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*Full Length Research Paper*

# An approach with distance-angle based search algorithm in route design for minimizing earthwork

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**The two dimensional (2D) planimetric view of the axis of the transportation modals on the ground is called route. The process of the connecting two given points (start and end points of the route) generates various alternative paths based on geometric standards of the design of a route, in particular based on an appropriate longitudinal grade of vertical profile. In this article, a method that connects two given points is presented for corridor planning with resulting route elements based on a parameter estimation of geometric modeling. In this approach, the methodology offers an automatic connecting algorithm of two given points due to azimuth angle of the searched points, step interval and horizontal coordinate differences to draw the zero line, which helps to perform cut fill balance due to minimizing earthwork cost. By running the algorithm, the intermediate points between the two given points are obtained as polygon. The geometric elements of the result route, which is like a polygon route, are generated by parameter estimation, firstly, by applying polynomial regression and then fitting the route element estimation process. The feasibility and validity of the proposed method was shown in a case study with two different data sources, which also includes a processing stage for evaluating the horizontal alignment geometric elements. The results showed that the proposed methodology could be used as a guide, which helped to minimize the total earthwork balance in route planning facilities.**

**Key words:** Route planning, earthwork cost, horizontal geometry, vertical geometry, optimization.

## INTRODUCTION

In route planning studies, to connect two given (3 dimensional coordinates of the start and end points of route) points with desired route geometric features, several alternative paths are generated. In traditional transportation design solutions, engineers start by selecting several candidate corridors through given datasets, and then they select most suitable one depend on their experiences. Moreover, manually designed solution may be short of an optimal solution (Jong et al., 2000). The route alignment optimization involves finding the best highway alternative between a pair of points (Jong et al., 2000; Jha, 2003; Jong and Schonfeld, 2003; Kim et al., 2004; Jha and Schonfeld, 2004) that is stated in Kim et al. (2005) as given two end points in the study area and allowing the existing conditions of the study area changeable, find the best alignment among alternatives to optimize a specified objective function, while considering needed structures and satisfying design and operational requirements.

Several optimization techniques have been developed and implemented on both horizontal and vertical or simultaneously for horizontal and vertical alignment design emphasized (Kim et al., 2005). The models for optimizing horizontal alignment are more complex and require substantially more data than those for optimizing vertical alignment (OECD, 1973). In the literature, the models for horizontal alignment optimization can be listed as calculus of variations (Wan, 1995; Howard et al., 1968; Thomson and Sykes, 1988; Shaw and Howard, 1981, 1982), network optimization (OECD, 1973; Turner and Miles, 1971; Athanassoulis and Calogero, 1973; Parker, 1977; Trietsch, 1987a, b) and dynamic programming (Hogan, 1973; Nicholson et al., 1976). Dynamic programming has been used in several studies for optimizing vertical alignment to determine the vertical highway alignment with minimum earthwork cost (Puy Huarte, 1973; Murchland, 1973; Goh et al., 1988). Linear programming has also been used to select roadway

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grades that minimize the cost of earthworks (Easa, 1988; Chapra and Canale, 1988). Chew et al. (1989) proposed solving a 3-dimensional (3D) highway route selection problem, where the optimal horizontal and vertical alignments are determined simultaneously. The last developments on route planning are introduced by genetic algorithms with 3 dimensional alignment definition integrated by geographic information systems (GIS) (Jong, 1998; Jha, 2000; Kim, 2001).

In this study, an approach has been developed and represented for route planning due to the topographic features of the study area. The proposed method automates connecting of the two given points in horizontal plan due to vertical alignment longitudinal grade condition, which helps to minimize earthwork cost. Step interval, azimuth angle and coordinate differences of the points are the decision variables of the path to reach the end point. By running the algorithm, the intermediate points between the two given points are obtained as polygon. The geometric elements of the resulting route are generated by parameter estimation, firstly, by applying polynomial regression and then fitting the route element estimation process.

## MATERIALS AND METHODS

### Search with angle and distance on circles

In this study, a model that selects intermediate points of the horizontal route due to gradient and gradient changes of the points obtained from vertical surface that minimize earthwork cost while satisfying the vertical geometric specifications, is presented. The model includes two stages: (1) determine the intermediate point's locations with the decision variables namely, gradient and gradient changes, (2) approximate the final alignment obtained in stage 1 by a set of polynomial functions for smoothed alignment estimation.

The main idea of this proposed method employed in stage 1 is to generate a search space with step interval and direction selection to reach the given end point of the alignment by successive iterations in which intermediate points of the route are selected automatically. Software with graphical user interface has been developed to perform evaluations using Matlab R2007b. The preferred data format employed in this software is grid data format because of its easy computational structure. Accordingly, high-resolution field data selected from the region of interest are modeled by a chosen interpolation method, and converted into grid data format with appropriate grid intervals. In the proposed method that connect two given points with constraints that minimize the earthwork cost, selecting a search direction limit that is altered by the location of the intermediate points, is the beginning part of stage 1. In the first iteration, the start point coordinates and end point coordinates are initially used for selecting the next intermediate point which may be a feasible route point. Then, in the second and following iterations, the found intermediate point that will change in each iteration and end point are employed to the searching process repeatedly until the alignment reaches the end point of the route. This search for nodes is applied by step intervals over a half or quarter circle, and the direction of the circle is altered due to the location of the points. Therefore, on each node, a circle is defined depending on the given conditions. The direction selection is then performed by calculation of the azimuth angle between the start and following points (or between successive points) that may direct the search circle to the desired end point. Not only the azimuth

