International Journal of the Physical Sciences Vol. 6(3), pp. 495-499, 4 February, 2011
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS10.514

Full Length Research Paper

Alternative approach for production lot size problem with scrap and cost reduction distribution policy

Fan-Yun Pai[1] and Chia-Kuan Ting[2]*

[1]Department of Business Administration, National Changhua University of Education, Changhua 500, Taiwan.
[2]Department of Industrial Engineering and Management, Chaoyang University of Technology
Wufong, Taichung 413, Taiwan.

Accepted 10 January, 2011

Conventional approach for solving replenishment lot size problem is by using the differential calculus with the need of applying the first-order and second-order differentiations to the long-run average production-inventory cost. A recent published paper used conventional method to determine optimal lot size for an integrated production-delivery system with scrap and cost lessening distribution policy. This paper uses an alternative algebraic approach to reexamine the aforementioned problem without referring to derivatives. As a result, optimal lot size and a simpler expression for the long-run average cost are derived, and they are identical to what were obtained by using the conventional method.

Key words: Lot size, optimization, scrap, multiple distribution policy, algebraic approach, production planning.

INTRODUCTION

Taft (1918) first introduced the concept and computation of the most economical production lot (also known as economic production quantity (EPQ) model) several decades ago, to assist manufacturer firm in minimizing total production-inventory costs. EPQ model implicitly assumes that all items manufactured are of perfect quality. However, in real life production environments, due to many factors, generation of defective items is inevitable. Hence, during past decades many studies have been carried out to address the imperfect productions and their related issues (Rosenblatt and Lee, 1986; Henig and Gerchak, 1990; Tersine, 1994; Cheung and Hausman, 1997; Salameh and Jaber, 2000; Chiu, 2003; Chiu and Chiu, 2006; Chiu et al., 2006; Inderfurth et al., 2007; Chiu et al., 2008; Baten and Kamil, 2009; Chiu et al., 2010a; Wazed et al., 2010a; Chiu et al., 2010b; Wazed et al., 2010b). Continuous inventory issuing policy is another unpractical assumption of the classic EPQ model. In an integrated production-shipment system, multiple or periodic deliveries policy is commonly used.

Goyal (1977) studied an integrated inventory-shipment model for the single supplier-single customer problem. He proposed a method that is typically applicable to those inventory problems where a product is procured by a single customer from a single supplier. Example with analysis was provided to illustrate his method. Many studies have since been carried out to address issue of various aspects of supply chain optimization (Schwarz et al., 1985; Banerjee, 1986; Goyal and Gupta, 1989; Kohli and Park, 1994; Hill, 1996; Viswanathan and Piplani, 2001; Diponegoro and Sarker, 2006; Yao and Chiou, 2004; Ertogral et al., 2007; Chiu et al., 2009; Abolhasanpour et al., 2009; Chen et al., 2010a; Chiu et al., 2010c). Chen et al. (2010b) combined a cost reduction inventory distribution policy into a production system with random scrap rate with the purpose of lessening producer's inventory holding cost in Chiu et al. (2009).

They used mathematical modeling and differential calculus to derive the optimal replenishment lot size for an imperfect EPQ model under (n+1) delivery policy.

Grubbström and Erdem (1999) recently presented an algebraic approach to solve the economic order quantity (EOQ) model with backlogging, without reference to the use of derivatives. For studies that used the same or similar method according to Chiu et al. (2007), Lin et al. (2008), and Chiu et al. (2010d). This paper applies the same alternative approach to a specific EPQ model examined by Chen et al. (2010b). We show that the optimal lot size and the long-run average cost can both be derived without using differential calculus.

*Corresponding author. E-mail: djk@cyut.edu.tw.

