International Journal of the Physical Sciences Vol. 6(3), pp. 340-357, 4 February, 2011
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS10.270
ISSN 1992 - 1950 ©2011 Academic Journals

Review

Multi-Dimensional coordinate spaces

Mario Arturo Ruiz Estrada

Faculty of Economics and Administration, University of Malaya, 50603 Kuala Lumpur, Malaysia.
E-mail: marioruiz@um.edu.my. Tel: +006012-6850293.

Accepted 05 November, 2010

This paper introduces a new group of multi-dimensional coordinate spaces. The main objective is to visualize n-dimensions in the same graphical space and time. This paper is divided into three parts. The first part shows the literature review about dimension and coordinate systems (Cartesian plane and coordinate space). The second part will present how the multi-dimensional coordinate spaces works. The third part of this paper shows fourteen different multidimensional coordinate spaces which are: the pyramid coordinate space with five and infinite axes; the diamond coordinate space with ten and infinite axes; the 4-dimensional coordinate space in vertical and horizontal position; the 5-dimensional coordinate space in vertical and horizontal position; the infinity-dimensional coordinate space under general and specific approach; the inter-linkage coordinate space; the cube-wrap coordinate space; and the mega-surface coordinate space.

Key words: Econographicology, multi-dimensional graphs and multi-dimensional geometry.

A SHORT REVIEW ABOUT DIMENSION AND COORDINATE SYSTEMS

Initially, four basic definitions of dimension were reviewed. According to the first definition by Poincarė (1912), dimension is a space that is divided by a large number of sub-spaces. It meant “however we please” and “partitioned”. The second definition of dimension is given by Brouwer (1924). He defines dimension as “between any two disjoint compacta”. Brouwer makes references about the existence of a group of sub-sets in different sets. It is based on the uses of the topological dimension of a compact metric space based on the concept of cuts (Fedorchuk et al., 1999). The third definition of dimension is given by Urysohn and Menger (Shchepin, 1998), who define dimension as “between any compactum and a point not belonging to it”. These four authors share certain common concepts in their definitions of dimension, such as the uses of spaces, subspaces, sets, subsets, and as partitioned and cut concepts. This means that any dimension needs to be studied using the ideas of spaces/sets or sub-spaces/sub-sets or partitions/cuts.

The idea of dimension is complex and rather deep for the human mind to penetrate. This is because we need to often perform abstractions and parameterizations of time and space of any geometrical object that cannot be visualized in the real world (Inselberg and Dimsdale, 1994). Moreover, this part of my research proposes an alternative definition of dimension. According to this book, the term “dimension” can be defined “as the unique mega-space that is built by infinite general-spaces, sub-spaces and micro-spaces that are systematically inter-connected.” The process of visualizing different dimensions graphically is made possible by the use of coordinate systems. These coordinate systems can generate the graphical modeling frameworks to represent different dimension(s) in the same graphical space. The coordinate systems can be divided into two types:

Cartesian plane and coordinate space

The difference between Cartesian plane and coordinate space has its origins in the number of axes. The Cartesian plane type is based on the uses of two axes; the coordinate space type is based on the use of three or more axes. Therefore, the Cartesian plane and coordinate space types are available to generate an idea about dimension(s) graphically through the optical visualization of several lines in a logical order by length,

JEL classification: C02.

width and height. The concept of dimension can also be explained by Euclidian geometry under the uses of the Euclidian spaces. Euclidian geometry can be divided into 2-dimensional Euclidean geometry (plane geometry) and 3-dimensional Euclidean geometry (solid geometry).

Additionally, the study of Euclidian geometry also involves the examination of the n-dimensional space represented by R^n or E^n under the uses of the n-dimensional space and n-vectors respectively.

HOW DO MULTI-DIMENSIONAL COORDINATE SPACES WORK?

The main reason to apply multi-dimensional coordinate spaces is to study any economic phenomena from a multidimensional perspective. This is originated by the limitations that the 2-dimensional coordinate space shows at the moment when it comes to generating a multidimensional optical visual effect of any economic phenomena in the same graphical space. Hence, the multidimensional coordinate spaces leads to an alternative graphical modeling which is more flexible and innovative than the current 2-dimensional coordinate space to observe multi-variable data behavior. The study of multi-dimensional coordinate spaces requires basic knowledge about the “n-dimensional space“. The idea of the n-dimensional space originated with many Greek thinkers and philosophers such as: Socrates, Plato, Aristotle, Heraclitus and Euclid (father of geometry).

The great contribution of Euclid to geometry was the design of plane geometry under 2-dimensional Euclidean geometry and solid geometry under 3-dimensional Euclidean geometry. However, the n-dimensional space can be defined as a mental refraction through optical visualization and brain stimulation by several lines in a logical order by length, width, height and colors to represent the behavior of simple or complex phenomena in different periods of time in the same graphical space.

Usually, the study of n-dimensional space is based on the application of the “coordinate system“. In fact, the coordinate spaces can be classified into 2-dimensional coordinate space, 3-dimensional coordinate space and multi-dimensional coordinate space. The main role of the coordinate system is crucial in the analysis of the relationship between two or more variables such as exogenous variable(s) and endogenous variable(s) on the same graphical space. In fact, the Euclidean space is given only the mathematical theoretical framework, but not the graphical modeling to visualize the n-dimensions according to different mathematical theoretical research works.

On the other hand, Minkowski (Einstein, 1952) introduced the idea of the 4-dimensional space or the “world“. The world, according to Minkowski, is originated by the application of the 3-dimensional continuum (or space). The difference between the 4-dimensional space and the 3-dimensional space graphical model is that the first graphical model replaces (X, Y, and Z) with (X_1, X_2, X_3, and X_4), thus $X_1 = X$; $X_2 = Y$; $X_3 = Z$ and $X_4 = \sqrt{-1}$. X_4 is based on the application of the Lorenz transformation axiom. The 4-dimensional space by Minkowski fails to offer a specific graphical modeling or alternative Cartesian coordinate system to help visualize the 4-dimensional space; it only offers a mathematical theoretical framework to describe the idea of 4-dimensional space. Moreover, the application of multi-dimensional coordinate spaces offer a large possibility to adapt n-dimensions, sub-dimensions, micro-dimensions, nano-dimensions and ji-dimensions in the visualization of any economic phenomenon.

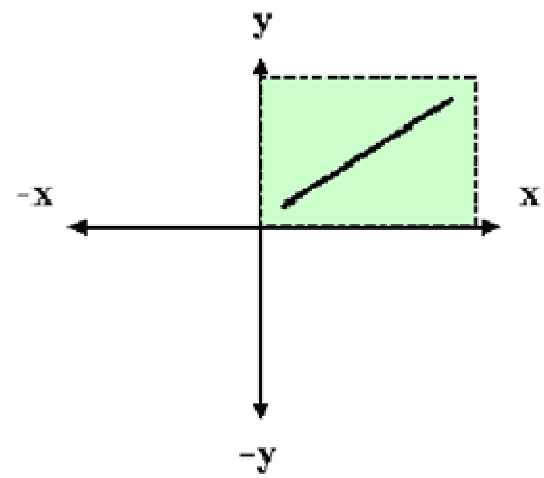


Figure 1. 2-Dimensional coordinate space.

Basically, the use of coordinate spaces by economists is based on plotting different dots that represent the relationship between two or more variables (endogenous and exogenous) in the first and fourth quadrants in the 2-dimensional coordinate space. Afterwards, they proceed to join all these dots by straight lines until is possible to visualize histograms, line graphs and scatter-plots (Figure 1). Hence, it is possible to observe the trend and behavior of different variables of any economic phenomenon. For example the relationship between unemployment/inflation, interest-rate/investment, prices/quantity demand and supply, and so on. From our point of view, each dot plotted on the 2-dimensional, 3-dimensional and multi-dimensional coordinate spaces represents a single rigid point. In fact, the plotting of a single rigid point in any coordinate space requires the application of two basic assumptions:

The first assumption is that two rigid points cannot occupy the same space at the same time; the second assumption is that different rigid point(s) deal in different n-dimensional spaces move under different speeds of time. The variable "time" in the case of multi-dimensional coordinate spaces needs to be classified by:

General time, partial time and constant time

General time runs in general-space, but sub-spaces, micro-spaces, nano-spaces all run under different partial times. In the case of JI-spaces, these are always fixed by constant time. Recently, a few economists have started to use the 3-dimensional coordinate space in economics by utilizing three axes:

"X-coordinate" (or exogenous variable), "Y-coordinate" (or exogenous variable) and the "Z-coordinate" (or endogenous variable)

This is based on the construction of surfaces or 3-dimensional manifolds to visualize multi-variable economic data behavior (Figure 2). According to our research the use of the 3-dimensional coordinate space is not so popular among economists and policy makers. Based on one thousand five hundred (1500) chapters published in twenty one (21) reputable economics journals[1] between the year 1939 and 2009 (JSTOR, 2009), it is possible to observe that the common types of graphical representations applied in the study of social sciences, especially in economics, were of the 2-dimensional coordinate space type. 99.5% of these chapters applied the 2-dimensional Cartesian coordinate system, and only 0.5% of them applied the 3-dimensional coordinate spaces. Additionally, this research will proffer several reasons as to why economists continue using the 2-dimensional coordinate space or sometimes the 3-dimensional coordinate space in the graphical representation of complex and dynamic economic phenomena. These reasons are listed as follows:

i) The 2-dimensional graphical models have been established over a long time, since the introduction of the 2-dimensional coordinate space by Descartes (Lafleur, 1960) up till today. The application of the 2-dimensional coordinate space in the economic graphical analysis has become a tradition.
ii) The 2-dimensional space is easy to apply in order to visualize basic trends or values in the same graphical space. A logical explanation about the common use of the 2-dimensional coordinate space is that it can be easily used to plot, draw and visualize any economic phenomenon. Therefore, the 2-dimensional coordinate space can generate a clear visual and mental reflection to understand complex and dynamic economic phenomena graphically in the same space and time.
iii) It is difficult to find alternative and suitable multi-dimensional graphical models to generate the transition from 2-dimensional coordinate space graphical modeling to multi-dimensional space graphical modeling. This research found some difficulties generating this crucial visual and mental transition from 2-dimensional coordinate space to multi-dimensional coordinate space. This could be due to the difficulty of plotting, drawing and visualizing multi-dimensional graphs.

