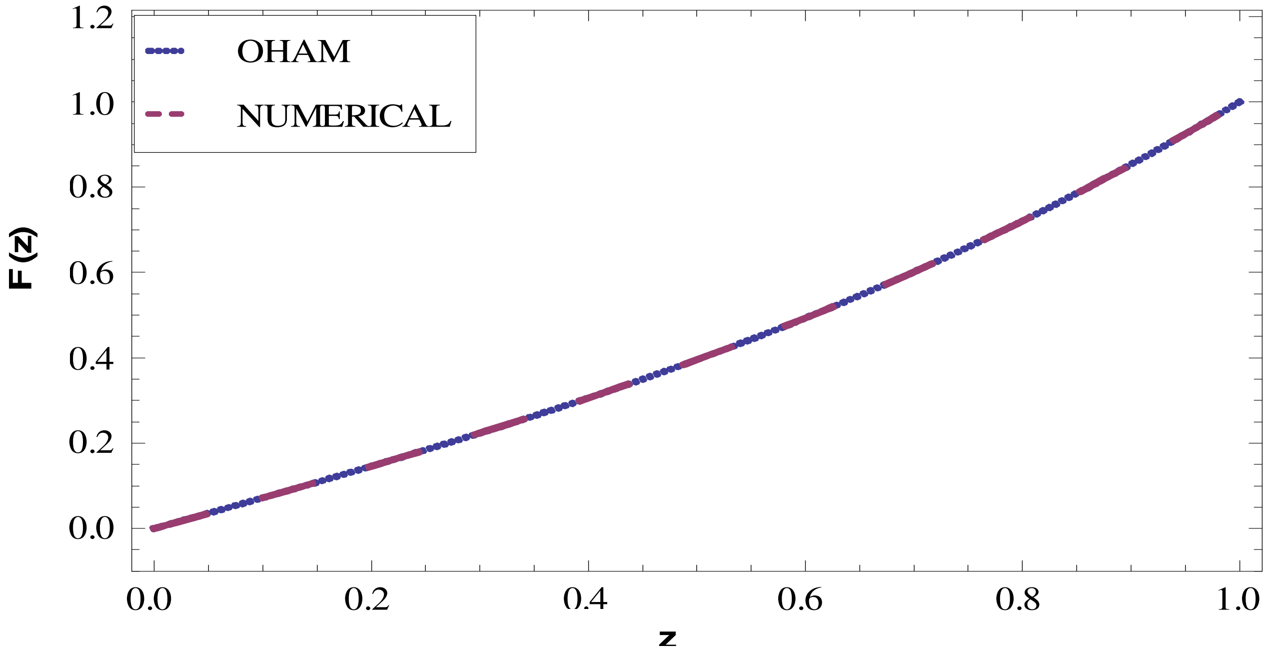

Figure 2. This plot represents the comparison between OHAM and numerical solutions.