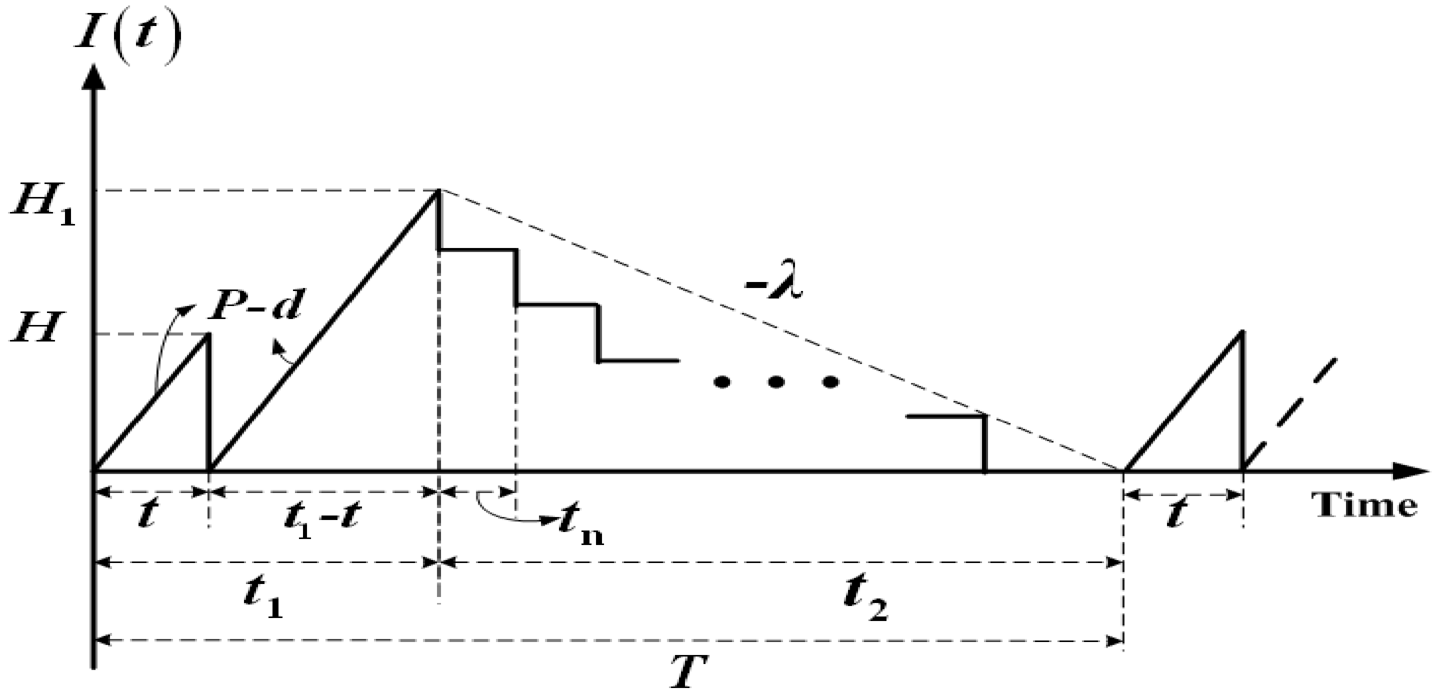

Figure 1. On-hand inventory of perfect quality items in EPQ model with random scrap rate and (n+1) delivery policy (Chen et al., 2010b).

METHODS

An algebraic approach is adopted in this paper to reexamine Chen et al.'s (2010b) model, as stated earlier. Description of this production system is as follows. Consider during the regular production uptime, it is assumed that there is an x portion of defective items produced randomly at a rate d. All nonconforming items produced are considered to be scrap items and are discarded at the end of each production. The constant production rate P is assumed to be larger than the sum of demand rate λ and production rate of defective items d. That is,
$(P-d-\lambda)>0$,

where $d=Px$.

Furthermore, a cost reduction (n+1) product distribution policy is used at the end of uptime. Under such a policy, an initial installment of finished items is delivered to customer for satisfying the product demand during the producer's production uptime. Fixed quantity n installments of finished items are then shipped to customer at a fixed interval of time during the downtime t_2. For the purpose of easing readability, this paper adopted the same notation and basic formulas as were used in Chen et al. (2010b) as follows:

H = the level of on-hand inventory in units for satisfying product demand during manufacturer's regular production time t_1,
H_1 = maximum level of on-hand inventory in units when regular production ends,
n = number of fixed quantity installments of the rest of finished batch to be delivered to customer during t_2,
t = the production time needed for producing enough perfect items for satisfying product demand during the production uptime t_1,
t_1 = the production uptime for the proposed EPQ model,
t_2 = time required for delivering the remaining perfect quality finished products,
t_n = a fixed interval of time between each installment of products delivered during t_2,
T = cycle length,
Q = production lot size to be determined for each cycle,
$I(t)$ = on-hand inventory of perfect quality items at time t,
K = setup cost per production run,
C = unit production cost,
h = unit holding cost,
K_1 = fixed delivery cost per shipment,
C_T = delivery cost per item shipped to customers,
$TC(Q)$ = total production-inventory-delivery costs per cycle for the proposed model,
$E[TCU(Q)]$ = the long-run average costs per unit time for the proposed model.