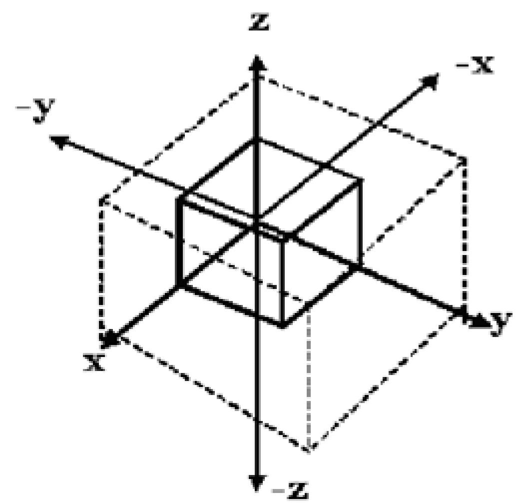


Figure 2. 3-Dimensional coordinate space.

Finally, a new set of multi-dimensional coordinate spaces is introduced in this document. The idea is to generate a new multidimensional optical visual effect to visualize complex economic phenomena. We can observe that the multidimensional coordinate spaces can incorporate a large number of exogenous variables that change constantly and directly affect the behavior of endogenous variable(s) in the same graphical space. These new types of multi-dimensional coordinate spaces are based on the pyramid coordinate space (five axes and infinite axes), the diamond coordinate space(ten axes and infinite axes), the 4-dimensional coordinate space (vertical position and horizontal position), the 5-dimensional coordinate space (vertical position and horizontal position), the infinity-dimensional coordinate space (general approach and specific approach), the inter-linkage coordinate space, the cube-wrap coordinate space, and the mega-surface coordinate space.

Being multi-dimensional, it enables economists, academics and policy makers to analyze economic phenomena from multidimensional perspectives across time and space.

THE CLASSIFICATION OF MULTI-DIMENSIONAL COORDINATE SPACE

Multi-dimensional coordinate spaces can be classified by the pyramid coordinate space (five axes and infinite

$(X_j{:}i, Y_i)$	$X_1{:}i$	$X_2{:}i$	$X_3{:}i$	$X_4{:}i$
Y_i	P1 = (3,2)	P2 = (2,2)	P3 = (1,2)	P4 = (3,2)

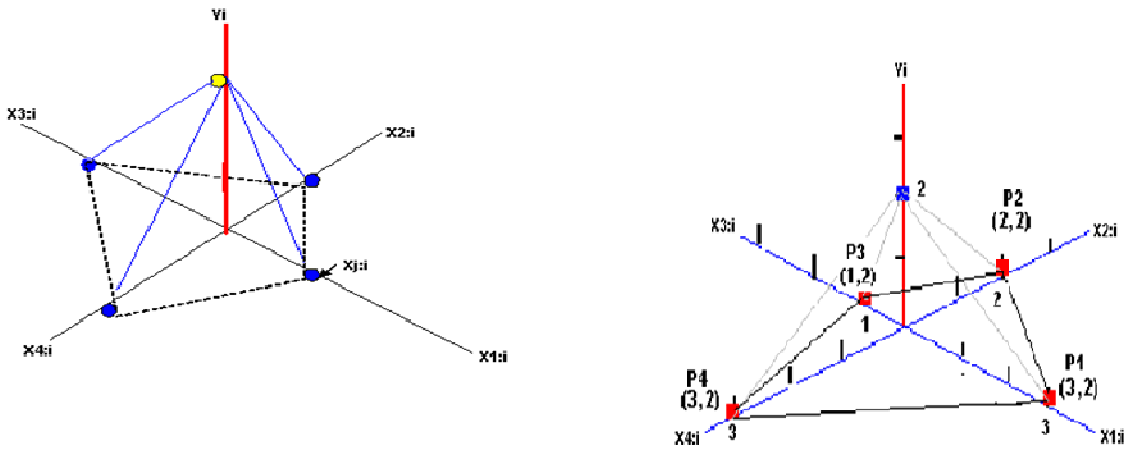


Figure 3. Pyramid coordinate space.

axes), the diamond coordinate space (ten axes and infinite axes), the 4-dimensional coordinate space (vertical position and horizontal position), the 5-dimensional coordinate space (vertical position and horizontal position), the infinity-dimensional coordinate space (general approach and specific approach), the inter-linkage coordinate space, the cube-wrap coordinate space, and the mega-surface coordinate space.

The pyramid coordinate space with five axes

The pyramid coordinate space with five axes consists of four independent axes (X_1, X_2, X_3, X_4) and one dependent axis (Y^*). The Y^* axis is positioned in the center part of this coordinate space among of the other four axes: X_1, X_2, X_3, X_4. The function used by the pyramidal coordinate space with five axes is fixed by $Y^* = f(X_1, X_2, X_3, X_4)$, where X_1, X_2, X_3, X_4, Y^* axes use only real positive numbers R_+ under the condition $0 \geq R_+ \leq +\infty$. The positive axes in the pyramidal coordinate space with five axes require the use of absolute values. The use of absolute values in each axis is based on the application of the non-negative property. Hence, all axes $/X_1/, /X_2/, /X_3/, /X_4/, /Y^*/$ always use values larger than or equal to zero. The pyramid coordinate space with five axes show clearly, within the same graphical space, any possible change(s) of any or all values plotted on each or all X_1, X_2, X_3, X_4 axes that can directly affect the behavior of the Y^* axis value.

In order to plot different values in each axis into the pyramid coordinate space with five axes, we need to plot each value directly on its axis line. At the same time, all values plotted on each axis line need to be joined together by straight lines until a pyramid-shaped figure with five faces can be visualized (Figure 3 and Prototype 1). Therefore, we have two possible graphical scenarios. The first graphical scenario:

If all or any X_1, X_2, X_3, X_4 axes values move from outside to inside, then the Y^* axis values move down. The second graphical scenario: if all or any X_1, X_2, X_3, X_4 axes values move from inside to outside, then the Y^* axis values move up. Basically, the pyramid coordinate system with five axes is represented (Prototype 1 and Figure 3):

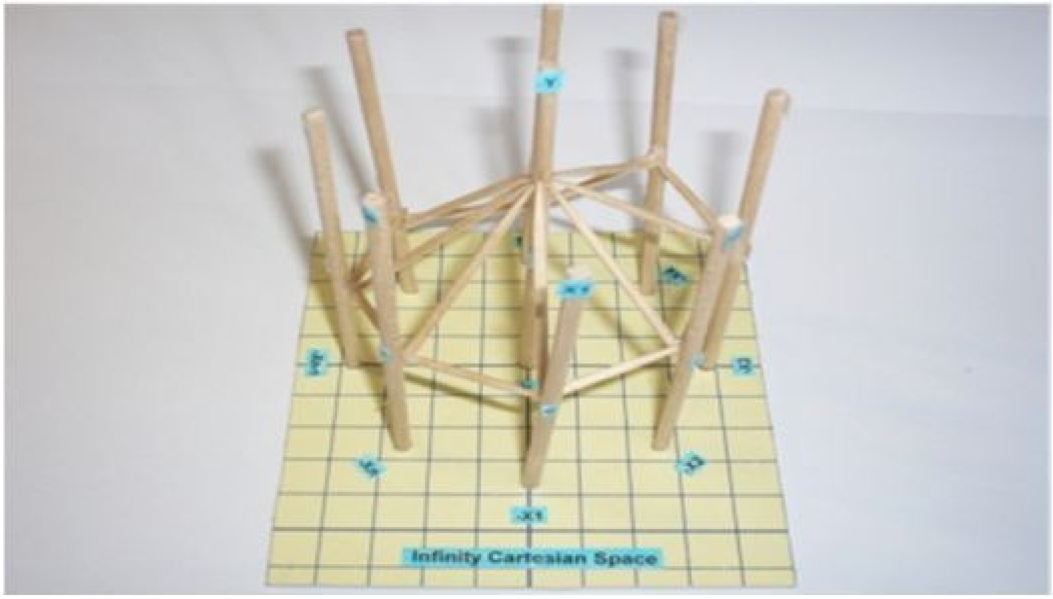
(1) $([X_1, X_2, X_3, X_4], Y^*)$

The pyramid coordinate space with infinite axes

The pyramid coordinate space with infinite axes consists of an infinite number of independent axes ($X_1, X_2, X_3, \ldots, X_\infty$) and one dependent axis (Y^*). The Y^* axis is positioned in the center part of this coordinate space among the other infinite axes:

$X_1, X_2, X_3, \ldots, X_\infty$. The function used by the pyramid coordinate space with infinite axes is fixed by $Y^* = f(X_1, X_2, X_3, \ldots, X_\infty)$, where $X_1, X_2, X_3, \ldots, X_\infty, Y^*$ axes use only real positive numbers R_+ under the condition $0 \geq R_+ \leq +\infty$.

The use of positive axes in the pyramidal coordinate space with infinite axes requires the use of absolute values. The use of absolute values in each axis is based on the application of the non-negative property. Hence, all axes $/X_1/, /X_2/, /X_3/, /X_4/, /Y^*/$ always use values larger than or equal to zero. The pyramid coordinate space with infinite axes show clearly, within the same graphical



Prototype 1. The pyramid coordinate space (P-Coordinate space) prototype.

