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Full Length Research Paper

Vibration reduction and stability study of a dynamical system under multi-excitation forces via active absorber

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An active vibration absorber for suppressing the vibration of the nonlinear system when subjected to external and parametric excitations forces will be studied. The aim of this paper is to study the effect of the nonlinear controller on the vibrating system. The approximate solutions up to the second order are derived using the method of multiple scale perturbation technique near the primary, principal parametric and internal resonance case. The stability of the solution is investigated using both phase plane methods and frequency response equations. The effects of different parameters on the vibration of the system are investigated. The reported results are compared to the available published work.

Key words: Active control method, perturbation method, stability, resonance.

INTRODUCTION

In weakly non-linear systems, internal resonances may occur if the linear natural frequencies are commensurate or nearly commensurate, and internal resonances provide coupling and energy exchange among the vibration modes (Nayfeh and Mook, 1979; Pai and Nayfeh, 1991). In the domain of mechanical vibration research, dynamic absorbers have extensive application in reducing vibrations of machinery. Nayfeh and Mook (1979) studied the saturation phenomenon when the natural frequencies of the system and controller are in the ratio 2:1. When the system is excited at a frequency near the high natural frequency, the structure responds at the frequency of the excitation and the amplitude of the response increases with the excitation amplitude (Golnaraghi, 1991). However, when the high-frequency model amplitude reaches a critical value, this mode saturates and all additional energy added to the system via increasing the excitation amplitude overflows into the low-frequency mode. Recently the use of internal resonance and saturation phenomenon in non-linear control has been extensively demonstrated by Park et al. (1993) and Oueini and Nayfeh (1996a). They used a second-order controller coupled to a linear vibrating system via quadratic or cubic terms. Oueini and Gonaraghi (1996) and Oueini et al. (1997) used the saturation phenomenon to successfully control the motion of a d. c. motor with a rigid beam attached. Pai et al. (2000) presented the study of controlling steady-state vibrations of a cantilever skewed plate using saturation phenomenon due to higher-order internal resonance. PZT patches were used as control actuators and sensors. Linear second order controllers were designed to couple with the plate via different orders of non-linear terms to establish energy bridges between the plate and controllers. Each linear second-order controller was designed to have 1:2 or 1:3 or 1:2:4 internal resonances with one of the plates vibration modes and hence was able to exchange energy with the plate around the specific modal frequency. Ashour and Nayfeh (2002) studied non-linear adaptive control of flexible structures using the saturation phenomenon. This phenomenon was utilized to suppress high-amplitude bending and torsional vibration modes of rectangular cantilever plates. A 2:1 internal resonance condition was maintained between the plant and the controller. Energy was transferred completely from one part to the other. When the plant was forced at resonance, this energy-transfer mechanism

