academicJournals

Vol. 8(34), pp. 1703-1715, 16 September, 2013
DOI: 10.5897/IJPS2013.3993
ISSN 1992 - 1950 © 2013 Academic Journals
http://www.academicjournals.org/IJPS

International Journal of Physical Sciences

Full Length Research Paper

Singulo oscillatory – stiff rational integrators

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Accepted 4 September, 2013

In this paper, we derive a general singulo oscillatory – stiff rational integrator of order (S+3) for s = 0, 1, 2, 3, 4,… for the solution of initial value problems in ordinary differential systems that are singular, oscillatory or stiff. We compared our integrators with certain maximum order second derivative hybrid multi-step methods, certain Tau and Euler methods, the adaptive implicit and classical Runge-Kutta methods and some existing conventional methods. Our results show good improvement over the existing methods compared with.

Key words: Rational integrator, initial value problems, stability, convergence, consistency.

INTRODUCTION

Many of the problems in real life from physical situation, chemical kinetics, engineering construction work, biological simulations, nuclear reactors and practical realities often lead to initial value ordinary differential equations (Kreider et al., 1968; Birkhoff and Rota, 1969; Burghes and Borries, 1981; Lambert, 1995; Bartels et al., 1996). These types of initial value problems (ivp) are stiff, singular or oscillatory (Luke et al., 1975; Fatunla, 1978, 1980; Niekerk, 1987; Aashikpelokhai, 1991, 2000; Abhulimen and Otunta, 2007; Ademiluyi et al., 2007; Aashikpelokhai and Momodu, 2008; Egbetade et al., 2008). The solutions of these problems sometimes results in large set of linear equations that generate mystical entities called Matrices (Kolman, 1980; Atkinson, 1985; Jacques and Judd, 1987; Billingsley, 1989; Health, 1997). The work of Health (1997) and Parasuram (2001) shows that natural modes and frequencies of vibration of a structure and their stabilities are determined by the locations of the eigenvalues of an appropriate matrix hence their computation is of critical interest in this study.
The problem for this research work is to find a suitable numerical solution to the ivp.

$$y^1 = f(x, y), y(x_0) = y_0, \ a \le x \le b \tag{1}$$

where f(x, y) is defined and continuous in a region D ⊂ [a,b] of the real line.

This work was motivated by the research work of Lambert and Shaw (1965), Aashikpelokhai (1991) and Momodu (2006). Essentially, Momodu (2006) extended the work of Lambert and Shaw (1965) from linear denominator to Quadratic denominator. Our work here is an extension of Momodu (2006) from Quadratic denominator to a polynomial of degree three. We however used the method of derivation as found in Aashikpelokhai (1991). According to Luke et al. (1975), this area has not been attractive because of the difficulty in deriving these types of integrators. This statement was recently re-echoed by Agbeboh et al. (2007). However because of the excellent accurate results so far recorded in the area and with Aashikpelokhai (2010) hope for attention of more researchers in this area, we are encouraged to extend Momodu (2006) to a polynomial of degree three.

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2010 AMS Subject Classification: 65Lxx

THE SINGULO OSCILLATORY – STIFF RATIONAL INTEGRATORS

Basic expansion

Let the operator $U: R \to C^{m+2}(x)$ be defined by

$$U(x)\ Q_3(x) = P_m(x) \tag{2}$$

where $U(x) = \sum_{r=0}^{\infty} c_r x^r$ (3)

is the Taylor's' series of the function whose approximant is sought,

$$P_m(x) = \sum_{i=0}^{m} p_i x^i \tag{4}$$

$$Q_3(x) = 1 + \sum_{i=1}^{3} q_i x^i \tag{5}$$

For the purpose of our integration, U(x) is subject to the constraint

$$U(x_{n+i}) = \begin{cases} y(x_{n+i}) & \text{for } i = 0 \\ y_{n+i} & \text{for } i = 0, 1, 2 \end{cases} \tag{6}$$

On expansion we have the

$$\left.\begin{aligned} p_0 &= c_0 \\ p_1 &= c_0 q_1 + c_1 \\ p_2 &= c_0 q_2 + c_1 q_1 + c_2 \\ p_3 &= c_0 q_3 + c_1 q_2 + c_2 q_1 + c_3 \\ &\vdots \\ &\vdots \end{aligned}\right\} \tag{7}$$

$$p_m = c_{m-3} q_3 + c_{m-2} q_2 + c_{m-1} q_1 + c_m, \quad \forall\ m \geq 3 \tag{8}$$

By definition of $p_m(x)$ we have $p_s = 0 \ \ \forall \ \ s > m$

That is $p_{m+1} = p_{m+2} = \ldots = p_s = 0 \ \ if \ \ s > m.$

We shall be interested in the first three consecutive terms for our simultaneous linear equation to yield $q_1,\ q_2,\ q_3$.

Thus for $p_{m+1} = p_{m+2} = p_{m+3} = 0$ we then have,

$c_{m-2} q_3 + c_{m-1} q_2 + c_m q_1 = -c_{m+1}$
$c_{m-1} q_3 + c_m q_2 + c_{m+1} q_1 = -c_{m+2}$
$c_m q_3 + c_{m+1} q_2 + c_{m+2} q_1 = -c_{m+3}$

Rearranging and putting in matrix form, we have

$$\begin{bmatrix} c_{m+2} & c_{m+1} & c_m \\ c_{m+1} & c_m & c_{m-1} \\ c_m & c_{m-1} & c_{m-2} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = -\begin{bmatrix} c_{m+3} \\ c_{m+2} \\ c_{m+1} \end{bmatrix} \tag{9}$$

where (i) $m \geq 0$ (ii) $C_\alpha = 0$ whenever $\alpha < 0$,

(iii) $c_r = \dfrac{h^r y_n^{(r)}}{r! x_{n+1}^r}, \quad r = 0, 1, 2, \ . \ . \ .$ (10)

Reduction to matrix – free form

Employing Crammers rule to obtain q_1, q_2, q_3 we have from Equation 9

$$\text{Det } \mathbf{c} = |\mathbf{c}| = \begin{vmatrix} c_{m+2} & c_{m+1} & c_m \\ c_{m+1} & c_m & c_{m-1} \\ c_m & c_{m-1} & c_{m-2} \end{vmatrix}$$

$$\therefore \quad |\mathbf{c}| = c_{m+2} c_m c_{m-2} - c_{m+2} c_{m-1}^2 - c_{m+1}^2 c_{m-2} + 2 c_{m+1} c_m c_{m-1} - c_m^3 \tag{11}$$

Also, to obtain values of q_i, i = 1, 2, 3 we have from Crammers rule

$$q_i = \frac{|\mathbf{c}(q_i)|}{|\mathbf{c}|} \qquad \text{for } i = 1, 2, 3.$$

Hence,

$$\therefore \quad q_1 = \frac{c_{m+3} c_{m-1}^2 - c_{m+3} c_m c_{m-2} + c_{m+2} c_{m+1} c_{m-2} - c_{m+1}^2 c_{m-1} - c_{m+2} c_m c_{m-1} + c_{m+1} c_m^2}{c_{m+2} c_m c_{m-2} - c_{m+2} c_{m-1}^2 + c_{m+1}^2 c_{m-2} + 2 c_{m+1} c_m c_{m-1} - c_m^3} \tag{12}$$

Similarly,

$$\therefore q_2 = \frac{c_{m+2} c_{m+1} c_{m+1} - c_{m+2}^2 c_{m-2} + c_{m+3} c_{m+1} c_{m-2} - c_{m+3} c_m c_{m-1} - c_m c_{m+1}^2 + c_{m+2} c_m^2}{c_{m+2} c_m c_{m-2} - c_{m+2} c_{m-1}^2 + c_{m+1}^2 c_{m-2} + 2 c_{m+1} c_m c_{m-1} - c_m^3} \tag{13}$$

Also,

$$\therefore q_3 = \frac{c_{m+2}^2 c_{m-1} + c_{m+1}^3 - 2 c_{m+2} c_{m+1} c_m - c_{m+3} c_{m+1} c_{m-1} + c_{m+3} c_m^2}{c_{m+2} c_m c_{m-2} - c_{m+2} c_{m-1}^2 + c_{m+1}^2 c_{m-2} + 2 c_{m+1} c_m c_{m-1} - c_m^3} \tag{14}$$

The integration formula

From the works so far, we move on to derive our integration formula from the basic form:

$$y_{n+1} = P_m(x_{n+1}) \left[1 + q_1 x_{n+1} + q_2 x_{n+1}^2 + q_3 x_{n+1}^3\right]^{-1} \tag{15}$$

into a form that would not require users to compute for the parameters. Our first step is to recall that the governing equation in matrix-free forms gave rise to:

$$c_r = \begin{cases} \dfrac{h_r y_n^{(r)}}{r! x_{n+1}^{r}} & r = 0,1,2 \ldots m \\ 0 & r < 0 \end{cases} \qquad (16)$$

The explicit formula we derive is the one that would come handy from the values obtained for $p_0, p_1, \ldots p_m$ where m ≥ 1 as given in Equation 7.

