

Full Length Research Paper

The stability of mesons near event horizon of black hole

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In this paper we assert that mesons remain stable near the black hole. Because there is not any effect of quarks inside the black hole on density matrix of individual quarks within the meson. Next we calculate the total cross section for mesons produced from one black hole. We conclude that the more the mass of the quark within the hadrons, the lower is the cross section. Finally we calculate the resulting information transformation from the collapsing matter to the state of outgoing Hawking radiation for mesons.

Key words: Mesons, black hole, information loss.

INTRODUCTION

Hawking has shown that quantum black holes can emit particles. These particles which emitted from a black hole at temperature inversely proportional to its mass are known as hawking radiation (Hawking, 1974). Hawking radiation may continue until black hole evaporates completely (Hawking, 1973). Obviously, if the whole process is governed by a unitary transformation as demanded by quantum mechanics, no information can be lost (Hawking, 1974; Hawking, 1973; Hawking, 1976). On the other hand since an observer outside the event horizon of black hole has no access to the fields inside the horizon, incomplete information about the fields and therefore unitarity is lost. Information loss is very important for considering black hole production at high energy. The process of electron positron annihilation at TeV centre of mass energy forms a quantum black hole without any color or charge $e^+ + e^- \rightarrow QBH$. Possible quantum black hole Matter states of this interaction is $(0, q\bar{q})$.

Recently Horowitz and Maldacena have suggested a mechanism to reconcile the unitarity of black hole evaporation (Horowitz and Maldacena, 2004). Their mechanism was very simple. They did not use of curved space- time. In this proposal there are three different Hilbert spaces that belong to matter degrees of freedom,

incoming and outgoing radiation. The total state of black hole is a direct product of matter state and entangled state of inside and outside states of event horizon. When black hole begins evaporating, matter state will be in a maximally entangled state with incoming Hawking radiation. For describing the unknown effects of quantum gravity an additional unitary transformation S is introduced. In some notation S is acted on matter states alone. The outside state of black hole is easily obtained by projecting this entangled state on the total state of black hole (HM mechanism) (Horowitz and Maldacena, 2004).

A few years ago radiation of Scalar and Dirac fields in three dimensional black holes was considered. It has been shown that Hawking temperature for Dirac fields agrees with the one obtained from scalar fields (Ahn, 2006; Ge and Shen, 2005; Ahn et al., 2008).

For scalar fields we have (Ahn, 2006):

$$|\phi_0\rangle_{out} = \frac{1}{\cosh^2(r_\omega)} \sum_M e^{-8\pi M \omega n} \langle n | s | \phi \rangle_M | n \rangle_{out} \quad (1)$$

$$\tanh(r_\omega) = e^{-4\pi M_{scalar}\omega}, \cosh(r_\omega) = (1 + e^{-8\pi M_{scalar}\omega})^{-\frac{1}{2}}$$

Where $|\phi\rangle_M$ is initial quantum state of scalar matter, S is

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a unitary transformation and ω is related to scalar field's energy.

Similarly for Dirac fields we derive (Ge and Shen, 2005; Ahn et al., 2008):

$$|\psi_{out}\rangle = [\cos^2(r_\omega) \sum_{l=0}^1 \tan^{2l}(r_\omega) \langle l | s | \psi \rangle | l \rangle] \quad (2)$$

Where $\tan(r_\omega) = e^{-4\pi M_{BH}\omega}$, $\cos(r_\omega) = (1 - e^{-8\pi M_{BH}\omega})^{-\frac{1}{2}}$, S is a unitary transformation that acts on Dirac matter and $|\psi\rangle$ is the initial quantum state of Dirac matter.

At this stage we try to work with entropy, to compare final states in Dirac fields as well as in scalar fields as the following.

