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Relativistic causality versus superluminal communication: Is the quantum mechanics a semi-empirical theory?

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The work analyzes the compatibility between the classical freedom, the local relativistic causality and the non-local behavior of quantum mechanics in the frame of the stochastic approach of the quantum hydrodynamic analogy (SQHA). The work describes the role of the quantum potential in generating the quantum non-local dynamics in a fluctuating environment. The analysis shows that it is possible to maintain the concept of classical freedom between far away weakly bounded systems (moderate non-locality) as well as to make compatible the uncertainty principle with the relativistic postulate of invariance of light speed. The work shows that the paradox of instantaneous quantum non local behavior at infinite distances of the standard formalism is an artifact due to the non-relativistic non-stochastic ambit of such theory where the light speed is infinite and the non-local interaction owns an infinite range of action. The work envisages that the SQHA can possibly lead to a fully theoretically self-standing quantum mechanics where the wave function collapse, during a measurement process, can be described by the theory itself without empirical postulates. Under this light the paper discusses the need of searching for (both local and non-local) hidden variables quantum mechanics as well as the need of superluminal communications in quantum experiments. The analysis shows that all these hypotheses are attempts of interpreting the outputs of quantum measurements that cannot be fully explained by the semi-empirical formalism of quantum mechanics, based on the statistical postulates of the measuring process as well as the existence of a classical observer. A two photon experiment is discussed to the light of the SQHA approach.

Key words: Quantum non-locality, superluminal transmission of quantum information, classical freedom, local relativistic causality, Einstein, Podolsky, and Rosen (EPR) paradox, macroscopic quantum decoherence, Bell's inequalities, quantum hydrodynamic analogy.

INTRODUCTION

The conflict between the quantum non-locality and the local character of the classical macroscopic experience is one of the most intriguing problems of the modern physics (Schrödinger, 1935; Einstein et al., 1935; Bell, 1964; Greenberger et al., 1990).

This fact has lead to many logical paradoxes that contrast with our sense of reality (Schrödinger, 1935; Einstein et al., 1935; Bell, 1964). The most known quantitative tentative to investigate the problem is given by Bell (1964) in response to the so called EPR paradox (Einstein et al., 1935) a critical analysis of the quantum non-locality respect to the notion of macroscopic classical freedom and local relativistic causality.

The central point of the problem is the leaking of the theoretical connection between the quantum mechanics and the classical one that would explain how the laws of physics pass from the quantum behavior to the classical one. The disconnection between the two theories leaves open the question about the hierarchy between them. The quantum mechanics, on the base of its semi-empirical statistical approach, needs the classical mechanics (that is, the classical observer) to be defined, while the quantum one seems to be the basic one from which the classical mechanics can stem out in the macroscopic limit where $\hbar$ tends to zero (Bialyniki-Birula et al., 1992).

One current of thought is represented by the "deterministic" approach to quantum mechanics that analyzes how the quantum equations are the generalization of the classical one (Bialyniki-Birula et al., 1992; Bohm, 1952; Madelung, 1926; Jánossy, 1962; Jánossy, 1962; Wyatt, 2005; Nelson, 1967, 1985; Guerra and Ruggero, 1973; Parisi and Wu, 1981) where the non-locality is introduced in various ways, the Madelung quantum potential (Bialyniki-Birula et al., 1992; Madelung, 1926; Jánossy, 1962), the Nelson's osmotic potential, the Bohm-Hylei quantum potential or the Paris and Wu fifth-time parameter.

A great help in explaining the origin of the non-locality of quantum mechanics comes from the QHA equations (Bialyniki-Birula et al., 1992; Madelung, 1926; Jánossy, 1962) that shows how the non-local restrictions come in the playing from the quantization of vortexes (Bialyniki-Birula et al., 1992) and by the elastic-like energy arising by the quantum pseudo-potential. On the contrary, the Schrödinger equation is a differential equation where the non-local character of evolution is determined by the initial and boundary conditions that must be defined for describing a physical problem and that are apart from the equation.

In the case of charged particles, the non-local properties of the Schrödinger equation come also from the presence of the electromagnetic (EM) potentials that depend by the intensities of EM fields in a non-local way (e.g., Aharonov –Bohm effect) (Wyatt, 2005). In the corresponding hydrodynamic equations (Bialyniki-Birula et al., 1992) the EM potentials do not appear but only in local way through the strength of the EM fields. The mathematically more clear origin of the quantum restrictions in the QHA make it suitable for the achievement of the connection between quantum concepts (probabilities) and classical ones (e.g., trajectories) (Wyatt, 2005) helping in overcoming the contrast between the quantum non-local behavior and our classical sense of reality.

The deterministic approach of the QHA and similar theories gains interest in the physics community due to the fact that it helps in explaining quantum phenomena that cannot be easily described by the usual formalism. They are multiple tunneling (Jona et al., 1981), critical phenomena at zero temperature (Ruggiero and Zannetti, 1981), mesoscopic physics (Ruggiero and Zannetti, 1983a, b; Chiarelli, 2013a), numerical solution of the time-dependent Schrödinger equation (Weiner and Askar, 1971; Weiner and Forman, 1974; Terlecki et al., 1982), quantum dispersive phenomena in semiconductors (Gardner, 1994), quantum field theoretical regularization procedure (Breit et al., 1984) and the quantization of Gauge fields, without gauge fixing and without ensuing the Faddeev-Popov ghost (Zwanziger, 1984).

On the theoretical point of view, one of the most promising aspect of this model is helping in investigating the quantum mechanical problems using efficient mathematical technique such as the stochastic calculus, the numerical approach and the supersymmetry.

A more recent and sophisticated approach is given by t'Hooft (1988, 1996, 1999). He proposes the obtaining of the quantum mechanics through a process of loss of information by using outputs coming from the black-hole thermodynamics and by the so called holographic principle (Susskind et al., 1993; Bousso, 2002).

A parallel current of thought, investigates the possibility of obtaining the classical state through the loss of quantum coherence in classically chaotic systems due to the presence of stochastic fluctuations (Cerruti et al., 2000; Calzetta and Hu, 1995; Wang et al., 2008; Lombardo and Villar, 2005; Mariano et al., 2001). In this case, most of the results are obtained by numerical and semi-empirical approaches, leaking of global theoretical view.

The present paper investigates the non-local property of quantum mechanics and its decoherence as a consequence of fluctuations by using the QHA (Madelung, 1926; Jánossy, 1962; Wyatt, 2005) implemented with the stochastic calculus. This strategy is supported by the advantage of the QHA in managing the non-local quantum dynamics in system larger than a single atom when fluctuations becomes important (Bousquet et al., 2001; Morato and Ugolini, 2011; Chiarelli, 2013b) and by its completeness respect to the Bohmian mechanics (Chiarelli, 2012; Bohm and Vigier, 1954).

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Stochastic generalization of the quantum hydrodynamic analogy

The QHA-equations are based on the fact that the Schrödinger equation, applied to a wave function $\psi_{(q,t)} = |\psi|_{(q,t)} \; exp\,[\frac{i}{\hbar} S_{(q,t)}]$, is equivalent to the motion of a particle density $n_{(q,t)} = |\psi|^2_{(q,t)}$ with velocity $\dot{q} = \frac{\nabla S_{(q,t)}}{m}$ (Bialyniki-Birula et al., 1992). In presence of stochastic noise $\eta_{(q,t,T)}$, that for the sufficiently general case, to be of practical interest, can be assumed Gaussian with null correlation time, the stochastic partial differential conservation equation for $n_{(q,t)}$ reads (Chiarelli, 2013b):

$$\partial_t n_{(q,t)} = -\nabla \bullet (n_{(q,t)} \dot{q}) + \eta_{(q,t,T)} \qquad (1)$$

$$< \eta_{(q_\alpha,t)}, \eta_{(q_\beta+\lambda,t+\tau)} > = < \eta_{(q_\alpha)}, \eta_{(q_\beta)} > G(\lambda)\delta(\tau)\delta_{\alpha\beta} \qquad (2)$$

$$\dot{p} = -\nabla (V_{(q)} + V_{qu(n)}), \qquad (3)$$

$$\dot{q} = \frac{\nabla S}{m} = \frac{p}{m}, \qquad (4)$$

$$V_{qu} = -(\frac{\hbar^2}{2m}) n^{-1/2} \nabla \bullet \nabla n^{1/2}. \qquad (5)$$

$$S = \int_{t_0}^{t} dt (\frac{p \bullet p}{2m} - V_{(q)} - V_{qu(n)}) \qquad (6)$$

$$\lim_{T \to 0} G(\lambda) = exp[-(\frac{\lambda}{\lambda_c})^2]. \qquad (7)$$

where T is the noise amplitude parameter (e.g., the temperature of an ideal gas thermostat put in equilibrium with the system (Chiarelli, 2013b) and $G(\lambda)$ is the shape of the spatial correlation function of η.