By considering Equations 16, 7, 8, 12, 13 and 14, we obtain values for $\left(p_i x_{n+1}^i\right)_{i=0}^m$, $\left(q_i x_{n+1}^i\right)_{i=1}^3$ which when substituted into Equation 15 yields:

$$y_{n+1} = \frac{\sum_{r=0}^{s} \frac{h^r y_n^{(r)}}{r!} + A\sum_{r=0}^{s-1} \frac{h^r y_n^{(r)}}{r!} + B\sum_{r=0}^{s-2} \frac{h^r y_n^{(r)}}{r!} + C\sum_{r=0}^{s-3} \frac{h^r y_n^{(r)}}{r!}}{1 + A + B + C} \qquad (17)$$

where,

$$A = q_1 \chi_{n+1} = h \frac{\begin{bmatrix} (s^3+s^2) y_n^{(s-2)^2} y_n^{s+3} - (s^3-s) y_n^{(s)} y_n^{(s-2)} y_n^{s+3} + (s^3+2s^2-3s) y_n^{(s-2)} y_n^{(s+1)} y_n^{(s+2)} - \\ (s^3+5s^2+6s) y_n^{(s+1)^2} y_n^{(s-1)} - (s^3-4s^2+3s) y_n^{(s+2)} y_n^{(s)} y_n^{s-1} + (s^3+6s^2+11s+6) y_n^{(s+1)} y_n^{(s)^2} \end{bmatrix}}{(s+3)\begin{bmatrix} (s^3-s) y_n^{(s+2)} y_n^{(s)} y_n^{(s-2)} - (s^3+s^2) y_n^{(s+2)} y_n^{(s-2)^2} - (s^3+s^2-2s) y_n^{(s+1)^2} y_n^{(s-2)} \\ + 2(s^3+3s^2+2s) y_n^{(s+1)} y_n^{(s)} y_n^{(s-1)} - (s^3+4s^2+5s+2) y_n^{(s)^3} \end{bmatrix}} \qquad (18)$$

$$B = q_2 x_{n+1}^2 = h^2 \frac{\begin{bmatrix} (s^3+5s^2+6s) y_n^{(s+2)} y_n^{(s+1)} y_n^{(s-1)} - (s^3+2s^2-3s) y_n^{(s+2)^2} y_n^{(s-2)} + (s^3+s^2+2s) y_n^{(s+3)} y_n^{(s+1)} y_n^{(s-2)} - \\ (s^3+3s^2+2s) y_n^{(s+3)} y_n^{(s)} y_n^{(s-1)} - (s^3+7s^2+16s+12) y_n^{(s+2)^2} y_n^{(s)} + (s^3+6s^2+11s+6) y_n^{(s)^2} y_n^{(s+2)} \end{bmatrix}}{(s+2)(s+3)\begin{bmatrix} (s^3-s) y_n^{(s+2)} y_n^{(s)} y_n^{(s-2)} - (s^3-s^2) y_n^{(s+2)} y_n^{(s-1)^2} - (s^3+s^2-2s) y_n^{(s+1)^2} y_n^{(s-2)} + \\ 2(s^3+3s^2+2s) y_n^{(s+1)} y_n^{(s)} y_n^{(s-1)} - (s^3+4s^2+5s+2) y_n^{(s)^3} \end{bmatrix}} \qquad (19)$$

$$C = q_3 x_{n+1}^3 = h^3 \frac{\begin{bmatrix} (s^3+4s^2+3s) y_n^{(s+2)^2} y_n^{(s-1)} - 2(s^3+6s^2+11s+6) y_n^{(s+2)} y_n^{(s+1)} y_n^{(s)} + (s^3+7s^2+16s+12) y_n^{(s+1)^3} - \\ (s^3+3s^2+2s) y_n^{(s+3)} y_n^{(s+1)} y_n^{(s-1)} + (s^3+4s^2+5s+2) y_n^{(s+3)} y_n^{(s)^3} \end{bmatrix}}{(s+1)(s+2)(s+3)\begin{bmatrix} (s^3-s) y_n^{(s+2)} y_n^{(s)} y_n^{(s-2)} - (s^3+s^2) y_n^{(s+2)} y_n^{(s-2)^2} - (s^3+s^2-2s) y_n^{(s+1)^3} y_n^{(s-2)} + \\ 2(s^3+3s^2-2s) y_n^{(s+1)} y_n^{(s)} y_n^{(s-1)} - (s^3+4s^2+5s+2) y_n^{(s)^3} \end{bmatrix}} \qquad (20)$$

For s = 0, 1, 2, 3, . . . and $y_n^{(\alpha)} = \begin{cases} y_n^{(\alpha)} & \text{if } \alpha \geq 0 \\ 0 & \text{otherwise} \end{cases}$

CONVERGENCE AND CONSISTENCY

To establish convergence and consistency, the work of Lambert (1974, 1976, 1995) is usually called into play. Lambert (1976, 1995) asserts that convergence is a minimal property of any given numerical integrator and that convergence must take place for all initial value problems. Lambert (1976, 1995) reports that one-step method is said to be convergent if, for all initial value problem satisfying the Lipschitz then,

$$\underset{h \to 0}{\text{Limit}} \max_{0 \leq n \leq N} \left\| y(x_n) - y_n \right\| = 0$$

Lambert (1995) went further to conclude that every one-step method is convergent if and only if the one step method is consistent. On consistency, a one-step method is said to be consistent if the increment function is consistent with the initial value problem, that is, $\Phi(x_n, y_n; 0) = f(x_n, y_n)$. Armed with these facts we state and prove the convergence and consistency of our rational general integrator.

Theorem

The general one-step rational integrator

$$y_{n+1} = \frac{\sum_{r=0}^{s} \frac{h^r y_n^{(r)}}{r!} + A\sum_{r=0}^{s-1} \frac{h^r y_n^{(r)}}{r!} + B\sum_{r=0}^{s-2} \frac{h^r y_n^{(r)}}{r!} + C\sum_{r=0}^{s-3} \frac{h^r y_n^{(r)}}{r!}}{1 + A + B + C}$$

where the function A, B and C, are specified by Equations 18, 19 and 20 respectively is consistent and convergent.