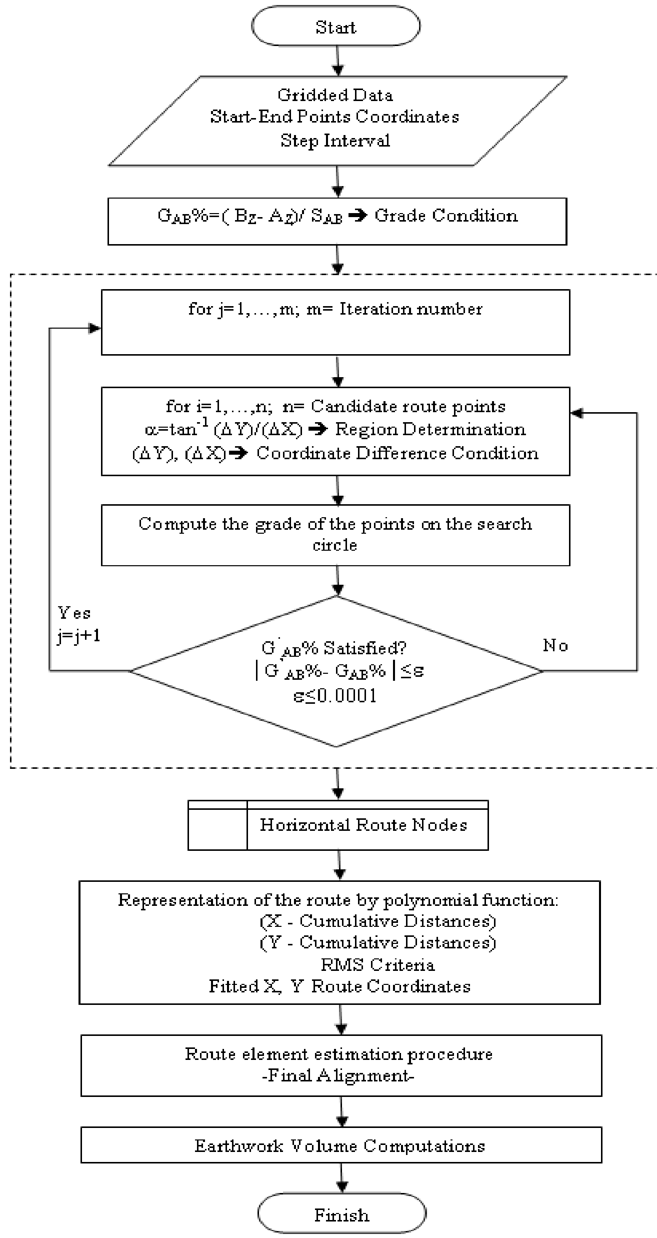



**Figure 1.** Flowchart of the proposed model.

angle but also horizontal coordinate differences are also required to avoid the backtracking alignment sections along the route. Figure 1 represents the running algorithm of the model.

As illustrated in Figure 2, the search circle is defined by the lower andupper limits, in which the structural behavior of the searching process is represented. Another factor for definition of the search circle is the step interval, which is also the radius of the mentioned circle. To select the step interval depending on the grid interval, which is applied in the gridding process by interpolation method, gives more satisfactory solutions for gradient calculations. As noted that the start point three-dimensional coordinates are known, the elevations of the end point of the lines on the circle are interpolated from vertical surface. Accordingly, the gradient and gradient changes between these two points are calculated for choosing the appropriate nodes due to desired conditions. Therefore, if the step interval is chosen bigger than the grid interval for

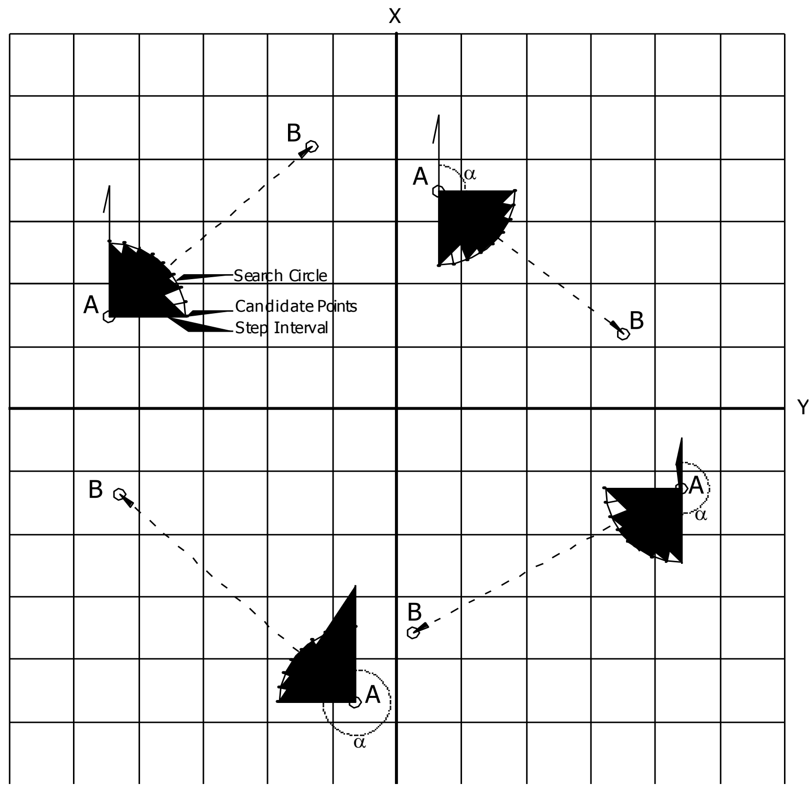



**Figure 2.** Representation of the lower and upper search limits of the program.

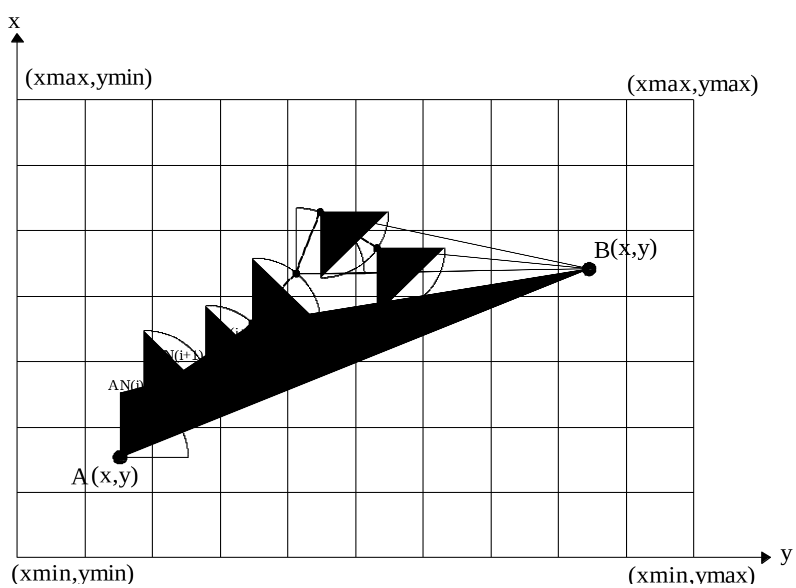



**Figure 3.** Illustration of the proposed model.

determining the next intermediate point location, the calculated gradient values give more reliable solutions while the processing time of the algorithm shortens. Since each step interval is a constant value at each iteration, determination of the next point, whether appropriate node for route analysis, is based on the constant gradient principle that minimize the earthwork cost. If the alignment follows these intermediate points found via topographic features of the field by means of the constant gradient condition, the resulting alignment will minimize the earthwork cost due to the longitudinal grade of the profile.