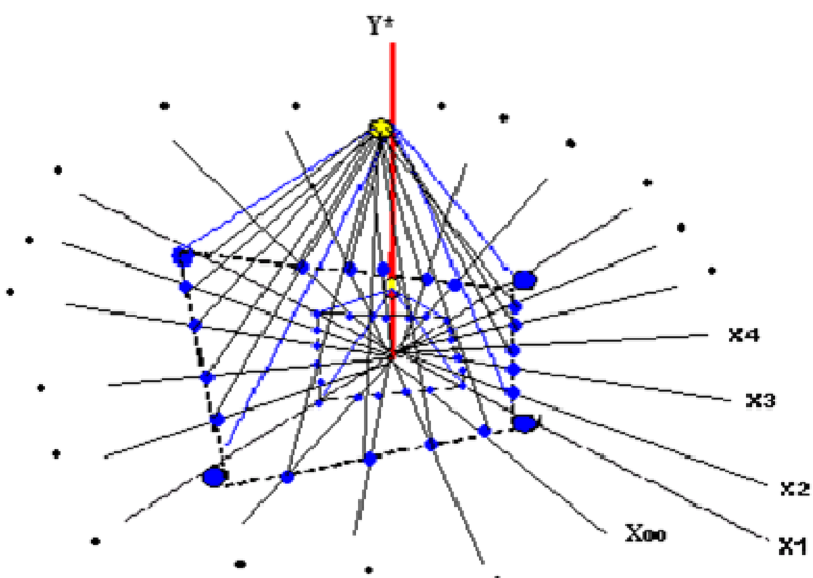
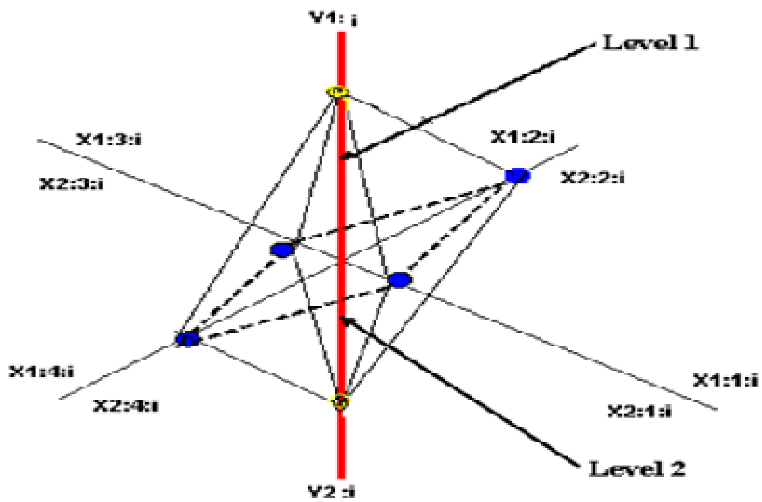


Figure 4. The pyramid coordinate space with infinite axes.

space, any possible change(s) of any or all values plotted on each or all $X_1, X_2, X_3,\ldots, X_\infty$ axes that can directly affect the behavior of the Y^* axis value. In order to plot different values in each axis into the pyramid coordinate space with infinite axes, each value must be plotted directly on its axis line. At the same time, all values plotted on each axis line need to be joined together by straight lines until a pyramid-shaped figure with infinite faces is built (Figure 4).

Therefore, we have two possible graphical scenarios: First, if all or any $X_1, X_2, X_3,\ldots, X_\infty$ axes values move from outside to inside, then the Y^* axis values move down; secondly, if all or any $X_1, X_2, X_3,\ldots, X_\infty$ axes values move from inside to outside, then the Y^* axis values move up. The pyramid coordinate system with infinite axes is represented by:

(2) $([X_1, X_2, X_3,\ldots, X_\infty], Y^*)$



$(X_{L}\ i, Y_{L;}\ i)$	X1:1 = 3	X1:2 = 2	X1:3 = 1	X1:4 = 1
Y1: i = 2	P1 = (3,2)	P2= (2,2)	P3 = (1,2)	P4= (1,2)
	X2:1 = 2	X2:2 = 3	X2:3 = 2	X2:4 = 2
Y2: i = 3	P5= (2,3)	P6 = (3,3)	P7 = (2,3)	P8 = (2,3)

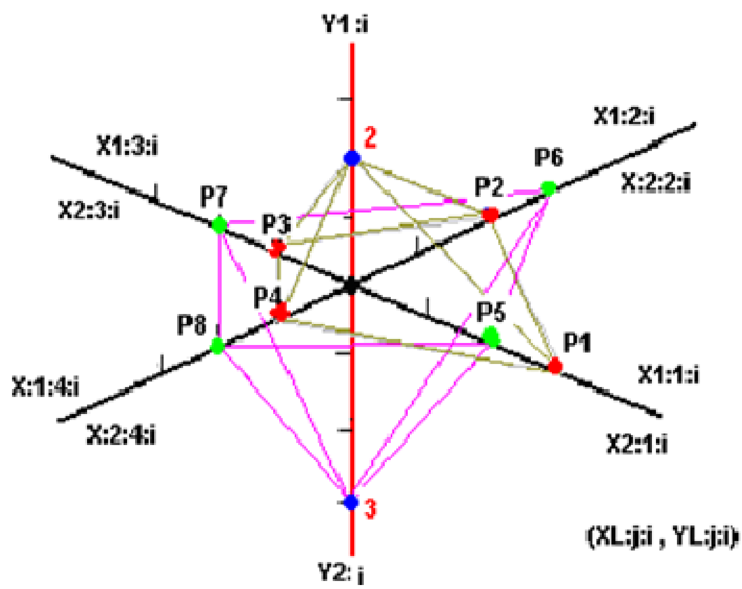
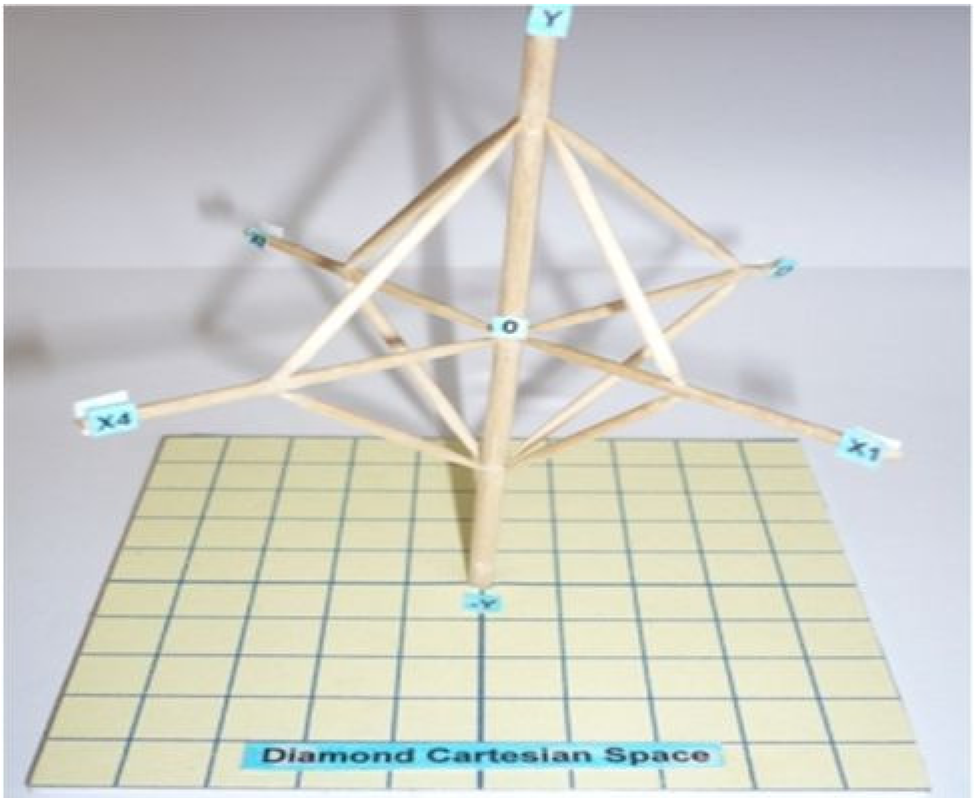


Figure 5. Diamond coordinate space.

The diamond coordinate space with ten axes

The diamond coordinate space with ten axes has two levels of analysis and ten axes. Each level of analysis is represented by *($X_{L:i}$, $Y_{L:i}$)*, where "*L*" represents the level of analysis, in this case either level one (L_1) or level two (L_2); "*i*" represents the quadrant level of analysis (in this case, quadrant 1, 2, 3 or 4). In order to plot different values in each axis into the diamond coordinate space with ten axes, each value is plotted directly on its respective axis line. At the same time, all values plotted on each axis line need to be joined together by straight lines until a diamond-shaped figure with eight faces is built (Figure 5 and Prototype 2). It is important to mention



Prototype 2. Diamond coordinate space prototype.

at this juncture that the first level (L_1) has five axes represented by $X_{1:1}$, $X_{1:2}$, $X_{1:3}$, $X_{1:4}$, $Y_{1:}$ four independent axes represented by $X_{1:1}$, $X_{1:2}$, $X_{1:3}$, $X_{1:4}$ and one dependent axis fixed by Y_1 respectively.

The second level (L_2) has five axes represented by $X_{2:1}$, $X_{2:2}$, $X_{2:3}$, $X_{2:4}$, Y_2. We assume that no relationship exists between level one (L_1) and level two (L_2) of analysis. The common issue between these two levels of analysis is that both levels use the same $X_{L:i}$ axes in the diamond coordinate space. However, level one (L_1) of the analysis cannot affect level two (L_2) of the analysis, and vice versa. If different levels of analysis are drawn in the diamond coordinate space, two different scenarios can be visualized and compared in the same diamond coordinate space at the same time (Figure 5). It is crucial to mention at this point that the fifth and tenth axes (Y_1 *and* Y_2) are positioned in the center part of the diamond coordinate space among the other eight axes: $X_{1:1}$, $X_{1:2}$, $X_{1:3}$, $X_{1:4}$, $X_{2:1}$, $X_{2:2}$, $X_{2:3}$, $X_{2:4}$. We assume that both Y_L (Y_1, Y_2) use only real positive numbers R_+. Therefore, in the diamond coordinate space, all $X_{1:1}$, $X_{1:2}$, $X_{1:3}$, $X_{1:4}$, Y_1, $X_{2:1}$, $X_{2:2}$, $X_{2:3}$, $X_{2:4}$, Y_2 axes are grouped together on the positive side of their respective axes. The only use of positive axes in the diamond coordinate space requires the use of absolute values. The use of absolute values in each axis is based on the application of the non-negative properties. Hence, all axes $/X_{1:1}/$, $/X_{1:2}/$, $/X_{1:3}/$, $/X_{1:4}/$, $/Y_1/$, $/X_{2:1}/$, $/X_{2:2}/$, $/X_{2:3}/$, $/X_{2:4}/$, $/Y_2/$ always use values larger than or equal to zero.