The noise spatial correlation function (7), is a direct consequence of the derivatives present into the quantum potential that give rise to an elastic-like contribution to the system energy that reads (Weiner, 1983):

$$\overline{H}_{qu} = \int_{-\infty}^{\infty} n_{(q,t)} V_{qu(q,t)} dq = -\int_{-\infty}^{\infty} n_{(q,t)}^{1/2} (\frac{\hbar^2}{2m}) \nabla \bullet \nabla n_{(q,t)}^{1/2} dq \qquad (8)$$

where a large "curvature" of $n_{(q,t)}$ leads to high quantum potential energy. This can be easily checked by calculating the quantum potential of the wave function $\psi = cos \frac{2\pi}{\lambda} q$ that reads:

$$V_{qu} = -(\frac{\hbar^2}{2m}) \left(cos^2 \frac{2\pi}{\lambda} q \right)^{-1/2} \nabla \bullet \nabla \left(cos^2 \frac{2\pi}{\lambda} q \right)^{1/2} = \frac{\hbar^2}{2m} \left(\frac{2\pi}{\lambda} \right)^2 \qquad (9)$$

Showing that the energy increases as the inverse squared of the distance λ between two adjacent peaks (that is, the wave length). In the stochastic case, given Gaussian fluctuations with correlation distance λ, (9) represents the energy of the frequency mode associated to the closest independent fluctuations.

Therefore, independent fluctuations of particle density (PD) very close each other (that is, $\lambda \to 0$), generating very high curvature on the density $n_{(q,t)}$, can lead to a whatever large quantum potential energy even in the case of vanishing fluctuations amplitude (that is, $T \to 0$).

In this case, fluctuations with infinitesimal amplitude (that is, $T \to 0$) and diverging energy content, can lead to a finite quantum potential energy contribution even in the limit of T=0 forbidding the convergence of equations (1-7) to the deterministic limit (Bialyniki-Birula et al., 1992) (that is, the standard quantum mechanics).

Therefore, in order to eliminating these unphysical solutions, the additional conditions (7) come into the set of the equations leading to physically coherent stochastic generalization of quantum mechanics (Chiarelli, 2013b).

If we require that $\overline{H}_{qu} < \infty$ (following the criterion that higher is the energy lower is the probability to reach the corresponding state (that is, state with infinite energy have zero probability to realize itself) it follows that independent fluctuations of the density $n_{(q,t)}$ on shorter and shorter distance are progressively suppressed (that is, have lower and lower probability of happening). This physical effect due to the quantum potential (that confers to the particle density function the elastic behavior like a membrane, very rigid against short range curvature) imposes a finite correlation length to the possible physical fluctuations.

In the small noise limit (Chiarelli, 2013b) the suppression of PD fluctuations on very short distance, due to the finite energy requirement, brings to a restriction on the correlation length of the noise itself λ_c in (7) (Chiarelli, 2013b) that reads:

$$\lim_{T \to 0} \lambda_c = 2 \frac{\hbar}{(2mkT)^{1/2}}, \qquad (10)$$

leading to explicit form of the variance (2) (Chiarelli, 2013b).

$$\lim_{T\to 0} < \eta_{(q_\alpha ,t)}, \eta_{(q_\beta + \lambda, t+\tau)} > = \mu \frac{kT}{2\lambda_c^{\,2}} exp[-(\frac{\lambda}{\lambda_c})^2] \delta(\tau)\delta_{\alpha\beta} \quad (11)$$

Where μ is a constant with the dimension of a migration coefficient.

Furthermore, in the case of very small noise amplitude, due to the constraints (11), the action (6), can be re-cast in the form (Chiarelli, 2013b):

$$\begin{aligned} S &= \int_{t_0}^{t} dt(\frac{p \bullet p}{2m} - V_{(q)} - V_{qu(n)}) \\ &= \int_{t_0}^{t} dt(\frac{p \bullet p}{2m} - V_{(q)} - V_{qu(n_0)} - \delta V_{qu}), \quad (12) \\ &= S_0 + \delta S \end{aligned}$$

Where δS is a vanishing small fluctuating quantity (Chiarelli, 2013b).

Non-local property of quantum potential in presence of noise

The quantized action depends by the values of the quantum potential related to the corresponding eigenstates (that is, stationary states). On the other hand, the eigen values are determined by the quantum potential that has to neutralize the force deriving by the Hamiltonian potential (Appendix A). Since this condition must happen in all points of the space the dynamics of a generic quantum state is clearly non-local.

If we consider a bi-dimensional space, the quantum potential makes the particle density function acting like an elastic membrane that becomes quite rigid against ripples with very short wave length.

Given that the force of the quantum potential in a point depends by the state of the system around it, it introduces the non-local character into the motion equations. For this reason, the quantum non-local properties can be very well identified and studied by means of the analytical mathematical investigations of the property of the quantum potential in Equation (5).

In order to analytically detail what happens in the macroscopic case, mathematically speaking, we observe that the quantum force (equal to minus the gradient of the quantum potential) cannot be taken out by the deterministic limit of Equation (1) as intuitively proposed by many authors (Bialyniki-Birula et al., 1992; Weiner, 1983) because this operation will wipe out the quantum stationary states (that is, quantum eigenstates) deeply changing the structure of such equation.

The presence of the QP is needed for the realization of the quantum eigenstates that happen when the force of the QP exactly balances the Hamiltonian one. On the contrary, in the stochastic case, when we deal with large-scale systems with physical length $\mathcal{L} >> \lambda_c$ submitted to fluctuations, in weakly interacting systems we can have a vanishing small quantum force at large distances (Appendix B) (Chiarelli, 2013a, b) that, becoming much smaller than fluctuations, can be correctly neglected in the motion equations.

It must be underlined that not all types of interactions lead to a vanishing small quantum force at large distance (a straightforward example is given by linear systems where the quantum potential owns a quadratic form (Appendix B, sections B.1-B.2) (Chiarelli, 2013a, b, c, d).

Nevertheless, there exists a large number of non-linear long-range weak potentials (e.g., Lennard Jones types) where the quantum potential tends to zero (Appendix B) at infinity and can be neglected (Chiarelli, 2013c). In this case a rarefied gas of such particles having the mean particle distance much larger that the quantum potential range of interaction (Chiarelli, 2013a, b) behaves as a classical phase.

Following we analyze the large scale form of the SPDE (1) for asymptotically vanishing quantum potential with finite range of interaction λ_q (24, B.5).

In order to investigate this point, let's consider a system whose Hamiltonian which reads:

$$H = \frac{p^2}{2m} + V_{(q)}, \quad (13)$$

in this case the QHA equations (Bialyniki-Birula et al., 1992;) (that is, the deterministic limit of (17)) can be derived by the following phase-space equation:

$$\partial_t \rho_{(q,p,t)} + \nabla \bullet (\rho_{(q,p,t)} (\dot{x}_H + \dot{x}_{qu})) = 0 \quad (14)$$

Where

$$n_{(q,t)} = \iiint \rho_{(q,p,t)} d^{3n} p \quad (15)$$

$$\dot{x}_H = (\partial_p H, -\nabla H) \quad (16)$$

$$\dot{x}_{qu} = (0, -\nabla V_{qu}) \quad (17)$$

by integrating equation (14) over the momentum *p* with the conditions that $\lim_{|p| \to \infty} \rho_{(q,p,t)} = 0$, with the constraint on the quantum phase space density ρ (a Wigner-like function):

$$\rho_{(q,p,t)} = n_{(q,t)} \delta(p - \nabla S). \quad (18)$$

The factor $\delta(p - \nabla S)$ warrants the wave-particle equivalence in the quantum mechanics limit and the correspondence rule:

$$p = m\dot{q} = \nabla S \tag{19}$$

between the quantum hydrodynamic model and the Schrödinger equation (Bialyniki-Birula et al., 1992; Weiner, 1983).