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limited the response of the plant. Sayam et al. (2005) studied a numerical simulation of the response of a uniform, cantilever beam subjected to base excitation. A saturation absorber was implemented to control the beam properties that were introduced when piezoelectric actuators were bonded to the uniform beam. The resulting coupling between uniform, cantilever beam modes was fully included in the analytical model. It was shown that this model coupling had a significant effect on the beam response, which was not present when modal coupling was neglected. Maccari (2006) applied a new vibration control method for time delay non-linear oscillators to the principle resonance of a parametrically excited Lienard system under state feedback control with a time delay. Vibration control can be successfully performed. He demonstrated that the time delay and the feedback gains can enhance the control performance, reduce the amplitude peak and suppress the quasi-periodic motion. Awrejcewicz and Pyryev (2006e) studied a mathematical model of a two-degree-of-freedom system. A novel thermodynamic model of frictional self-excited strike-slip vibrations is proposed. A mathematical system consisting of two masses which are coupled by an elastic spring and moving vertically between two walls is considered. The applied friction force depends on the relative velocity of the sliding bodies. Stability of stationary solutions is considered. Ji and Hansen (2006) investigated the effect of time delay involved in a non-linear feedback control on the stability of the trivial equilibrium of a vander Pol-Duffing oscillator. They have been discussed using linear stability analysis, center manifold technique, normal forms as well as a perturbation method. El-Bassiony (2006) used a non-linear elastromeric damper or absorber to control the torsional vibrations of the crank shaft in internal combustion engines when subjected to both external and parametric excitation torque. He deduced that the damping coefficients of the crankshaft greatly affects system behavior. The smaller or more negative damping factor, the worst behavior of the system, as it leads to larger steady-state amplitudes and dynamic chaos or instability for both crankshaft and absorber. Larger magnitudes of absorber non-linearities reduce the absorber effectiveness. Jun et al. (2007), introduced the nonlinear saturation-based control strategy for the suppression of the self-excited vibration of a van der Pol oscillator. It is demonstrated that the saturation-based control method is effective in reducing the vibration response of the self-excited plant when the absorber's frequency is exactly tuned to one-half the natural frequency of the plant. Eissa and Sayed (2006a, b; 2008), studied the effects of different active controllers on simple and spring pendulum at the primary resonance via negative velocity feedback or its square or cubic. Eissa et al. (2005a, b), studied mathematically the vibrations of a cantilever beam or the aircraft wing and investigated the saturation phenomena that suppresses these vibrations at one of the extracted resonance cases. Also, Eissa et al. (2005c; 2006a, d), have studied both passive and active controllers of the vibrating systems of an aircraft wing. Furthermore, active control method has been applied to suppress steady state vibrations of different dynamical systems at the primary and different internal resonance ratios (Eissa et al., 2006; El-Serafi et al., 2006c). Sayed and Kamel (2011a) investigated the effect of linear absorber on the vibrating system and the saturation control of a linear absorber to reduce vibrations due to rotor blade flapping motion. The stability of the obtained numerical solution is investigated using both phase plane methods and frequency response equations. Variation of some parameters leads to the bending of the frequency response curves and hence to the jump phenomenon occurrence. Sayed and Hamed (2011b) investigated the response of a two-degree-of-freedom system with quadratic coupling under parametric and harmonic excitations. The method of multiple scale perturbation method is applied to obtain approximate solutions up to and including the second-order approximations. Sayed and Kamel (2012) applied active control for suppressing the vibration of the non-linear plant when subjected to external and parametric excitations in the presence of 1:2 and 1:3 internal resonance. The method of multiple scale perturbation technique is applied to determine four first-order non-linear ordinary differential equations that govern the modulation of the amplitudes and phases in the presence of internal resonance of the two systems with quadratic and cubic order of control. These equations were used to determine the steady state solutions and their stability. The stability study of non-linear periodic solution for the two considered resonance cases and the stability of the obtained numerical solution are investigated using frequency, force–response curves and phase-plane method.

In this paper we added an active non-linear vibration controller to the system with internal resonances case using quadratic and cubic terms. The multiple time scale perturbation technique is applied throughout. An approximate solution is derived. The stability of the system is investigated applying both frequency response functions (FRFs) and phase-plane methods. The effects of the controller on the system behavior are studied numerically. Optimum working conditions of the system are obtained when applying active control method.

MATHEMATICAL ANALYSIS

The considered equation is the modified non-linear differential equation describing the vibration of an air craft wing which is given by:

$$\left.\begin{aligned} &\ddot{X}_1 + 2\varepsilon\zeta_1\omega_1\dot{X}_1 + \omega_1^2 X_1 = \varepsilon\beta_1 X_1^2 X_2 \\ &\ddot{X}_2 + 2\varepsilon\zeta_2\omega_2\dot{X}_2 + \varepsilon\alpha_1 X_2^2 + \varepsilon\alpha_2 X_2^3 + \omega_2^2 X_2 = \varepsilon\beta_2 X_1^3 + \varepsilon F\cos(\Omega t) + \varepsilon x_2 F_1 \sin(\Omega_1 t) \end{aligned}\right\} \quad (1)$$

where X_1 denotes the response of a second-order controller, ω_1 is the natural frequency of the controller, ζ_1 is the damping coefficient of the controller, X_2 represents one of the modal co-ordinates of the wing, ω_2 is the modal frequency, ζ_2 is the damping ratio, β_1 and β_2 are positive gains constants, F, F_1 are the amplitudes of the external excitation force, Ω and Ω_1 are the external excitation frequencies, α_1 and α_2 are quadratic and cubic non-linear parameters respectively, and t is the time. We seek a second-order approximate solution of Equation (1) applying multiple time scales method in the form

$$\left.\begin{aligned} X_1(t;\varepsilon) &= x_{10}(T_0,T_1,T_2)+\varepsilon x_{11}(T_0,T_1,T_2)+\varepsilon^2 x_{12}(T_0,T_1,T_2) \\ X_2(t;\varepsilon) &= x_{20}(T_0,T_1,T_2)+\varepsilon x_{21}(T_0,T_1,T_2)+\varepsilon^2 x_{22}(T_0,T_1,T_2) \end{aligned}\right\}, \quad (2)$$

