

Full Length Research Paper

Parametric investigation on an off-centre braced frame system's stiffness

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In this paper, a particular off-centre braced (OCB) system subjected to lateral load, which induced compressive force to the brace elements is examined. This bracing system consists of three members, where the diagonal member is not straight and it is connected to the corner of the frame by a third member. The out-of-straightness of the diagonal members will introduce eccentricity to the system. This system improves the energy dissipation due to earthquake as well as its eccentricity allows architects to have more openings in the panel areas. In this regard, the location of connection point of the three braced elements that is the eccentricity, considering the opening dimensions has significant effect on the stiffness of the system. In order to assess the influence of the connection position and other parameters such as cross-section area of the brace elements and span/height ratio of the frame on the stiffness of the system, analytical studies to obtain the stiffness equations have been developed. The results indicate that as the eccentricity increasing (connection point moves closer to the corner of the frame), the frame's stiffness decreases. Also, the cross-section area of the third member has a significant role on the stiffness of this system and can make up the stiffness elimination due to increasing eccentricity of the connection point. In addition, a range of values in locating the brace elements connection point is introduced, which could be helpful for designers.

Key words: Brace, eccentricity, connection, earthquake.

INTRODUCTION

In regions subjected to seismic activities, the occurrence of severe earthquakes can cause serious damage to properties and loss of lives if buildings are not provided with seismic resistance capability. The need to have buildings suitably designed against earthquake is even more necessary in highly urbanized areas with many tall buildings where economical and human loss is of major concern to the community. As the nature and occurrence of earthquakes are random, it is necessary to consider different levels of earthquake intensity in the design of earthquake resistance structures. The design of seismic resistant buildings in seismic regions should satisfy two criteria. First, under frequent and low to moderate earthquakes, the structure should have sufficient strength and stiffness to control deflection and prevent any

structural damage. Second, under rare and severe earthquakes the structures must have sufficient ductility to prevent collapse (Roeder and Popov, 1977).

For high and medium rise buildings, structural steel has been used extensively due to its excellent strength and ductility properties. In seismic design of such steel frames, there are some common conventional seismic resistant approaches, where braced frames are among the most common steel structures for resisting lateral loads. In general, they are divided into two groups: concentric and eccentric. Concentric braced systems are more desirable because of relative good stiffness, along with their easy construction and economy aspects. On the other hand, eccentric braces need more construction accuracy, thereby it decreases construction speed and needs more cost in spite of better stiffness performance and higher energy dissipation because they mainly yield in bending (Popov et al., 1985; IF et al., 1988; Tsai et al., 1993; Kim and Choi, 2005; Moghaddam et al., 2005).

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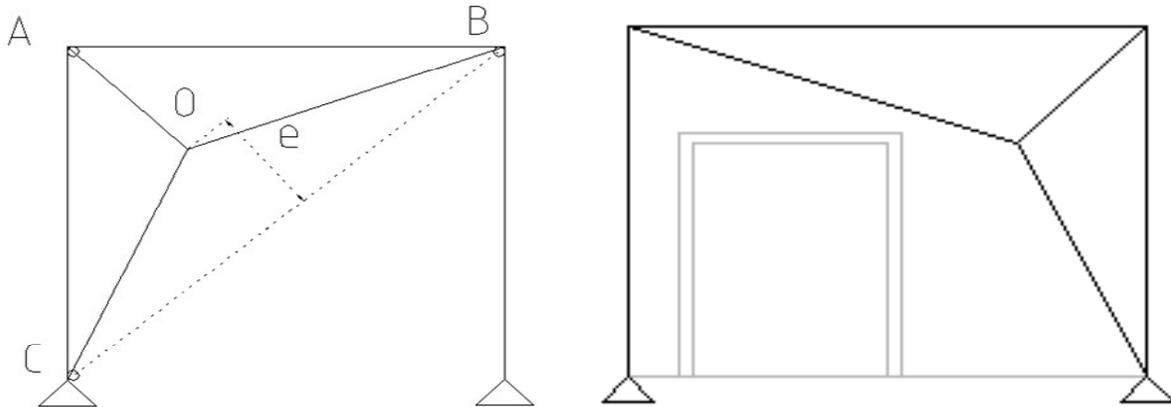


Figure 1. An Off-Centre brace system.