The illustration of the model can be seen in Figure 3. As seen in the figure, the longitudinal grade value computed from the start and end points coordinates, are involved as constant variable into the developed software and through the defined lower and upper limits of the search circle on each node, the grade change constraint is searched by moving on this circle segment to capture the appropriate value. The algorithm captures the most appropriate node as intermediate point for the alignment, and skips the next iteration. The developed software requires two different data sources: (1) field data, which should be interpolated by selected interpolation method in grid data format (2) start and end points of the alignment with their three-dimensional coordinates.

In the developed software, an alternating search circle depending on the location of the intermediate points is achieved in each iteration. This computed circle direction is performed by the evaluation of the azimuth angle of the two points. The definition of the circle direction is to provide backtrackings along the horizontal alignment. Here, it is required to determine the tending direction of the route. The tending direction is specified by the horizontal coordinate differences of the start and end points in two-dimensional system.

For notational compatibility, in the expression, A (xa, ya, za) and B (xb, yb, zb) denote start and end points with their three dimensional coordinates, respectively. $A_x, A_y, A_z$ are the coordinates of the start point and $B_x, B_y, B_z$ are the coordinates of the end point. Iterative search approach for stage 1 is started by satisfying the calculation steps and conditions as follows.

***Step: Determination of the first direction selection***

**Calculation 1:** Compute the azimuth angle between A and B points, respectively, for region definition on the coordinate axis:

$$\alpha_{AB} = a\tan\left(\frac{By - Ay}{Bx - Ax}\right) \tag{1}$$

**Calculation 2:** Determination with coordinate differences:

$$\begin{aligned} \Delta y &= |By - Ay| \\ \Delta x &= |Bx - Ax| \end{aligned} \tag{2}$$

**Calculation 3:** Determination with the location of the A and B points:

$$S_{AB0} = ((A_x - B_x)^2 + (A_y - B_y))^{0.5} \tag{3}$$

$$S_{AB} = \eta * S_{AB0} \tag{4}$$

$$G_{AB}\% = (B_z - A_z)/S_{AB} \tag{5}$$

It should be noted that due to the prior experiences obtained from trials of the algorithm, it is seen that the distance between A and B points, namely $S_{AB0}$, calculated by the horizontal coordinates of A and B, which is used in evaluation of the longitudinal grade to connect these two points on the third dimension (profile generation), is not sufficient for the iterative searching. Because the route is formed by the several straight lines, so the needed gradient is greater than the theoretical gradient value. Therefore, an iteratively found coefficient, namely $\eta$ greater than 1, for calibrating this difference is multiplied by the $S_{AB0}$. Here, IP is the abbreviation of the intermediate point for each iteration.

**Criteria:** To start iterative search:

Condition 1: For $0^g < \alpha_{AB} < 100^g$, let i=1:101 and angle = [0:1:100];

$$IP(x(i)) = Ax + s \times \cos((angle(i)) \tag{6a}$$

$$IP(y(i)) = Ay + s \times \sin((angle(i)) \tag{6b}$$

Condition 2: For $100^g < \alpha_{AB} < 200^g$, let i=1:101 and angle = [0:1:100];

$$IP(x(i)) = Ax + s \times \cos((angle(i) + 100^g) \tag{7a}$$

$$IP(y(i)) = Ay + s \times \sin((angle(i) + 100^g) \tag{7b}$$

Condition 3: For $200^g > \alpha_{AB} > 300^g$, $|Ay - By| > |Ax - Bx|$, let i=1:101 and angle1= [100:1:200];

$$IP(x(i)) = Ax + s \times \cos((angle1(i) + 100^g) \tag{8a}$$

$$IP(y(i)) = Ay + s \times \sin((angle1(i) + 100^g) \tag{8b}$$

Condition 4: For $200^g > \alpha_{AB} > 300^g$, $|Ay - By| < |Ax - Bx|$, let i=1:101 and angle1= [100:1:200];

$$IP(x(i)) = Ax + s \times \cos((angle1(i)) \tag{9a}$$

$$IP(y(i)) = Ay + s \times \sin((angle1(i)) \tag{9b}$$

Condition 5: For $300^g > \alpha_{AB} > 400^g$, let i=1:101 and angle= [0:1:100];

$$IP(x(i)) = Ax + s \times \cos((angle(i) + 300^g) \tag{10a}$$

$$IP(y(i)) = Ay + s \times \sin((angle(i) + 300^g) \tag{10b}$$

Figure 4 shows the backtracking point, which occurs without using coordinate differences criteria. The searching process ends up when the last intermediate point reaches to the end point (B) in a distance, which should be chosen as an approach value. This approach distance shown in Figure 5a, denoted as d, should be added to the developed software to prevent missing the end point. Because the step interval at each iteration is constant and in the last iteration capturing the point B is not always achieved, exactly. Figure 5a and b show the running algorithm without stopping criteria and with stopping criteria cases, respectively.