where $T_n = \varepsilon^n t$ we have $n=(0,1,2)$ where T_0 is the fast time scale and T_1, T_2 are the slow time scales respectively. The derivatives will be in the form

$$\begin{aligned} \frac{d}{dt} &= D_0+\varepsilon D_1+\varepsilon^2 D_2, \\ \frac{d^2}{dt^2} &= D_0^2+2\varepsilon D_0 D_1+\varepsilon^2(D_1^2+2D_0D_2), \end{aligned} \quad (3)$$

where $D_n = \frac{\partial}{\partial T_n}, n=0,1,2$. Substituting Equations (2) and (3) into Equation (1) and equating the coefficients of similar powers of ε, one obtain the following set of ordinary differential equations:

$O(\varepsilon^0)$:

$$(D_0^2+\omega_1^2)x_{10}=0 \quad (4)$$

$$(D_0^2+\omega_2^2)x_{20}=0 \quad (5)$$

$$O(\varepsilon^1): (D_0^2+\omega_1^2)x_{11} = -2D_0D_1x_{10}-2D_0\zeta_1\omega_1x_{10}+\beta_1x_{10}^2x_{20} \quad (6)$$

$$(D_0^2+\omega_2^2)x_{21} = -2D_0D_1x_{20}-2D_0\zeta_2\omega_2x_{20}-\alpha_1x_{20}^2-\alpha_2x_{20}^3+\beta_2x_{10}^2+F\cos(\Omega T_0) +x_{20}F_1\sin(\Omega_1 T) \quad (7)$$

The general solution of Equations (4) and (5) are given in the form

$$\left.\begin{aligned} x_{10} &= A_1\exp(i\,\omega_1T_0)+cc, \\ x_{20} &= A_2\exp(i\,\omega_2T_0)+cc \end{aligned}\right\}, \quad (8)$$

where A_1 and A_2 are a complex function in T_1 and cc represents the complex conjugate. Substituting Equation (8) into Equations (6) and (7), yields

$$\begin{aligned} (D_0^2+\omega_1^2)x_{11} &= -2i\,\omega_1D_1A_1\exp(i\,\omega_1T_0)-2i\,\omega_1^2\zeta_1A_1\exp(i\,\omega_1T_0) \\ &+\beta_1\left[2A_1\bar{A}_1A_2\exp(i\,\omega_2T_0)+A_1^2A_2\exp(i\,(2\omega_1+\omega_2)T_0) \right. \\ &\left. +A_1^2\bar{A}_2\exp(i\,(2\omega_1-\omega_2)T_0)\right]+cc \end{aligned} \quad (9)$$

$$\begin{aligned} (D_0^2+\omega_2^2)x_{21} &= -2i\,\omega_2D_1A_2\exp(i\,\omega_2T_0)-2i\,\omega_2^2\zeta_2A_2\exp(i\,\omega_2T_0) \\ &-\alpha_1A_2^2\exp(2i\,\omega_2T_0)-\alpha_2A_2^3\exp(3i\,\omega_2T_0)-3\alpha_2A_2^2\bar{A}_2\exp(i\,\omega_2T_0) \\ &-\alpha_1A_2\bar{A}_2+\beta_2\left[A_1^3\exp(3i\,\omega_1T_0)+3A_1^2\bar{A}_1\exp(i\,\omega_1T_0)\right]+\frac{F}{2}\exp(i\,\Omega T_0) \\ &+\frac{A_2F_1}{2i}\exp(i\,(\Omega_1+\omega_2)T_0)+\frac{\bar{A}_2F_1}{2i}\exp(i\,(\Omega_1-\omega_2)T_0)+cc \end{aligned} \quad (10)$$