Regardless of their benefits, designers occasionally use some alternative expedient to solve significant problems in opening especially in northern and southern faced of the buildings.

One of the solutions is to adopt Off-centre Braced system (OCB) which is also called Non-geometric bracing system in some regions. This braced system does not have limitation compared to the previous types of braces in making openings. Therefore, a suitable plan of Off-centre brace can be used extensively for more possibilities to make openings concurrent with its significant capabilities in energy dissipation. This braced system consists of three members as shown in Figure 1. In this system, the two diagonal elements are not straight and connected at point O. The out-of-straightness introduces an eccentricity, e to the system and the third member is connected from this point to the corner of the frame. This bracing system is commonly adopted in the seismic region. Several studies on the behaviour of this system considering the brace elements under tension have been reported (Moghaddam and Estekanchi 1995; 1999). However, evidence from Bam earthquake in the late 2003 shown that the out-of-plane buckling of the brace elements under compression is more critical. Currently, this behaviour is not considered in the design because of two dimensional assumption of this system by designers, thereby it caused severe damages to the structure. In order to address this issue, investigation on the OCB system subjected to lateral load, which induced compression forces on the brace elements is examined here.

The focus of this paper is to investigate the influence of the connection point's location on the frame stiffness. Computation of the frame stiffness is carried out in MATLAB computer program (MATLAB(R2007a) 2007). Some recommendations regarding the selection of the connection point, the effect of the cross-section area of the third member and the frame aspect ratio are presented.

CONVENTIONAL SEISMIC RESISTANCE METHODS

Moment Resisting Frames (MRFs)

During a major earthquake, energy dissipation in an MRF is mainly obtained through inelastic action in the beam-column joints, and such frames generally have considerable ductility if the beams and columns are proportioned to meet the so-called strong column –weak girder design concept. The deformation mechanism of MRF consists of three major components: (i) due to column flexure, (ii) due to beam flexure, and (iii) due to panel zone distortion (Black et al., 1980 to 10; Popov, et al., 1985; Yanga and Yang, 2009). Concentrically braced frames (CBFs); This system is one of the most desirable methods for designing structures to resist the lateral load because of the relative good stiffness, along with their easy construction and economy aspects. Hence, these important criteria made this system more common than the previous method.

In this system, a vertical truss system is formed utilizing a set of diagonal braces placed concentrically at the joints. The braces provide effective resistance against lateral loading during minor earthquakes. In general, CBFs have sufficient stiffness to limit elastic interstory drift, without the costs involved with MRFs. However, when overloading occurs due to a major earthquake, conventionally designed braces can buckle exhibiting the plastic mechanism. In addition, experimental studies on the behaviour of brace struts under cyclic axial loading indicate that their load carrying capacity can significantly decrease under severe cyclic loads, resulting in a large loss of energy dissipation capacity. This behavior directly affects CBFs global behavior, in which the system become unstable for a relatively small inelastic deformation level. Also, their lateral load carrying capacity dramatically decrease as the number of cyclic excursions and/or cyclic displacements increased (Masion and Popove, 1980; Kasai and Popov, 1986;

Kim and Choi, 2005; Moghaddam et al., 2005; Davaran and Hoveidae, 2009; Yanga and Yang, 2009).

In order to improve the performance of CBFs, some alternative design schemes have been proposed such as by making the slenderness ratio of the brace larger, which leads to the reduction of loss in the compressive strength of brace under cyclic loading.

Eccentrically braced frames (EBFs)

The two basic requirements for seismic design are high stiffness at working load level and large ductility at severe overloadings. These requirements are difficult to be satisfied when the above conventional frames are used. On the contrary, Eccentrically Braced Frames (EBFs) offer an economical steel framing system satisfying both requirements. In all types of this system, the vertical components of axial forces in the braces are held in equilibrium by shear and bending moments in short beams of lengths, which is the active links. Active links are designed to remain elastic at working loads and deform inelastically on over loading of structure, thereby dissipating large amount of energy. In this system the hazardous brace buckling can be entirely prevented since the link acts as fuse to limit the brace axial force. Also this frame has a much greater lateral resisting capacity than that of an MRF if the beam section used are the same (Roeder and Popov, 1977; Manheim and Popov, 1982; Tsai et al., 1993; Özhendekci and Özhendekci, 2008; Bosco and Rossi, 2009; Mastrandrea and Piluso, 2009; Mastrandrea and Piluso, 2009).