### Geometric definition of the alignment

#### *Polynomial fitting*

The obtained alignment from the model expressed on the previously is constituted by the consecutive straight lines linked by the intermediate points assuming that they are a set of feasible nodes along the horizontal alignment. Horizontal alignment is composed of straight lines, circular elements and transition curves. However, the obtained horizontal alignment may not represent a satisfied result. For the fact

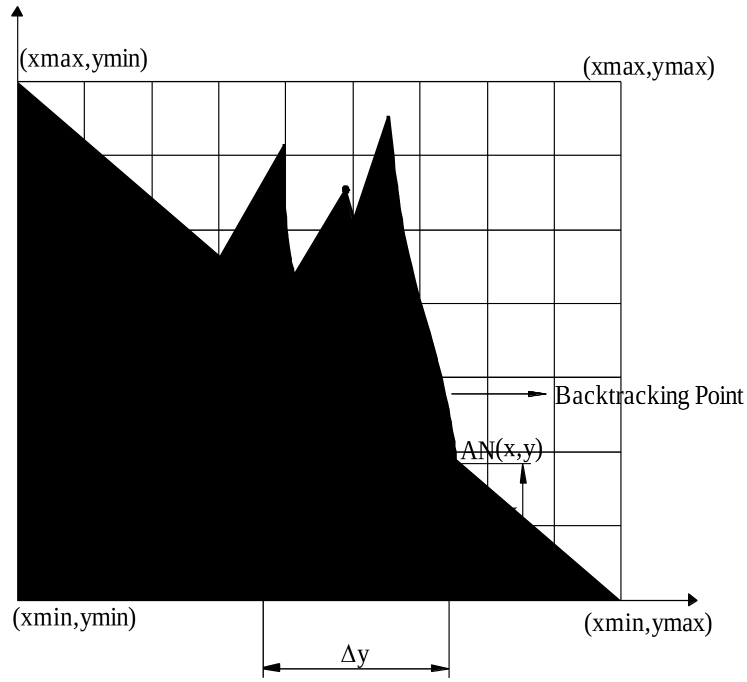



**Figure 4.** Backtracking of the alignment.

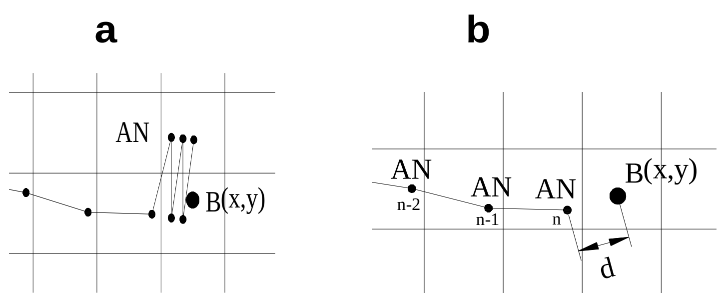



**Figure 5.** (a) Without stopping criteria (b) with stopping criteria.

that it may not always represent a horizontal alignment, the vertical alignment is sufficient. So, a polynomial function is employed on the horizontal alignment between the consecutive nodes into three stages: (1) implement polynomial function between cumulative distance to X coordinate (2) implement polynomial function between cumulative distance to Y coordinate (3) represent the horizontal alignment by means of fitted X and Y coordinates obtained from stage 1 and 2.

The general equation of the polynomial function can be seen in Equation 11a and b;

$$f(s,x) = a_0 + a_1 x + a_2 x^2 + \dots\dots + a_{n-1} x^{n-1} + a_n x^n \quad (11a)$$

$$f(s,y) = b_0 + b_1 y + b_2 y^2 + \dots\dots + b_{n-1} y^{n-1} + b_n y^n \quad (11b)$$

Equation 11 coefficients can be solved by the least squares method (LSM), as follows; assume that the interested polynomial regression model is as given in Equation 12.

$$o = A\beta + e \quad (12)$$

Here, o=n×1 is referred to as vector of observation; A=n×u is the coefficient matrix; β=vector of unknown parameters; e= vector of normally distributed random errors with mean zero; n= number of observation and u=number of unknown parameters. The general form of LSM is $\hat{\boldsymbol{\beta}} = (\mathbf{X}^{\mathbf{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{o}$. The residual vector can be obtained by $\mathbf{v} = \mathbf{o} - \mathbf{X}\hat{\boldsymbol{\beta}}$. The root mean squares (RMS) values can be evaluated by Equation 13;

$$RMS = \sqrt{\frac{vv}{n}} \quad (13)$$

***Route element estimation***

The aforementioned process provides a smooth alignment for the obtained successive route points. To convert the smooth alignment into the final alignment, a route element estimation procedure is implemented. That is, the horizontal route definition achieved in two stages: (1) straight line definition, (2) circular curve definition. To fit the straight lines and circular curves, the route data points are separated into the related sections. Thus, in stage 1, the deflection angles (Equation 15) from azimuths (Equation 14) of each consecutive point are calculated to figure out the tangent sections of the routes. The differences (Equation 16) between the consecutive deflection angles are taken into consideration for the procedure of related sections points' selections. Since the absolute differences of the deflection angles are equal to zero within some tolerance, the points up to here are included in straight-line sections. After the determination of the tangent section points, namely straight line-section, the least-squares method for linear regression is applied. For i=1,…, n; n=candidate points of the route on search circle:

$$\alpha_i = \tan^{-1}((y_{i+1} - y_i)/(x_{i+1} - x_i)) \tag{14}$$

$$\Delta_i = \alpha_i - \alpha_{i+1} \tag{15}$$

$$\left|\Delta_i - \Delta_{i+1}\right| \leq \varepsilon \tag{16}$$

In stage 2 for horizontal route definition, the remaining points between two straight lines, determined in stage 1, are evaluated for circular curve fitting procedure. For the circular curve sections, the remaining points between two straight lines are fitted using Newton's method with an iterative process by solving non linear equations. The equation of a circle is as given in Equation 17:

$$R^2 = (xi - xm)^2 + (yi - ym)^2 \tag{17}$$

where (xm, ym) are the coordinates of the centre of the circle and R is the radius of the circle.

$$\begin{bmatrix} x_i & y_i & 1 \\ x_{i+1} & y_{i+1} & 1 \\ x_{i+2} & y_{i+2} & 1 \\ .... & ........ & ... \\ x_n & y_n & 1 \end{bmatrix} * \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -x_i^2 - y_i^2 \\ -x_{i+1}^2 - y_{i+1}^2 \\ -x_{i+2}^2 - y_{i+2}^2 \\ ................ \\ -x_n^2 - y_n^2 \end{bmatrix} \tag{18}$$

where a, b and c is denoted as:

$$\begin{aligned} a &= 2 \times xm \\ b &= 2 \times ym \\ c &= xm^2 + ym^2 - R^2 \end{aligned} \tag{19}$$