For a bounded solution of Equations (9) and (10), the coefficients of the secular terms should be eliminated, and then the non-homogeneous solutions are given by:

$$\begin{aligned} x_{11} &= \frac{2\beta_1A_1\bar{A}_1A_2}{\omega_1^2-\omega_2^2}\exp(i\,\omega_2T_0)+\frac{\beta_1A_1^2A_2}{\omega_1^2-(2\omega_1+\omega_2)^2}\exp(i\,(2\omega_1+\omega_2)T_0) \\ &+\frac{\beta_1A_1^2\bar{A}_2}{\omega_1^2-(2\omega_1-\omega_2)^2}\exp(i\,(2\omega_1-\omega_2)T_0)+cc \end{aligned} \quad (11)$$

$$\begin{aligned} x_{21} &= \frac{\alpha_1A_2^2}{3\omega_2^2}\exp(2i\,\omega_2T_0)+\frac{\alpha_2A_2^3}{8\omega_2^2}\exp(3i\,\omega_2T_0)-\frac{\alpha_1A_2\bar{A}_2}{\omega_2^2} \\ &+\frac{\beta_2A_1^3}{\omega_2^2-9\omega_1^2}\exp(3i\,\omega_1T_0)+\frac{3\beta_2A_1^2\bar{A}_1}{\omega_2^2-\omega_1^2}\exp(i\,\omega_1T_0)+\frac{F}{2(\omega_2^2-\Omega^2)}\exp(i\,\Omega_1T_0) \\ &+\frac{A_2F_1}{2i\,(\omega_2^2-(\Omega_1+\omega_2)^2)}\exp(i\,(\Omega_1+\omega_2)T_0)-\frac{\bar{A}_2F_1}{2i\,(\omega_2^2-(\Omega_1-\omega_2)^2)}\exp(i\,(\Omega_1-\omega_2)T_0)+cc \end{aligned} \quad (12)$$

From the derived solutions (11 and 12), the resonance cases are reported:

1. Primary resonance case: $\Omega \cong \omega_2$, $\Omega_1 \cong \omega_2$.
2. Sub-harmonic resonance case: $\Omega_1 \cong 2\omega_2$, $\Omega_1 \cong 3\omega_2$, $\Omega_1 \cong 4\omega_2$, $\Omega \cong 2\omega_2$, $\Omega \cong 3\omega_2$.
3. Internal resonance case: $\omega_2 \cong \omega_1$, $\omega_2 \cong 3\omega_1$, $\omega_1 \cong 2\omega_2$, $3\omega_2 \cong \omega_1$, $\omega_2 \cong 2\omega_1$.
4. Simultaneous resonance case: Any combination of the above resonance cases is considered as simultaneous or incident resonance case.

STABILITY OF THE SYSTEM

After studying numerically the different resonance cases, one of the worst cases has been chosen to study the system stability. The selected resonance case is primary case where $\Omega \cong \omega_2, \Omega_1 \cong 2\omega_2$ and internal resonance $\omega_2 \cong 3\omega_1$. In this case we introduce detuning parameters σ_1 and σ_2 such that

$$\omega_2 = 3\omega_1 + \varepsilon\sigma_1, \ \Omega = \omega_2 + \varepsilon\sigma_2 \text{ and } \Omega_1 = 2\omega_2 + 2\varepsilon\sigma_2 \quad (13)$$

Eliminating the secular terms of Equations (9) and (10), leads to the solvability conditions for the first order approximation as follows

$$-2i\,\omega_1 D_1 A_1 - 2i\,\omega_1^2 \zeta_1 A_1 + \beta_1 \bar{A}_1^2 A_2 e^{i\sigma_1 T_1} = 0, \quad (14)$$

$$-2i\,\omega_2 D_1 A_2 - 2i\,\omega_2^2 \zeta_2 A_2 - 3\alpha_2 A_2^2 \bar{A}_2 + \beta_2 A_1^3 e^{-i\sigma_1 T_1} + \frac{F}{2} e^{i\sigma_2 T_1} + \frac{\bar{A}_2 F_1}{2i} e^{2i\sigma_2 T_1} = 0 \quad (15)$$

Putting

$$A_n = \frac{a_n}{2} e^{i\gamma_n}, \ n = 1, 2., \quad (16)$$

where a_n and γ_n are the steady state amplitudes and the phases of the motions respectively. Substituting Equation (16) into Equations (14) and (15) and equating the real and imaginary parts we obtained:

$$a_1' = -\zeta_1 \omega_1 a_1 + \frac{\beta_1 a_1^2 a_2}{8\omega_1} \sin\theta_1, \quad (17)$$

$$\left(\frac{\theta_1' + \theta_2'}{3}\right) a_1 = \left(\frac{\sigma_1 + \sigma_2}{3}\right) a_1 + \frac{\beta_1 a_1^2 a_2}{8\omega_1} \cos\theta_1, \quad (18)$$