When a spatially distributed random noise is present, Equation (17) has the corresponding phase space SPDE that reads

$$\partial_t \rho_{(q,p,t)} + \nabla \bullet (\rho_{(q,p,t)}(\dot{x}_H + \dot{x}_{qu})) = \eta_{(q,t,T)} \delta(p - \nabla S), \tag{20}$$

whose zero noise limit is the deterministic PDE (14). Near the deterministic limit, in the case of Gaussian noise (2), it is possible to re-cast (20) as:

$$\partial_t \rho_{(q,p,t)} + \nabla \bullet (\rho_{(q,p,t)}(\dot{x}_H + \dot{x}_{qu(\rho_0)})) = -\nabla \bullet (\rho_{(q,p,t)} \delta \dot{x}_{qu}) + \eta_{(q,t,T)} \delta(p - \nabla S), \tag{21}$$

where ρ_0 is the solution of the deterministic QHA equations and where $\delta \dot{x}_{qu} = (0, -\nabla \delta V_{qu})$, where:

$$\begin{aligned} \delta V_{qu} &= -(\frac{\hbar^2}{2m}) \{ n^{-1/2} \nabla \bullet \nabla n^{1/2} - n_0^{-1/2} \nabla \bullet \nabla n_0^{1/2} \} \\ &= V_{qu(n)} - V_{qu(n_0)} \end{aligned} \tag{22}$$

where $n_{0(q,t)} = \iiint \rho_{0(q,p,t)} d^{3n} p$.

Thanks to conditions (7, 10) (Chiarelli, 2013b), closer and closer we get to the deterministic limit (that is, $\frac{\lambda_c}{\mathcal{L}} \to \infty$, where $\mathcal{L}$ is the physical length of the system), smaller and smaller is the amplitude of the random term on the right side of (21):

$$-\nabla \bullet (\rho_{(q,p,t)} \delta \dot{x}_{qu}) + \eta_{(q,t,T)} \delta(p - \nabla S) = X_{(q,t)} \tag{23}$$

When $\frac{\lambda_c}{\mathcal{L}} \to \infty$ the standard quantum mechanics is achieved and the quantum potential cannot be disregarded from the hydrodynamic quantum motion equations.

Large-scale classical behavior in non-linear asymptotically weakly-bonded systems

On the contrary, when $\lambda_c << \mathcal{L}$, in weakly bounded system when the force steaming from the quantum potential at large distance tends to zero it is possible to coherently define a measure of the quantum potential range of interaction λ_q that reads (Chiarelli, 2013b):

$$\lambda_q = 2\lambda_c \frac{\int_0^\infty | q^{-1} \frac{\partial V_{qu}}{\partial q} | dq}{| \frac{\partial V_{qu}}{\partial q} |_{(q=\lambda_c)}}, \tag{24}$$

When $\lambda_q < k \in \Re$ the quantum potential becomes much smaller than its fluctuations at large distance ad it can correctly be disregarded by the equation of motion.

Thence, when $\frac{\lambda_q}{\mathcal{L}} \to 0$ it follows that:

$$\dot{x}_{qu(\rho_0)} << \delta \dot{x}_{qu} \tag{25}$$

Where Equation (25) expresses the fact that the quantum potential force $\dot{x}_{qu(\rho_0)} = (0, -\nabla V_{qu(\rho_0)})$ is much smaller than its fluctuations $\delta \dot{x}_{qu(\rho_0)} = (0, -\nabla \delta V_{qu})$ (that is, $| \nabla V_{qu(n_0)} | << | \nabla \delta V_{qu(n)} |$).

For sake of completeness, we observe that close to the deterministic limit (that is, to the quantum mechanics) when $\mathcal{L} < \lambda_c$ the quantum potential cannot be disregarded even if it is vanishing small, therefore the quantum potential range of interaction λ_q is physically meaningful if, and only if, $\lambda_q > \lambda_c$. For $\lambda_q < \lambda_c$ the quantum potential range of interaction must be retained equal to λ_c.

Introducing (25) into equation (21), for $\mathcal{L} >> \lambda_q \geq \lambda_c$, it follows that:

$$\partial_t \rho_{(q,p,t)} + \nabla \bullet (\rho_{(q,p,t)}(\dot{x}_H)) \cong -\nabla \bullet (\rho_{(q,p,t)} \delta \dot{x}_{qu}) + \eta_{(q,t,T)} \delta(p - \nabla S) \tag{26}$$

and that

$$\lim_{\frac{\lambda_c}{\mathcal{L}} \to 0} < \eta_{(q_\alpha,t)}, \eta_{(q_\beta+\lambda,t+\tau)} > = \mu (\frac{kT}{2m})^{\frac{1}{2}} \delta(\lambda) \delta(\tau) \delta_{\alpha\beta} \tag{27}$$

Given that for $\mathcal{L} >> \lambda_q \geq \lambda_c$ the noise amplitude results $T >> T_c = \frac{\hbar^2}{2mk\mathcal{L}^2}$ (where for $\mathcal{L} = 3 \times 10^{-6}$ cm and m

equal to the proton mass T_C can be as low as 3°K) in weakly bounded with system $\lambda_q \approx \lambda_c$ (Chiarelli, 2013a) the stochastic phase space PDE (26) reads:

$$\partial_t \rho_{(q,p,t)} + \nabla \bullet (\rho_{(q,p,t)} (\dot{x}_H)) = X_{(q,t)} . \quad (28)$$

where $X_{(q,t)}$ is a stochastic (sufficiently small quantity) giving rise to classically fluctuating dynamics that do not own eigenstates.

Physically speaking, the central point in weakly quantum entangled systems, whose characteristic length $\mathcal{L}$ is much bigger than the quantum potential range of interaction λ_q, is that the stochastic sequence of fluctuations of the quantum potential does not allow the coherent reconstruction of the superposition of state since they are much bigger than the quantum potential itself. In this case (especially in classically chaotic systems) the effect of the quantum potential with fluctuations on the dynamics of the system is not equal to the effect of its average (even in the unlikely case of fluctuations have a null time mean).

If the quantum potential can be disregarded in the large scale description, the action (12) reads:

$$\begin{aligned} S &= \int_{t_0}^{t} dt \left(\frac{p \bullet p}{2m} - V_{(q)} - V_{qu(n_0)} - \delta V_{qu} \right) \\ &\cong \int_{t_0}^{t} dt \left(\frac{p \bullet p}{2m} - V_{(q)} - \delta V_{qu} \right) \\ &= S_{cl} + \delta S \end{aligned} \quad (29)$$

and hence, the momentum of the solutions given by the δ-function in Equation (18) (that is, $\delta(p - \nabla (S_{cl} + \delta S))$ approaches the classical value (plus a fluctuation) and reads:

$$p = \nabla (S_{cl} + \delta S) = p_{cl} + \delta p \quad (30)$$

Observing that the quantum coherence length λ_c results by the geometrical mean of the stochastic length $\frac{\hbar c}{kT}$ (of order of unity or less, (about 1,44 cm at 1°K)) and the Compton length $l_C = \frac{\hbar}{mc}$ (the reference length for the standard quantum mechanics) it follows that the description of a macroscopic system (with a resolution Δq such as $\lambda_c , l_C , \lambda_q < \Delta q << \mathcal{L}$) can behave classically stochastic at laboratory scale, even at low temperature, since for T_c as small as the temperature of the background radiation 2,725°K, it results $\lambda_c \propto (\frac{l_C \hbar c}{kT})^{1/2} = 2.8 x 10^{-8} m$ for a particle of proton mass (or $\lambda_c \approx 3 x 10^{-9} m$ at a temperature of 300°K)).

Even if the condition $\lambda_c < \lambda_q < \Delta q$ is usually satisfied for macroscopic objects constituted by Lennard-Jones interacting particles, there also exists (at laboratory condition) the possibility to have $\Delta q < \lambda_q$ and, hence, to detect quantum phenomena.

The most direct and immediate example is given by observables depending by molecular properties of solid crystals that, due to the linearity of the particles interaction, can own a very large quantum potential range of action λ_q (that may result of order of ten times of the atomic distances (Chiarelli, 2013a).

Another possibility is to refrigerate a fluid below its critical density (if it does not undergo solidification) in order to obtain that the mean molecular distance becomes smaller than λ_p or/and λ_c (Chiarelli, 2013b).

Even if the linear systems are the most studied and known ones, those characterized by non-linear weak interactions, to which equation (28) can apply, are more wide-spread in nature.

For instance, equation (28) can apply to the case of a rarefied gas phase of Lennard-Jones potential interacting particles where the mean inter-particle distance d is much bigger than λ_q and λ_c (for instance for the helium at room temperature it results $\lambda_c \approx \lambda_q \approx 0.6 x 10^{-8} cm$ and $d \approx 6 x 10^{-7} cm$). In this case, the quantum superposition of states of molecules (or group of them) does not exist so that the macroscopic gas system behaves classically.