Proof

From the integrator (17) we have

$$y_{n+1} - y_n = \frac{\sum_{r=1}^{s} \frac{h^r y_n^{(r)}}{r!} + A\sum_{r=1}^{s-1} \frac{h^r y_n^{(r)}}{r!} + B\sum_{r=1}^{s-2} \frac{h^r y_n^{(r)}}{r!} + C\sum_{r=1}^{s-3} \frac{h^r y_n^{(r)}}{r!}}{1 + A + B + C} \qquad (21)$$

hence,

$$\frac{y_{n+1} - y_n}{h} = \frac{\sum_{r=1}^{s} \frac{h^{r-1} y_n^{(r)}}{r!} + A\sum_{r=1}^{s-1} \frac{h^{r-1} y_n^{(r)}}{r!} + B\sum_{r=1}^{s-2} \frac{h^{r-1} y_n^{(r)}}{r!} + C\sum_{r=1}^{s-3} \frac{h^{r-1} y_n^{(r)}}{r!}}{1 + A + B + C} \qquad (22)$$

$$= \frac{y_n^{(1)}[1 + A + B + C] + \sum_{r=2}^{s} \frac{h^r y_n^{(r)}}{r!} + A\sum_{r=2}^{s-1} \frac{h^r y_n^{(r)}}{r!} + B\sum_{r=2}^{s-2} \frac{h^r y_n^{(r)}}{r!} + C\sum_{r=2}^{s-3} \frac{h^r y_n^{(r)}}{r!}}{1 + A + B + C} \qquad (23)$$

Note that from Equations 18, 19 and 20

$$\underset{h \to 0}{\text{Limit}} \ A = 0, \qquad \underset{h \to 0}{\text{Limit}} \ B = 0, \qquad \underset{h \to 0}{\text{Limit}} \ C = 0$$

Hence $\underset{h \to 0}{\text{Limit}} \left[\dfrac{y_{n+1} - y_n}{h}\right] = \dfrac{y_n^{(1)}[1+0+0+0] + 0+0+0+0}{1+0+0+0}$

$$= y_n^{(1)}$$

$$\therefore \quad \underset{h \to 0}{\text{Limit}} \left[\frac{y_{n+1} - y_n}{h}\right] = y_n^{(1)} = f(x_n, y_n)$$

as required.

Thus our new rational integrator (17) is consistent with the initial value problem (1). But (1) was chosen arbitrarily. Hence our integrator is convergent (Lambert, 1976).

STABILITY CONSIDERATION

By employing $y^1 = \lambda y \ and \ \bar{h} = \lambda h$ we obtain our stability function as

$$\varsigma(\bar{h}) = \frac{(s+1)(s+2)(s+3)\sum_{r=0}^{s}\frac{h^r}{r!} - 3(s+1)(s+2)\sum_{r=0}^{s-1}\frac{h^{r+1}}{r!} + 3(s+1)\sum_{r=0}^{s-2}\frac{h^{r+2}}{r!} - \sum_{r=0}^{s-3}\frac{h^{r+3}}{r!}}{(s+1)(s+2)(s+3) - 3(s+1)(s+2)\bar{h} + 3(s+1)\bar{h}^2 - \bar{h}^3}$$

for s = 0, 1, 2, 3, . . .
In analyzing our stability functions to determine the type of stability properties they posses, we shall analyse the first three cases that is for s = 0, 1, 2.

Case 1: s = 0

The Stability function for s = 0 on simple substitution is given as

$$\zeta(\bar{h}) = \frac{6}{6 - 6\bar{h} + 3\bar{h}^2 - \bar{h}^3}$$

By setting $\bar{h} = u + iv, \ i^2 = -1$, we get $\left| \zeta(\bar{h}) \right| \le 1$

this holds after expansion, simplification, rearranging and collecting terms $\Leftrightarrow$

$$(u^2+v^2)^3 - 6u(u^2+v^2)^2 + 9(u^2+v^2) + 12(u^2+v^2)(u^2-v^2) - 24u\,(2u^2-3u+3) \ge 0$$

Observe that:

i. Our preference is for: $(u^2+v^2)^3, \ 9(u^2+v^2)^2$
$\forall \ u, \ v \ we \ have$
$(u^2+v^2)^3 \ge 0, \ 9(u^2+v^2)^2 \ge 0$
Hence Region of Instability (RIS) from this set is empty, by this contribution.
ii. Preference is for:

$$-6u(u^2+v^2)^2 \ and \ 24u\,(2u^2-3u+3)$$
$$\forall \ u \le 0, and \ \ \forall \ v$$
$$-6u\,(u^2+v^2)^2 \ge 0$$
$$-24u(2u^2-3u+3) \ge 0$$

Hence our RIS from this set is the right half of the complex plane, by this our RAS will be the entire left half of the complex plane.
iii. Preference is for:

$$12(u^2+v^2)(u^2-v^2)$$

(i) $\forall \ u, \ v \ we \ have:$
$$(u^2+v^2) \ge 0,$$
(ii) $(u^2-v^2) \ge 0$
$$\Leftrightarrow u \ge v \ge 0 \ or \ u \le v \le 0,$$
$$Hence \ 12(u^2+v^2)(u^2-v^2) \ge 0 \ \forall \ u \le 0$$
For easy actualization of the nature of the RAS we shall employ the polar form where,

$$u = R\ Cos\ \theta, v = R\ Sin\theta,$$
$$u^2 + v^2 = R^2(Cos^2\theta + Sin^2\theta) = R^2,$$
$$u^2 - v^2 = R^2(Cos^2\theta - Sin^2\theta) = R^2 Cos2\theta$$

Consequently our inequality becomes,
$$R^6 - 6R^5Cos\theta + 9R^4 + 12R^3Cos2\theta - 24RCos\theta(2R^2Cos^2\theta - 3RCos\theta + 3) \ge 0$$
Which gives a vast span of Region of Absolute stability on plotting $\theta \in [0^0, 360^0]$

Figure 1 shows that our integrator is A-stable and so the Region of Absolute Stability of the integrator is the entire left – half of the complex plane and the exterior part of the region of Instability shaded in the figure. Our Singulo Oscillatory – Stiff integrator is L- Stable since by direct substitution of $\zeta(\bar{h})$,

$$\underset{\mathrm{Re}(\bar{h}) \to -\infty}{Limit} \left| \zeta(\bar{h}) \right| = 0$$

Case 2: s = 1

Our stability function is given as

$$\zeta(\bar{h}) = \frac{24 + 6\bar{h}}{24 - 18\bar{h} + 6\bar{h}^2 - \bar{h}^3}$$

By setting $\bar{h} = u + iv, \ i^2 = -1$, we get $\left| \zeta(\bar{h}) \right| \le 1 \Leftrightarrow$

$$(u^2+v^2)^3 - 12u(u^2+v^2)^2 + 72u^2(u^2+v^2) - 72u(u^2+v^2) - 48u(4u^2 - 12u + 24) \ge$$
Observe that:

i. Our preference is for: $(u^2+v^2)^3, \ 9(u^2+v^2)^2$
$\forall \ u, \ v \ we \ have$
$$(u^2+v^2)^3 \ge 0 \text{ and } 72u^2(u^2+v^2) \ ^3\ge 0$$
Hence RIS for this set is empty.
ii. Preference is for

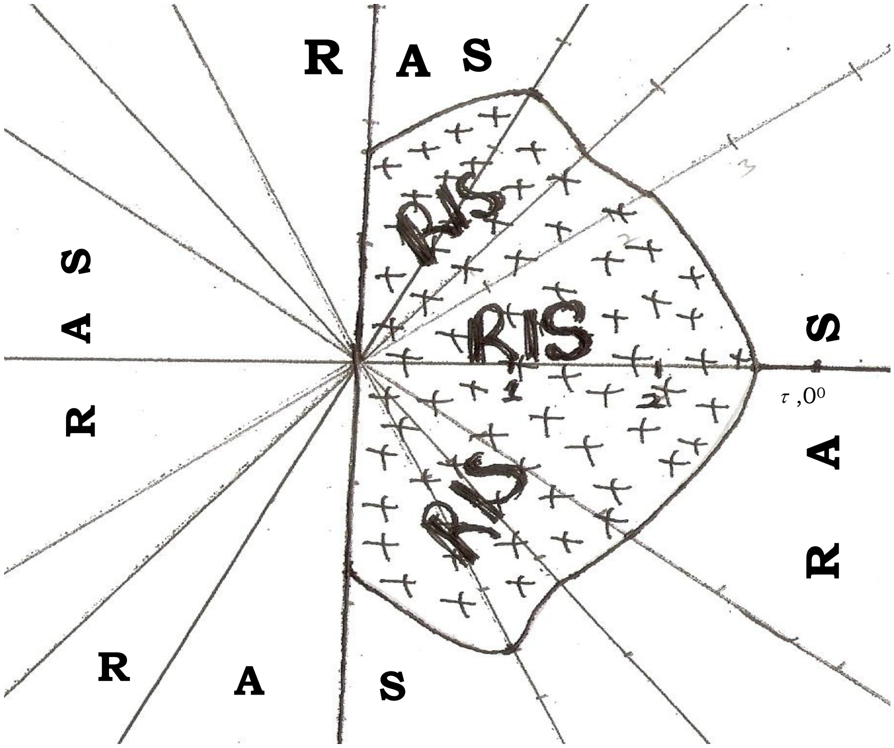

Figure 1. Region of absolute stability (RAS) for our new method for s = 0, $\tau = 2.55$.

$$-12u\left(u^2+v^2\right)^2-72u\left(u^2+v^2\right)-48u\left(4u^2-12u+24\right)$$

$\forall\ u \le 0\ and\ \forall\ v$ we have that

$$-12u\left(u^2+v^2\right)^2 \ge 0,$$
$$-72u\left(u^2+v^2\right)^2 \ge 0$$
$$-48u\left(4u^2-12u+24\right) \ge 0$$

Hence our RAS will be the entire left half of the complex plan.