The on-hand inventory of perfect quality items of the proposed model is illustrated in Figure 1 (Chen et al., 2010b). Total production-inventory-distribution costs per cycle $TC(Q)$ consists of the variable manufacturing cost, setup cost, disposal cost for scrap items, (n+1) fixed distribution costs and variable delivery cost, holding cost for perfect quality items during production uptime t_1, holding cost for defective items during t_1, and holding cost for finished goods during the delivery time t_2 where n fixed-quantity installments of the finished batch are delivered at a fixed interval of time.

$$TC(Q)=CQ+K+C_S[xQ]+(n+1)K_1+C_TQ(1-x)$$
$$+h\left[\frac{H}{2}(t)+\frac{H_1}{2}(t_1-t)+\frac{dt_1}{2}(t_1)\right]+h\left[\left(\frac{n-1}{2n}\right)H_1t_2\right] \quad (1)$$

With further derivations, one has $TC(Q)$ as (Chen et al., 2010b).

$$TC(Q) = CQ + K + (n+1)K_1 + C_S(xQ) + C_T\left[Q(1-x)\right] + \frac{hQ^2}{2}\left\{\begin{array}{l} \dfrac{2\lambda^2}{P^3(1-x)} - \dfrac{\lambda}{P^2} + \dfrac{(1-x)^2}{\lambda} - \dfrac{(1-2x)}{P} \\ -\left(\dfrac{1}{n}\right)\left[\dfrac{(1-x)^2}{\lambda} - \dfrac{2(1-x)}{P} + \dfrac{\lambda}{P^2}\right] \end{array}\right\} \tag{2}$$

The defective rate x is assumed to be a random variable with a known probability density function, taking into account of the randomness, one can use the expected values of x in cost analysis and obtains E[$TCU(Q)$] as follows (Equation (11) in Chen et al., 2010b).

$$E\left[TCU(Q)\right] = \frac{C\lambda}{1-E(x)} + \frac{\left[(n+1)K_1 + K\right]\lambda}{Q\left[1-E(x)\right]} + \frac{C_S E(x)\lambda}{1-E(x)} + C_T\lambda + \frac{hQ}{2}\left\{\begin{array}{l} \dfrac{2\lambda^3}{P^3\left[1-E(x)\right]}E\left(\dfrac{1}{1-x}\right) - \dfrac{\lambda^2}{P^2\left[1-E(x)\right]} \\ +\left[1-E(x)\right] - \dfrac{\lambda\left[1-2E(x)\right]}{P\left[1-E(x)\right]} \\ -\left(\dfrac{1}{n}\right)\left[\left[1-E(x)\right] - \dfrac{2\lambda}{P} + \dfrac{\lambda^2}{P^2\left[1-E(x)\right]}\right] \end{array}\right\} \tag{3}$$

Production lot sizing using an algebraic derivations

Algebraic approach is employed in this section for deriving the optimal replenishment lot size solution. First let u_0, u_1, u_2 and u_3 denote the following:

$$u_0 = \frac{C\lambda}{1-E(x)} \tag{4}$$

$$u_1 = \frac{C_S E(x)\lambda}{1-E(x)} + C_T\lambda \tag{5}$$

$$u_2 = \frac{\left[(n+1)K_1 + K\right]\lambda}{\left[1-E(x)\right]} \tag{6}$$

$$u_3 = \frac{h}{2}\left\{\begin{array}{l} \dfrac{2\lambda^3}{P^3\left[1-E(x)\right]}E\left(\dfrac{1}{1-x}\right) - \dfrac{\lambda^2}{P^2\left[1-E(x)\right]} \\ +\left[1-E(x)\right] - \dfrac{\lambda\left[1-2E(x)\right]}{P\left[1-E(x)\right]} \\ -\left(\dfrac{1}{n}\right)\left[\left[1-E(x)\right] - \dfrac{2\lambda}{P} + \dfrac{\lambda^2}{P^2\left[1-E(x)\right]}\right] \end{array}\right\} \tag{7}$$

Then Equation (3) can be expressed as:

$$E\left[TCU(Q)\right] = (u_0 + u_1) + u_2 \cdot Q^{-1} + u_3 \cdot Q \tag{8}$$

With further rearrangement of Equation (8), one obtains:

$$E\left[TCU(Q)\right] = (u_0 + u_1) + Q\left[u_2 \cdot Q^{-2} + u_3\right] \tag{9}$$

or

$$E\left[TCU(Q)\right] = (u_0 + u_1) + Q\left[\left(\sqrt{u_2}\cdot Q^{-1}\right)^2 + \left(\sqrt{u_3}\right)^2 - 2\left(\sqrt{u_2}\cdot Q^{-1}\right)\left(\sqrt{u_3}\right) + 2\left(\sqrt{u_2}\cdot Q^{-1}\right)\left(\sqrt{u_3}\right)\right] \tag{10}$$