A deeper analysis (Chiarelli, 2013a), shows that the classical behavior of molecules of a real gas is maintained down to the density of liquids. On the contrary, due to the linearity of intermolecular forces in solid crystals, λ_q becomes bigger than the mean inter-particle distance (Chiarelli, 2013a) and the quantum behavior of groups of atoms is maintained. Nevertheless, since the linear interaction of solids ends over a certain distance, the quantum behavior survives just in phenomena depending by the molecular scale (e.g., Bragg's diffraction).

The quantum macroscopic state of a body made of weakly interacting particles like ordinary molecules does not have any physical existence in a noisy environment.

COMPATIBILITY BETWEEN THE LOCAL RELATIVISTIC CAUSALITY AND THE (NON-LOCAL) QUANTUM UNCERTAINTY RELATIONS IN THE FRAME OF THE SQHA

If in the classical macroscopic reality we try to detect microscopic variables, below a certain point the wave-particle dual properties of bodies emerge thanks to the quantum potential effect. In the classical approach the particle concept owns the characteristic that position and velocity are perceived as independent. On the other hand, on microscopic scale the wave property of the matter (e.g., the impossibility to interact just with a part of the system without entirely perturbing it) leads to the coupling between conjugated variables such as position and velocity (Oppenheim and Wehner, 2007). If we increase of spatial confinement of the wave function, an increase of the quantum potential energy (due to the overall increase of derivatives of $n^{1/2}$) is produced. This fact leads to possible higher particle momentum values in the following measurements.

The scale-dependence of the quantum potential interaction leads the classical perception of the reality until the resolution size Δq is at least larger than the quantum coherence length λ_c .

Moreover, we observe that higher is the amplitude of the noise T , smaller is the length λ_c and, hence, higher is the attainable degree of spatial precision within the classical scale. On the other hand, higher is the amplitude of noise, higher are the fluctuations of observables such as the velocity and/or energy. In the frame of the SQHA, it is straightforward to show that these mutual opposite effects on conjugated variables are the basis of the Heisenberg principle of uncertainty. In fact, by using the quantum stochastic hydrodynamic model, it is possible to derive the relation between the time interval Δt of a measurement and the related variance of the energy on a particle of mass *m*.

If on distances smaller than λ_c any system behave in quantum mode (as a wave) so that any its sub-parts cannot be perturbed without disturbing all the entire system, it follows that the independence between the measuring apparatus and the measured system (*classical freedom*) requires that they must be far apart, at least, more than $\frac{\lambda_c}{2}$ and hence for the finite speed of propagation of interactions and information (*local relativistic causality* (LRC)) the measure process must last longer than the time $\tau = \frac{\lambda_c}{c}$.

Moreover, given that the noise $\eta(q,t,T)$ in (13) in the small noise limit (that is, T sufficiently small) leads to Gaussian energy fluctuations (Chiarelli, 2013b), it follows that the mean value of the energy fluctuation for each degree of freedom of a particle is $\Delta E_{(T)} = \frac{1}{2}kT$ (Ozawa, 2003) and thence, in the non-relativistic limit ($mc^2 >> kT$) for a particle of mass *m*, the energy variance ΔE reads:

$$\Delta E \approx (<(mc^2 + \Delta E_{(T)})^2 - (mc^2)^2>)^{1/2} \cong (<(mc^2)^2 + 2\Delta E_{(T)} - (mc^2)^2>)^{1/2} \cong (2mc^2 < \Delta E_{(T)}>)^{1/2} \cong (2mc^2 kT)^{1/2} \quad (31)$$

from which it follows that (Chiarelli, 2013b; Ozawa, 2003)

$$\Delta E \Delta t > \Delta E \Delta \tau = \frac{(2mc^2 kT)^{1/2} \lambda_c}{2c} = \hbar \cdot \quad (32)$$

It is worth noting that the product $\Delta E \Delta \tau$ is constant since the growing of the energy variance with the square root of the temperature $\Delta E \approx (2mc^2 kT)^{1/2}$ is exactly compensated by the decrease of the minimum time of measurement:

$$\tau \propto \frac{\hbar}{(2mc^2 kT)^{1/2}} \quad (33)$$

furnishing an elegant physical explanation why the Heisenberg relations exist in term of a physical constant.

The same result is achieved if we derive the uncertainty relation between the position and the momentum of a particle of mass *m*.

If we measure the spatial position of a particle with a precision of $2\Delta L > \lambda_c$ so that we do not perturb its quantum wave function (that, due to environmental fluctuations, is spontaneously localized on a spatial domain of order of λ_c) the variance Δp of the modulus of its relativistic momentum $(p^\mu p_\mu)^{1/2} = mc$ due to the vacuum fluctuations reads:

$$\Delta p \approx (<(mc + \frac{\Delta E_{(T)}}{c})^2 - (mc)^2>)^{1/2} \cong (<(mc)^2 + 2m\Delta E_{(T)} - (mc)^2>)^{1/2} \cong (2m < \Delta E_{(T)}>)^{1/2} \cong (2mkT)^{1/2} \quad (34)$$

leading to the uncertainty relationship

$$\Delta L \Delta p > \frac{\lambda_c}{2} \Delta p = \frac{\lambda_c}{2} (2mkT)^{1/2} = \hbar \quad (35)$$

If we impose measuring the spatial position with a precision $2\Delta L < \lambda_c$, we have to localize the quantum state of the particle more than what is spontaneously is.

Since quantum potential realizes the particle-wave equivalence, the wave-function localization and momentum variance are submitted to the properties of the Fourier transform relationships (holding for any wave system): The uncertainty relations remain satisfied anyway we try to localize the wave function (either by environmental fluctuations or by physical means (that is, external potentials).

Connections between the uncertainty relations and local relativistic causality

In the frame of the SQHA, particles are necessarily correlated each other until they are separated by a distance smaller than λ_c, the distance over which the wave function is governed by quantum law (they still may present quantum correlations (stochastic influenced) until they are separated by a distance up to λ_q, but in this case we do not have quantum entanglement as described by the standard (deterministic) quantum mechanics).

If two particles are quantum entangled, when the measurement on one of the two is performed (so that the global wave-function collapses to an eigenstate) in the context of the SQHA model, we are in presence of a kinetic (irreversible) evolution toward a stationary state (eigenstate) (SQE) with its characteristic (not null) time τ_c.

If we assume the Copenhagen interpretation of quantum mechanics, so that the measurement process ends when the wave function is collapsed to the eigenstate, the "quantum relaxation" interval of time τ_c represents the minimum time of measurement. In this case, the compatibility of the SQHA (that is, of the quantum mechanics) with the *local relativistic causality* implies that it must be $\tau_c > \tau = \frac{\lambda_c}{c}$ (or at least $\tau_c > \frac{\lambda_c}{2c}$ if the wave function decoherence starts from the center toward the border).

From experimental point of view, in order to demonstrate that the *local relativistic causality* (LRC) breaks down in quantum processes, it needs to demonstrated that the decoherence time τ_c is so short that the wave function collapse to the eigenstate is faster than the light to travel the radius $\frac{\lambda_c}{2}$ over which the quantum entangled state extends itself and hence, it is sufficient to demonstrate that $\tau_c < \frac{\lambda_c}{2c}$.

Given that, by introducing (10) in (31) in presence of environmental energy fluctuations it holds:

$$\lambda_c = \frac{2\hbar c}{\Delta E} \tag{36}$$

and hence $\Delta E \tau_c < \hbar$, it follows that, in the SQHA model (that is, low speed limit), the violation of the Heisenberg uncertainty principle necessarily involves the LRC breaking and (for microscopic systems with characteristic length $\mathcal{L} < \lambda_c$) vice versa.

The same conclusion is achieved if, by using external means, we confine the wave function in a region of length $\Delta L < \lambda_c$.

RELATIVISTIC APPROACH

The SQHA approach is the classical limit of the corresponding relativistic model. In such low velocity limit model, the light speed goes to infinity and hence the compatibility with the RLC can be checked just showing that the uncertainty relations are compatible with the requirement of finite speed of interactions.