Experimental results showed that an EBF can be almost as stiff as a CBF when an appropriate eccentricity is assigned for the braces, and yet behaves in a very ductile manner at large cyclic over loadings. The active links will dissipate great amount of energy by inelastic action (Engelhardt and Popov, 1989).

Knee bracing frame system (KBF)

In a KBF, the diagonal brace provides most of the elastic lateral stiffness to eliminate pinching in the hysteresis of structures. The brace is designed to resist compression without buckling. The KBF system is different from that proposed by Aristizabal-Ochoa (Fujimoto et al., 1972; Aristizabal-Ochoa, 1986) where the brace was designed for tension only. The knee element is designed to yield in flexural during severe seismic excitation. The size of the knee element is chosen such that it yields before the beam and columns do. In this manner the knee element function like a ductile fuse that dissipates energy in the event of a severe earthquake. The yield of knee element limits the brace load thereby preventing the brace from

buckling.

The knee element can be replaced easily after earthquake since it is a secondary member in the structure. As the knee element serves like an anchor to the brace it is also known as a ductile knee anchor. Because of its good characteristics, many studies have been conducted to improve this system behaviour (Balendra et al., 1990; Balendra et al., 1994; Lotfollahi and Mofid, 2006; Mofid and Lotfollahi, 2006).

PRIMARY CHARACTERISTICS OF OFF-CENTRE BRACED FRAME (OCB) SYSTEM

In this particular bracing system which is capable of providing certain amount of seismic isolation to the structure, the tensile diagonal strut is not straight (Moghaddam and Estekanchi, 1995). Because of cyclic nature of earthquake load, this kind of braces are used in double bay rather than single bay as shown in Figure 2 and general configuration of OCB system for multistory building is shown in Figure 3. The behavior of this system against lateral load is one of the most important characteristics of OCB systems. As shown in Figure 4, by imposing a lateral load to this braced frame, the three braced members who are connected at point O will be in tension. Simultaneously in the other bay, the three braced members are in compression. Evidence from Bam earthquake shows that compressive forces imposed on point O in Figure 4 lead to an out-of-plane buckling of the system.

Generally, most studies on this braced system focused on the behaviour of brace elements under tensile forces (Moghaddam and Estekanchi, 1995; Moghaddam and Estekanchi, 1999). However, when these members are under compressive load, lesser load can be sustained because of weakness in their connections. In some occasions, instability in out-of-plane can be observed which limit the capability of braced elements to bear tensile axial force in the next cycle. Some engineers do not pay attention to its instability because of two dimensional imaginations. This can cause a lot of damages during the severe earthquakes. Catastrophic earthquake in Bam (central city of Iran) in 2003 demonstrate these defaults in designing OCB system.

DERIVATION OF FRAME STIFFNESS

The Off-Centre Braced frame as shown in Figure 5 is assumed as a truss system and the location of brace members connection, point O, is introduced by two parameters, m and n which are coefficients of width and height of the frame respectively. With above explanations, the geometric and trigonometric

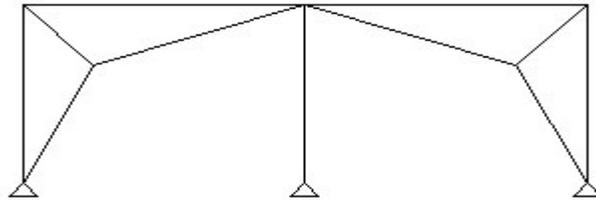


Figure 2. A common type of OCB in seismic areas.

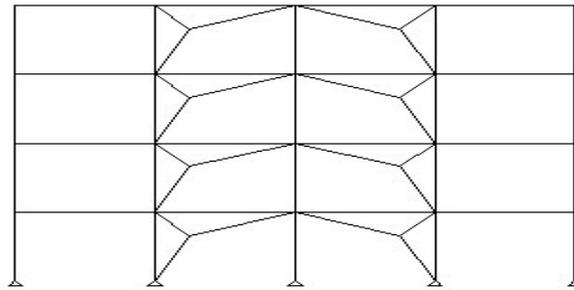


Figure 3. A multistory building with OCB system.