For ground state in scalar fields we have:

$$\rho = |\phi_{out}\rangle \langle \phi_{out}| = \frac{1}{\cosh^4(r_\omega)} \sum e^{-16\pi M\omega} |n\rangle \langle n| \quad (3)$$

$$S_{scalar} = \frac{8\pi M\omega}{\ln 2} + \frac{4 \ln(\cosh(r_\omega))}{\ln 2 \cosh^2(r_\omega)} \quad (4)$$

Similarly for Dirac fields we derive.

$$\rho = |\psi_{out}\rangle \langle \psi_{out}| = \cos^4(r_\omega) \sum \tan^4(r_\omega) |l\rangle \langle l| \quad (5)$$

$$S_{Dirac} = \frac{\cos^4(r_\omega) 8\pi M\omega}{\ln 2 (1 - \tan^2(r_\omega))^2} + \frac{\cos^4(r_\omega)}{1 - \tan^2(r_\omega)} \log_2(\cos^4 r_\omega) \quad (6)$$

We observe that these entropies are independent of unitary transformation S . This means that interactions between particles in matter and inside of black hole don't have any effect on the entropy of outside states of black hole.

To obtain the total cross section for scalar field and Dirac field from one black hole we need to multiply the black hole production cross section with the number of scalar fields or Dirac fields produced from a single black hole (Chamblin et al., 2009).

$$\sigma_{scalar} = N_{scalar} \sigma_{BH} \quad (7)$$

$$\sigma_{dirac} = N_{dirac} \sigma_{BH} \quad (8)$$

Where N_{scalar} and N_{dirac} are calculated in similar to

(Chamblin et al., 2009):

$$N_{scalar} = \int_0^{t_f} dt \int_0^{M_{BH}} dp \frac{C_s \sigma_s}{8\pi^2} \frac{P^2}{\sqrt{p^2 + M_{scalar}^2}} \frac{1}{(e^{\frac{p}{T_{BH}}} + 1)} \quad (9)$$

$$N_{dirac} = \int_0^{t_f} dt \int_0^{M_{BH}} dp \frac{C_s \sigma_s}{8\pi^2} \frac{P^2}{\sqrt{p^2 + M_{dirac}^2}} \frac{1}{(e^{\frac{p}{T_{BH}}} + 1)} \quad (10)$$

$$\text{In which } t_f = \frac{C_s}{M_p} \left(\frac{M_{BH}}{M_p} \right)^{\frac{d+3}{d+1}}$$

Where d is the number of extra dimensions, σ_s is the grey body factor (Chamblin et al., 2009; Giddings and Thomas, 2002; Cooper and Nayak, 2004; Cooper et al., 2003), M_p is the plank mass and C_s depends on the extra dimensions and also on the polarization degrees of freedom and T_{BH} is the temperature of black hole.

Here we only consider the color part of the wave functions $\psi_{M,S}$ and $\psi_{M,S}^*$ coming from the representation $3 \otimes 3^*$ for the color singlet wave functions of the mesons (Millers, 2004; Dimopoulos and Landsberg, 2001; Argyres et al., 1998).

$$|\psi_{M,t}\rangle = \frac{1}{\sqrt{3}} (|1_A 1_A^*\rangle + |1_B 1_B^*\rangle + |1_C 1_C^*\rangle) \quad (11)$$

Where A, B and C is related to three color of quarks.

Previously it has been shown that the Minskowski state will evolve into a state, called the Unruh state, which can be formulated as (Unruh, 1976; Ge and Shen, 2005; Ahn et al., 2008):

$$|1\rangle_M = |1_k\rangle_{out}^+ |0_{-k}\rangle_{in}^- \quad (12)$$

Where $|0\rangle_{in}$, $|1\rangle_{out}$ denotes the state inside and outside of black hole and $|1\rangle_M$ denotes the Minskowski state.

With using equation (11) and equation (12) we can easily derive:

$$|\psi_{M,S}\rangle = \frac{1}{\sqrt{3}} (|1_A 1_A^*\rangle_{out} |0_A 0_A^*\rangle_{in} + |1_B 1_B^*\rangle_{out} |0_B 0_B^*\rangle_{in} + |1_C 1_C^*\rangle_{out} |0_C 0_C^*\rangle_{in}) \quad (13)$$

and keeping the left to right order of the quark and

antiquark for its conjugate wave function

$${}_{M,S}\langle\psi| = \frac{1}{\sqrt{3}}(\langle 1_A 1_A^* |_{in} \langle 0_A 0_A^* | + \langle 1_B 1_B^* |_{in} \langle 0_B 0_B^* | + \langle 1_C 1_C^* |_{in} \langle 0_C 0_C^* |) \quad (14)$$