The final result is that, if the two levels of analysis are joined, it is possible to visualize a diamond-shaped figure. The diamond coordinate system is represented by:

(3.1.) $([X_{1:1}, X_{1:2}, X_{1:3}, X_{1:4}], Y_1)$
(3.2.) $([X_{2:1}, X_{2:2}, X_{2:3}, X_{2:4}], Y_2)$

The diamond coordinate space with infinite axes

The diamond coordinate space with infinite axes has two levels of analysis and infinite axes. Each level of analysis is represented by $(X_{L:i}\,,\ Y^*_{L:i}\,)$, where "*L*" represents the level of analysis, in this case either level one (L_1) or level two (L_2); "*i*" represents the quadrant level of analysis (in this case, quadrant 1, 2, 3,…, ∞). In order to plot different values in each axis into the diamond coordinate space with infinite axes, each value is plotted directly on its respective axis line. At the same time, all values plotted on each axis line need to be joined together by straight lines until it forms a diamond-shaped figure with infinite faces (Figure 6). It is important to mention at this juncture that the first level (L_1) has infinite axes represented by $X_{1:1}$, $X_{1:2}$, $X_{1:3}$,…, $X_{1:\infty}$, $Y^*_{1:}$ infinite independent axes represented by $X_{1:1}$, $X_{1:2}$, $X_{1:3}$,…,$X_{1:\infty}$ and one dependent

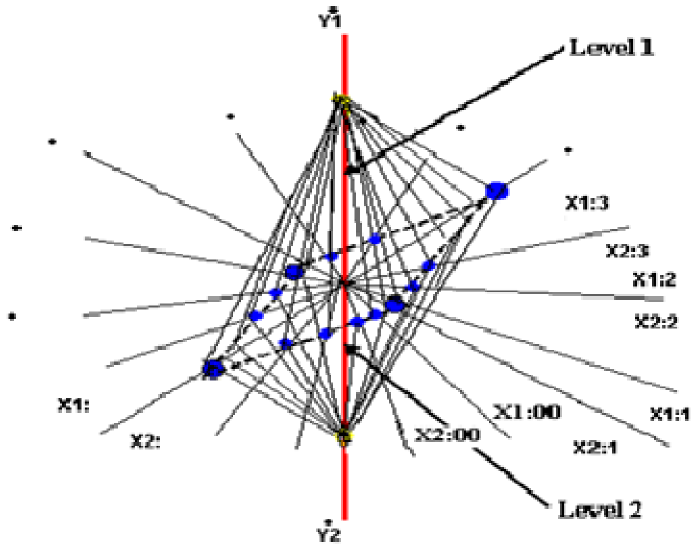


Figure 6. The diamond coordinate space with infinite axes.

axis fixed by Y^*_1 respectively.

The second level (L_2) has infinite axes represented by $X_{2:1}$, $X_{2:2}$, $X_{2:3}, \ldots, X_{2:\infty}, Y^*_2$. We assume that no relationship exists between level one (L_1) and level two (L_2) of analysis.

The common factor between these two levels of analysis is that both levels use the same $X_{L:i}$ axes in the diamond coordinate space with infinite axes. However, level one (L_1) of the analysis cannot affect level two (L_2) of the analysis, and vice versa. If we draw different levels of analysis in the diamond coordinate space with infinite axes, we can visualize and compare two different scenarios in the same diamond coordinate space at the same time. It is crucial to mention at this point that the Y^*_1-axis *and* Y^*_2-axis is positioned in the center part of the diamond coordinate space with infinite axes (among the other infinite axes $X_{L:i}$). We assume that both Y^*_L (Y^*_1, Y^*_2) use only real positive numbers R_+. Therefore, in the diamond coordinate space, all $X_{1:1}$, $X_{1:2}$, $X_{1:3}, \ldots, X_{1:\infty}, Y^*_1$, $X_{2:1}$, $X_{2:2}$, $X_{2:3}, \ldots, X_{2:\infty}, Y^*_2$ axes are grouped together on the positive side of their respective axes. The only use of positive axes in the diamond coordinate space with infinite axes requires the use of absolute values. The use of absolute values in each axis is based on the application of the non-negative properties. Hence, all axes $/X_{1:1}/$, $/X_{1:2}/$, $/X_{1:3}/, \ldots, /X_{1:\infty}/$, $/Y^*_1/$, $/X_{2:1}/$, $/X_{2:2}/$, $/X_{2:3}/, \ldots, /X_{2:\infty}/$, $/Y^*_2/$ always use values larger than or equal to zero.

The final result is that if the two levels of analysis are joined, it is possible to visualize a diamond-shaped figure. The diamond coordinate system is represented by:

(4.1.) $([X_{1:1}, X_{1:2}, X_{1:3}, \ldots, X_{1:\infty}], Y^*_1)$
(4.2.) $([X_{2:1}, X_{2:2}, X_{2:3}, \ldots, X_{2:\infty}], Y^*_2)$

The 4-dimensional coordinate space: Vertical position

The 4-dimensional coordinate space in vertical position offers four axes:

Xv_1, Xv_2, Xv_3, and Yv.

All these four axes are distributed by three independent axes:

Xv_1, Xv_2, Xv_3

And one dependent axis:

Yv.

The Xv_1, Xv_2, Xv_3, Yv axes fix positive and negative real numbers $R_{+/-}$.

In order to plot different values in each axis into the 4-dimensional coordinate space in vertical position, each value is plotted directly on its axis line. All values plotted on each axis line need to be joined together by straight lines until a pyramid-shaped figure is formed, with four faces in vertical position (Figure 7 and Prototype 3). Additionally, the Yv axis is positioned in the center part of the 4-dimensional coordinate space in vertical position (among the other three axes). It is the convergent point of all the other three axes:

Xv_1, Xv_2, Xv_3.

In other words, all Xv_1, Xv_2, Xv_3 axes always converge in

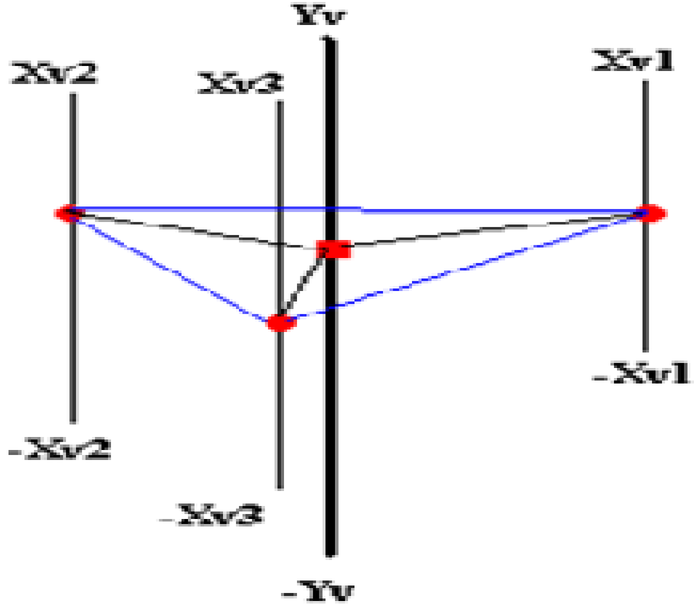
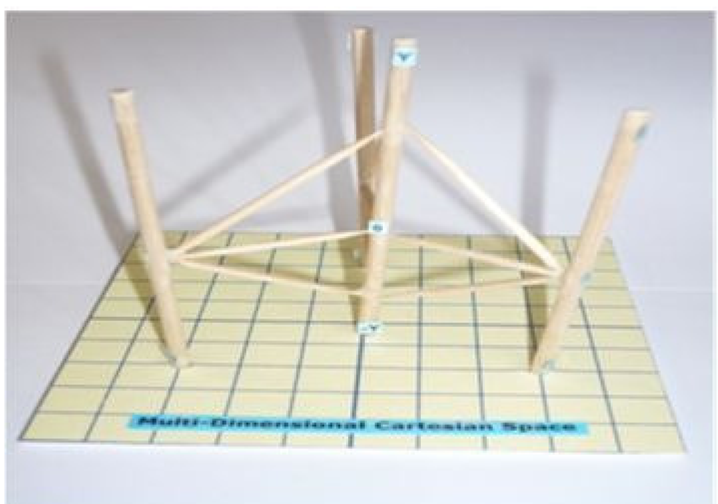


Figure 7. The 4-dimensional coordinate space in vertical position.



Prototype 3. The 4-dimensional coordinate space: Vertical position.

the *Yv* axis. The 4-dimensional coordinate system in vertical position is represented by:

(5.) $([Xv_1, Xv_2, Xv_3], Yv)$

The 4-dimensional coordinate space in horizontal position

The 4-dimensional coordinate space in horizontal position offers four axes:

Xh_1, Xh_2, Xh_3, *and* Yh.

All these four axes are distributed by three independent axes:

Xh_1, Xh_2, Xh_3 and one dependent axis: Yh.

The Xh_1, Xh_2, Xh_3, Yh axes are fixed positive and negative real numbers $R_{+/-}$. In order to plot different values in each axis into the 4-dimensional coordinate space in horizontal position, each

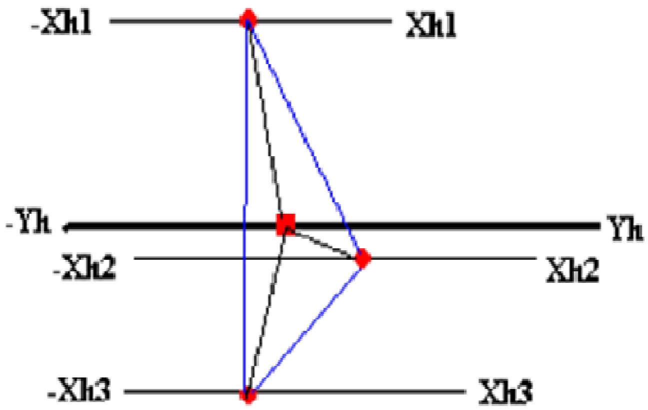


Figure 8. The 4-dimensional coordinate space in horizontal position.

value is plotted directly on its axis line. All values plotted on each axis line need to be joined together by straight lines until a pyramid-shaped figure is built, with four faces in horizontal position (Figure 8). Additionally, the *Yh* axis is positioned in the center part of the 4-dimensional coordinate space in horizontal position (among the other three axes). It is the convergent point of all the other three axes:

Xh_1, Xh_2, and $Xh3$.

In other words, all Xh_1, Xh_2, Xh_3 axes always converge in the *Yh* axis. The 4-dimensional coordinate system in horizontal position is represented by:

(6) $([Xh_1, Xh_2, Xh_3], Yh)$

The 5-dimensional coordinate space in vertical position

The 5-dimensional coordinate space in vertical position consists of five vertical axes:

Xv_1, Xv_2, Xv_3, Xv_4, and Yv.

All these five axes are distributed by four independent axes:

Xv_1, Xv_2, Xv_3, Xv_4 and one dependent axis Yv.