Even if the stochastic generalization of the quantum relativistic hydrodynamic approach is still not available, from the hydrodynamic representation of the Dirac equation (Chiarelli, 2014a) we can inspect the Lorentz invariance of the relativistic quantum potential that can enforce the hypothesis of compatibility between the LRC and the quantum non-locality. The relativistic quantum potential allows verifying if the non-local interactions that it introduces into the quantum equation of motion propagate themselves compatibly with the postulate of the relativity about the invariance of light speed as the fastest way to which signals and interactions can be transmitted.

Since the invariance of light speed is the generating property of the Lorentz transformations, the co-variant form (that is, invariant 4-scalar product) of quantum potential that reads (Chiarelli, 2014b):

$$V_{qu} = -\frac{\hbar}{2i}\left[\dot{q}^{\mu}\,\partial_{\mu}\, ln[\frac{\boldsymbol{R}}{\overline{\boldsymbol{R}}}]\right] \tag{37}$$

where

$$ln[\frac{\boldsymbol{R}}{\overline{\boldsymbol{R}}}] = \left(ln[\frac{|\Psi_1|}{|\Psi_3|}],\ ln[\frac{|\Psi_2|}{|\Psi_4|}],\ ln[\frac{|\Psi_3|}{|\Psi_1|}],\ ln[\frac{|\Psi_4|}{|\Psi_2|}] \right) \tag{38}$$

$$\dot{q}^{\mu} = \frac{\overline{\Psi} c\gamma^{\mu}\Psi}{|\Psi|^2} = \frac{\Psi^* c\gamma^0\gamma^{\mu}\Psi}{|\Psi|^2} \tag{39}$$

where Ψ_i are the components of the bispinor 4-dimensional wave function

$$\Psi = (\Psi_1, \Psi_2, \Psi_3, \Psi_4),$$

and where γ^{μ} are the 4x4 matrices derived by the 2x2

Pauli matrices (Bialyniki-Birula et al., 1992), united to the property of the 4-dimensional wave function Ψ that changes accordingly with the Lorentz transformation, allows affirming that the quantum non-local behavior (deriving by the quantum potential) is compatible with such a postulate of the relativity.

In fact, whatever inertial system we choose moving with velocity $v < c$, the quantum potential expression (37) describes the quantum dynamics as realize themselves in such new reference system (where the light speed is always c and hence not attainable). This fact forbids that in any inertial system the time difference between the initial conditions (e.g., starting of measurement (that is, cause)) and the final one (wave collapse (that is, effect)) is null (or negative) so that the quantum-potential action on the whole wave function cannot realize itself in a null time.

This result enforces the hypothesis that any measurable quantum non-local process (even involving a large distance) is compatible with the postulate of invariance of light speed as the fastest way to which signals and interactions can be transmitted.

The paradox of instantaneous non local quantum action at infinite distances is, hence, an artifact that appears in the non-relativistic non-stochastic theories of quantum mechanics due to the fact that the light speed tends to infinity and the non-local interaction own an infinite range of action.

COMMENTS ABOUT THE SQHA MODEL

In the frame of the stochastic QHA the achievement of the classical characteristics of physical reality (Einstein et al., 1935) such as the classical freedom and local relativistic causality is realized as a large-scale effect in systems of asymptotically weakly bounded particles.

As far as the resolution limit of the classical description is much larger than the length over which the wave (quantum) properties of the matter can be detected, the classical concepts are not contradicted. When we deal with observables of microscopic systems, the quantum properties arise since the quantum potential (that is, the wave property of the matter) comes into effect.

The SQHA shows that the *classical freedom* principle (independence between systems) and the local relativistic causality are compatible with the quantum mechanics in the frame of a unique theory.

The possibility of *classical freedom* comes from the fact that, in fluctuating environment asymptotically weakly bounded systems can disentangle themselves at large distance (beyond the quantum coherence lengths λ_c and quantum potential range of action λ_q).

It also noteworthy that in the frame of the SQHA model, linear system (or more tightly bounded ones) do not disentangle themselves even at large distance forbidding the realization of the large-scale classical behavior (so that the classical universe as we know is a direct consequence of the electrical and gravitational forces that goes to zero at infinity).

The recovering of the quantum mechanics as the deterministic limit of a stochastic theory (that is, the SQHA) fulfills the philosophical need of determinism (Schrödinger, 1935; Einstein et al., 1935; Bell, 1964). In the SQHA model the quantum mechanics represent the deterministic limit of a stochastic theory. In this picture, the deterministic quantum distribution functions can be thought as a sort of “mechanical-like” distributions (not statistical) whose evolution is determined and well defined once the initial distributions and boundary conditions are defined.

The statistical variability and hence the indeterminism of the system evolution is introduced by the environmental fluctuations.

In the context of the SQHA model, the large-scale classical freedom allows the realization of statistical measurements so that, in principle, the description of the measurement process (as the interaction with a classical observer system) can find its description inside the theory itself.

In the SQHA, the wave-function collapse to an eigenstate (due to the interaction with a large scale apparatus in a classical fluctuating environment) can descend by the irreversible dynamics of the stochastic motion equations as a kinetic process to a stationary state (eigenstate).

This fact leads to a quantum theory with the conceptual property of a complete theory (that does not need additional postulates) able to describe the quantum evolution even during irreversible quantum processes such as the measurements.

In the frame of the SQHA model, a non-local based theory with the property of large scale *local freedom* compatible with the relativistic postulate of maximum speed of light and information transmission (*local relativistic causality*) has no necessity to postulate superluminal transmission of information to explain the result of quantum experiments obtained at large distance.

To this end, in the final part of this work we want to examine the logical consequences of assuming the existence of superluminal transmission of information in quantum experiments.

ARE SUPERLUMINAL INFORMATION EXCHANGED DURING QUANTUM EXPERIMENTS?

The attempts of solving the problem of quantum correlations between experiments at large distance (that dates back to the foundation of the quantum physics) have followed various ways: The local treatment of quantum mechanics possibly with the help of hidden (local) variables. This possibility has been shown to be not realizable by the violation of Bell's inequalities (Bell, 1964).

The establishing of non-local theory compatible with the classical physics: The completion of quantum mechanics by using non-local hidden variables. This hypothesis argues that the fully quantum evolution is determined by information that cannot be obtained by the observer (von Neumann, 1932/1955). The Bohmian mechanics furnishes an example of this completion where the hidden variables are non local (Maudlin, 1994).

The assumption that there is a sort of quantum kinetic synchronization among quantum entangled particles that is maintained along irreversible processes (such as the measurement ones) happening in presence of fluctuations and involving large-scale classical objects. This point of view basically hypothesizes that exists a more general quantum theory able to comprehend the classical, relativistic and irreversible phenomena. Superluminal information are exchanged during quantum measurements.

The cases 2A and 2B even similar, differ each other: the first one considers that the quantum mechanics is a complete theory for describing the system evolution but it has a leaking of information about additional (non-local) variables that the classical observer cannot achieve; the second one is based upon the assumption that the theoretical quantum equations themselves do not allow the complete description of the evolution.

The latter hypothesis can be justified by fact that the standard quantum mechanics is a semi-empirical theory (needing additional empirical postulates) that is not able to describe the quantum irreversible evolution of the wave function collapse to an eigenstate during a measure.

On the other hand, if the decay to an eigenstate cannot be determined by the quantum motion equation, but only in a probabilistic way, generally speaking, this means that we do not have all the complete "machinery" to describe the quantum evolution of a system.

About the existence of superluminal communications in quantum experiments, Clauser Horne, Shimony and Holt (CHSH) (Clauser et al., 1969) have shown that in quantum mechanics experiments the Tirelson's limit of the correlation coefficient $S_{CHSH} = 2\sqrt{2}$ cannot be overcame, while Popescu and Rohrlich (Popescu and Rohrlich, 1994) showed that superluminal communication is not necessary for correlated experiments with $S_{CHSH} \le 4$ and hence, in principle, they may be not needed in quantum mechanics.

In order to analyze the problem under the light of the SQHA model, we discuss below the output of a two entangled photons experiment traveling in opposite direction in the state:

$$|\psi>= \frac{1}{\sqrt{2}}|H,H> + e^{i\phi}|V,V> \tag{40}$$

that cross polarizers oriented in the same direction following the scheme in Figure 1.

The assumption that the state of the photon is defined only after the measurement has taken place leads to accept that the photon superposition state interacts with the polarizer, but it is still not fully collapsed neither to $|H>$ nor to $|V>$ until it is adsorbed by the polarizer or by the photon counter.