$$a_2' = -\zeta_2 \omega_2 a_2 + \frac{F}{2\omega_2} \sin\theta_2 - \frac{a_2 F_1}{4\omega_2} \cos 2\theta_2 - \frac{\beta_2 a_1^3}{8\omega_2} \sin\theta_1, \quad (19)$$

$$\theta_2' a_2 = \sigma_2 a_2 - \frac{3\alpha_2 a_2^3}{8\omega_2} + \frac{F}{2\omega_2} \cos\theta_2 + \frac{a_2 F_1}{4\omega_2} \sin 2\theta_2 + \frac{\beta_2 a_1^3}{8\omega_2} \cos\theta_1, \quad (20)$$

where $\theta_1 = \sigma_1 T_1 - 3\gamma_1 + \gamma_2$ and $\theta_2 = \sigma_2 T_1 - \gamma_2$.

STEADY STATE SOLUTIONS

The periodic motions are obtained when $a_n' = \theta_n' = 0$. Hence, the fixed points of Equations (17) to (20) are given by

$$-\zeta_1 \omega_1 a_1 + \frac{\beta_1 a_1^2 a_2}{8\omega_1} \sin\theta_1 = 0, \quad (21)$$

$$\left(\frac{\sigma_1 + \sigma_2}{3}\right) a_1 + \frac{\beta_1 a_1^2 a_2}{8\omega_1} \cos\theta_1 = 0, \quad (22)$$

$$-\zeta_2 \omega_2 a_2 - \frac{\beta_2 a_1^3}{8\omega_2} \sin\theta_1 + \frac{F}{2\omega_2} \sin\theta_2 - \frac{a_2 F_1}{4\omega_2} \cos 2\theta_2 = 0, \quad (23)$$

$$\sigma_2 a_2 - \frac{3\alpha_2 a_2^3}{8\omega_2} + \frac{\beta_2 a_1^3}{8\omega_2} \cos\theta_1 + \frac{F}{2\omega_2} \cos\theta_2 + \frac{a_2 F_1}{4\omega_2} \sin 2\theta_2 = 0 \quad (24)$$

In Equations (21) to (24) two cases ($a_1 = 0, \ a_2 \neq 0$) were used, the frequency response equation was obtained in the following form;

$$\left(\sigma_2 - \frac{3\alpha_2 a_2^2}{8\omega_2}\right)^2 + \left(\zeta_2 \omega_2\right)^2 - \left(\frac{F}{2\omega_2 a_2}\right)^2 - \left(\frac{F_1}{4\omega_2}\right)^2 = 0. \quad (25)$$

NUMERICAL SOLUTION

The Runge-Kutta of fourth order method was applied to determine the numerical solution of the given system of (1). Figure 1 shows that the steady state response without controller at primary resonance $\Omega \cong \omega_2$ is about 9 times of the maximum amplitude F, the system is stable with limit cycle.

Effects of the controller

Figure 2 illustrates the results when the controller is effective, when $\Omega \cong \omega_2, \Omega_1 \cong 2\omega_2$ and $\omega_2 \cong 3\omega_1$. The effectiveness of the controller is E_a (E_a = steady state amplitude of the main system without controller / steady state amplitude of the main system with controller) is about 6. The oscillation of the system becomes tuned with multi limit cycle and the oscillation of the controller becomes stable with limit cycle.

Frequency response curve

In this section, the steady state response of the given system at various parameters near the simultaneous primary, parametric and internal resonance case is investigated and studied. Results are presented in graphical forms as steady state amplitudes against detuning parameters for both the system and the controller.

For the case, $a_2 \neq 0$ and $a_1 = 0$, Figure 3 shows that the steady state amplitude of the main system against the detuning parameter σ_2 as a basic case. Figure 4 shows that the steady state amplitude is monotonic decreasing function in damping coefficient ζ_2 and the curve bent to the right leading to the occurrence of the jump phenomena. The steady state amplitude of the main