The Xv_1, Xv_2, Xv_3, Xv_4, Yv axes fix positive and negative real numbers $R_{+/-}$. In order to plot different values in each axis into the 5-dimensional coordinate space in vertical position, each value is plotted directly on its axis line. All values plotted on each axis line need to be joined together by straight lines until a pyramid-shaped figure is built, with five faces in vertical position (Figure 9 and Prototype 4). Therefore, the *Yv* axis is positioned in the center of the 5-dimensional coordinate space in vertical position (among the other four vertical axes). The *Yv* axis is the convergent axis of all the other four vertical axes:

Xv_1, Xv_2, Xv_3, and Xv_4.

The 5-dimensional coordinate system in horizontal position is represented by:

(7) $([Xv_1, Xv_2, Xv_3, Xv_4], Yv)$

The 5-dimensional coordinate space in horizontal position

The 5-dimensional coordinate space in horizontal position consists of five vertical axes:

Xh_1, Xh_2, Xh_3, Xh_4, Yh.

All these five axes are distributed by four independent axes:

Xh_1, Xh_2, Xh_3, Xh_4 and one dependent axis Yh.

The Xh_1, Xh_2, Xh_3, Xh_4, Yh axes fix positive and negative real numbers $R_{+/-}$. In order to plot different values in each axis into the 5-dimensional coordinate space in horizontal position, each value is plotted directly on its axis line. All values plotted on each axis line need to be joined together by straight lines until a pyramid-shaped figure with five faces in horizontal position is formed (Figure 10). Therefore, the *Yh* axis is positioned in the center of the 5-dimensional coordinate space in horizontal position (among the other four horizontal axes). The *Yh* axis is the convergent axis of all the other four vertical axes:

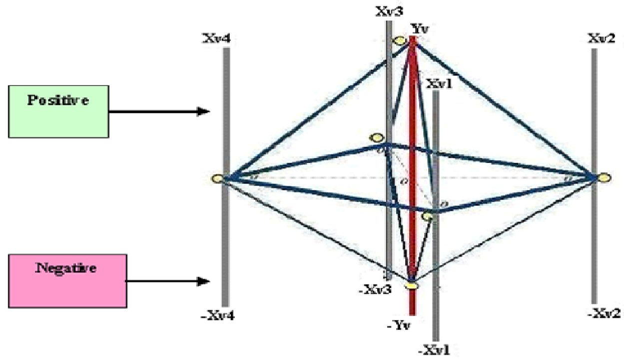
Xh_1, Xh_2, Xh_3, Xh_4.

The 5-dimensional coordinate system in horizontal position is represented by:

(8) $([Xh_1, Xh_2, Xh_3, Xh_4], Yh)$

The infinity dimensional coordinate space under the general approach

The infinity dimensional coordinate space under the general approach shows a series of n-number of sub-cylinders "*C*" located in the same general cylinder; each sub-cylinder in the same general cylinder is fixed by its Level "*L*" respectively, where $L = \{1, 2, 3,...,k\}$, $k \rightarrow \infty$…Plotting values into the different sub-cylinders in the same general cylinder is based on the sub-cylinder location, axis position and ratio. $X_{C:L}$ is the independent variable in sub-cylinder "*C*" at level "*L*" lying in position



(Xj:i , Y:i)	(X1:i or -X1:i)	(X2:i or -X2:i)	(X3:i , -X3:i)	(X4:i , -X4:i)
(Y1:i or -Y1:i)	(5,1)	(-2,1)	(6,1)	(-5,1)

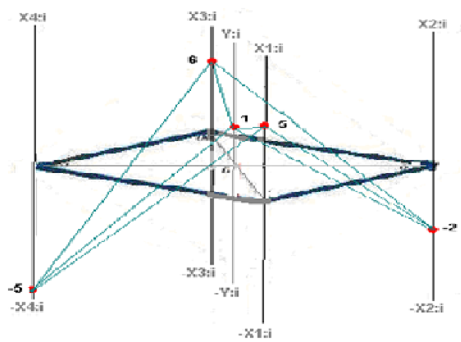
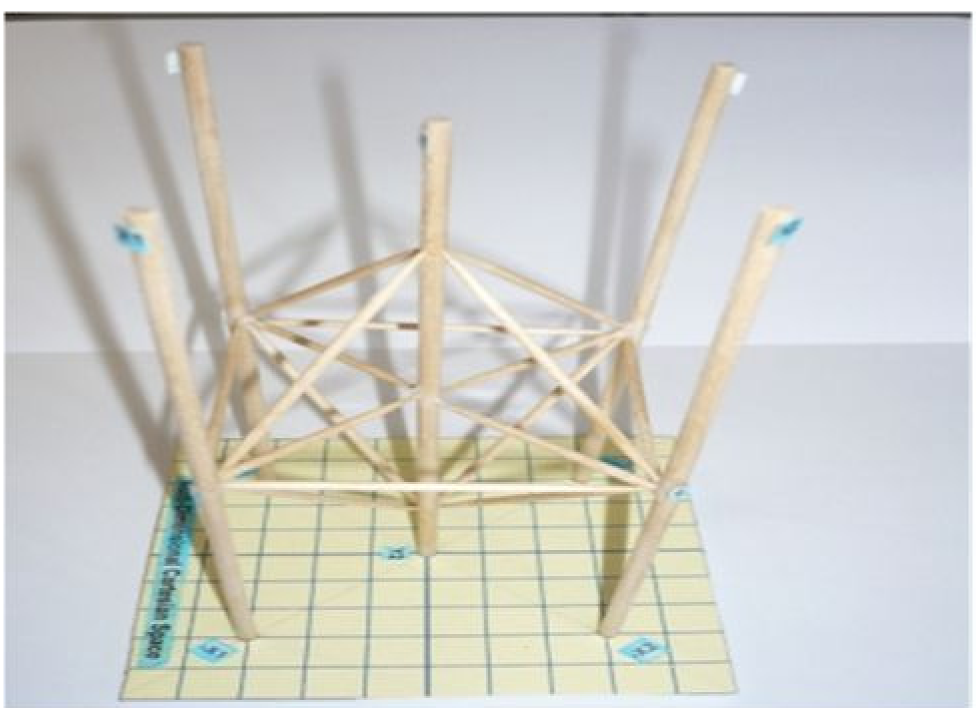


Figure 9. The 5-dimensional coordinate space in vertical position.



Prototype 4. The 5-dimensional coordinate space: Vertical position.

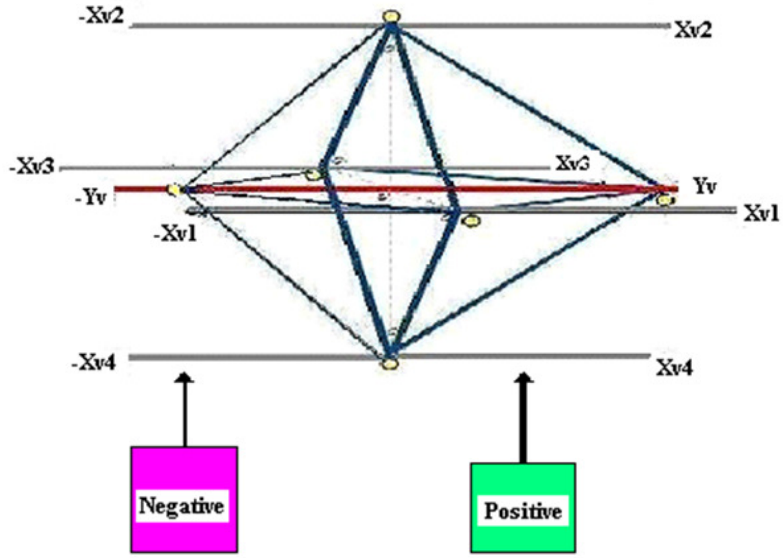


Figure 10. The 5-dimensional coordinate space in horizontal position.

$P_{C:L}$ with value $R_{C:L}$. The position is based on $P_{C:L}$ by $0^{\circ} \leq P_{C:L} \leq 360^{\circ}$, the position of $X_{C:L}$ in cylinder "*C*" at level "*L*". And finally the ratios location under the $R_{C:L}$ is the radius corresponding to the $X_{C:L}$ in cylinder "*C*" at level "*L*". Finally, the $Y_{C:L}$ is the dependent variable at level "*L*". The values of the independent axes $X_{C:L}$ affect $Y_{C:L}$ simultaneously.

The infinity dimensional coordinates space under the general approach; its function is given below:

(9) $Y_{C:L} = f(X_{C:L}, P_{C:L}, R_{C:L})$

For example, the value of a specific independent axis at time point 1, say $X_{1:1:1}$ is plotted as $R_{1:1:1}$; the radius pictured lying on a flat surface at angle $P_{1:1:1}$ is measured from 0° line used for its reference line. The points from the end of the radii are joined to meet in a single point on the top of each sub-cylinder at height $Y_{1:1}$, the level "L". The diameter of the sub-cylinder is twice the maximum radius. In order to plot different values in each axis into the infinity dimensional coordinate space under the general approach, each value needs to be plotted directly on its axis line. All values plotted on each axis line need to be joined together by straight lines until a cone-shaped figure in vertical position is formed (Figure 11 and Prototype 5).

The infinity dimensional coordinate space under the specific approach

Basically, the infinity dimensional coordinate space under the specific approach offers a new coordinate system according to expression 10. The basic coordinate space system is formed by three levels of analysis: general-space (i); sub-space (j); micro-space (k). In the case of plotting into this coordinate space, start with defining the specific general-space (i), sub-space (j), micro-space (k), alpha-space (α) and beta-space (β) respectively:

(10) $(\alpha_{<i:j:k>}, \beta_{<i:j:k>})$

The infinity dimensional coordinate space under the specific approach is able to show different dimensions that cannot be observed in the classic 2-dimensional Cartesian coordinate plane and 3-dimensional coordinate space. Hence, the 2-dimensional Cartesian coordinate plane and 3-dimensional coordinate space can be considered as sub-axes systems within the infinity dimensional coordinate space under the specific approach. The structure of the infinity dimensional coordinate space under the specific approach is formed by infinite general-spaces (i), sub-spaces (j) and micro-spaces (k). These are distributed into different places along the general cylinder (Figure 12). Therefore, the infinity coordinate space under the specific approach starts from the general space zero (i_0) until the general space infinity (i_{∞}). And each sub-space starts from sub-space zero (j_0) until sub-space infinity (j_{∞}). Finally, the micro-space starts from micro-space zero (K_0) until micro-space infinity (K_{∞}) (Expression 11).