## RESULTS AND DISCUSSION

### Evaluating the route points

A case study was performed to achieve the alignment optimization by using proposed approach, and to represent the resulting alignments with their geometric design elements. Two types of data were used in this study, including cartographic digitized sheet map and LIDAR dataset. Figure 6 shows the contour map of the regions of interest with given start and end points, denoted as A and B, respectively, and statistical summaries about related regions are given in Table 1. Figure 6a and b shows the data produced by the cartographic digitized sheet map and LIDAR data, respectively. The Kriging interpolation method was used for both datasets and converted into gridded data format to obtain inputs as the gridded data. The data shown in Figure 6a produced from 1/5000 scaled sheet map by point-to-point digitizing. Although, the smoothed contour levels were obtained due to the digitizing process for cartographic digitization in Figure 6a, rough contours were obtained from LIDAR dataset. To overcome this difficulty, some processes may be implemented on LIDAR data e.g. filtering procedure on raw data to obtain smooth surface, data reduction before interpolation and selecting large grid intervals. In this study, large grid interval selection and smoothed contour levels were applied to the LIDAR data set.

The method has been implemented in Matlab software to obtain the advantages of the matrix calculation facilities. In order to make calculations, the field data is stored in three matrices as X, Y and Z data, and given points three-dimensional coordinates are stored in two matrices. In each iteration, the candidate point's information are stored and checked with the desired grade condition. If the appropriate point is found in any iteration, developed software takes it as intermediate point of the route and goes through the next iteration unless it finds the last point by the help of stopping criteria. Once this algorithm for all points in each iteration is realized, the coordinates of the corresponding points of the route are placed in a matrix. The elevations of the route points are also stored for subsequent evaluations. The resulting alignments of the routes can be seen in Figure 7 obtained from running the algorithm.

### Representation of route by polynomial fitting

The horizontal route involves straight lines, circular curves and transition curves. The points, whose horizontal coordinates have been determined after running the proposed algorithm, may not always follow an exact path which expresses a smooth route with geometric elements. Search circle angle intervals and the changed topographic features along the route have been affecting the resulting route. If the search circle has more candidate points by dividing more intervals than proposed intervals on circle, it may give smoother route. However, this process requires more data storage capacity. In order to overcome this problem, polynomial function has been chosen to define the centerline of the route. The fitting procedure is implemented on X coordinates to distance from start point and Y coordinates the distance from start point, respectively. The selection of the polynomial degree was done by evaluating the minimum

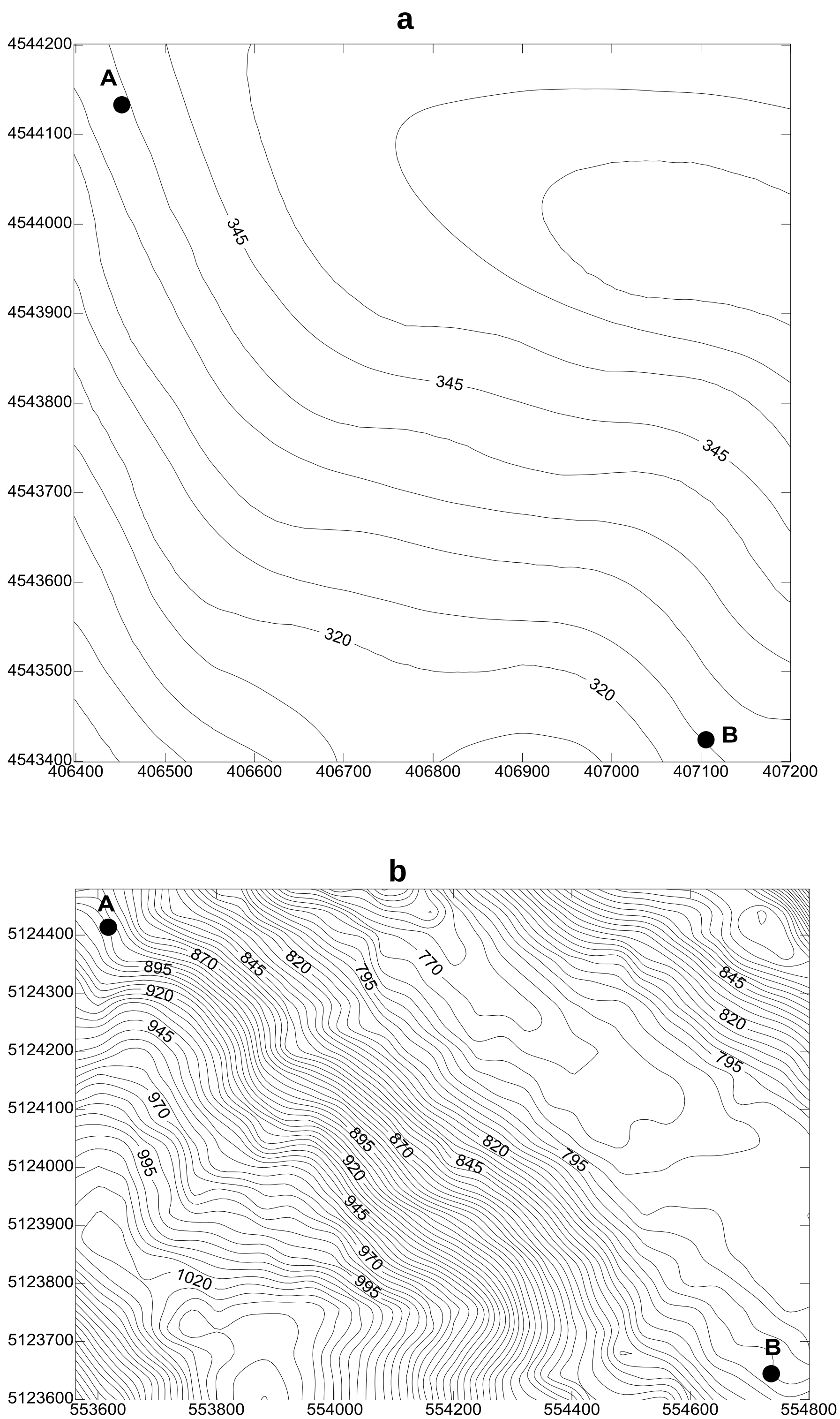



**Figure 6**. Representation of the regions of interest with contour maps. (a) RI_1 (Region of Interest-1), (b) RI_2 (Region of Interest-2).