Given that in the SQHA approach the wave function collapse is not instantaneous (but takes a time interval that we name Δt_1 and Δt_2 for the two photons, respectively), the measurement time τ_m starts at the arrival of the first entangled photon at its polarizer-photon counter system (at the time t_1) and ends when the other entangled one is detected at the second polarizer-photon counter system (at the time $t_2 = t_1 + \tau_m$) (the contemporary detection of the photons at the two photon-counters systems when placed at the same distance from the source for instance, does not imply that the duration of the measurement process is null).

The better way to perform the experiment is to increase as much as possible the distance between the two polarizer-photon counter systems $\mathcal{L}$. The best possibility is to have such a distance that spans over a cosmological length. To comply with this condition, we can think to have the source on the Earth, one polarizer-photon counter system on the Moon and the other on Mars. We can also suppose that the Moon, the Earth and the Mars are aligned each others. In this case , it follows that the distance between the two polarizer-photon counter systems $\mathcal{L} = \mathcal{D}_{e-mo} + \mathcal{D}_{e-ma}$ and that $t_1 = \frac{\mathcal{D}_{e-mo}}{c}$, $t_2 = \frac{\mathcal{D}_{e-mo}}{c} + \tau_m$, where $\mathcal{D}_{e-mo} \cong 3{,}84\, x10^8\, m$ and $\mathcal{D}_{e-ma}$ are the Earth-Moon and the Earth-Mars distances respectively.

If we assume that the quantum potential (QP) propagates itself at the speed of light for bringing the information about the first photon detection to the second one, it follows that the measurement time lasts longer than $\tau_m = \Delta t_1 + \Delta t_2 + \frac{\mathcal{L}}{c}$.

Thence, the time delay Δt between the arrival of the second photon to mars and its detection must result:

$$\Delta t = t_2 - \frac{\mathcal{D}_{e-ma}}{c} = \frac{\mathcal{D}_{e-mo}}{c} + \tau_m - \frac{\mathcal{D}_{e-ma}}{c} = 2\frac{\mathcal{D}_{e-mo}}{c} + \Delta t_1 + \Delta t_2 \tag{41}$$

Thence if $\Delta t < 2\frac{\mathcal{D}_{e-mo}}{c} + \Delta t_1 + \Delta t_2$ or better, $\Delta t < 2\frac{\mathcal{D}_{e-mo}}{c} \cong 2.55\, s$ (and, hence, $\tau_m < \frac{\mathcal{L}}{c}$) the photon wave function collapse on mars has happened before the arrival of the quantum potential signal coming from the first photon detection on the Moon. In this case there is no possibility of transferring quantum information between the two photons without violating the RLC. Therefore two alternative possibilities remain:

1. Superluminal transmission of information during the experiment, and
2. Intrinsic dynamical synchronization fully describeable via a complete relativistic quantum stochastic theory.

The two possibilities exclude themselves each other: if we own the complete quantum model we would be able to describe any physical event without additional hypotheses. On the other hand if we do not have it, we need a surrogate hypothesis, to fill the gap that in this case consists in hypothesizing the superluminal transmission of information. In this case, we have to define the kind and the characteristics of such an interaction and its "mechanics" since it is not contained in the quantum one.

Conclusion

The work analyzes the non-local property of quantum mechanics in the frame of the stochastic QHA model and shows that it can have a finite range of action, allowing in weakly bounded systems the realization of the classical mechanics on large scale limit.

The analysis shows that it is possible to maintain the concept of *freedom* of the classical reality between far away systems beyond the range of interaction of quantum potential as well as to make compatible the *local relativistic causality* with the uncertainty principle, one of the most relevant manifestations of the non-local behavior of the quantum mechanics.

The *moderate non-locality* of the SQHA approach can be compatible with the assumption that the speed of light is the maximum velocity of transmission of information and interactions. This is confirmed by the relativistic QHA approach that shows that the quantum potential propagates the non-local quantum interaction accordingly with the relativistic postulate of light speed invariance as the maximum velocity of transmission.

The model shows that the paradox of instantaneous quantum action at infinite distances is an artifact that appears in the non-relativistic non-stochastic limit of quantum mechanics where the light speed goes to infinity and non-locality becomes a global property.

The SQHA model shows that is possible to have a theory where *moderate non-locality*, *classical freedom* and *relativistic causality* can cohabit together showing that there is no need for searching a local quantum mechanics (giving a theoretical support to the Bell's inequality violations).

On the base of a simple two photon experiment, the paper shows that the intellectual necessity of postulating that superluminal information are exchanged during non-local quantum experiments may be due to the leaking of theoretical completeness of the standard quantum formalism.

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APPENDIX A

In the QHA the eigenstates can be identified by the their stationarity that happens due the fact that the force generated by the quantum potential exactly counterbalance that one due to the Hamiltonian potential (with the initial condition $\dot{q} = 0$).

Since the quantum potential changes with the state of the system, more than one stationary state (each one with its own $V_{qu}{}^{n}$) is possible and more than one quantized eigenvalues of the energy may exist with the corresponding action values:

$$S_0{}^n = \int_{t_0}^{t} dt\left(\frac{p \bullet p}{2m} - V_{(q)} - V_{qu_0}{}^n\right) \qquad \text{(A.0)}$$

The above statements can be straightforwardly checked in the case of a linear system. For a harmonic oscillator described by the Hamiltonian $H = \frac{p^2}{2m} + \frac{m\omega^2}{2}q^2$, whose generic *n*-th eigenstate reads:

$$\psi_{n(q)} = n^{\frac{1}{2}}{}_{(q,t)} \exp\left[\frac{i}{\hbar}S_{(q,t)}\right] = H_{n\left(\frac{m\omega}{2\hbar}q\right)} \exp\left(-\frac{m\omega}{2\hbar}q^2\right), \qquad \text{(A.1)}$$

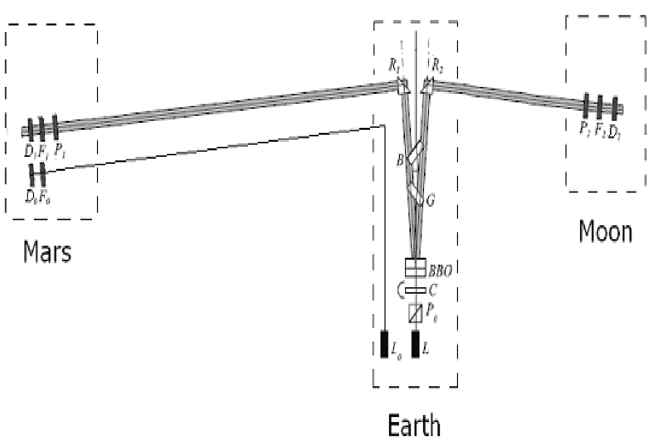

Figure 1. Schematic drawing of the experimental apparatus.

(where $H_{n(x)}$ represents the *n*-th Hermite polynomial) the density $n_{(q,t)}$ and the action $S_{(q,t)}$ respectively read:

$$n^{1/2}{}_{(q,t)} = H_{n\left(\frac{m\omega}{2\hbar}q\right)} \exp\left(-\frac{m\omega}{2\hbar}q^2\right) \qquad \text{(A.2)}$$

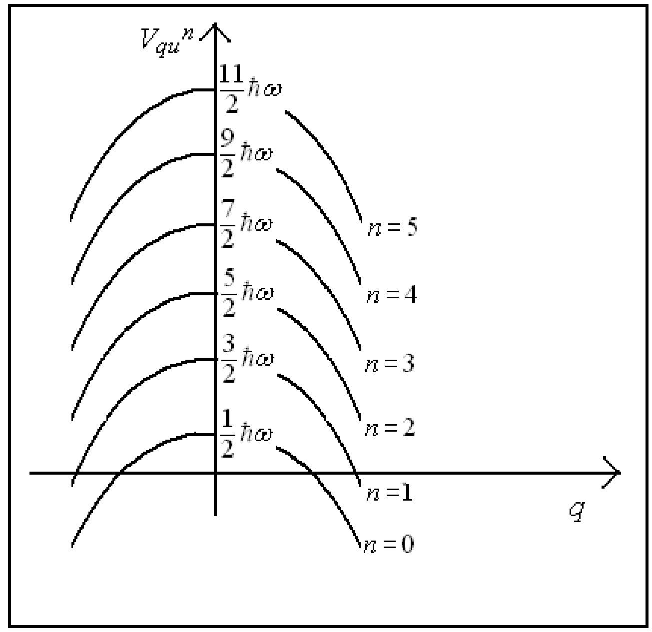

Figure 2. The repulsive quantum potential for the firsts five eigenstates of a harmonic oscillator.

