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Short Communication

Some properties of a class of fuzzy neural networks

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This paper investigates some properties of Takagi-Sugeno (T-S) fuzzy Hopfield neural networks. First, we prove that there exists a unique solution of the T-S fuzzy Hopfield neural network. Second, we determine a condition for input-to-state stability (ISS) of the T-S fuzzy Hopfield neural network. These results will be useful to analyze dynamic behavior of fuzzy neural networks.

Key words: Unique solution, input-to-state stability (ISS), fuzzy neural networks.

INTRODUCTION

In this paper we investigate some properties of the following Takagi-Sugeno (T-S) fuzzy Hopfield neural network:

Fuzzy rule i:

$IF\ \omega_1\ is\ \mu_{i1}\ and \ldots \omega_s\ is\ \mu_{is}\ THEN$

$$\dot{x}(t) = A_i x(t) + W_i \phi(x(t)) + J(t), \quad (1)$$

where $x(t) = [x_1(t) \ldots x_n(t)]^T \in R^n$ is the state vector, $A_i = diag\{-a_{(i,1)}, \ldots, -a_{(i,n)}\} \in R^{n\times n}$ $(a_{(i,k)} > 0, k = 1, \ldots, n)$ is the self-feedback matrix, $W_i \in R^{n\times n}$ is the connection weight matrix, $\phi(x(t)) = [\phi_1(x(t)) \ldots \phi_n(x(t))]^T : R^n \to R^n$ is the nonlinear function vector satisfying the global Lipschitz condition with Lipschitz constant $L_\phi > 0$, $J(t) \in R^n$ is an external input vector, $\omega_j \quad (j = 1, \ldots, s)$ is the premise variable, $\mu_{ij} \quad (i = 1, \ldots, r, j = 1, \ldots, s)$ is the fuzzy set that is characterized by membership function, r is the number of the IF-THEN rules, and s is the number of the premise variables. Using a singleton fuzzifier, product fuzzy inference, and weighted average defuzzifier, the system (1) is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(\omega)[A_i x(t) + W_i \phi(x(t)) + J(t)], \quad (2)$$

where $\omega = [\omega_1, \ldots, \omega_s]$, $h_i(\omega) = w_i(\omega) / \sum_{j=1}^{r} w_j(\omega)$, $w_i : R^s \to [0,1] \quad (i = 1, \ldots, r)$ is the membership function of the system with respect to the fuzzy rule i. h_i can be regarded as the normalized weight of each IF-THEN rule and it satisfies $h_i(\omega) \geq 0, \sum_{i=1}^{r} h_i(\omega) = 1$. Basically, the Takagi-Sugeno (T-S) fuzzy models are based on using a set of fuzzy rules to describe nonlinear systems in terms of a set of local linear models that are smoothly connected by fuzzy membership functions (Takagi and Sugeno, 1985).The T-S fuzzy models can be used to represent some complex nonlinear systems by having a set of neural networks as its consequent parts. Some stability problems for T-S fuzzy neural networks have been investigated (Huang et al., 2005; Ali and Balasubramaniam, 2009; Li et al., 2009a, b; Ahn, 2010, 2011a, 2011b; Balasubramaniam and Chandran, 2011). In this paper, we present some properties of T-S fuzzy

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Abbreviations: T-S, Takagi-Sugeno; **ISS,** input-to-state stability.

Hopfield neural networks. We show that the T-S fuzzy Hopfield neural network has a unique solution. In addition, a new input-to-state stability (ISS) condition is derived for this neural network. The presented analysis opens a path for application of fuzzy neural networks to nonlinear control.

EXISTENCE AND UNIQUENESS OF SOLUTION

In this section, we show the existence and uniqueness of the solution of the T-S fuzzy Hopfield neural network (2) in the following theorem:

Theorem 1. The T-S fuzzy Hopfield neural network (2) with the initial state $x(0)$ has a unique solution.