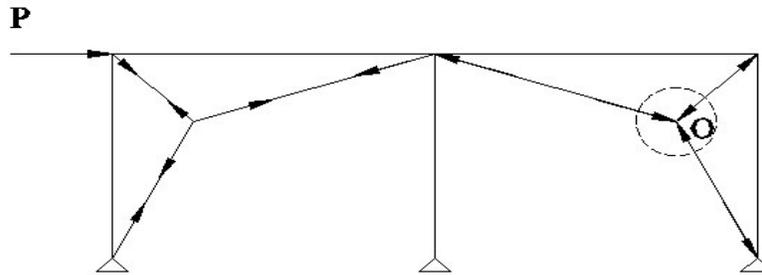


Figure 4. Axial forces in OCB frame due to applied lateral load, P.

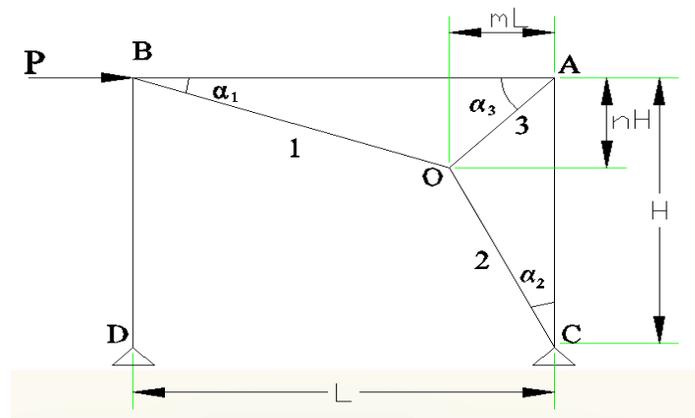


Figure 5. A model of OCB frame.

parameters of this brace model can be determined as follows:

$$L_1 = \sqrt{(nH)^2 + (L(1-m))^2} \tag{1}$$

$$L_2 = \sqrt{(mL)^2 + (H(1-n))^2} \tag{2}$$

$$L_3 = \sqrt{(nH)^2 + (mL)^2} \tag{3}$$

Where L_1 , L_2 and L_3 are length of brace elements 1, 2 and 3 respectively.

By solving equilibrium equations of statically determinate pin-jointed frame, axial forces of these brace members can be obtained as follows:

$$F_1 = \frac{-PH}{L \sin \alpha_1} \tag{4}$$

$$F_2 = \frac{-P}{\sin \alpha_2} \tag{5}$$

$$F_3 = \frac{-P}{\sin \alpha_3} \left(\frac{\cos \alpha_2}{\sin \alpha_2} - \frac{H}{L} \right) \tag{6}$$

Frame displacement is computed by structural equation as follow:

$$\Delta = \sum_{i=1}^3 \frac{P_i P_{i1} L_i}{E_i A_i} \tag{7}$$

Where i is the number of members, P_i is the axial force of member i due to lateral force P , P_{i1} is the axial force of member i due to unit lateral force, E_i and A_i are respectively modulus of elasticity and cross section area of brace member i . In all computations, modulus of elasticity is assumed constant and equal to E . Hence:

$$\Delta = \frac{PH^2 L_1}{EA_1 L^2 \sin^2 \alpha_1} + \frac{PL_2}{EA_2 \sin^2 \alpha_2} + \frac{PL_3 (\cot \alpha_2 - \frac{H}{L})^2}{EA_3 \sin^2 \alpha_3} \tag{8}$$

Based on the strength of materials principles, the relation between cross section areas of brace elements can be expressed by the proportion of brace member axial forces, therefore:

$$A_3 = \frac{A_1 L \sin \alpha_1}{H \sin \alpha_3} \left(\cot \alpha_2 - \frac{H}{L} \right) \tag{9}$$

$$A_2 = \frac{A_1 L \sin \alpha_1}{H \sin \alpha_2} \tag{10}$$

By replacing the two aforementioned equations and trigonometric parameters of this frame, equation 8 is simplified as:

$$\Delta = \frac{P((nH)^2 + (L(1-m))^2)^{\frac{1}{2}}}{nL^2 EA} \times \frac{((m-m^2)L^2 + (n-n^2)H^2)}{mn} \tag{11}$$