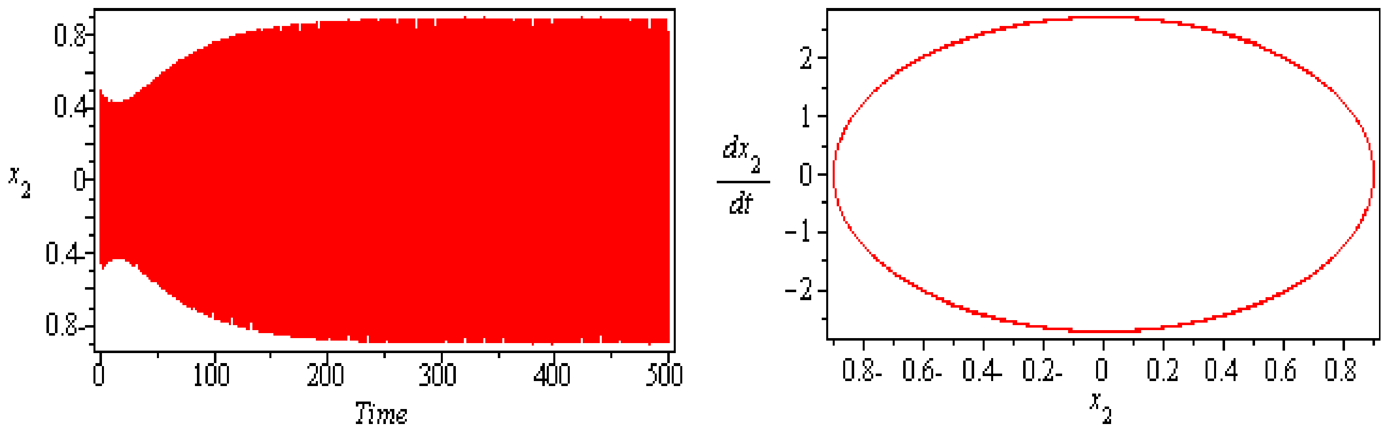

Figure 1. System behavior without controller at simultaneous primary and principle parametric resonance $\Omega \cong \omega_2, \Omega_1 \cong 2\omega_2$.