$$S_{(q,t)} = S_{(t)}, \qquad \text{(A.3)}$$

leading to the quantum potential of the *n*-th eigenstate (Figure 2)

$$\begin{aligned} V_{qu}{}^n &= -\left(\frac{\hbar^2}{2m}\right) n^{1/2} \nabla \bullet \nabla n^{1/2} \\ &= -\frac{m\omega^2}{2}q^2 + \left[n\left(\frac{\frac{m\omega}{\hbar}H_{n-1} - 2(n-1)H_{n-2}}{H_n} \right) + \frac{1}{2} \right]\hbar\omega \\ &= -\frac{m\omega^2}{2}q^2 + \left(n + \frac{1}{2}\right)\hbar\omega \end{aligned} \qquad \text{(A.4)}$$

where it has been used the recurrence formula of the Hermite polynomials

$$H_{n+1} = \frac{m\omega}{\hbar}qH_n - 2nH_{n-1}, \qquad \text{(A.5)}$$

that gives the following energy eigenvalues

$$\begin{aligned} E_n &= \langle \psi_n | H | \psi_n \rangle = \int_{-\infty}^{\infty} n_{(q,t)} \left[\frac{m\omega^2}{2}(q-\underline{q})^2 + V_{qu}{}^n + \frac{m}{2}|\dot{q}|^2 \right] \\ &= \int_{-\infty}^{\infty} n_{(q,t)} \left[\frac{m\omega^2}{2}(q-\underline{q})^2 + V_{qu}{}^n + \frac{1}{2m}|\nabla S_{(q)}|^2 \right] \\ &= \int_{-\infty}^{\infty} n_{(q,t)} \left[\frac{m\omega^2}{2}(q-\underline{q})^2 - \frac{m\omega^2}{2}(q-\underline{q})^2 + \left(n+\frac{1}{2}\right)\hbar\omega \right] = \left(n+\frac{1}{2}\right)\hbar\omega \end{aligned} \qquad \text{(A.6)}$$

as well as:

$$\dot{p} = -\nabla (H + V_{qu}) = -\nabla ((n + \frac{1}{2})\hbar \omega) = 0 \,, \qquad \text{(A.7)}$$

$$\dot{q} = \frac{\nabla S_{(q,t)}}{m} = 0 \,, \qquad \text{(A.8)}$$

$$S_0{}^n = \int_{t_0}^{t} dt(\frac{p \bullet p}{2m} - V_{(q)} - V_{qu_0}{}^n) = E_n (t - t_0) \qquad \text{(A.9)}$$

APPENDIX B

Large-distance quantum force

To obtain the macro-scale form of equations (47) we need to evaluate the large-scale limit of the quantum force $\dot{x}_{qu} \equiv -\nabla_q V_{qu}$ in it. The behavior of $n^{1/2}$ determines the quantum potential (QP) in (5). For sake of simplicity, we discuss the one-dimensional case of localized state with $n^{1/2}$ that at large distance goes like:

$$\lim_{|q| \to \infty} n^{1/2} \propto exp[-P^k (q)] \qquad \text{(B.1)}$$

where $P^k{}_{(q)}$ is a polynomial of degree equal to *k*, $z_q = \gamma^{-1} q$ is the macroscopic variable (where $\gamma = \frac{\Delta q}{\lambda_q}$, where Δq is the macro-scale resolution) and λ_q is the range of the QP interaction. By using (B.1), the QP (5) at large scale reads:

$$\lim_{\gamma \to \infty} V_{qu} = \lim_{\gamma \to \infty} -k^2 \gamma^{-\phi} z_q{}^{1-\phi} + k(k-1)\gamma^{-(1.5+\phi)} z_q{}^{-(3+\phi)/2} \qquad \text{(A.2)}$$

where $\phi = 3 - 2k$.

Thence, for $k < \frac{3}{2}$ (that is, $\phi > 0$) $\forall$ $z_q \neq 0$ finite , the quantum force $-\nabla_q V_{qu}$ at large scale (that is, $\gamma \to \infty ,\ q = \gamma z_q \to \infty$) reads:

$$\lim_{\gamma \to \infty (z_q \, finite)} -\nabla V_{qu} = \lim_{q \to \infty} 2k^2 (k-1) (\gamma z_q)^{-\phi} + k(k-1)(k-2) (\gamma z_q)^{-\frac{1}{2}(3+2\phi)} z_q{}^{\frac{1}{2}\phi} \approx 2k^2 (k-1) (\gamma z_q)^{-\phi} = 0 \qquad \text{(B.3)}$$

Moreover, since the integral:

$$\int_0^{\infty} | q^{-1} \nabla V_{qu} | dq \cong Const + \alpha \int_{q_0}^{\infty} | \frac{1}{q^{1+\phi}} | dq < \infty \qquad \text{(B.4)}$$

converges for $\phi > 0$, (B.4) tells us if the QP force is negligible on large scale as given by (B.3). Therefore, finite values of the mean weighted distance:

$$\lambda_q = 2 \frac{\int_0^{\infty} | q^{-1} \frac{\partial V_{qu}}{\partial q} | dq}{\lambda_c{}^{-1} | \frac{\partial V_{qu}}{\partial q} |_{(q = \lambda_c)}} \,, \qquad \text{(B.5)}$$

warrants the vanishing of QP at large distance and, hence, it can be assumed as an evaluation of the quantum potential range of interaction.

It is worth mentioning that condition (B.4) is not satisfied by linear systems whose eigenstates have $\phi = -1$ (22), so that $\lambda_q = \infty$ and they cannot admit the classical limit.

It is also worth noting that condition (B.4), obtained for $n^{1/2}$ owing the form (B.1), also holds in the case of oscillating wave functions whose modulus is of type:

$$\lim_{|q| \to \infty} n^{1/2} = | q^m \sum_n a_n \, exp[iA_n{}^p{}_{(q)}] \} exp[-P^k{}_{(q)}] | \qquad \text{(B.6)}$$

where $A_n{}^p{}_{(q)}$ are polynomials of degree equal to *p*. In this case, in addition to the requisite $0 \leq k < \frac{3}{2}$, the conditions $m \in \Re$ and $p \leq 1$ are required to warrant (B.4) (38).

For instance, the Lennard-Jones-type potentials holds $\lim_{|q| \to \infty} A_n{}^p{}_{(q)(q)} \propto q$ and, hence, they own λ_q finite.

In the multidimensional case, λ_q depends by the path of integration Σ and (B.5) reads:

$$\lambda_q = 2 \frac{\int_{\Sigma_{r=0}}^{\Sigma_{r=\infty}} r_{(\Sigma)}{}^{-1} | \frac{\partial V_{qu}}{\partial q_i} | \bullet d\Sigma_i}{\lambda_c{}^{-1} | \frac{\partial V_{qu}}{\partial r} |_{(r_{(\Sigma)} = \lambda_c)}} \qquad \text{(B.7)}$$

where $r = |q|$ and $d\Sigma_i$ is the incremental vector tangent to Σ.

Since, the physical meaning of λ_q must be independent by the path of integration (we know that $\frac{\partial V_{qu}}{\partial q_i}$ is integrable but do we do not know nothing about the integrability of $r_{(\Sigma)}^{-1} | \frac{\partial V_{qu}}{\partial q_i} |$) in order to well define λ_q the fixation of the integral path is needed. If we choose the integration path $\Sigma = r m_i$ where m_i is a generic versor, λ_q reads:

$$\lambda_{q(m_i)} = 2 \frac{\int_{r=0}^{\infty} r^{-1} | \frac{\partial V_{qu}}{\partial r} |_{(q = r m_i)} dr}{\lambda_c^{-1} | \frac{\partial V_{qu}}{\partial r} |_{(q = \lambda_c m_i)}} \tag{B.8}$$

Moreover, since in order to evaluate at what distance the quantum force becomes negligible whatever is the direction of the versor m_i, among the values of (B.8) we must consider the maximum one so, finally, λ_q reads:

$$\lambda_q = Max \left\{ 2 \frac{\int_{r=0}^{\infty} r^{-1} | \frac{\partial V_{qu}}{\partial r} |_{(q=rm_i)} dr}{\lambda_c^{-1} | \frac{\partial V_{qu}}{\partial r} |_{(q=\lambda_c m_i)}} \right\} \tag{B.9}$$

B.1: QUANTUM potential characteristics

In order to elucidate the interplay between the Hamiltonian potential and the quantum potential, that together define the quantum evolution of the particle, we observe that the quantum potential is primarily defined by the PD.