Therefore, one obtains:

$$E\left[TCU(Q)\right] = (u_0 + u_1) + Q\left[\left(\sqrt{u_2}\cdot Q^{-1}\right) - \sqrt{u_3}\right]^2 + 2Q\left(\sqrt{u_2 \cdot u_3}\right)\cdot Q^{-1} \tag{11}$$

E[$TCU(Q)$] is minimized, if the square term in Equation (11) equals zero. That is,

$$\left[\left(\sqrt{u_2}\cdot Q^{-1}\right) - \sqrt{u_3}\right] = 0 \tag{12}$$

or

$$Q = \sqrt{\frac{u_2}{u_3}} \tag{13}$$

Substituting Equations (4) and (5) in Equation (13), with further derivations, one has the optimal lot size as:

$$Q^* = \sqrt{\frac{2\lambda\left[(n+1)K_1 + K\right]}{h\left\{\begin{array}{l} \dfrac{2\lambda^3}{P^3}E\left(\dfrac{1}{1-x}\right) - \dfrac{\lambda^2}{P^2}\left(1+\dfrac{1}{n}\right) - \dfrac{\lambda\left[1-2E(x)\right]}{P} \\ +\left[1-E(x)\right]^2\left(1-\dfrac{1}{n}\right) + \left(\dfrac{1}{n}\right)\left[\dfrac{2\lambda\left[1-E(x)\right]}{P}\right] \end{array}\right\}}} \tag{14}$$

One notes that Equation (14) is identical to the optimal replenishment lot size Q^* given in Equation (17) of Chen et al. (2010b), which is derived by the use of the conventional differential calculus method.

It follows that if the optimal lot size Q^* is used, then equation (11) the long-run average cost E[$TCU(Q^*)$] will be simplified as:

$$E\left[TCU(Q)\right] = (u_0 + u_1) + 2Q\left(\sqrt{u_2 \cdot u_3}\right)\cdot Q^{-1} \tag{15}$$

or

$$E\left[TCU(Q)\right] = (u_0 + u_1) + 2\left(\sqrt{u_2 \cdot u_3}\right) \tag{16}$$

Numerical example and verification

Research results obtained from previous section are verified here using the same numerical example as in Chen et al. (2010b). Consider a manufactured item that can be produced at an annual rate of 60,000 units and this item has experienced a flat annual demand rate of 3,400 units. There is a defective rate x, which follows a uniform distribution over the range (0 and 0.3) during the production process. All nonconforming items are considered to be scrap. Additional parameters include the set up cost K = \$20,000; unit manufacturing cost C = \$100; CS = \$20 per scrap item; unit holding cost h = \$20 per year; the fixed delivery cost K1 = \$4,400 per shipment; and unit delivery cost CT = \$0.1. To verify the research results derived by the proposed algebraic approach, the following two scenarios are considered respectively (Chen et al., 2010b):

Scenario 1: Total number of deliveries (n+1) = 4 and the lot-size solution Q = 4,768. By applying Equation (3) one obtains the long-run average cost function E[TCU(4,768)] = \$470,263.

Scenario 2: Suppose that total number of deliveries (n+1) is 4. By applying Equations (14) and (16), one obtains Q* = 5,214 and E[TCU(5214)] = \$470,032, respectively. It is noted that the resulting numbers from both scenarios are the same as that in Chen et al. (2010b).

CONCLUSIONS

Chen et al. (2010b) used the mathematical modeling and differential calculus to derive the optimal lot size for a production lot size problem with scrap and cost lessening distribution policy. This paper reexamines their model by using an alternative algebraic approach in lieu of their differential calculus. As a result, the optimal replenishment lot size Q* as well as the long-run average production-inventory- delivery cost E[TCU(Q*)] can be derived without derivatives, respectively. The proposed alternative method is a straightforward algebraic approach; it allows those students or practitioners who lack for sufficient knowledge of calculus to learn or deal with such a specific EPQ model with ease.

ACKNOWLEDGEMENTS

The authors greatly appreciate the support of the National Science Council of Taiwan under grant #: NSC 99-2410-H-018-036.

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