On employing the polar form where u = R cos θ, v = R sin θ our inequality becomes

$$R^6-12R^5\cos\theta+72R^4\cos^2\theta-72R^3\cos\theta-48R\cos\theta\left[4R^2\cos^2\theta-12R\cos\theta+24\right] \ge 0$$

On plotting $\theta \in \left[0^0, 360^0\right]$ we obtain our Region of Absolute Stability as given in Figure 2, which shows that the integrator is A-stable and the Region of Absolute Stability is the entire left – half of the complex plane and the space outside the Region of Instability.

The Integrator is L – Stable as by direct substitution of $\zeta\left(\bar{h}\right)$

$$\underset{\mathrm{Re}\left(\bar{h}\right)\to-\infty}{Limit}\ \left|\zeta\left(\bar{h}\right)\right| = 0$$

Case 3: s = 2

Our stability function as $\zeta\left(\bar{h}\right)=\dfrac{60+24\bar{h}+3\bar{h}^2}{60-36\bar{h}+9\bar{h}^2-\bar{h}^3}$

By setting $\bar{h}=u+iv,\ i^2=-1$, we get $\left|\zeta\left(\bar{h}\right)\right| \le 1 \Leftrightarrow$

$$\left(u^2+v^2\right)^3-18u\left(u^2+v^2\right)^2+144u^2\left(u^2+v^2\right)-432u\left(u^2+v^2\right)-480u\left(u^2-3u+15\right) \ge 0\ \forall\ u<0$$
$$ie\ \left|\varsigma(u,v)\right| \le 1\ \forall\ u,v \in \left\{(u+iv): u<0\right\}$$

Observe that

i. $\left(u^2+v^2\right)^3 \ge 0,\ 144u^2\left(u^2+v^2\right) \ge 0\ \forall\ u,v$

ii. $-18u\left(u^2+v^2\right)^2 \ge 0, -432u\left(u^2+v^2\right) \ge 0,\ -480u\left(u^2-3u+15\right) \ge 0\ \forall\ u<0$

Since our Stability function for S = 2 is equivalent to Aashikpelokhai (1991) for k = 3, the polar form and RAS will be the same. We simply quote the polar form and hence the RAS as follows:

$$R^6-18R^5\cos\theta+144R^4\cos^2\theta-48R^3\cos\theta\left(19\cos^2\theta+9\sin^2\theta\right)+1440R^2\cos^2\theta-7280R\cos\theta \ge 0$$

The RAS is given in Figure 3 which is equivalent to Aashikpelokhai (1991) for k = 3. Hence the integrator is A-stable and so the Region of Absolute Stability of the integrator is the entire left – half of the complex plane.

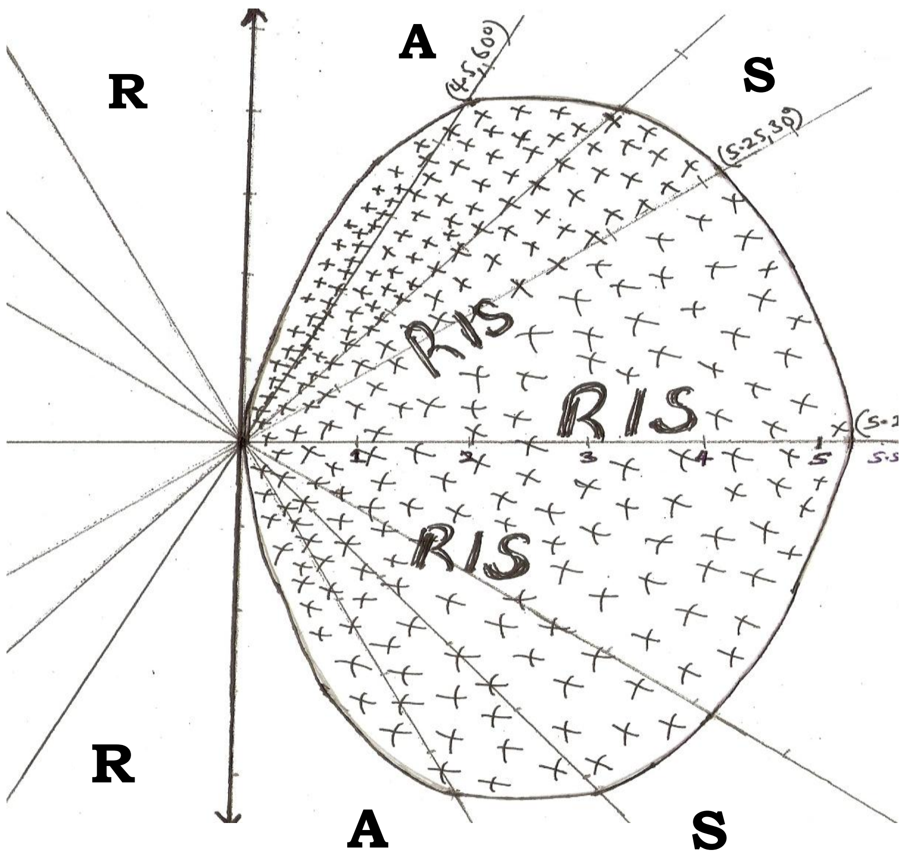

Figure 2. Showing RAS for our new method for s = 1.

We easily confirm that the method is L – Stable, since on simple application of $\zeta(\bar{h})$,

$$\underset{\mathrm{Re}(\bar{h})\to -\infty}{Limit} \left|\zeta(\bar{h})\right| = 0$$

NUMERICAL EXPERIMENT

We shall be concerned with the application of our new integrator in solving a number of problems in this section. Our interest is to select problems in the three classes of problems, this integrator have been designed for. A number of comparisons are made with the respective exact solution where such exact solutions exist and also compared with recent research work in that area.

Example 1

$y^1 = -100y$, $y(x_0) = 1$, $x \in [0, 1]$ Exact solution $y(x) = \ell^{-100x}$ Ademiluyi and Kayode (2001). A variable Step Size was used starting with h = 0.10.

Table 1 shows the performance of our integrator. The convergence rate is very high as s increases from 0 to 2 and then from s = 5 the convergence is excellent. Our new integrator performs better than the maximum order second derivative hybrid multi-step methods of Ademiluyi and Kayode (2001) and converges quickly to the analytic solution than its counterpart.

Example 2

$y^1(x) + 2xy(x) = 0$, $0 \le x \le 1$, $y(0) = 1$ Exact solution $y(x) = \ell^{-x^2}$ Egbetade et al. (2008).

Uniform mesh-size h = 0.1 was used. Table 2 shows that our new numerical integrator gives a better approximation than the Euler but next to Tau for case s = 0 and s = 1. For s = 2, our integrator gives a better approximation than Tau and Euler. The table shows that our integrator converges quickly to the analytic solution at each corresponding mesh point than Tau and Euler methods.

Example 3

$2(1 + x)\, y^1(x) + y(x) = 0$, $0 \le x \le 1$, $y(0) = 1$ Exact solution $y(x) = (1 + x)^{-1/2}$ Egbetade et al. (2008).