The infinity dimensional coordinate space under the specific approach can connect a large number of micro-spaces (k) distributed into the same sub-space (j) and

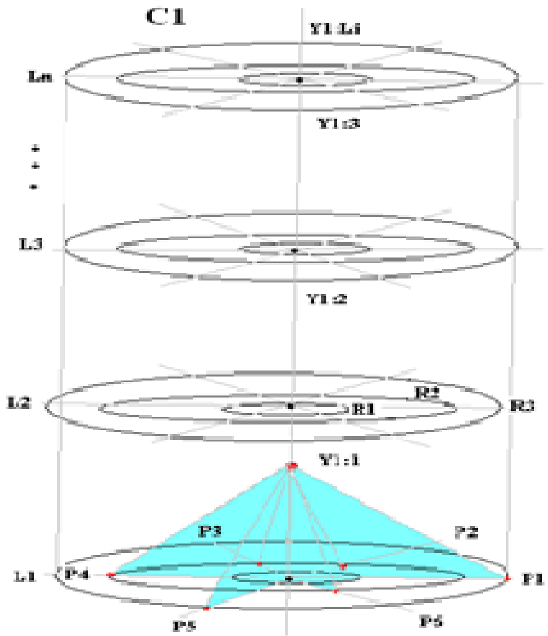
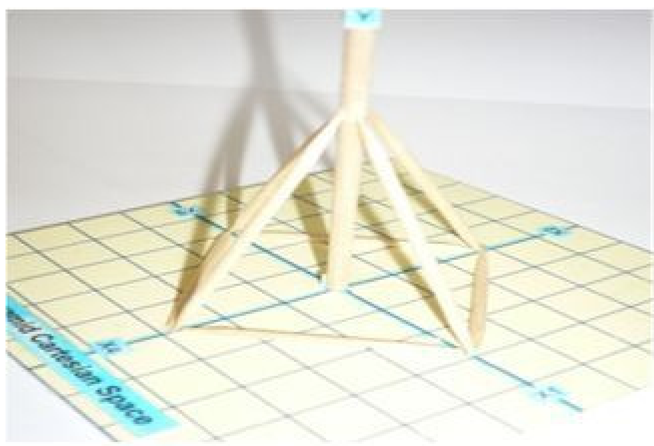


Figure 11. The infinity dimensional coordinate space (general approach).



Prototype 5. The infinity dimensional coordinate space (general approach).

general space (i) by the application of the inter-linkage connectivity of micro-spaces (╦). At the same time, the infinity dimensional coordinate space under the specific approach can also connect a large number of general spaces in the same coordinate space. It is based on the application of the inter-linkage connectivity of general-spaces (╬).

(11) ($\alpha_{<0:0:0>}$, $\beta_{<0:0:0>}$) ╦ ($\alpha_{<0:0:1>}$, $\beta_{<0:0:1>}$) ╦ . . . ╦ ($\alpha_{<0:0:\infty>}$, $\beta_{<0:0:\infty>}$)╬

($\alpha_{<0:1:0>}$, $\beta_{<0:1:0>}$) ╦ ($\alpha_{<0:1:1>}$, $\beta_{<0:1:1>}$) ╦ . . . ╦ ($\alpha_{<0:1:\infty>}$, $\beta_{<0:1:\infty>}$)╬

($\alpha_{<0:2:0>}$, $\beta_{<0:2:0>}$) ╦ ($\alpha_{<0:2:1>}$, $\beta_{<0:2:1>}$) ╦ . . . ╦ ($\alpha_{<0:2:\infty>}$, $\beta_{<0:2:\infty>}$)╬.╬

($\alpha_{<0:\infty:0>}$, $\beta_{<0:\infty:0>}$) ╦ ($\alpha_{<0:\infty:1>}$, $\beta_{<0:\infty:1>}$) ╦ . . . ╦ ($\alpha_{<0:\infty:\infty>}$, $\beta_{<0:\infty:\infty>}$)╬

($\alpha_{<\infty:0:0>}$, $\beta_{<\infty:0:0>}$) ╦ ($\alpha_{<\infty:0:1>}$, $\beta_{<\infty:0:1>}$) ╦ . . . ╦ ($\alpha_{<\infty:0:\infty>}$, $\beta_{<\infty:0:\infty>}$)╬

($\alpha_{<\infty:1:0>}$, $\beta_{<\infty:1:0>}$) ╦ ($\alpha_{<\infty:1:1>}$, $\beta_{<\infty:1:1>}$) ╦ . . . ╦ ($\alpha_{<\infty:1:\infty>}$, $\beta_{<\infty:1:\infty>}$)╬

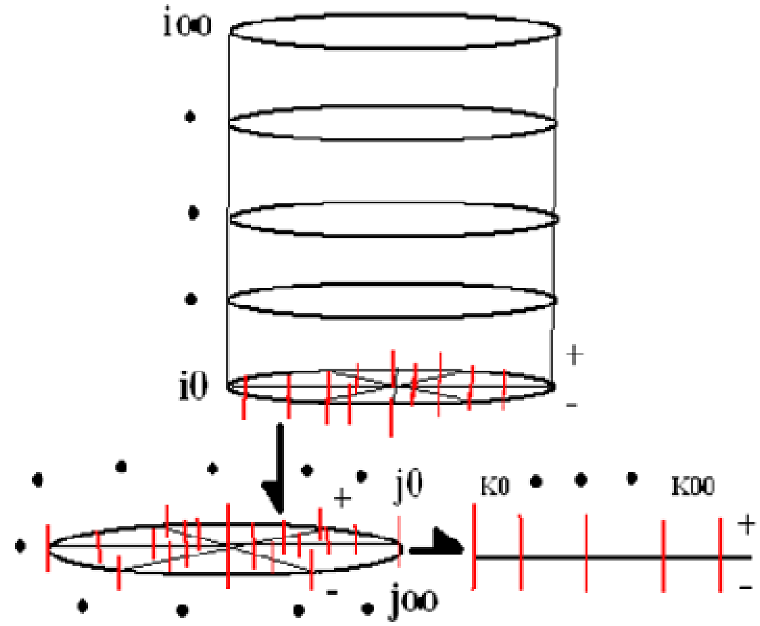


Figure 12. The Infinity dimensional coordinate space under the specific approach.

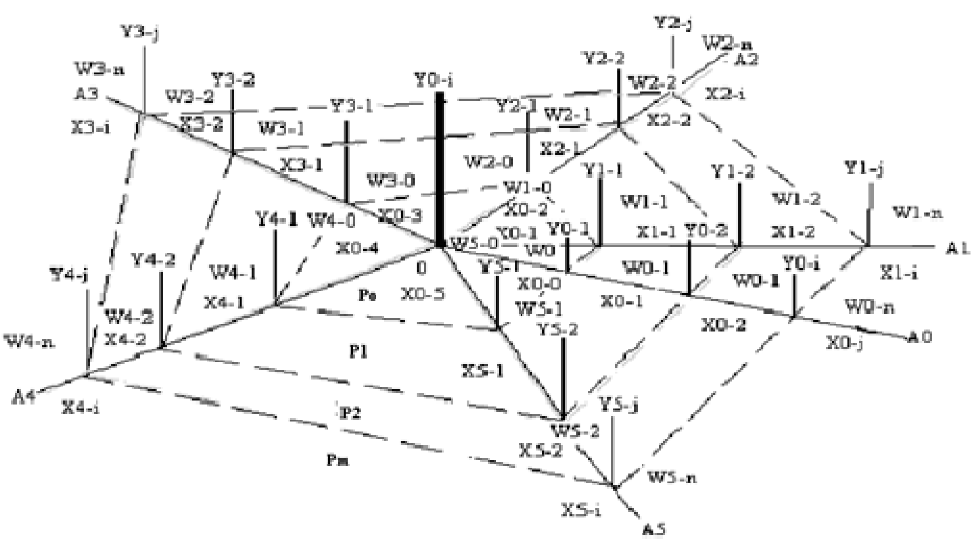
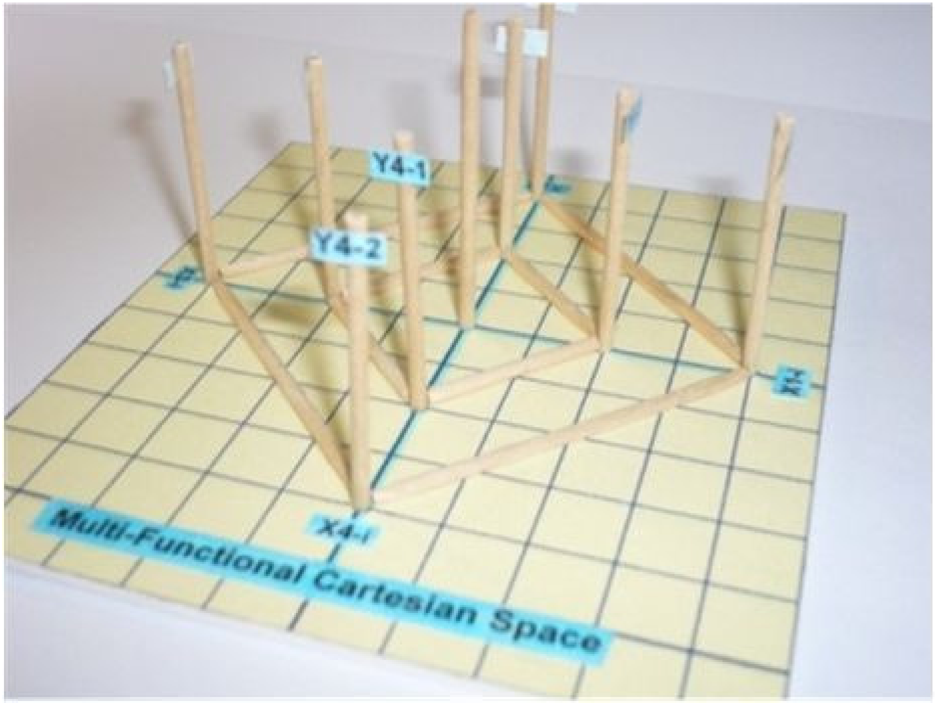


Figure 13. The inter-linkage coordinate space.