Based on Hooke's law the stiffness of system can be obtained as:

$$K = \frac{nL^2 EA}{((nH)^2 + (L(1-m))^2)^{\frac{1}{2}}} \times \frac{mn}{((m-m^2)L^2 + (n-n^2)H^2)} \tag{12}$$

Parameters H , L , m , n , E and A are varied to investigate the effect of changing the eccentricity of the diagonal members on the frame displacement and stiffness.

The eccentricity of point O (as illustrated in Figure 6) can be further defined by introducing parameters e_1 and e_2 (Moghaddam and Estekanchi, 1999) where:

$$e_1 = \frac{OH}{AH} \tag{13}$$

$$e_2 = \frac{CH}{CB} \tag{14}$$

Here, e_1 represents the deviation of the members BOC from the diagonal BC and e_2 defines the position of the projection of O on the diagonal BC. By replacing the dimension of the members' length above, e_1 and e_2 are obtained as:

$$e_1 = 1 - m - n \tag{15}$$

$$e_2 = \frac{(1-n)H^2 + mL^2}{\sqrt{H^2 + L^2}} \tag{16}$$

By considering the two above equations and simplifying them, two formulas for parametric computation of m and n parameters are obtained:

$$m = e_2 - e_1 \left(\frac{H}{\sqrt{H^2 + L^2}} \right)^2 \tag{17}$$

$$n = 1 + e_1 \left(\frac{H^2}{\sqrt{H^2 + L^2}} - 1 \right) + e_2 \tag{18}$$

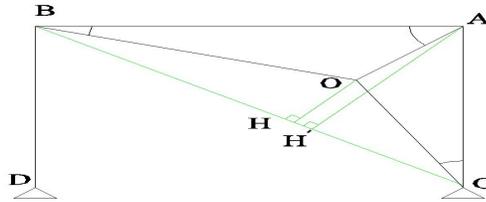
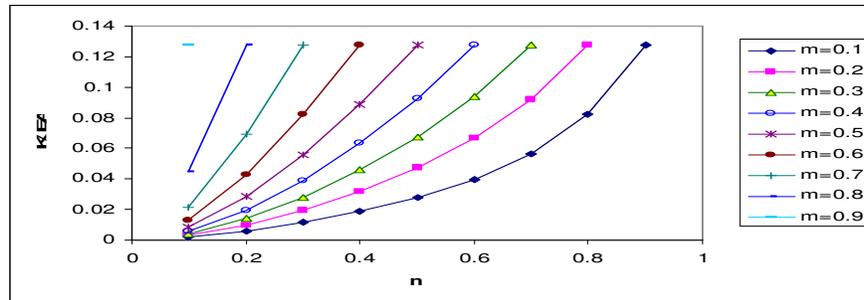
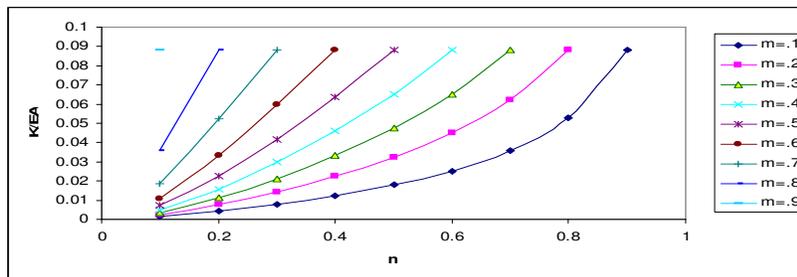


Figure 6. An OCB frame with new definition of eccentricity.



(a) H/L=3/4



(b) H/L=4/4

Figure 7. Effect of connection point on stiffness of OCB frame.

According to equation 15, any connection point lies on the diagonal of the frame has the eccentricity e_1 equal zero, consequently:

$$\text{If } e_1 = 0 \Rightarrow m + n = 1 \tag{19}$$

This indicates that the maximum amount of the coefficient sum occurs when point O is located on the diagonal member BC. The influence of these parameters on the frame displacement and stiffness are described in the next section.