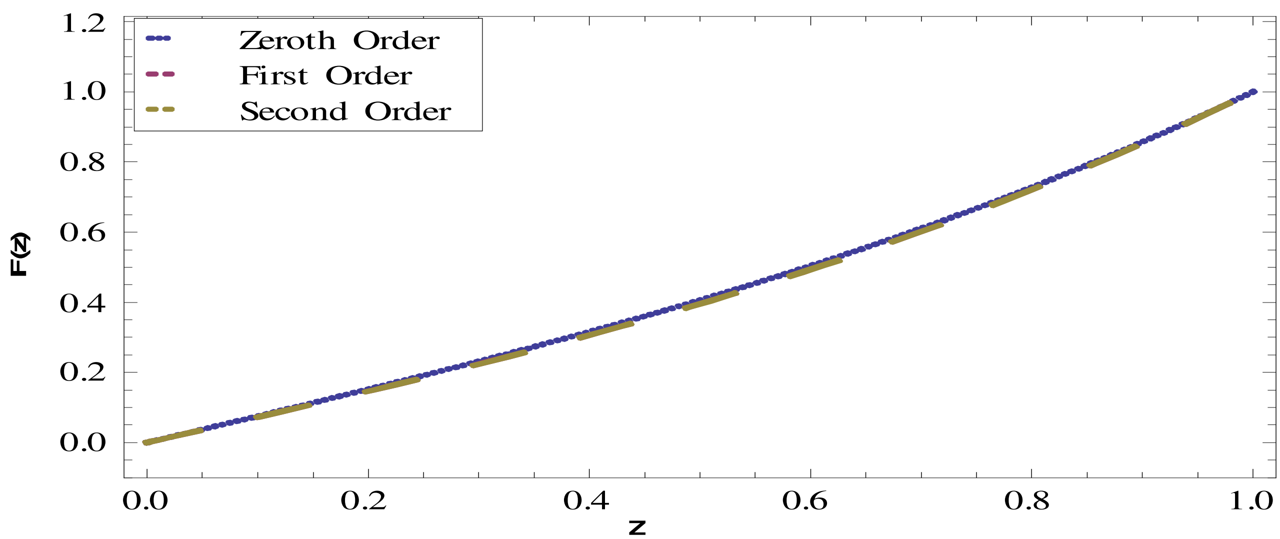

Figure 3. Plot shows zeroth order, first order and second order OHAM solution.

RESULTS AND DISCUSSION

Figure 2 shows the excellent agreement between OHAM and numerical. Figure 3 shows the fast convergence of OHAM through zeroth order, first order and second order OHAM solution. Figure 4 represents error analysis by residual curve. The solution curve is very smooth and is amenable for any investigation and interpretation. In Table 1, we compare the OHAM solution (21) to the numerical method solution based on approximants.

Conclusion

In this letter, the optimal homotopy asymptotic method (OHAM) is directly applied to derive approximate solutions of the title problem with slip boundary conditions illustrate our method. As a result:

1. We obtain the approximate solutions of the title problem with good accuracy.
2. This approach is simple in applicability, as it does not require discretization like other numerical and approximate

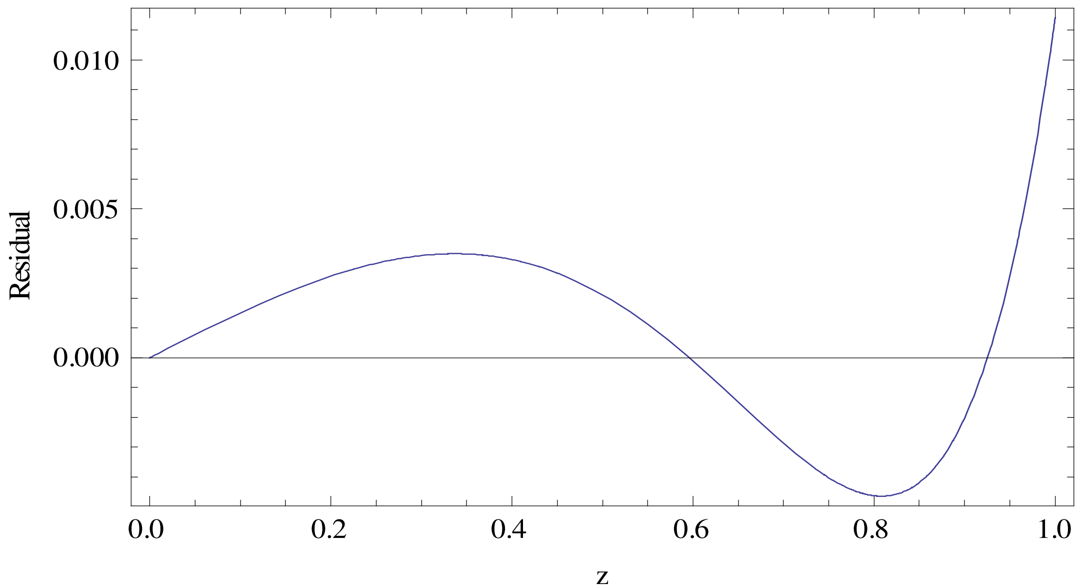

Figure 4. Plot shows error analysis by residual curve.

Table 1. Comparison of OHAM with numerical solutions.

x	Numerical	OHAM
0.0	0.0	0.0
0.1	0.0721879	0.0721919
0.2	0.146122	0.146129
0.3	0.223535	0.223544
0.4	0.306136	0.306146
0.5	0.395594	0.395603
0.6	0.493531	0.493538
0.7	0.601502	0.601507
0.8	0.720989	0.720993
0.9	0.853389	0.853391
1.0	1.0	1.0

methods.

3. Moreover, this technique is fast converging to the exact solution and requires less computational work.

4. This confirms our credence that the efficiency of the OHAM gives much wider applicability.

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