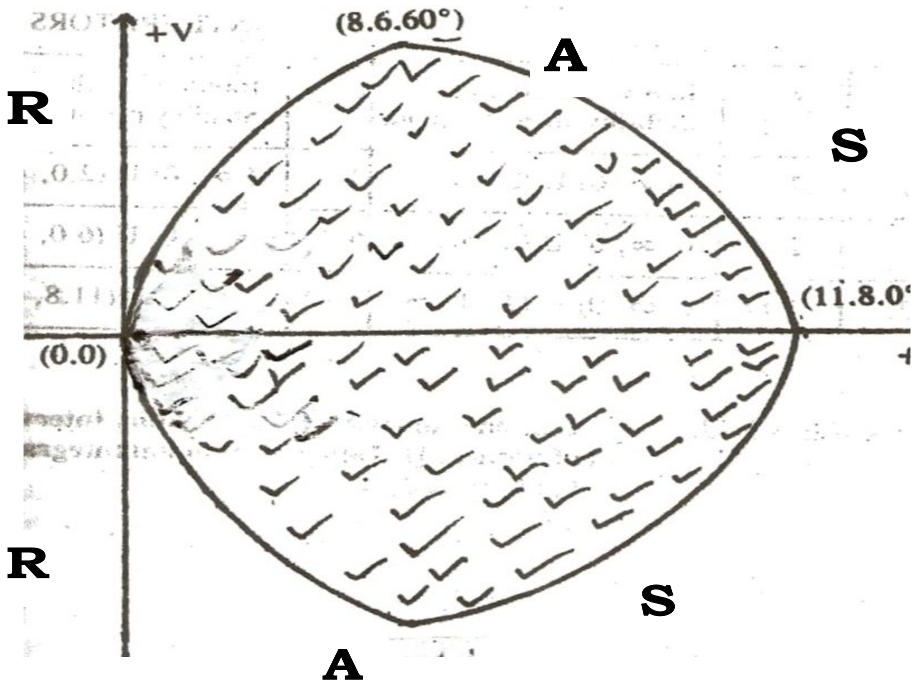

Figure 3. Shows RAS for our new method of s = 2 $\equiv$ Aashikpelokhai (1991) for k = 3.

Uniform mesh-size h = 0.1 was used. Table 3 shows that our new integrator performs better than the Tau and Euler methods proposed by Egbetade et al. (2008).

The Table shows that our integrator converges quickly to the analytic solution at each corresponding mesh point than the given Tau and Euler method.

Example 4

$y^1 = 1 + y^2$, $y(0) = 1$ $\quad 0 \le x \le 1$ Exact solution $y = \tan\left(x + \frac{\pi}{4}\right)$ Aashikpelokhai (1991) and Fatunla (1982).

Uniform mesh – size h = 0.05 was used. From Table 4, the trend shows that for a given mesh point, the errors in numerical integrator decreases as s increases from 0 to 2 and also shows that the accuracy in the computed results falls as we approach the point of singularity. Our new numerical integrator compete fairly well with known conventional methods like Lambert and Shaw (1965), Fatunla (1982), Niekerk (1987) and Aashikpelokhai (1991). Our integrator converges quickly to the analytic solutions as s increases and performs better than those compared with.

Example 5

$x^1 = 5x - 2y, \quad x(0) = 1$

$y^1 = 3x \qquad y(0) = 2, \; h = 0.01$

Exact solutions: $\begin{matrix} x = 2e^{2t} - e^{3t} \\ y = 3e^{2t} - e^{3t} \end{matrix}$ Ademiluyi et al. (2007).

Tables 5 and 6 shows that our integrators compete favourably well with Ademiluyi et al. (2007).

Example 6

$$y_1^{(1)} = -2000y_1 + 999.75y_2 + 1000.25$$
$$y_2^{(1)} = y_1 - y_2$$
$$y_1(0) = 0$$
$$y_2(0) = -2 \qquad x \in [0,10] \quad h = 0.50$$

Ademiluyi and Kayode (2001).

Exact solutions: $\begin{matrix} y_1 = -1.499875\exp(-0.5x) + 0.499875\exp(-20005x) + 1 \\ y_2 = -2.99975\exp(-0.5x) - 0.00025\exp(-20005x) + 1 \end{matrix}$

Tables 7 and 8 show the performance of our integrators compared with Ademiluyi and Kayode (2001).

Example 7

$y' = \begin{bmatrix} 0 & 1 \\ -1 & 5(1 - y_1^2) \end{bmatrix} y, \; y(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \; 0 \le x \le 1.$ Fatunla (1980) and Aashikpelokhai (1991).

Exact solutions: Unknown.

Table 1. Error in Numerical Integrators on Example 1.

h	Value of x	Analytic solution	Ademiluyi, and Kayode (2001)	Rational integrator, order = S + 3						
				S = 0	S = 1	S = 2	S = 3	S = 4	S = 5	S = 6
1.00000D-01	1.00000D-01	-1.68748D+02	2.14454D-02	-4.3469870000D-03	2.0001050000D-02	-5.1678738001D-02	1.68748D+02	-1.11022D-16	0.00000D+00	0.00000D+00
5.00000D-02	1.50000D-01	-5.09314D-04	2.00536D-02	-8.4833320000D-07	1.0061580000D-06	-8.4833318193D-07	5.09620D-04	1.11022D-16	-1.11022D-16	0.00000D+00
2.50000D-02	1.75000D-01	-1.01713D-07	1.97436D-02	-8.0351810000D-09	2.5665250000D-09	-7.4090876666D-10	1.26823D-07	5.96046D-08	0.00000D+00	0.00000D+00
1.25000D-02	1.87500D-01	6.97190D-09	1.68208D-02	-2.8626790000D-10	2.9542090000D-11	-3.3080616716D-12	2.22223D-10	1.11022D-16	-5.96046D-08	0.00000D+00
6.25000D-03	1.93750D-01	3.84976D-09	1.24074D-02	-1.4987770000D-11	6.0145620000D-13	-2.9159240580D-14	9.79550D-13	-5.96046D-08	5.96046D-08	0.00000D+00
3.12500D-03	1.96875D-01	2.81725D-09	7.84083D-03	-8.7327870000D-13	1.5429890000D-14	-3.4695157775D-16	7.59669D-15	0.00000D+00	5.96046D-08	0.00000D+00
1.56250D-03	1.98438D-01	2.40973D-09	4.43528D-03	-5.2832730000D-14	4.3859390000D-16	-4.7468516322D-18	-1.52056D-15	5.96046D-08	0.00000D+00	0.00000D+00
7.81250D-04	1.99219D-01	2.22864D-09	2.36270D-03	-3.2499840000D-15	1.3083900000D-17	-6.9469936565D-20	-8.87885D-16	-5.96046D-08	-1.11022D-16	0.00000D+00
3.90625D-04	1.99609D-01	2.14326D-09	1.21590D-03	-2.0153060000D-16	3.9957290000D-19	-1.0513465586D-21	1.33227D-15	1.11022D-16	0.00000D+00	0.00000D+00
1.95313D-04	1.99805D-01	2.10180D-09	6.19892D-04	-1.2546370000D-17	1.2344020000D-20	-1.6543612251D-23	6.66134D-16	1.11022D-16	0.00000D+00	0.00000D+00
9.76563D-05	1.99902D-01	2.08138D-09	Not stated	-7.8260720000D-19	3.9084280000D-22	7.0310352067D-24	-1.33227D-15	2.22045D-16	0.00000D+00	0.00000D+00

Table 2. Error in numerical integrators on Example 2.