with values of the sum of the residual errors with an upper degree limitation, which was 30 in this study to avoid ondulations along the route. Here, the observation weights are equal to each other and weights are 1. For RI_1, the order of the polynomial functions were found as 20 for both X-distance and Y-distance evaluations,

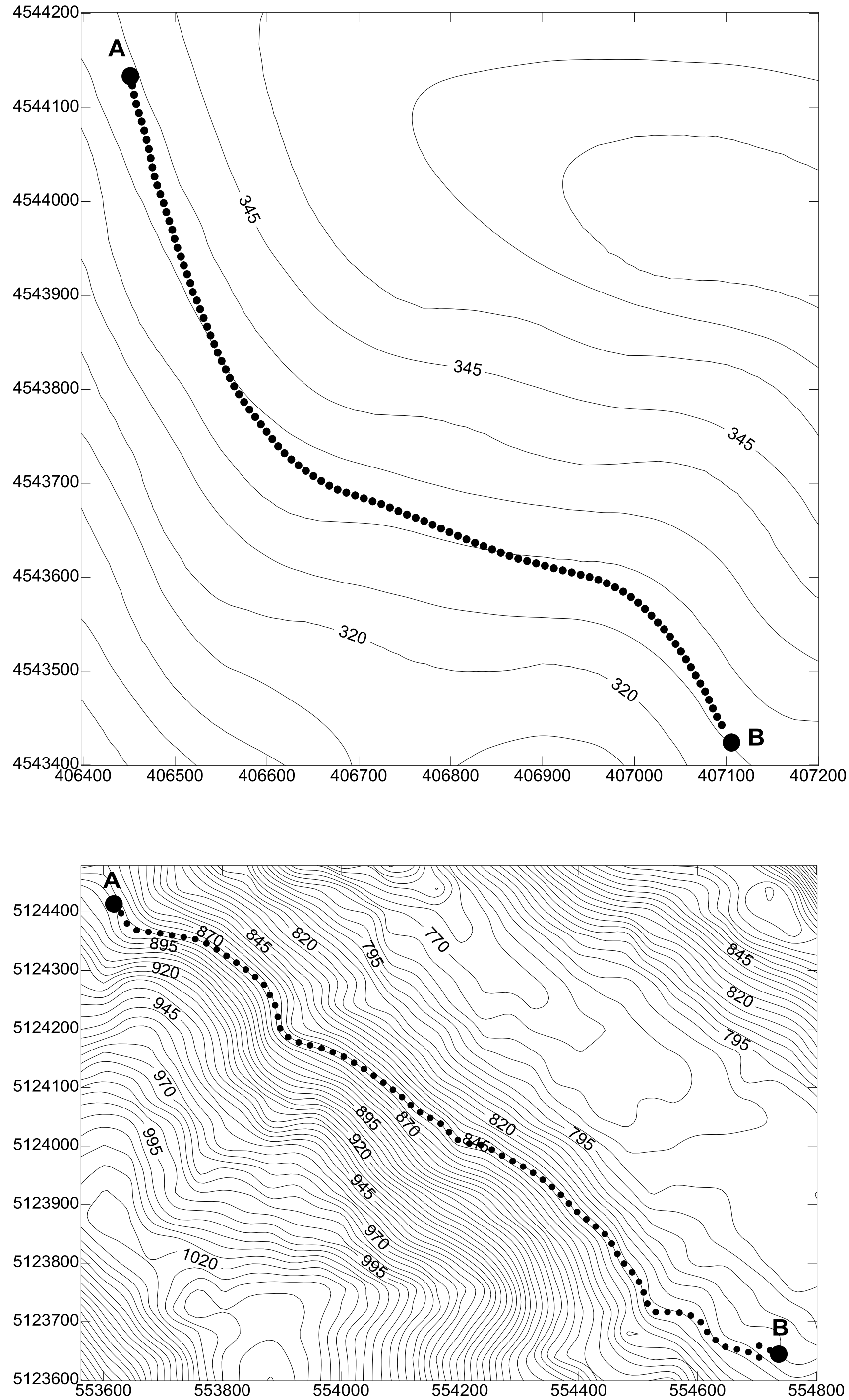



**Figure 7.** Obtained route points for RI_1 and RI_2, respectively.

0.2791 and 0.3127 m root mean squares (RMS) values, respectively. The degree of polynomial functions for RI_2 were also found 20 in respect to the X-distance and Y-distance evaluations with 1.7969 m and 2.8537 m root mean squares (RMS) values, respectively. Figure 8 shows the implementation of the polynomial functions for each dataset.

## Route element estimation procedure

After obtaining the fitted X and Y coordinates of the route points, the final alignment geometric estimation procedure are applied as explained in the route element estimation section. Figure 9 shows the final alignments with contour maps (a1 and a2) and perspective views (b1