PARAMETRIC INVESTIGATIONS

Location of connection point

The effect of eccentricity that is the location of connection

point O, from the diagonal member BC (Figure 6) is investigated. By using the derived equations, a computer programme based on MATLAB was developed in order to examine the effects of parameters mentioned in the previous section on the frame stiffness. For a particular height/span ratio ($H/L=3/4$) and by setting n values, the relationship of stiffness with increasing value of m can be obtained, as shown in Figure 7 (a). The curves show that the stiffness increases as the values of m increases (that is point O moves closer to diagonal BC). The same effect can be seen when m is constant and n increases. On the other hand, when the ratio of H/L is increases (that is $H/L=4/4$ as shown in Figure 7 (b), the stiffness decreases compared to the previous frame dimension. Overall, it is shown that the maximum value of stiffness which is identical for all values of n is obtained when point O is on the diagonal BC or in the other word $m + n = 1$.

The effect of eccentricities e_1 and e_2 on the stiffness of

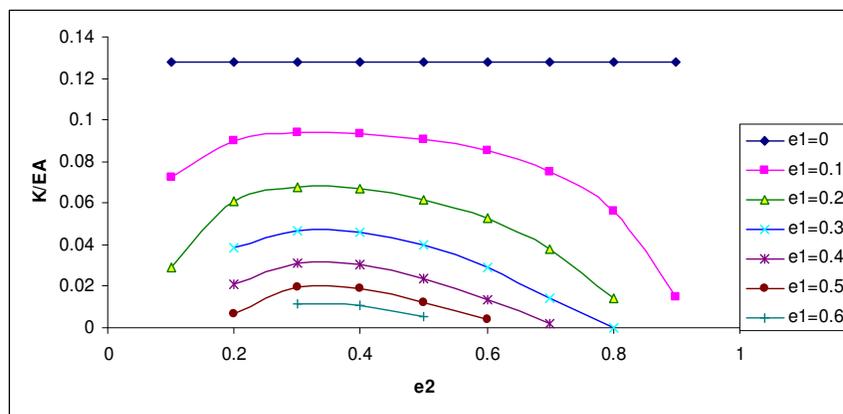
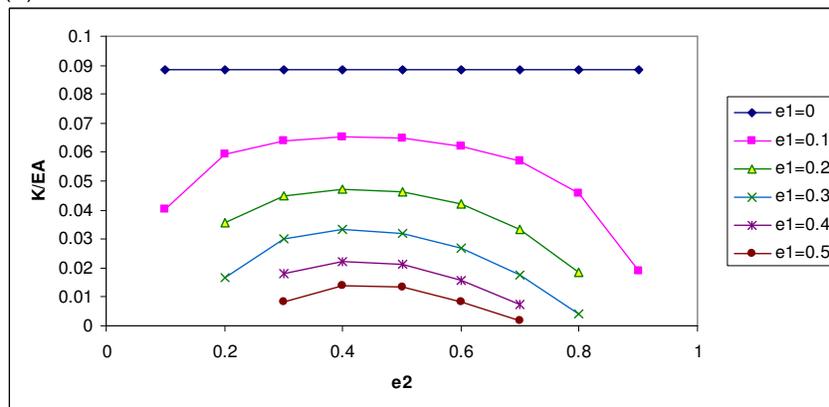
(a) $H/L=3/4$ (b) $H/L=4/4$

Figure 8. Effect of eccentricity e_1 and e_2 on stiffness of OCB frame.

the two frame dimensions is shown in Figure 8. This figure indicates that e_1 value has significant effect on system's stiffness and a designer must attempt to decrease this eccentricity in order to have higher stiffness. In addition, it can be concluded that for a constant value of e_1 , the stiffness has an almost parabolic relation with e_2 values. In this regard, maximum stiffness occurs when the eccentricity e_2 lies on the perpendicular line from frame corner to the frame diagonal. Also, the maximum stiffness of the frame is obtained when point O lies on the diagonal BC (that is $e_1 = 0$). In addition, by increasing the eccentricity and moving the connection point closer to the frame corner, the stiffness decreases. It means that the allowable lateral force decreases. These results are consistent with the findings from the previous studies (Moghaddam and Estekanchi 1995; Moghaddam and Estekanchi 1999).