Value of x	Analytic solution	Egbetade et al. (2008)		Rational integrator, order = S + 3		
		Tau	Euler	S = 0	S = 1	S = 2
0.10	9.9004983375D-01	4.72D-02	9.95D-03	-4.917615D-05	0.000000D+00	8.2505386967D-08
0.20	9.6078943915D-01	1.32D-02	1.92D-02	-4.960649D-05	1.577458D-06	8.4904185682D-08
0.30	9.1393118527D-01	2.17D-03	2.69D-02	-5.253809D-05	6.306041D-06	9.5526001109D-08
0.40	8.5214378897D-01	1.03D-03	3.22D-02	-5.735429D-05	3.684451D-06	1.1631798935D-07
0.50	7.7880078307D-01	1.48D-04	3.48D-02	-6.331738D-05	2.489074D-06	1.5519120644D-07
0.60	6.9767632607D-01	6.97D-05	3.45D-02	-6.963457D-05	1.85956D-06	2.4763663675D-07
0.70	6.1262639418D-01	6.03D-05	3.17D-02	-7.552774D-05	1.522491D-06	8.3757392144D-07
0.80	5.2729242404D-01	6.40D-06	2.69D-02	-8.030164D-05	1.353403D-06	-4.1282457863D-07
0.90	4.4485806622D-01	8.00D-07	2.06D-02	-8.340400D-05	1.280778D-06	-1.3945307575D-07
1.00	3.6787944117D-01	2.00D-07	4.51D-03	-8.447050D-05	1.257789D-06	-7.5853677450D-08

Problem type

Van Der Pol's oscillator problem. Nonlinear Stiff in some regions and non – stiff in other regions. Tables 9 and 10 shows the performance of our integrator at s = 2 when compared with Aashikpelokhai (1991), Fatunla (1978, 1980) and Norelli (1985).

CONCLUSION AND RECOMMENDATIONS

In this work, we have been able to derive a family of rational integrator having its members A-stable

Table 3. Error in numerical integrators on Example 3.

Value of x	Analytic solution	Egbetale et al. (2008)		Rational integrator, order = S + 3				
		Tau method	Euler method	S = 0	S = 1	S = 2	S = 3	S = 4
0.1	9.534626D-01	4.69D-03	3.40D-03	3.351720D-06	9.485746D-09	3.2235009217D-08	-9.5341701444D-01	-9.5341701444D-01
0.2	9.128709D-01	1.10D-04	6.05D-03	2.128150D-06	-7.834597D-08	-6.4574900649D-08	-4.3465059489D-05	-4.3465059489D-05
0.3	8.770580D-01	1.07D-04	8.02D-03	1.504906D-06	-3.893317D-09	4.8008580444D-09	-1.9801983859D-09	-1.9801983859D-09
0.4	8.451542D-01	3.26D-05	9.54D-03	1.070653D-06	7.889883D-09	1.3577405422D-08	-9.0168061594D-14	-9.0168061594D-14
0.5	8.164966D-01	3.05D-05	1.07D-02	7.475101D-07	-2.029751D-08	-1.6461896735D-08	-4.1041068680D-18	-4.1041068680D-18
0.6	7.905694D-01	1.55D-05	1.17D-02	5.984675D-07	3.145635D-08	3.4112142000D-08	-1.8674165387D-22	-1.8674165387D-22
0.7	7.669650D-01	3.30D-06	1.23D-02	3.980663D-07	-2.875809D-08	-2.6876332382D-08	-8.4946652258D-27	-8.4946652258D-27
0.8	7.453560D-01	2.20D-06	2.21D-02	3.223554D-07	-4.409917D-09	-3.0492284253D-09	-3.8632620036D-31	-3.8632620036D-31
0.9	7.254763D-01	8.00D-07	2.24D-02	2.867425D-07	3.280389D-08	3.3805727995D-08	-1.7566313688D-35	-1.7566313688D-35

Table 4. Error in numerical integrators on Example 4.

Value of x	Analytic solution	Lambert and Shaw (1965)	Luke et al. (1975)	Fatunla order 4 K = 4 (1982)	Niekerk Order 2 (1987)	Aashikpelokhai (1991)		Rational integrator, order = S + 3		
						Order 3 K - 2	Order 5 K = 3	S = 0	S = 1	S = 2
0.10	1.1053555921D+00	9(-9)	1(-5)	-	2(-6)	2(-6)	2(-10)	-2.3922899373D-05	3.4389746828D-08	-1.5467094272D-10
0.20	1.2230488842D+00	2(-7)	2(-5)	6(-5)	8(-6)	3(-6)	2(-10)	-2.1573281999D-05	9.7985064507D-09	-1.5691825617D-10
0.30	1.3560878681D+00	4(-7)	3(-5)	5(-5)	2(-5)	5(-6)	2(-10)	-1.9837158535D-05	1.6594276397D-08	-1.6103607337D-10
0.40	1.5084976569D+00	7(-7)	3(-5)	2(-4)	5(-5)	1(-5)	2(-10)	-1.8660567270D-05	-1.1544370793D-08	-1.6727819130D-10
0.50	1.6857964172D+00	1(-6)	7(-5)	2(-4)	1(-4)	2(-5)	2(-10)	-1.7899054741D-05	-8.7192120191D-10	-1.7604917524D-10
0.60	1.8957651776D+00	4(-6)	2(-4)	7(-4)	5(-4)	8(-5)	2(-10)	-1.7366474947D-05	2.0532583034D-07	-1.8799828361D-10
0.65	2.1497477742D+00	8(-6)	4(-3)	1(-3)	1(-3)	2(-4)	2(-10)	-1.7488961197D-05	1.9269808504D-07	-2.0412205259D-10
0.70	2.4649630098D+00	2(-5)	1(-2)	3(-3)	3(-3)	6(-4)	1(-9)	-1.8098347009D-05	1.9229756232D-07	-2.2602453242D-10
0.75	2.8688844682D+00	1(-4)	1(-1)	1(-2)	2(-2)	5(-3)	4(-9)	-1.9083213533D-05	4.5660165604D-07	-2.5640156665D-10

$a(b) = ax10^{b}$

and at least three of its members L-stable, leaving its users for easy choice of s for use as all the p_i and q_j, $(i = 1\ (1)\ s)$, $(j = 1\ (1)\ 3)$ have been fully derived. This integrator can cope effectively well with the three classes of problems, namely Stiff, mildly Stiff and non-stiff, singular and non-singular and oscillating and non-oscillatory initial value problems in ordinary differential equations.

The integrator compared favourably well with

Table 5. Error in numerical integrators first component on Example 5.