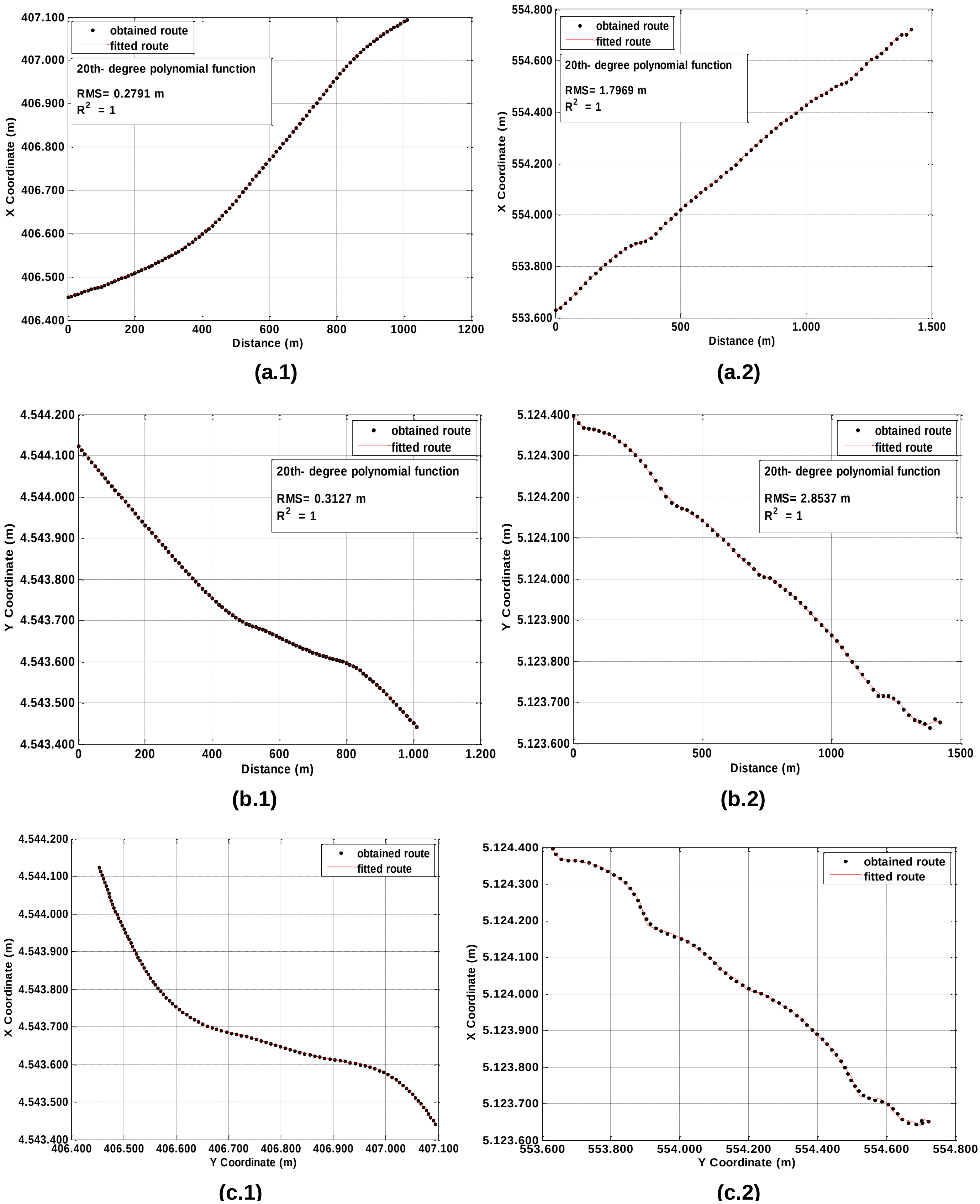



**Figure 8.** Polynomial function implementation: (a.1) and (a.2) for X to cumulative distance, (b.1) and (b.2) for Y to cumulative distance, (c.1) and (c.2) fitted X and Y coordinates, referring 1 to RI_1 and 2 to RI_2.

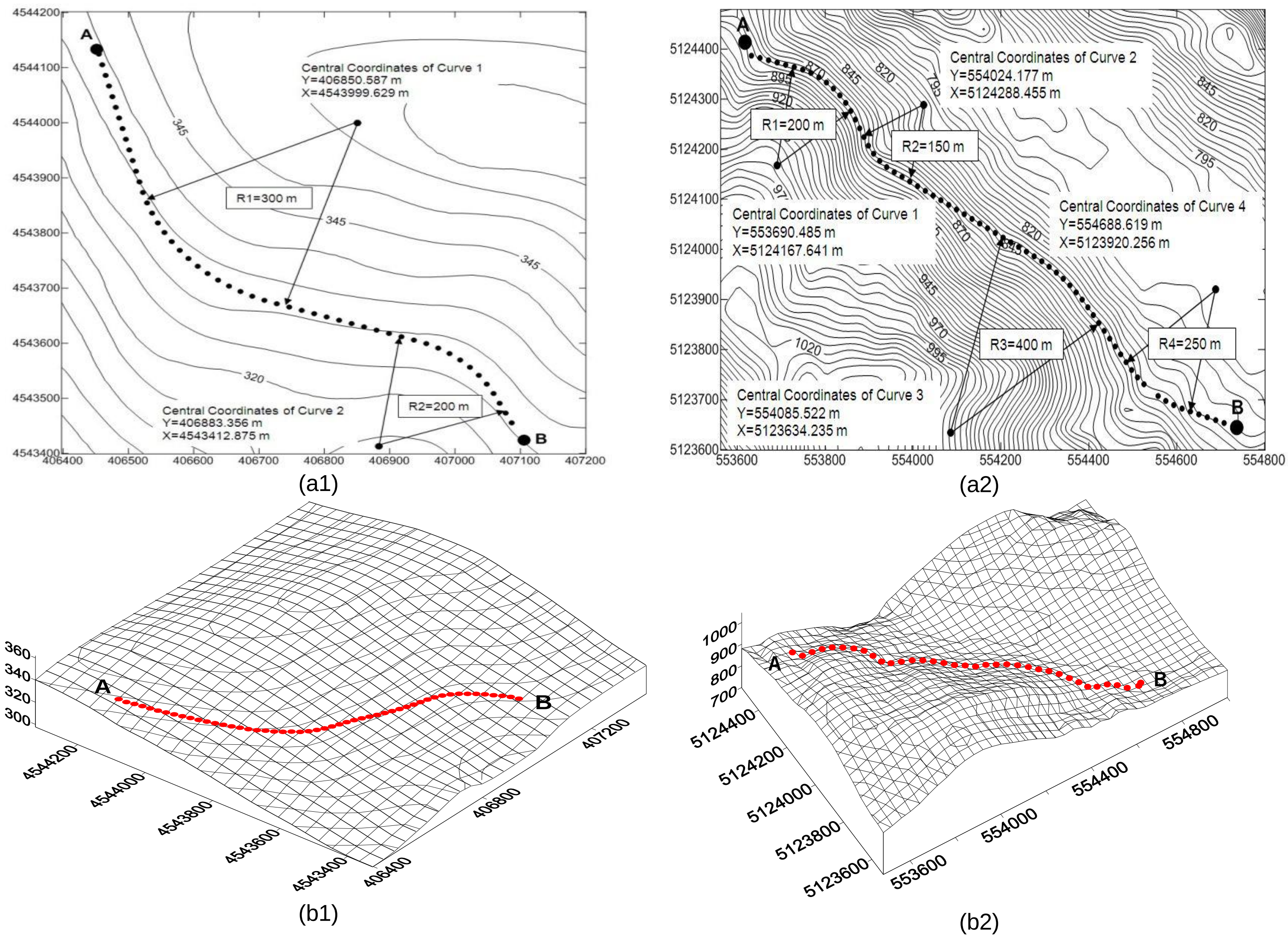



**Figure 9.** Horizontal route defined by route element estimation procedure (a.1) & (a.2) for contour map views, (b.1) & (b.2) for perspective views,