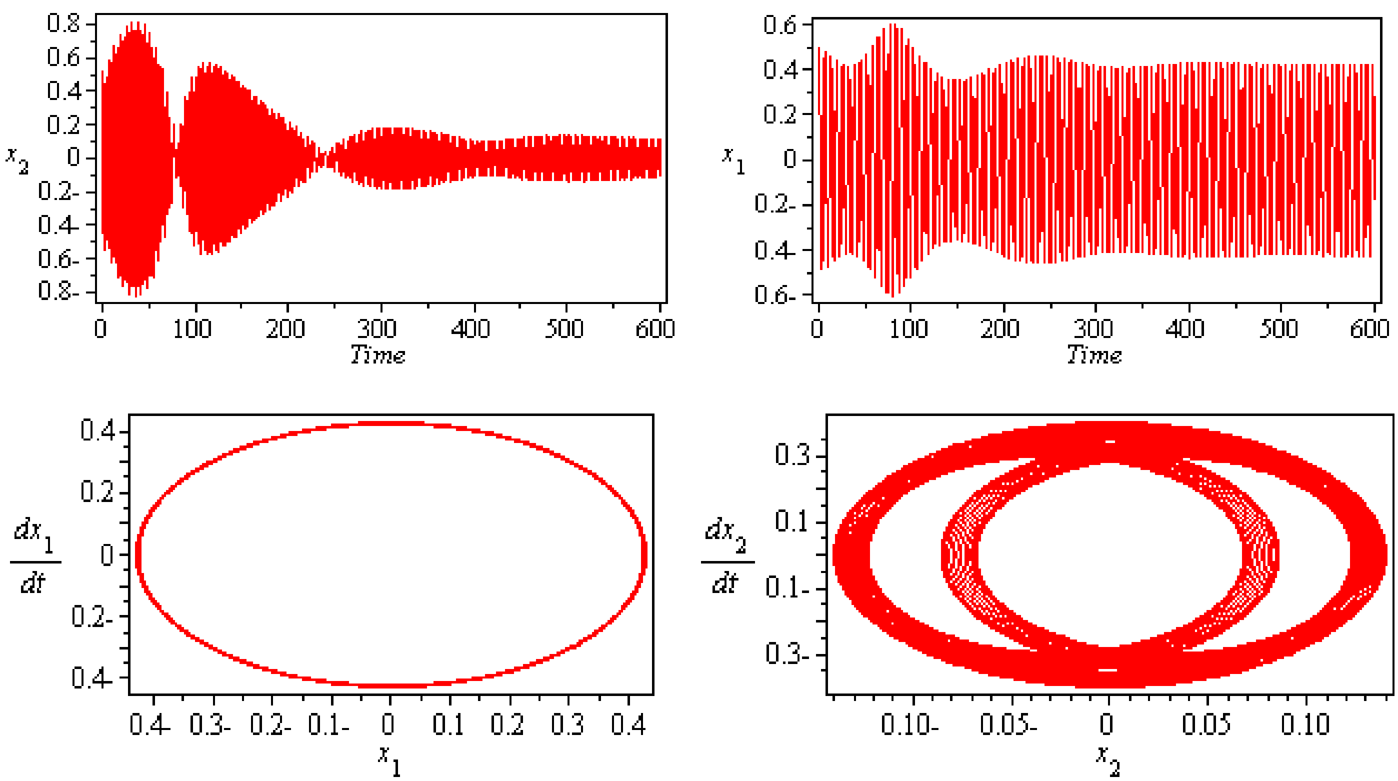

Figure 2. System behavior with controller at simultaneous primary, parametric and internal resonance $\Omega \cong \omega_2, \Omega_1 \cong 2\omega_2, \omega_2 \cong 3\omega_1$.

system is monotonic decreasing function in ω_2, also for the decreasing natural frequency ω_2, the curve bent to the right leading to the occurrence of the jump phenomena as shown in Figure 5. For the positive and negative values of α_2, produce either hard or soft spring respectively as the curve is either bent to the right or the left, leading to the appearance of the jump phenomena as in Figure 6. Figures 7(a) and (b) show that the steady state amplitude of the main system is a monotonic

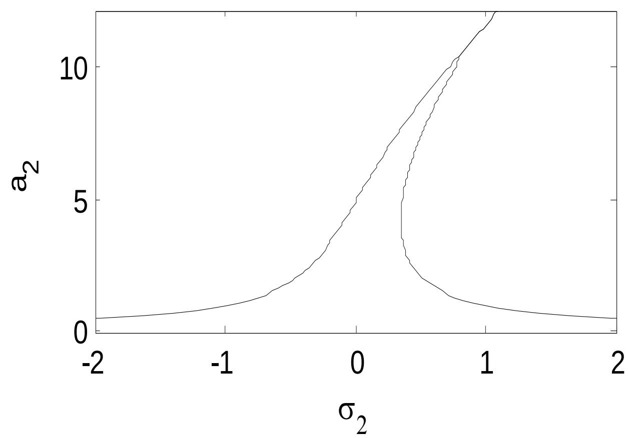

Figure 3. Effects of the detuning parameter σ_2 .

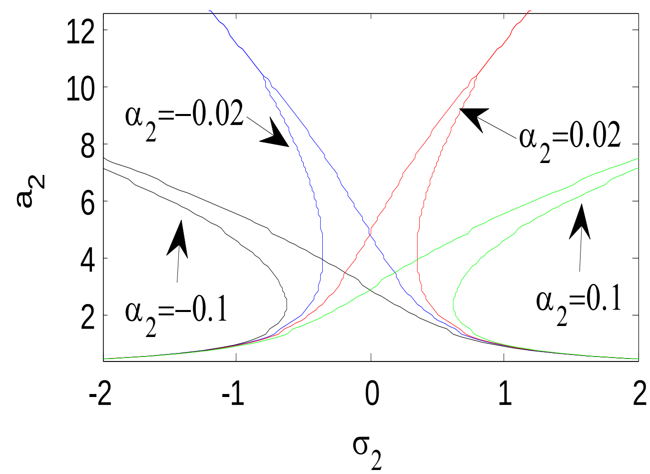

Figure 6. Effects of the non-linear parameter α_2

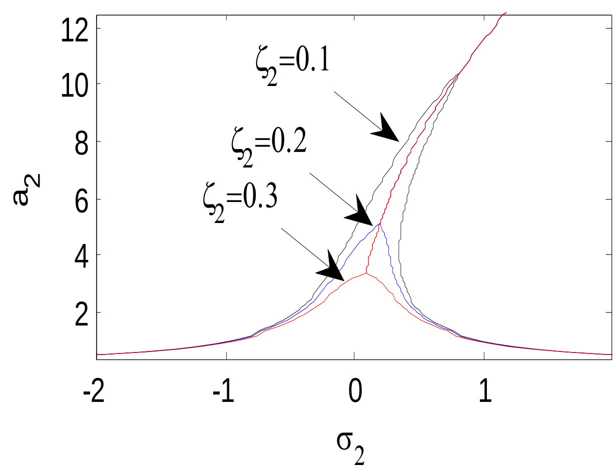

Figure 4. Effects of the damping coefficient ζ_2 .