Values of TI	Ademiluyi et al. (2007)	New Integrator (Order = S + 3)						
		S = 2	S = 3	S = 4	S = 5	S = 6	S = 7	S = 8
3.00000000D-02	0.3238292541D-04	-4.01897400D-10	-6.15099061D-09	-6.15099061D-09	-6.15099061D-09	-6.15099061D-09	-6.15099061D-09	-6.15099061D-09
6.00000000D-02	0.2995808673D-04	-3.56716000D-10	-1.11073899D-07	-1.11073899D-07	-1.11073899D-07	-1.11073899D-07	-1.11073899D-07	-1.11073899D-07
9.00000000D-02	0.2787104097D-04	-1.89672700D-09	-1.15763126D-07	-1.15763126D-07	-1.15763126D-07	-1.15763126D-07	-1.15763126D-07	-1.15763126D-07
1.20000000D-01	0.2605580892D-04	1.23133400D-09	-1.05589737D-09	-1.05589737D-09	-1.05589737D-09	-1.05589737D-09	-1.05589737D-09	-1.05589737D-09
1.50000000D-01	0.2446254221D-04	-4.03830900D-09	-1.62302885D-07	-1.62302885D-07	-1.62302885D-07	-1.62302885D-07	-1.62302885D-07	-1.62302885D-07
1.80000000D-01	0.2305287807D-04	1.01706500D-09	3.51731264D-08	3.51731264D-08	3.51731264D-08	3.51731264D-08	3.51731264D-08	3.51731264D-08
2.10000000D-01	0.2179681180D-04	8.98252400D-10	-4.33673466D-08	-4.33673466D-08	-4.33673466D-08	-4.33673466D-08	-4.33673466D-08	-4.33673466D-08
2.40000000D-01	0.2067053833D-04	7.26404000D-10	-6.53996413D-08	-6.53996413D-08	-6.53996413D-08	-6.53996413D-08	-6.53996413D-08	-6.53996413D-08
2.70000000D-01	0.196542975D-04	-1.23739400D-09	-1.62053748D-08	-1.62053748D-08	-1.62053748D-08	-1.62053748D-08	-1.62053748D-08	-1.62053748D-08
3.00000000D-01	0.1873444048D-04	3.30105100D-10	4.25262905D-08	4.25262905D-08	4.25262905D-08	4.25262905D-08	4.25262905D-08	4.25262905D-08
3.30000000D-01	0.1789630619D-04	6.88611400D-11	-1.37350189D-08	-1.37350189D-08	-1.37350189D-08	-1.37350189D-08	-1.37350189D-08	-1.37350189D-08
3.60000000D-01	0.1712994847D-04	-2.37488500D-10	-8.42760395D-08	-8.42760395D-08	-8.42760395D-08	-8.42760395D-08	-8.42760395D-08	-8.42760395D-08
3.90000000D-01	0.1712994847D-04	-5.64629500D-10	-9.86133362D-08	-9.86133362D-08	-9.86133362D-08	-9.86133362D-08	-9.86133362D-08	-9.86133362D-08
4.20000000D-01	0.1642652616D-04	-1.00651400D-09	-9.38846805D-08	-9.38846805D-08	-9.38846805D-08	-9.38846805D-08	-9.38846805D-08	-9.38846805D-08
4.50000000D-01	0.1577859276D-04	-1.43394400D-09	4.70074202D-10	4.70074202D-10	4.70074202D-10	4.70074202D-10	4.70074202D-10	4.70074202D-10
4.80000000D-01	0.1517983194D-04	-1.99811000D-09	-9.95356952D-08	-9.95356952D-08	-9.95356952D-08	-9.95356952D-08	-9.95356952D-08	-9.95356952D-08
5.10000000D-01	0.1462485089D-04	-2.54803700D-09	-5.62501530D-08	-5.62501530D-08	-5.62501530D-08	-5.62501530D-08	-5.62501530D-08	-5.62501530D-08
5.40000000D-01	0.1410901759D-04	9.74980300D-08	2.95151725D-09	2.95151725D-09	2.95151725D-09	2.95151725D-09	2.95151725D-09	2.95151725D-09
5.69999900D-01	0.13628331113D-04	1.17562200D-07	-2.83948820D-08	-2.83948820D-08	-2.83948820D-08	-2.83948820D-08	-2.83948820D-08	-2.83948820D-08
5.99999900D-01	0.1275894717D-04	1.40095800D-07	1.67908905D-08	1.67908905D-08	1.67908905D-08	1.67908905D-08	1.67908905D-08	1.67908905D-08
6.29999900D-01	0.1236456325D-04	1.65635400D-07	2.52512509D-08	2.52512509D-08	2.52512509D-08	2.52512509D-08	2.52512509D-08	2.52512509D-08
6.59999800D-01	0.1199382870D-04	1.94263200D-07	-1.15564580D-08	-1.15564580D-08	-1.15564580D-08	-1.15564580D-08	-1.15564580D-08	-1.15564580D-08
6.89999800D-01	0.1164467828D-04	2.26304200D-07	5.60328139D-09	1.53459467D-11	1.53459467D-11	1.53459467D-11	1.53459467D-11	1.53459467D-11
7.19999800D-01	0.1131528043D-04	2.62465000D-07	4.31058034D-09	4.31058034D-09	4.31058034D-09	4.31058034D-09	4.31058034D-09	4.31058034D-09
7.49999800D-01	0.1070939677D-04	3.02850300D-07	-1.91763583D-08	-1.91763583D-08	-1.91763583D-08	-1.91763583D-08	-1.91763583D-08	-1.91763583D-08
7.79999700D-01	0.1016509888D-04	3.47702700D-07	-1.72803656D-08	-1.72803656D-08	-1.72803656D-08	-1.72803656D-08	-1.72803656D-08	-1.72803656D-08
8.09999700D-01	0.9913183923D-05	3.98191800D-07	-4.51632189D-08	-4.51632189D-08	-4.51632189D-08	-4.51632189D-08	-4.51632189D-08	-4.51632189D-08
8.39999700D-01	0.9673451145D-05	4.54307000D-07	1.03405924D-08	1.03405924D-08	1.03405924D-08	1.03405924D-08	1.03405924D-08	1.03405924D-08
8.69999600D-01	0.9445040210D-05	5.17281900D-07	-1.83549945D-07	-1.83549945D-07	-1.83549945D-07	-1.83549945D-07	-1.83549945D-07	-1.83549945D-07
8.99999600D-01	0.1043015177D-04	5.86612800D-07	-4.18202735D-08	-4.18202735D-08	-4.18202735D-08	-4.18202735D-08	-4.18202735D-08	-4.18202735D-08

other conventional methods like Lambert and Shaw (1965), Luke et al. (1975), Fatunla (1978, 1980, 1982), Norelli (1985), Niekek (1987) and Aashikpelokhai (1991). The new integrator also compared favourably well with recent work in the area like Ademiluyi and Kayode (2001), Egbetale et al. (2008) and Ademiluyi et al. (2007).

The integrator can effectively cope with ivp arising from mechanical oscillation, chemical

Table 6. Error in numerical integrators second component on Example 5.