In general, it can be concluded that the stiffness of the frame is strongly depend on e_1 value, where smaller e_1 will guarantee higher stiffness of the frame and the

designer must adjust the value of e_2 to have enough area for opening. Consequently, a designer should try to select a connection point as close as possible to the diagonal member. For a constant value of e_1 , there is small variation of stiffness for an e_2 value in the range of 0.3 to 0.6. This guideline is helpful for designer to manage the opening in this restricted area.

Cross section area of the third member

Investigations on the effect of the brace elements' cross-section areas on the stiffness of OCB system are summarized in Tables 1 and 2. Two frame configurations are examined that is $H/L=3/4$ and $H/L=4/4$. Only e_1 value equal to 0.3 is considered here. The stiffness is presented as a coefficient of K/EA . In this study, the cross-section area are divided into three cases; (i) all members are having similar area, A (ii) member 3 (A3) has bigger cross section area than the other two diagonal members, and (iii) member 1 (A1) and 2 (A2) have bigger

Table 1. Stiffness coefficient (K/EA) of OCB frame with different member's cross section area ($H/L=3/4, e_1 = 0.3$).

e_2	A1=A2=A3=A	A1 and A2=A			A3=A		
		A3=1.2A	A3=1.5A	A3=2A	A1 and A2=1.2A	A1 and A2=1.5A	A1 and A2=2A
0.2	0.247	0.264	0.29	0.333	0.279	0.327	0.407
0.3	0.559	0.627	0.729	0.899	0.603	0.668	0.777
0.4	0.754	0.858	1.01	1.27	0.8	0.87	0.987
0.5	0.656	0.742	0.871	1.09	0.701	0.769	0.882
0.6	0.416	0.458	0.521	0.627	0.456	0.518	0.619
0.7	0.217	0.227	0.243	0.27	0.249	0.298	0.38
0.8	0.115	0.115	0.115	0.115	0.138	0.172	0.23

Table 2. Stiffness coefficient (K/EA) of OCB frame with different member's cross section area ($H/L=4/4, e_1 = 0.3$).

e_2	A1=A2=A3=A	A1 and A2=A			A3=A		
		A3=1.2A	A3=1.5A	A3=2A	A1 and A2=1.2A	A1 and A2=1.5A	A1 and A2=2A
0.3	0.228	0.248	0.279	0.33	0.253	0.29	0.353
0.4	0.404	0.456	0.533	0.662	0.434	0.477	0.551
0.5	0.496	0.565	0.668	0.84	0.526	0.572	0.648
0.6	0.404	0.456	0.533	0.662	0.434	0.477	0.551
0.7	0.228	0.248	0.279	0.33	0.253	0.29	0.353

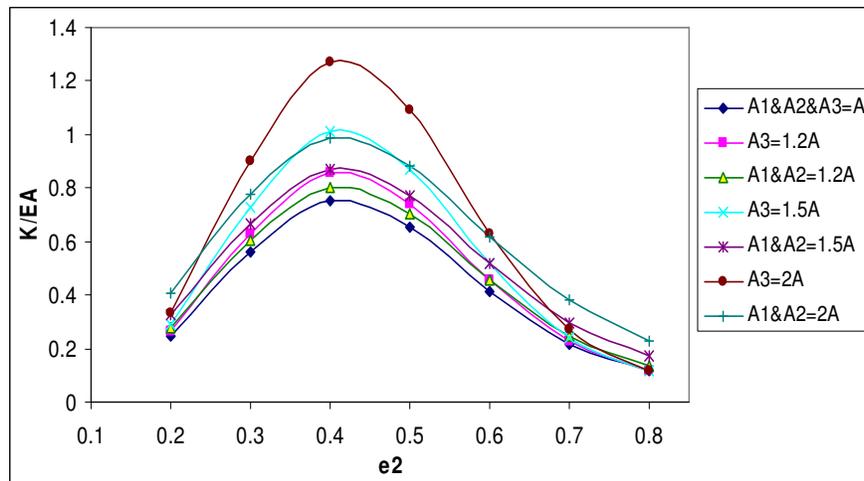


Figure 9. Effect of cross section area of brace member on stiffness of OCB frame according to Table 1.

cross-section area than member 3. It is shown that by increasing the cross-section area of the third member, the stiffness of the system can be increased without being necessary to change the braced elements connection point. This solution will be useful for designers in their design. However, by referring to Figures 9 and 10, as the connection point moves closer towards both ends

of the frame diagonal, opposite result is observed.