Fixed the PD at the initial time, then the Hamiltonian potential and the quantum one determine the evolution of the PD in the following instants that on its turn modifies the quantum potential.

A Gaussian PD has a parabolic repulsive quantum potential, if the Hamiltonian potential is parabolic too (the free case is included), when the PD wideness adjusts itself to produce a quantum potential that exactly compensates the force of the Hamiltonian one, the Gaussian states becomes stationary (eigenstates). In the free case, the stationary state is the flat Gaussian (with an infinite variance) so that any free Gaussian PD expands itself following the ballistic dynamics of quantum mechanics since the Hamiltonian potential is null and the quantum one is a quadratic repulsive one.

From the general point of view, we can say that if the Hamiltonian potential grows faster than a harmonic one, the wave equation of a self-state is more localized than a Gaussian one and this leads to a stronger-than a quadratic quantum potential.

On the contrary, a Hamiltonian potential that grows slower than a harmonic one will produce a less localized PD that decreases slower than the Gaussian one, so that the quantum potential is weaker than the quadratic one and it may lead to a finite quantum non-locality length (B.5).

More precisely, as shown above, the large distances exponential-decay of the PD given by (B.1) with *k*<3/2 is a sufficient condition to have a finite quantum non-locality length (20).

In absence of noise, we can identify three typologies of quantum potential interactions (in the uni-dimensional case): *k*> 2 strong quantum potential that leads to quantum force that grows faster than linearly and λ_q is infinite (*super-ballistic* expansion for the free particle PD) and reads:

$$\lim_{|q| \to \infty} \frac{\partial V_{qu}}{\partial q} \propto q^{1+\varepsilon} . \ (\varepsilon > 0) \tag{B.10}$$

k = 2 that leads to quantum force that grows linearly

$$\lim_{|q| \to \infty} \frac{\partial V_{qu}}{\partial q} \propto q \tag{B.11}$$

and λ_q is infinite (*ballistic* expansion for the free particle PD); 2 > *k* ≥ 3/2 "middle quantum potential"; the integrand of (B.4) will result:

$$Cons\ tan\ t > \lim_{|q| \to \infty} | q^{-1} \frac{\partial V_{qu}}{\partial q} | > q^{-1} . \tag{B.12}$$

The quantum force remains finite or even becomes vanishing at large distance but λ_q may be still infinite (*under-ballistic* expansion for the free particle PD).

k < 3/2 "week quantum potential" interaction leading to quantum force that becomes vanishing at large distance following the asymptotic behavior:

$$\lim_{|q| \to \infty} | q^{-1} \frac{\partial V_{qu}}{\partial q} | > q^{-(1+\varepsilon)} , \varepsilon > 0 \tag{B.13}$$

with a finite λ_q for T ≠ 0 (*asymptotically vanishing* expansion for the free particle PD).

B.2 Pseudo-Gaussian particle

Gaussian particles generate a quadratic quantum potential that is not vanishing at large distance and hence cannot lead to macroscopic local dynamics. Nevertheless, imperceptible deviation by the perfect Gaussian PD may possibly lead to finite quantum non-locality length. Particles that are inappreciably less localized than the Gaussian ones (let's name them as pseudo-Gaussian) own $\frac{\partial V_{qu}}{\partial q}$ that can sensibly deviate by the linearity so that the quantum non-locality length may be finite.
We have seen above that for *k* < 3/2 (when the PD decreases slower than a Gaussian) a finite range of interaction of the quantum potential λ_q is possible.

The Gaussian shape is a physically good description of particle localization, but irrelevant deviations from it, at large distance, are decisive to determine the quantum non-locality length.

For instance, let's consider the pseudo-Gaussian wave-function type:

$$n = n_0 \, exp[-\frac{(q-\underline{q})^2}{\underline{\Delta q}^2 [1+[\frac{(q-\underline{q})^2}{\Lambda^2 f(q-\underline{q})}]]}] \quad (B.14)$$

where $f(q-\underline{q})$ is an opportune regular function obeying to the condition:

$$\Lambda^2 f(0) >> \underline{\Delta q}^2 \text{ and } lim_{|q-\underline{q}|\to\infty} f(q-\underline{q}) << \frac{(q-\underline{q})^2}{\Lambda^2}. \quad (B.15)$$

For small distance it holds

$$(q-\underline{q})^2 << \Lambda^2 f(q-\underline{q}) \quad (B.16)$$

and the localization given by the PD is physically indistinguishable from a Gaussian one, while for large distance we obtain the behavior:

$$lim_{|q-\underline{q}|\to\infty} n = n_0 \, exp[-\frac{\Lambda^2 f(q-\underline{q})}{\underline{\Delta q}^2}] . \quad (B.17)$$

For instance, we may consider the following examples

a)

$$f(q-\underline{q}) = 1 \quad (B.18)$$

$$lim_{|q-\underline{q}|\to\infty} n = n_0 \, exp[-\frac{\Lambda^2}{\underline{\Delta q}^2}] ; \quad (B.19)$$

(b)

$$f(q-\underline{q}) = 1 + |q - \underline{q}| \quad (B.20)$$

$$lim_{|q-\underline{q}|\to\infty} n = n_0 \, exp[-\frac{\Lambda^2 |q-\underline{q}|}{\underline{\Delta q}^2}] ; \quad (B.21)$$

(c)

$$f(q-\underline{q}) = 1 + ln[1+|q-\underline{q}|^g] \approx ln[|q-\underline{q}|^g] \; (0<g<2) \quad (B.22)$$

$$lim_{|q-\underline{q}|\to\infty} n \approx n_0 |q-\underline{q}|^{-g\frac{\Lambda^2}{\underline{\Delta q}^2}} ; \quad (B.23)$$

(d)

$$f(q-\underline{q}) = 1 + |q-\underline{q}|^g \; (0< g <2) \quad (B.24)$$

$$lim_{|q-\underline{q}|\to\infty} n = n_0 \, exp[-\frac{\Lambda^2 |q-\underline{q}|^g}{\underline{\Delta q}^2}] \quad (B.25)$$

All cases (a-d) lead to a finite quantum non-locality length λ_q .

In the case (d) the quantum potential for $|q-\underline{q}| \to \infty$ reads:

$$lim_{|q-\underline{q}|\to\infty} V_{qu} = lim_{|q-\underline{q}|\to\infty} -(\frac{\hbar^2}{2m})|\psi|^{-1} \nabla_q \cdot \nabla_q |\psi| = -(\frac{\hbar^2}{2m})[\frac{\Lambda^4 g^2 (q-\underline{q})^{2(h-1)}}{(2\underline{\Delta q}^2)^2} - \frac{\Lambda^2 g(g-1)(q-\underline{q})^{g-2}}{2\underline{\Delta q}^2}] \quad (0<g<2) \; (B.26)$$

leading, for 0< *g* <2, to the quantum force:

$$lim_{|q-\underline{q}|\to\infty} -\nabla_q V_{qu} = (\frac{\hbar^2}{2m})[\frac{\Lambda^4 g^2 (2g-1)(q-\underline{q})^{2g-3}}{(2\underline{\Delta q}^2)^2} - \frac{\Lambda^2 g(g-1)(g-2)(q-\underline{q})^{g-3}}{2\underline{\Delta q}^2}] \quad (B.27)$$

that for *g* < 3/2 gives $lim_{|q-\underline{q}|\to\infty} -\nabla_q V_{qu} = 0$. It is interesting to note that for *g* =2 (linear case):

$$|\psi| = n^{1/2} = n_0^{1/2} \, exp[-\frac{(q-\underline{q})^2}{2\underline{\Delta q^2}}] \qquad \text{(B.28)}$$

the quantum potential is quadratic

$$lim_{|q-\underline{q}|\to\infty} V_{qu} = -(\frac{\hbar^2}{2m})[\frac{(q-\underline{q})^2}{(\underline{\Delta q^2})^2} - \frac{1}{\underline{\Delta q^2}}] \,, \qquad \text{(B.29)}$$

and the quantum force is linear (repulsive) and reads

$$lim_{|q-\underline{q}|\to\infty} -\nabla_q V_{qu} = (\frac{\hbar^2}{2m})[\frac{2(q-\underline{q})}{(\underline{\Delta q^2})^2}] \qquad \text{(B.30)}$$

The linear form of the force exerted by the quantum potential leads to the ballistic expansion (variance that grows linearly with time) of the free Gaussian quantum states.