Values of TI	Ademiluyi et al. (2007)	New integrator (Order = S + 3)						
		S = 2	S = 3	S = 4	S = 5	S = 6	S = 7	S = 8
3.00000000D-02	0.000000000D+00	-4.53041826D-09	2.97802411D-07	2.97718588D-07	2.97718588D-07	2.97718588D-07	2.97718588D-07	2.97718588D-07
6.00000000D-02	0.000000000D+00	3.74003140D-09	2.12429478D-08	2.11451257D-08	2.11451257D-08	2.11451257D-08	2.11451257D-08	2.11451257D-08
9.00000000D-02	0.000000000D+00	-6.74308609D-09	-9.77442065D-08	-9.78790471D-08	-9.78790471D-08	-9.78790471D-08	-9.78790471D-08	-9.78790471D-08
1.20000000D-01	0.000000000D+00	5.04410380D-09	-2.56280690D-08	-2.58062087D-08	-2.58062087D-08	-2.58062087D-08	-2.58062087D-08	-2.58062087D-08
1.50000000D-01	0.000000000D+00	-2.04006416D-08	1.23085502D-07	1.22857956D-07	1.22857956D-07	1.22857956D-07	1.22857956D-07	1.22857956D-07
1.80000000D-01	0.000000000D+00	4.88393992D-09	1.06220372D-07	1.05937690D-07	1.05937690D-07	1.05937690D-07	1.05937690D-07	1.05937690D-07
2.10000000D-01	0.000000000D+00	4.87973262D-09	-5.71917855D-08	-5.75347849D-08	-5.75347849D-08	-5.75347849D-08	-5.75347849D-08	-5.75347849D-08
2.40000000D-01	0.000000000D+00	4.90586283D-09	-1.05662416D-07	-1.06070213D-07	-1.06070213D-07	-1.06070213D-07	-1.06070213D-07	-1.06070213D-07
2.70000000D-01	0.000000000D+00	-4.79951656D-08	2.09875879D-07	2.09359915D-07	2.09359915D-07	2.09359915D-07	2.09359915D-07	2.09359915D-07
3.00000000D-01	0.1096978240D-05	4.91908336D-09	-1.54067350D-07	-1.54657053D-07	-1.54657053D-07	-1.54657053D-07	-1.54657053D-07	-1.54657053D-07
3.30000000D-01	0.1109145273D-05	4.90235097D-09	-1.97239157D-07	-1.97948710D-07	-1.97948710D-07	-1.97948710D-07	-1.97948710D-07	-1.97948710D-07
3.60000000D-01	0.1109145273D-05	4.91438845D-09	1.98260945D-08	1.89917504D-08	1.89917504D-08	1.89917504D-08	1.89917504D-08	1.89917504D-08
3.90000000D-01	0.1121585230D-05	4.85971352D-09	-1.44263610D-08	-1.53889994D-08	-1.53889994D-08	-1.53889994D-08	-1.53889994D-08	-1.53889994D-08
4.20000000D-01	0.1134307399D-05	4.74913797D-09	-1.46154218D-07	-1.47247327D-07	-1.47247327D-07	-1.47247327D-07	-1.47247327D-07	-1.47247327D-07
4.50000000D-01	0.1147321494D-05	4.63128025D-09	4.15880099D-08	4.04549403D-08	4.04549403D-08	4.04549403D-08	4.04549403D-08	4.04549403D-08
4.80000000D-01	0.1160637680D-05	4.45390658D-09	3.70378901D-07	3.69060326D-07	3.69060326D-07	3.69060326D-07	3.69060326D-07	3.69060326D-07
5.10000000D-01	0.1174266597D-05	4.23985869D-09	2.77000748D-07	2.75495756D-07	2.75495756D-07	2.75495756D-07	2.75495756D-07	2.75495756D-07
5.40000000D-01	0.1188219395D-05	-7.07429284D-08	-6.21336675D-08	-6.38247699D-08	-6.38247699D-08	-6.38247699D-08	-6.38247699D-08	-6.38247699D-08
5.69999900D-01	0.1217143933D-05	-6.10936928D-08	-1.18398513D-08	-1.37702192D-08	-1.37702192D-08	-1.37702192D-08	-1.37702192D-08	-1.37702192D-08
5.99999900D-01	0.1232140783D-05	-4.95526433D-08	-1.14634431D-07	-1.16802906D-07	-1.16802906D-07	-1.16802906D-07	-1.16802906D-07	-1.16802906D-07
6.29999900D-01	0.1247511805D-05	-3.58115599D-08	1.93723432D-07	1.91263741D-07	1.91263741D-07	1.91263741D-07	1.91263741D-07	1.91263741D-07
6.59999800D-01	0.1263271177D-05	-1.96332968D-08	-2.21608700D-08	-2.49093537D-08	-2.49093537D-08	-2.49093537D-08	-2.49093537D-08	-2.49093537D-08
6.89999800D-01	0.1279433808D-05	-6.77740974D-10	2.33782189D-08	2.02886157D-08	2.02886157D-08	2.02886157D-08	2.02886157D-08	2.02886157D-08
7.19999800D-01	0.1296015373D-05	2.13456812D-08	-7.31643777D-08	-7.65913257D-08	-7.65913257D-08	-7.65913257D-08	-7.65913257D-08	-7.65913257D-08
7.49999800D-01	0.1313032376D-05	4.66411394D-08	-4.04818419D-08	-4.42973263D-08	-4.42973263D-08	-4.42973263D-08	-4.42973263D-08	-4.42973263D-08
7.79999700D-01	0.1330502196D-05	7.59433001D-08	2.21412357D-07	2.17343559D-07	2.17343559D-07	2.17343559D-07	2.17343559D-07	2.17343559D-07
8.09999700D-01	0.1348443152D-05	1.09458130D-07	2.99723895D-07	2.95165076D-07	2.95165076D-07	2.95165076D-07	2.95165076D-07	2.95165076D-07
8.39999700D-01	0.1366874563D-05	1.47845335D-07	2.23335017D-07	2.18292594D-07	2.18292594D-07	2.18292594D-07	2.18292594D-07	2.18292594D-07
8.69999600D-01	0.1385816818D-05	1.91262203D-07	-1.04686314D-07	-1.10260577D-07	-1.10260577D-07	-1.10260577D-07	-1.10260577D-07	-1.10260577D-07
8.99999600D-01	0.1405291455D-05	2.40932396D-07	-1.40836560D-07	-1.46990015D-07	-1.46990015D-07	-1.46990015D-07	-1.46990015D-07	-1.46990015D-07

kinetics, electrical networks, nuclear reactor control, tunnel switching problems and reversible enzyme kinetics. Our RAS of the integrator covers the whole of the left half of the complex plane. Hence, this integrator is fully recommended for users who are currently working in this area of research.

Table 7. Error in numerical integrators first component on Example 6.

Values of TI	Ademiluyi and Kayode (2001)	New integrator (Order = S + 3)		
		S = 2	S = 3	S = 4
0.5	0.62546D+10	-1.8597990000D-01	-4.1791014417D+08	1.0733593395D+11
1.0	-0.164725D-4	-2.5982120000D-06	7.0350798400D-06	4.4114315832D-04
1.5	0.884654D-5	2.7616450000D-06	5.4594691825D-06	3.2616226040D-04
2.0	0.285650D-4	5.7546770000D-06	4.2456011141D-06	2.6516493362D-04
2.5	0.439218D-4	8.1974730000D-06	3.1322937279D-06	-1.7729331943D-05
3.0	0.558816D-4	7.6845330000D-06	2.8370901596D-06	4.3121567216D-04
3.5	0.651960D-4	7.4236880000D-06	2.2622175933D-06	4.2652807910D-04
4.0	0.724500D-4	6.7186670000D-06	1.7225585679D-06	2.3561118467D-04
4.5	0.780994D-4	5.7379170000D-06	1.1743983170D-06	1.0220427121D-04
5.0	0.824992D-4	5.1037650000D-06	1.2824719999D-06	4.2256810127D-04
5.5	0.859258D-4	4.2722520000D-06	1.0587345097D-06	3.9742962227D-04
6.0	0.885944D-4	3.4621620000D-06	6.4438750580D-07	1.0882681777D-04

Table 8. Error in numerical integrators second component on Example 6.

Values of TI	Ademiluyi and Kayode (2001)	New integrator (Order = S + 3)		
		S = 2	S = 3	S = 4
0.5	-0.195980D-3	-2.3362080000D+00	2.0796841379D+05	-5.3681289336D+07
1.0	-0.130935D-3	-1.4621490000D-03	7.9457824294D-06	-4.5847248398D-07
1.5	-0.802970D-4	5.8857820000D-03	5.6141249800D-06	-3.4633949753D-07
2.0	-0.406600D-4	9.3650730000D-03	4.7584613492D-06	-2.8558171494D-07
2.5	-0.101465D-4	1.7823120000D-02	-1.0350207275D-05	-2.4721998593D-07
3.0	0.137732D-4	-8.0727550000D-03	4.2436688444D-06	-1.6707486894D-07
3.5	0.32401D-4	-1.0290480000D-03	3.4515515805D-06	-9.4924783589D-08
4.0	0.46910D-4	3.7119820000D-04	2.5510474640D-06	-7.1556906067D-08
4.5	0.58208D-4	6.9258230000D-04	1.6931455414D-06	-9.4993801825D-08
5.0	0.670084D-4	3.2224520000D-04	1.7304789729D-06	-5.7660370367D-08
5.5	0.738615D-4	2.8508780000D-04	1.3426086373D-06	-2.8298192567D-08
6.0	0.791987D-4	4.1747570000D-04	9.9075178761D-07	-2.2527173571D-08

Table 9. Error in numerical integrators first component on Example 7.

h	S = 2	Aashikpelokhai (1991)	Fatunla (1980)	Fatunla (1978)	Norelli (1985)	N_I
0.0125	1.8695066367D+00	1.8694389	1.8694388	1.8694389	N = 5:1. 8692929	80
0.0250	1.8660314693D+00	1.8694387	1.8694389	1.8694387	N = 5:1.8691415	40
0.0500	1.8636132555D+00	1.8694846	1.8694380	1.8694357	N = 5:1.8688354	20
0.1000	1.8712469008D+00	1.8695057	1.8705973	1.8693953	N = 5:1.8682097	10

Table 10. Error in numerical integrators second component on Example 7.

h	S = 2	Aashikpelokhai (1991)	Fatunla (1980)	Fatunla (1978)	Norelli (1985)	N_I
0.0125	-1.4822680915D-01	-0.14823588	-0.14823588	-0.14823587	N = 5:-0.1495268	80
0.0250	-1.4870613284D-01	-0.14823589	-0.14823587	-0.14823589	N = 5:-0.1497309	40
0.0500	-1.4904503196D-01	-0.14822960	-0.14823599	-0.14823631	N = 5:-0.1501113	20
0.1000	-1.4735968362D-01	-0.14822671	-0.14610294	-0.14824187	N = 5:-0.1507630	10

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