We can now write down the density matrices ρ for the hadrons using the direct product of $|\psi\rangle$ and $\langle\psi|$

$$\rho_{M,S} = |\psi\rangle_{M,S} \langle\psi|_{M,S} \quad (15)$$

For the quarks we look at the one quark reduced density matrices, which give the statistical state of the individual quark within the hadron. In order to get the reduced density matrices for the mesons, we project out all the antiquark states $|i^*\rangle$ and $\langle j^*|$ by using the orthonormality and the completeness properties.

$$\rho_q = \frac{1}{3}(|1_{out} 0_{in}\rangle_{AA} \langle 1_{out} 0_{in}| + |1_{out} 0_{in}\rangle_{BB} \langle 1_{out} 0_{in}| + |1_{out} 0_{in}\rangle_{CC} \langle 1_{out} 0_{in}|) \quad (16)$$

This is the reduced density matrix for the quarks in the color singlet state. For calculating density matrices for outside state, we project out all the inside states $|0\rangle_{in}$ and $\langle 0|_{out}$ by using the orthonormality properties.

$$\rho_q^{out} = \frac{1}{3}(|1_{out}\rangle_{AA} \langle 1_{out}| + |1_{out}\rangle_{BB} \langle 1_{out}| + |1_{out}\rangle_{CC} \langle 1_{out}|) \quad (17)$$

This density matrix is similar to (Millers, 2004; Dimopoulos and Landsberg, 2001; Argyres et al., 1998). Thus we conclude that near black hole mesons are stable. Because there is not any effect of quarks inside the black hole on density matrix of individual quarks within the meson.

The entropy S of the quantum states by using this density matrix becomes $S = \ln 3$. This value of S is equal to entropy of quark density of meson in flat space. In fact near black hole there is not any decreasing in entanglement between quarks inside meson. For this reason mesons remain stable in curves space-time.

Now we want to calculate the total cross section for mesons as scalar fields produced from one black hole:

$$\sigma_{meson}^1 = \frac{1}{3}(N_{quark}^A N_{antiquark}^A + N_{quark}^B N_{antiquark}^B + N_{quark}^C N_{antiquark}^C) \sigma_{BH} \quad (18)$$

In which

$$N_{quark} = \int_0^{t_f} dt \int_0^{M_{BH}} dp \frac{C_S \sigma_S}{8\pi^2} \frac{P^2}{(e^{\frac{T_{BH}}{P^2 + M_{quark}^2}} + 1)} \quad (19)$$

$$N_{antiquark} = \int_0^{t_f} dt \int_0^{M_{BH}} dp \frac{C_S \sigma_S}{8\pi^2} \frac{P^2}{(e^{\frac{T_{BH}}{P^2 + M_{antiquark}^2}} + 1)} \quad (20)$$

By using the quark masses we can compare different production cross sections with each other:

$$\sigma_{u\bar{u}} \geq \sigma_{d\bar{d}} > \sigma_{s\bar{s}} > \sigma_{c\bar{c}} > \sigma_{b\bar{b}} > \sigma_{t\bar{t}} \quad (21)$$

Furthermore maybe some mesons like pions form in matter of black hole and then exit from black hole. Their cross sections are calculated as following:

$$\sigma_{meson}^2 = N_{meson} \sigma_{BH} \quad (22)$$

In which

$$N_{meson} = \int_0^{t_f} dt \int_0^{M_{BH}} dp \frac{C_S \sigma_S}{8\pi^2} \frac{P^2}{(e^{\frac{T_{BH}}{P^2 + M_{meson}^2}} + 1)} \quad (23)$$

It is clear from equation (21) and equation (18) that $\sigma_{meson}^2 > \sigma_{meson}^1$

This means that more mesons produce in the matter of black hole and then come out of black hole. However we should regard information loss in our calculation.