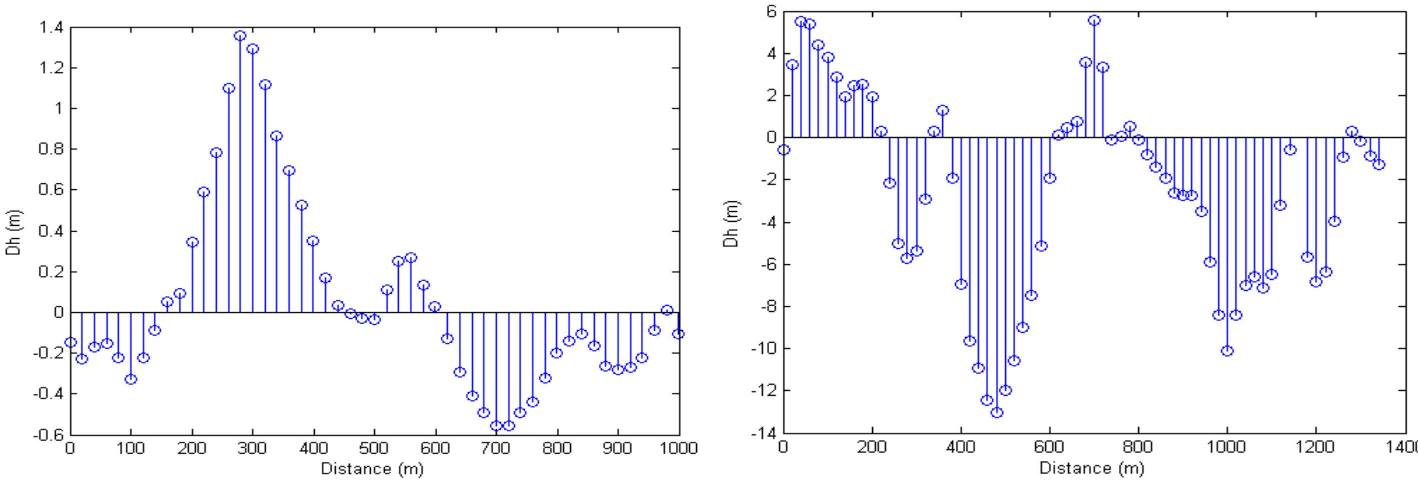
and b2). Here, 1 and 2 refer to RI_1 and RI_2, respectively. The fitted curve radius and the central coordinates of the curves obtained by the geometric element definition from the study areas were given for each region in Figure 9. The result alignments obtained by this procedure give satisfactory solution in terms of geometric design parameter and earthwork volumes. The curve radius are found, 300 and 200 m for first region, and 200, 150, 400 and 250 m for second region.

To check the validity of the proposed methodology, the drifting values between grade line and ground line were computed. The differences between grade line and the ground line were given in Figure 10. In Figure 10, the grade line represents the elevations found via longitudinal grade condition, and ground line indicates the horizontal route elevations of the field data. The maximum, minimum and mean differences for the first and second data groups between grade line and ground line were 1.36, -0.56, 0.06 m and 5.53, -13.04, -2.66 m, respectively. The differences for the second dataset were relatively higher than the first dataset because of the topographic feature differences of two regions.

Furthermore, the cumulative volumetric evaluations of routes for two regions were performed with different platform lengths in which the wide were ranged from 10 to 40 m longs with a 10 m increase. The proposed method finds the route centerline axis, thus a volumetric evaluation may give an idea about the topographic feature of the buffer of the route. The comparison for volumetric computation can be seen in Table 2.

## Conclusion

In this article, a fast and effective model for determining the horizontal and vertical alignment points of a route for




(a) (b)

**Figure 10.** Differennt graphics; referring (a) to RI_1 and (b) to RI_2.

minimizing the earthwork cost has been developed. The method is based on an iterative successive route point selection by implementing an approach and ulterior processing of this information. The points obtained by running the algorithm, which are corresponding to a path, can be addressed as a guide for designers. By considering the longitudinal grade of the route as a grade condition, the compatibility of the candidate points in each iteration are controlled and the most appropriate candidate points are taken as intermediate point of the route and, this process goes on unless capturing the last point with an approach distance.

The feasibility and validity of the proposed method was shown in a case study with two different data sources, which also included a processing stage for evaluating the horizontal alignment geometric elements. The volumetric computation was also performed for controlling the vertical alignment validity. The illustrated software algorithm is affected by the topographic features of the region of interest. Although, the first data group shown in this case study, namely RI_1, has a smooth hillside landform and the differences between grade line and ground line vary from 1.38 to -0.57 m, the second data group has a steep-rugged hillside, and the differences vary from 5.53 to -13.04 m. As shown in Table 2, volumetric evaluations were performed for different platform wide, which varied from 10 to 40 m with surface areas and volumes for cut and fill sections. The cut and fill balance approximation were satisfied for the first dataset more than the second dataset. Although the cumulative volumes for first region vary from 643.47 to 1988.58 $m^3$ for minimum and maximum platform wide, the cumulative volumes for second region vary from 36273.97 to 141888.70 $m^3$ for minimum and maximum. The data source is also a factor for affecting the resulting accuracy. So, high-resolution field data and smooth surface usage should also be recommended for implementing proposed methodology. Finally, this searching algorithm may allow an accurate determination of route points that may be used as a guide for route planning by additional evaluations to represent a real alignment after automatic evaluation.

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