Proof. For all $z_1(t) \in R^n$ and $z_2(t) \in R^n$, we have:

$$\left\| \sum_{i=1}^{r} h_i(\omega)[A_i z_1(t) + W_i \phi(z_1(t)) + J(t)] - \sum_{i=1}^{r} h_i(\omega)[A_i z_2(t) + W_i \phi(z_2(t)) + J(t)] \right\|$$
$$= \left\| \sum_{i=1}^{r} h_i(\omega) A_i (z_1(t) - z_2(t)) + \sum_{i=1}^{r} h_i(\omega) W_i (\phi(z_1(t)) - \phi(z_2(t))) \right\|$$
$$\leqq \sum_{i=1}^{r} \|h_i(\omega)\| \|A_i\| \|z_1(t) - z_2(t)\| + \sum_{i=1}^{r} \|h_i(\omega)\| \|W_i\| \|\phi(z_1(t)) - \phi(z_2(t))\|.$$

It is clear that:

$$0 \le h_i(\omega) \le 1, \qquad i = 1, \ldots, r.$$

Thus, we have:

$$\left\| \sum_{i=1}^{r} h_i(\omega)[A_i z_1(t) + W_i \phi(z_1(t)) + J(t)] - \sum_{i=1}^{r} h_i(\omega)[A_i z_2(t) + W_i \phi(z_2(t)) + J(t)] \right\|$$
$$\leqq \sum_{i=1}^{r} \|A_i\| \|z_1(t) - z_2(t)\| + \sum_{i=1}^{r} \|W_i\| L_\phi \|z_1(t) - z_2(t)\|$$
$$= \left\{ \sum_{i=1}^{r} \|A_i\| + L_\phi \sum_{i=1}^{r} \|W_i\| \right\} \|z_1(t) - z_2(t)\|. \tag{3}$$

Let $f(x(t),t) = \sum_{i=1}^{r} h_i(\omega)[A_i x(t) + W_i \phi(x(t)) + J(t)]$. Then, the relation (3) becomes:

$$\|f(z_1(t),t) - f(z_2(t),t)\| \le \left\{ \sum_{i=1}^{r} \|A_i\| + L_\phi \sum_{i=1}^{r} \|W_i\| \right\} \|z_1(t) - z_2(t)\|. \tag{4}$$

Since:

$$\left\{ \sum_{i=1}^{r} \|A_i\| + L_\phi \sum_{i=1}^{r} \|W_i\| \right\} > 0,$$

$f(x(t),t)$ is a global Lipschitz function. According to Theorem 3.2 in (Khalil, 2002), the T-S fuzzy Hopfield neural network (2) has a unique solution. This completes the proof.

ISS CONDITION

We introduce the following definitions:

Definition 1. *A function* $\gamma : R_{\ge 0} \to R_{\ge 0}$ *is a* K function if it is continuous, strictly increasing and $\gamma(0) = 0$. It is a K_∞ function if it is a K function and also $\gamma(s) \to \infty$ as $s \to \infty$.

Definition 2. A function $\beta : R_{\ge 0} \times R_{\ge 0} \to R_{\ge 0}$ is a KL function if, for each fixed $t \ge 0$, the function $\beta(\cdot, t)$ is a K function, and for each fixed $s \ge 0$, the function $\beta(s, \cdot)$ is decreasing and $\beta(s,t) \to 0$ as $t \to \infty$.

The notion of ISS can be described as follows:

Definition 3. The system $\dot{x}(t) = f(x(t), u(t))$, where $x(t) \in R^n, u(t) \in R^m$, is said to be input-to-state stable if there exist a K function $\gamma(\cdot)$ and a $\mathrm{K}L$ function $\beta(\cdot,\cdot)$, such that, for each input $u(t)$ and each initial state $x(0)$, it holds that (Sontag, 1990; Jiang et al., 1994; Christofides and Teel, 1996; Sontag, 1998; Angeli and Nesic, 2001):

$$\|x(t)\| \le \beta(\|x(0)\|, t) + \gamma\left(\sup_{0 \le \mu \le t} \|u(\mu)\| \right), \tag{5}$$

for each $t \ge 0$. Now we derive an ISS condition of the T-S fuzzy Hopfield neural network (2) in the following theorem:

Theorem 2. The T-S fuzzy Hopfield neural network (2) is input-to-state stable if;

$$\|W_i\| < \frac{1}{L_\phi} \sqrt{\frac{\gamma_i - 2\|P\|}{\|P\|}}, \qquad \|P\| < \frac{\gamma_i}{2}, \qquad \gamma_i > 0, \qquad P = P^T > 0, \tag{6}$$

where P satisfies the Lyapunov equation:

$A_i^T P + P A_i = -\gamma_i I$ for $i = 1, \ldots, r$.

Proof. We consider the function $V(t) = x^T(t) P x(t)$,

$P = P^T > 0$. Its time derivative along the trajectory of (2) is given as:

$$\dot{V}(t) = \sum_{i=1}^{r} h_i(\omega)\Big\{ -\gamma_i x^T(t)x(t) + 2x^T(t)PW_i\phi(x(t)) + 2x^T(t)PJ(t)\Big\}. \tag{7}$$

By Young's inequality (Arnold, 1989), we have:

$$\begin{aligned}2x^T(t)PW_i\phi(x(t)) &\le x^T(t)Px(t) + (PW_i\phi(x(t)))^T P^{-1}(PW_i\phi(x(t))) \\ &\le \|P\|\|x(t)\|^2 + \|P\|\|W_i\|^2\|\phi(x(t))\|^2 \\ &\le \|P\|\|x(t)\|^2 + L_\phi^2\|P\|\|W_i\|^2\|x(t)\|^2\end{aligned} \tag{8}$$

and

$$\begin{aligned}2x^T(t)PJ(t) &\le x^T(t)Px(t) + (PJ(t))^T P^{-1}(PJ(t)) \\ &\le \|P\|\|x(t)\|^2 + \|P\|\|J(t)\|^2.\end{aligned} \tag{9}$$

Substituting (8) and (9) into (7), we finally obtain:

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{r} h_i(\omega)\Big\{ -\left(\gamma_i - 2\|P\| - L_\phi^2\|P\|\|W_i\|^2\right)\|x(t)\|^2 + \|P\|\|J(t)\|^2\Big\} \\ &= -\sum_{i=1}^{r} h_i(\omega)\left(\gamma_i - 2\|P\| - L_\phi^2\|P\|\|W_i\|^2\right)\|x(t)\|^2 + \|P\|\|J(t)\|^2.\end{aligned} \tag{10}$$

Defining:

$$\alpha(r) = \sum_{i=1}^{r} h_i(\omega)\left(\gamma_i - 2\|P\| - L_\phi^2\|P\|\|W_i\|^2\right) r^2,$$

$$\theta(r) = \|P\| r^2,$$

then $\dot{V}(t) \le -\alpha(\|x(t)\|) + \theta(\|J(t)\|)$. $V(t)$ is an ISS-Lyapunov function (Sontag and Wang, 1995) if $\alpha(\cdot)$ and $\theta(\cdot)$ are class K_∞ functions. As defined, $\theta(\cdot)$ satisfies this condition. Hence, for the system (2) to be ISS, it is required that $\left(\gamma_i - 2\|P\| - L_\phi^2\|P\|\|W_i\|^2\right) > 0$, which implies:

$$\|W_i\|^2 < \frac{\gamma_i - 2\|P\|}{L_\phi^2\|P\|}, \qquad \|P\| < \frac{\gamma_i}{2}, \tag{11}$$

for $i = 1,\ldots,r$. This completes the proof.

CONCLUSION

In this paper, we prove the uniqueness of the solution of T-S fuzzy Hopfield neural networks. Furthermore, we establish a condition for the weight of the connection matrix of T-S fuzzy Hopfield neural networks, in order to guarantee ISS. It is expected that these results can be extended to a general class of neural networks.

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