Gottesman and PresKill (2004) introduce one unitary transformation U to describe interactions between matter and inside of horizon (Daniel, 2004). In their fanciful language, the transformation U can be interpreted as an interaction between the information's past and future. Various authors (Deutsch, 1991) have pointed out that such interactions can cause a breakdown of unitarity. Indeed, Schumacher and Bennett (2007) have observed that time travel can be simulated by combining quantum entanglement with post selection, and they have studied the departures from unitarity that result when interactions are also included (Ahn, 2007). By using of their method we can calculate the resulting information transformation from the collapsing matter to the state of outgoing Hawking radiation for mesons:

$$|meson\rangle_{in \otimes out} = \frac{1}{\cosh(r_\omega)} \sum e^{-4\pi M \omega n} |n\rangle_{in} \otimes |n\rangle_{out} \quad (24)$$

$$\langle meson|_{M \otimes in} = \frac{1}{\cosh(r_\omega)} \sum e^{-4\pi M \omega n} \langle n|_M \otimes \langle n|_{in} (s \otimes I) U \quad (25)$$

$$T_{meson} = \langle M \otimes_{in} meson | meson \otimes_{out} \rangle \quad (26)$$

$$f = \sum | \langle n | T_{meson} | m \rangle |^2 = \frac{1}{\cosh^2(r_\omega)}$$

If information transformation from the collapsing matter to the state of outgoing Hawking radiation be complete the value of f should be one. Thus we observe that due to the deviation of the f from unity, some of information is lost in black holes. This information loss is depended on the mass of meson.

SUMMARY AND CONCLUSION

In this paper first we compared entropies of final states in Dirac fields as well as in scalar fields. Next we concluded that near black hole mesons are stable. After that we compared the total cross section for mesons produced from one black hole. We saw that the more the mass of the quark within the hadrons, the lower is the cross section. Finally we calculated the resulting information transformation from the collapsing matter to the state of outgoing Hawking radiation for mesons. We observe that due to the deviation of the f from unity, some of information is lost in black holes.

REFERENCES

- Ahn D (2006). Final state boundary condition of the Schwarzschild black hole. Phys. Rev. D. 74, 084010.
- Ahn D (2007). Control of black hole evaporation? JHEP. 0703: 021.
- Ahn D, Moon YH, Mann RB, Fuentes-Schuller I (2008). The Black hole final state for the Dirac fields In Schwarzschild spacetime. JHEP. 0806: 062.
- Argyres PC, Dimopoulos S, March-Russell J (1998). Black Holes and Sub-millimeter Dimensions. Phys. Lett., B441: 96.
- Chamblin A, Cooper F, Nayak GC (2009). Top Quark Production from Black Holes at the CERN LHC. Phys. Lett., B672: 147-151.
- Cooper F, Mottola E, Nayak GC (2003). Minijet Initial Conditions For Non-Equilibrium Parton Evolution at RHIC and LHC. Phys. Lett. B, 555: 181.
- Cooper F, Nayak GC (2004). Interaction of a TeV scale black hole with the quark-gluon plasma at CERN LHC. Phys. Rev. D. 69, 065010.
- Deutsch D (1991). Quantum mechanics near closed timelike lines. Phys. Rev. D., 44: 3197.
- Dimopoulos GC, Landsberg G (2001). Black Holes at the Large Hadron Collider. Phys. Rev. Lett., 87: 161602.
- Ge HX, Shen GY (2005). Reconsidering the black hole final state in Dirac fields. Phys. Lett., B612: 61-64.
- Giddings SB, Thomas S (2002). High energy colliders as black hole factories: The end of short distance physics. Phys. Rev. D., 65: 056010.
- Gottesman D, Preskill J (2004). Comment on "The black hole final state". JHEP., 0403: 026.
- Hawking SW (1973). Particle creation by black holes. Common. Math. Phys., 43: 199.
- Hawking SW (1974). Black-hole evaporation. Nature, 248: 30.
- Hawking SW (1976). Breakdown of predictability in gravitational collapse. Phys. Rev. D., 14: 2466.
- Horowitz GT, Maldacena J (2004). The black hole final state. J. High. Energy. Phys., 02: 008.
- Millers DE (2004). Entropy for $3+1$ Quark States. Eur. Phys. J., C34: 435-437.
- Schumacher B, Bennett CH (2007). unpublished. See <http://qip-server.tcs.tifr.res.in/qip/HTML/Courses/Bennett/TIFR5.pdf>, slides 39-47.
- Unruh WG (1976). Notes on Black Hole Evaporation. Phys. Rev. D, 14: 870.