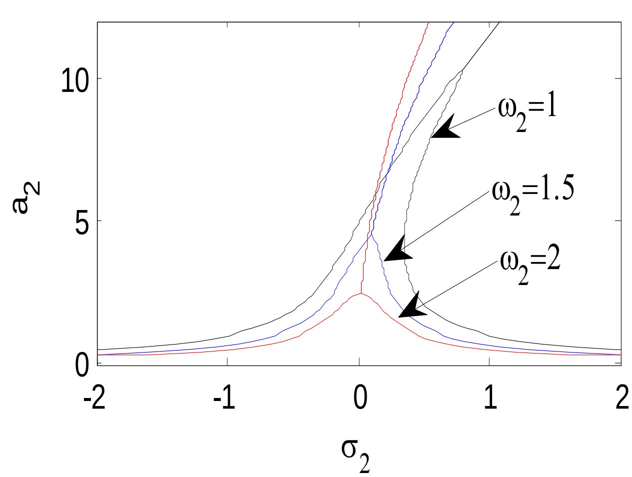

Figure 5. Effects of the natural frequency ω_2 .

increasing function in the excitation amplitudes f and f_1.

Conclusions

The main system of non-linear differential equation a single-degree-of-freedom system subjected to parametric and external excitations is considered with controller $\omega_2 : \omega_1 = 3:1$ and solved using multiple time scale perturbation method. The numerical solution is derived up to the third order approximation. The stability is obtained and studied applying both FRF and phase-plane methods. From this study the following may be concluded.

1. The steady state amplitude is a monotonic decreasing function in the damping coefficients ζ_1 and the natural frequencies ω_2 .

2. The positive and negative values of α_2 , produce either hard or soft spring respectively as the curve is either bent to the right or the left, leading to the appearance of the jump phenomenon.

3. The steady state amplitude is monotonic increasing function in the excitation amplitudes F and F_1.

4. The steady state response without controller at primaryresonance $\Omega \cong \omega_2$, is about 9 times of the maximum amplitude f and the system is stable and free of dynamic chaos.

5. The effectiveness of the absorber is about $E_a = 6$ at primary resonance $\Omega = \omega_2$, sub-harmonic resonance $\Omega_1 = 2\omega_2$ and internal resonance $\omega_2 = 3\omega_1$ this mean that the vibration of the main system can be reduced via active control.

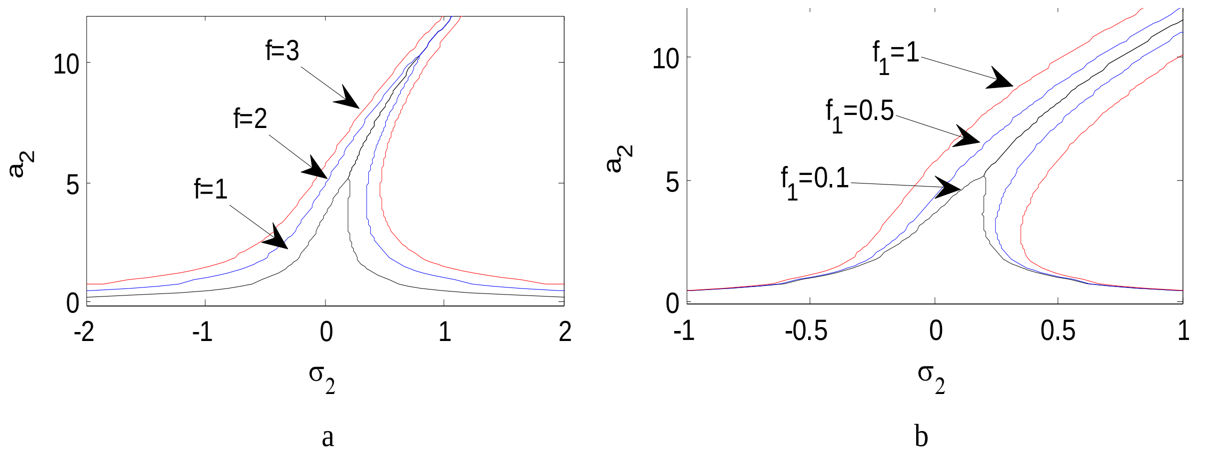

Figure 7. Effects of the excitation amplitude f (a) and f_1 (b).

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