$(\alpha_{<\infty:2:0>}, \beta_{<\infty:2:0>})$ ╦ $(\alpha_{<\infty:2:1>}, \beta_{<\infty:2:1>})$ ╦ . . . ╦ $(\alpha_{<\infty:2:\infty>}, \beta_{<\infty:2:\infty>})$╬╬
$(\alpha_{<\infty:\infty:0>}, \beta_{<\infty:\infty:0>})$ ╦ $(\alpha_{<\infty:\infty:1>}, \beta_{<\infty:\infty:1>})$ ╦ . . . ╦ $(\alpha_{<\infty:\infty:\infty>}, \beta_{<\infty:\infty:\infty>})$

The inter-linkage coordinate space

The inter-linkage coordinate space is formed by an infinite number of general axes (A_0, A_1,…, A_n …), perimeter levels (L_0, L_1,…, L_n …) and windows refraction (W_0, $W_{1,\ldots,}W_{n\ldots}$) (Figure 13 and Prototype 6). Each window refraction is based on joining its sub-x axis ($X_{A\text{-}L}$) with its sub-y axis ($Y_{A\text{-}L}$) respectively. Therefore, the window refraction (W_0, $W_{1\ldots}W_{n\ldots}$) is followed by the coordinate space ($X_{A\text{-}L}$,$Y_{A\text{-}L}$). All windows refraction on the same general axis (A_0, A_1 ,…, A_n …) will be joined together under the application of the inter-linkage connectivity of



Prototype 6. The inter-linkage coordinate space.

windows refraction represented by "®". The inter-linkage connectivity of windows refraction is represented by the symbol "®". The inter-linkage connectivity of windows refraction "®" will inter-connect all windows refraction (W_0, W_1,…, W_n …) on the same general axis (A_0, A_1,…, A_n …) but at different perimeter levels (L_0, L_1 ,…, L_n …). Moreover, the inter-linkage coordinate system is represented by (Expression 12):

(12) Perimeter level P_0 ® Perimeter level P_1 ® … ® Perimeter level P_n
General Axis 0 (A_0): $W_{0-0} = (x_{0-0,}y_{0-0})$ ® $W_{0-1} = (x_{0-1,,}y_{0-1})$ ®…® $W_{0-\infty} = (x_{0-\infty}, y_{0-\infty})$
General Axis 1 (A_1): $W_{1-0} = (x_{1-0,,}y_{1-0})$ ® $W_{1-1} = (x_{1-1,,}y_{1-1})$ ®…® $W_{1-\infty} = (x_{1-\infty}, y_{1-\infty})$
General Axis 2 (A_2): $W_{2-0} = (x_{2-0,,}y_{2-0})$ ® $W_{2-1} = (x_{2-1,,}y_{2-1})$ ®…® $W_{2-\infty} = (x_{2-\infty}, y_{2-\infty})$
General Axis 3 (A_3): $W_{3-0} = (x_{3-0,,}y_{3-0})$ ® $W_{3-1}= (x_{3-1,,}y_{3-1})$ ®…® $W_{3-\infty} = (x_{3-\infty}, y_{3-\infty})$
General Axis 4 (A_4): $W_{4-0} = (x_{4-0,,}y_{4-0})$ ® $W_{4-1}= (x_{4-1,,}y_{4-1})$ ®… ® $W_{4-\infty} = (x_{4-\infty},y_{4-\infty})$
General Axis 5 (A_5): $W_{5-0} = (x_{5-0,,}y_{5-0})$ ® $W_{5-1} = (x_{5-1,,}y_{5-1})$®… ® $W_{5-\infty} = (x_{5-\infty}, y_{5-\infty})$

General Axis n (A_∞): $W_{\infty-0} = (x_{\infty-0}, y_{\infty-0})$ ®
® $W_{\infty-\infty} = (x_{\infty-\infty}, y_{\infty-\infty})$

Finally, the inter-linkage coordinate space is able to fix a large number of different functions located in different windows refraction (W_0, W_1,…, W_n …), perimeter levels (L_1, L_2 ,…, L_n …) and general axes (A_1, A_2 ,…, A_n …) (Expression 13):

(13) Perimeter level P_0 ® Perimeter level P_1 ® … ® Perimeter level P_n

General Axis 0 (A_0): $y_{0-0} = f(x_{0-0})$ ® $y_{0-1} = f(x_{0-1})$ ®…….® $y_{0-\infty}= f(x_{0-\infty})$
General Axis 1 (A_1): $y_{1-0} = f(x_{1-0})$ ® $y_{1-1} = f(x_{1-1})$ ®…….® $y_{1-\infty} = f(x_{1-\infty})$
General Axis 2 (A_2): $y_{2-0} = f(x_{2-0})$ ® $y_{2-1} = f(x_{2-1})$ ®……..® $y_{2-\infty} = f(x_{2-\infty})$
General Axis 3 (A_3): $y_{3-0} = f(x_{3-0})$ ® $y_{3-1} = f(x_{3-1})$ ®…….® $y_{3-\infty} = f(x_{3-\infty})$
General Axis 4 (A_4): $y_{4-0} = f(x_{4-0})$ ® $y_{4-1} = f(x_{4-1})$ ®…….. ® $y_{4-\infty} = f(x_{4-\infty})$
General Axis 5 (A_5): $y_{5-0} = f(x_{5-0})$ ® $y_{5-1} = f(x_{5-1})$®……….® $y_{5-\infty} = f(x_{5-\infty})$

General Axis n (A_∞): $y_{\infty-0} = f(x_{\infty-0})$ ® ……………………….® $y_{\infty-\infty}= f(x_{\infty-\infty})$

The cube-wrap coordinates space

The cube-wrap coordinate space offers an alternative coordinate space. The main objective of the cube-wrap coordinate space is to show unknown dimensions that cannot be visualized by the 2-dimensional Cartesian plane and 3-dimensional coordinate space. Initially, the

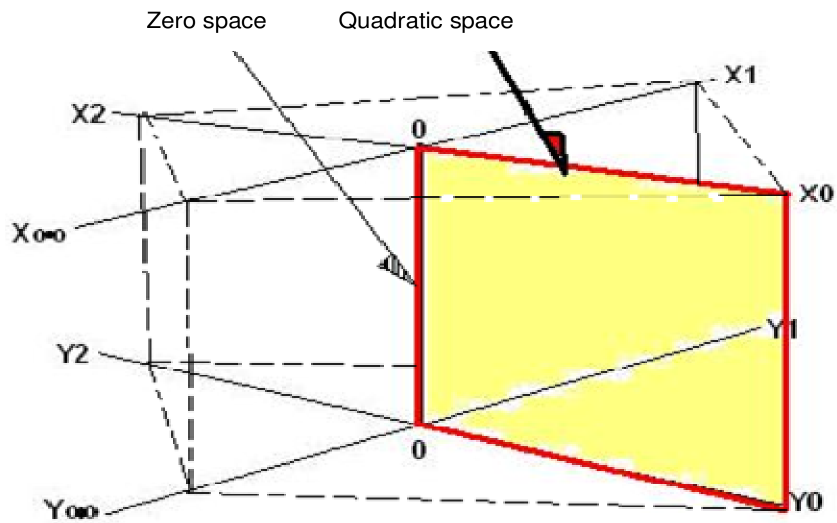


Figure 14. The cube-wrap coordinate space and the cube-wrap space.

cube-wrap coordinate space is divided into two quadrants. The first quadrant is located on the top of the cube-wrap coordinate space; this represents all $/X_i/$-axes. The second quadrant is located under the button of the cube-wrap coordinate space; this represents all $/Y_j/$-axes (Figure 14). In the process to plot values on this coordinate space, start by plotting each value on its respective axis line. The space that exists between $/X_i/$-axes and $/Y_j/$-axes will be called the "quadratic-space refraction" because each $/X_i/$-axis has its respective $/Y_j/$-axis. The construction of the quadratic space refraction is based on two basic steps. The first step is to plot each value on the $/X_i/$-axis line and $/Y_j/$-axis line, for which we suggest the application of the inter-linkage connectivity of micro-spaces (╤) (Expression 14). The second step is to join the values located on $/X_i/$-axis and $/Y_j/$-axis by a single straight vertical line.

(14) ($/X_i/$ ╤ $/Y_j/$)

We assume that between X_i–axes and Y_j–axes exists a common single straight vertical line that joins both set of axes. This common single straight line is called the zero space. Hence, the cube-wrap coordinate space starts from the quadratic space refraction zero (L_0) and extends to the quadratic space refraction infinity (L_∞). The cube wrap coordinate space requires the application of absolute values $/R_{+/-}/$ because the cube wrap coordinate space only works with positive real numbers R_+. The final coordinate system to build the cube-wrap coordinate space is represented by Expression 15.

(15) CW = $(S_0) = (/X_{00}/$ ╤ $/Y_{00}/)$ ╬ $(S_1) = (/X_{01}/$ ╤ $/Y_{01}/)$ ╬ … ╬ $(S_\infty) = (/X_{\infty\infty}/$ ╤ $/Y_{\infty\infty}/)$

The final stage of analysis in the cube-wrap space is based on its size. There are three possible stages that the cube-wrap space can experience at any time:

(16) If all values are growing constantly in $/X_i/$ and $/Y_j/$ then the cube-wrap is experiencing an expansion stage.
(17) If all values are decreasing constantly in $/X_i/$ and $/Y_j/$ then the cube-wrap is experiencing a contraction stage.
(18) If all values are keeping constant in $/X_i/$ and $/Y_j/$ then the cube-wrap is experiencing a static stage.

The mega-space coordinate space

The mega-space coordinate space is formed by an infinite number of axes in vertical position. Each vertical axis (X_{ij}) shows positive integer numbers on the top and negative integer numbers on the bottom in the same vertical axis. At the same time, all the vertical axes can be located by row number (i) and column number (j) in the mega-space coordinate space (Expression 19). The idea to apply the mega-surface coordinate space is to build the mega-surface.

The mega-surface shows how a large number of variables behave together in the same graphical space. Initially, the construction of the mega-surface is started by joining each vertical axis value by straight lines with its neighboring vertical axis: front side; left side; right side; back (Figure 15).

It is necessary to join all the axes in order to apply the inter-linkage connectivity condition (╬) on all vertical axes simultaneously.

(19) X_{ij} => ╬X_{11} ╬ X_{12} ╬ .... ╬$X_{1\infty}$
╬X_{21} ╬ X_{22} ╬ .... ╬$X_{2\infty}$
╬X_{31} ╬ X_{32} ╬ .... ╬ $X_{3\infty}$
╬$X_{\infty:1}$ ╬ $X_{\infty:2}$╬ .... ╬ $X_{\infty\infty}$

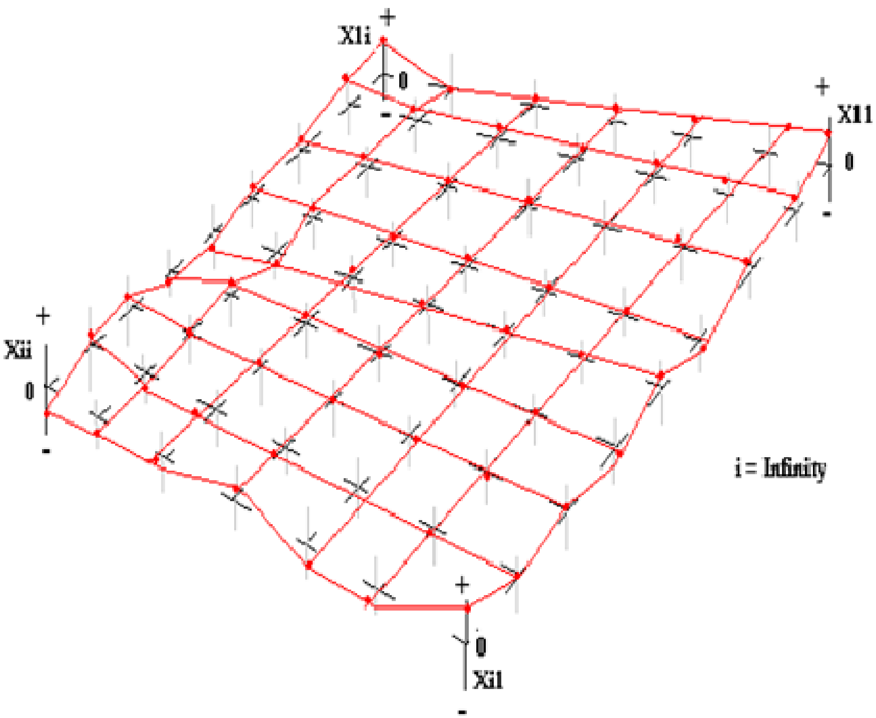


Figure 15. The mega-space coordinate space and the mega-surface.

The final analysis of the mega-surface is based on its location within the mega-space coordinate space. These are the possible stages that the mega-surface can experience:

(20) If all X_{ij} values > 0 then the mega-surface shows an expansion stage.
(21) If all X_{ij} values = 0 then the mega-surface shows a stagnation stage.
(22) If X_{ij} values < 0 then the mega-surface shows a contraction stage.
(23) If some X_{ij} shares positive, negative or zero values in different vertical axes, then the mega-surface shows an unstable performance stage.

The cubes coordinate space

The cubes coordinate space is formed by an infinite number of general axes ($A_0, A_1, \ldots, A_n$), where each axis shows different levels ($L_0, L_1, \ldots, L_n$), perimeters ($P_0, P_1, P_2 \ldots P_n$), and cubes with different sizes and colors ($C_{0/\beta}, C_{1/\beta} \ldots C_{n/\beta}$). Therefore, the coordinate system of the cubes-coordinate space is represented by $S_{A:L:P:C} = (A_i, L_j, P_k, C_{s/\beta})$ respectively, where *i, j, k* and *s* represent different values between 0 and ∞…. and β represents the different colors of each cube at different levels ($L_0, L_1, \ldots, L_n$). All the cubes with different sizes and colors in the same axis under the same level ($L_0, L_1, \ldots, L_n$) and different perimeters ($P_0, P_1, P_2 \ldots P_n$) will be joined together, based on the application of the concept called "macroeconomics links structures" represented by the symbol "@". Moreover, the cubes-coordinate space coordinate system is expressed by the following (Expression 24 and Figure 16):

(24) Level P_0 @………………. @ Level P_n

A_0: $S_{0:0:0:C(\alpha/\beta)} = (A_0, L_0, P_0, C_{\alpha/\beta})$ @ ……. .. @ $S_{0:0:\lambda:C(\alpha/\beta)} = (A_0, L_0, P_\lambda, C_{\alpha/\beta})$ @
@ $S_{0:1:0:C(\alpha/\beta)} = (A_0, L_1, P_0, C_{\alpha/\beta})$ @ ……….. @ $S_{0:1:\lambda:C(\alpha/\beta)} = (A_0, L_1, P_\lambda, C_{\alpha/\beta})$ @
@ . . @ @ $S_{0:\theta:\lambda:C(\alpha/\beta)} = (A_0, L_\theta, P_\lambda, C_{\alpha/\beta})$ @ ……….. @ $S_{0:1:\lambda:C(\alpha/\beta)} = (A_0, L_1, P_\lambda, C_{\alpha/\beta})$ @
@A_1: $S_{1:0:0:C(\alpha/\beta)} = (A_1, L_0, P_0, C_{\alpha/\beta})$ @ ……. .. @ $S_{1:0:\lambda:C(\alpha/\beta)} = (A_1, L_0, P_\lambda, C_{\alpha/\beta})$ @
@ $S_{1:1:1:C(\alpha/\beta)} = (A_1, L_1, P_0, C_{\alpha/\beta})$ @ ……….. @ $S_{1:\theta:\lambda:C(\alpha/\beta)} = (A_1, L_1, P_\lambda, C_{\alpha/\beta})$ @
@ . . @
@ $S_{1:\theta:\lambda:C(\alpha/\beta)} = (A_1, L_\theta, P_\lambda, C_{\alpha/\beta})$ @ ……….. @ $S_{1:\theta:\lambda:C(\alpha/\beta)} = (A_1, L_\theta, P_\lambda, C_{\alpha/\beta})$ @ @
A_n: $S_{n:0:0:C(\alpha/\beta)} = (A_n, L_0, P_0, C_{\alpha/\beta})$ @ ……. .. @ $S_{n:0:\lambda:C(\alpha/\beta)} = (A_n, L_0, P_\lambda, C_{\alpha/\beta}$ @
$S_{n:1:1:C(\alpha/\beta)} = (A_n, L_1, P_0, C_{\alpha/\beta})$ @ ……….. @ $S_{n:1:\lambda:C(\alpha/\beta)} = (A_n, L_1, P_\lambda, C_{\alpha/\beta})$ @
@ . . @
@ $S_{\theta:\lambda:C:\alpha/\beta} = (A_n, L_\theta, P_\lambda, C_{\alpha/\beta})$ @ …………….. @ $S_{n+1:\theta+1:\lambda+1:C:\alpha+1/\beta} = (A_{n+1}, L_{\theta+1}, P_{\lambda+1}, C_{\alpha+1/\beta})$

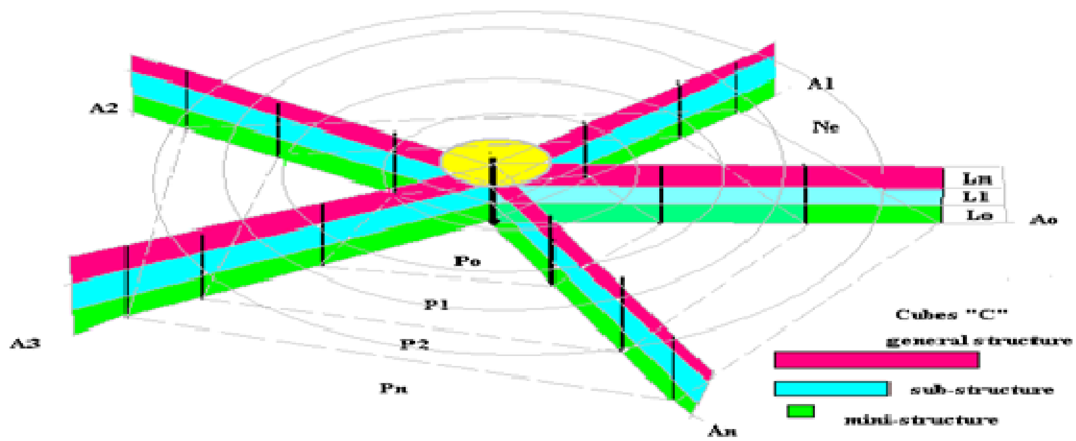


Figure 16. The cubes-coordinate coordinate space.

Finally, the cubes-coordinate space shows a general function where the dependent variable is identified by the national economy base "N_e". N_e is the final result from ten macroeconomic structures. It is based on the linking of all macroeconomics structures (S_0, S_1,…, S_n) under different axes (A_1, A_2 ,…, A_n), levels (L_1, L_2 ,…, L_n), perimeters (P_0, P_1, P_2…P_n) and cubes with different sizes and colors ($C_{0/\beta}$, $C_{1/\beta}$… $C_{n/\beta}$) in the same sub-coordinate space respectively. It is expressed by the following: (Expression 25)

(25) $/\Delta N_e/ = [/\Delta A_o/ \text{ @ } /\Delta A_1/ \text{ @ } \ldots \text{@ } /\Delta A_n/]$

CONCLUDING REMARKS

According to this paper the study of dimension(s) is necessary to be studied by general-dimensions, sub-dimensions and micro-dimensions. At the same time, we need to assume that time in each dimension is moving in different levels of speed. Hence, the 2-dimensional Cartesian coordinate plane and the 3-dimensional coordinate space can show certain limitations to visualize complex dimensions simultaneously in the same graphical space and time. In fact, the multi-dimensional coordinate spaces can open the possibility to offer an alternative graphical modeling to visualize unknown dimensions in the same graphical space and time according to this research.

ACKNOWLEDGEMENT

This work was supported by the Social and Behavioral Cluster of University of Malaya (UM).

REFERENCES

Brouwer J (1924). Über den natürlichen Dimensionsbegriff, J. Reine. Angew. Math., 142: 146-152.
Einstein A (1952). Relativity: The Special and the General Theory. New York: Three Rivers Press, pp. 78-91.
Fedorchuk VV, Levin M, Shchepin EV (1999). On Brouwer's Definition of Dimension. Russ. Math. Surv., 54: 432-433.
Inselberg A, Dimsdale B (1994). Multidimensional Lines I: Representation. SIAM J. Appl. Math., 54(2): 559-577.
Lafleur L (1960). Discourse on Method, Optics, Geometry, and Meteorology (translation from French to English from Rene Descartes -1637-). New York: The Liberal Arts Press, pp. 100-300.
Poincaré H (1912). The Foundations of Sciences: Sciences and Hypothesis, the Value of Sciences, Sciences and Method. New York: The Sciences Press, pp. 741-763.
Shchepin EV (1998). Topology of limit spaces with uncountable inverse spectra, Uspehi Mat. Nauk., 31(5-191): 191-226.