This effect is not significant because of locating the connection point in this range is impractical. Therefore, it can be concluded that by increasing the cross-section area of the third member, more economic and higher stiffness design can be obtained because the longer elements (members 1 and 2) have smaller cross-section

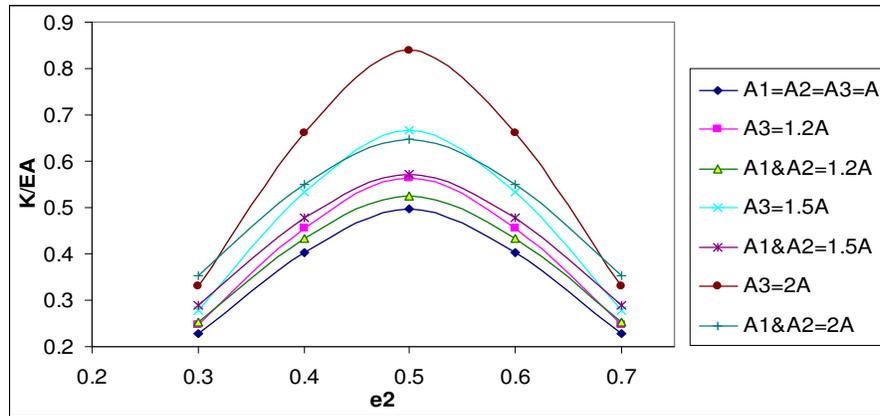
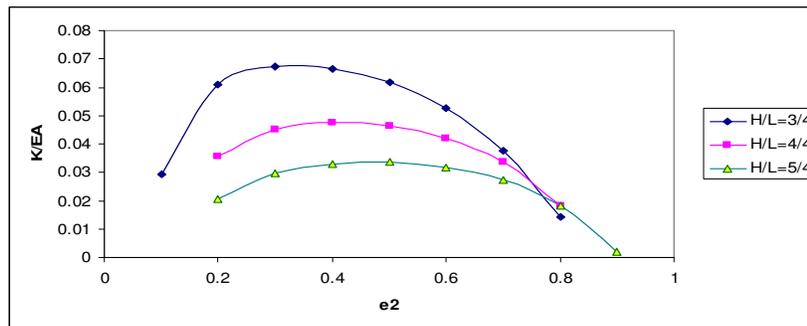
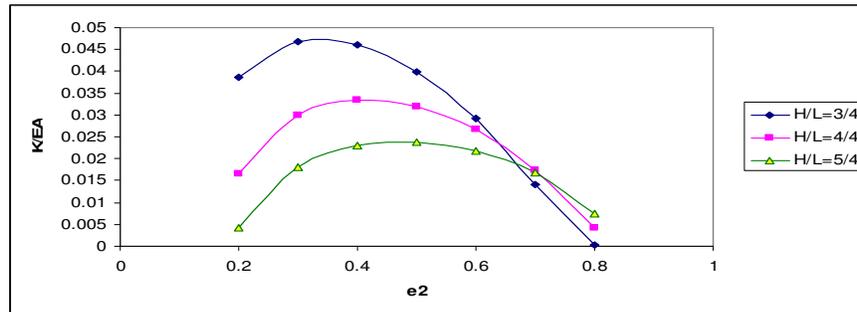


Figure 10. Effect of cross section area of brace member on stiffness of OCB frame according to Table 2.



(a) $e_1 = 0.2$



(b) $e_1 = 0.3$

Figure 11. Effect of different aspect ratio on stiffness of OCB frame.

area compared to the shortest brace element.

Frame aspect ratio

In order to investigate the effect of height/span ratio on the stiffness of this braced system, three frame configurations are examined that is $H/L=3/4$, $4/4$ and $5/4$. For this study, the cross-section areas of the brace

elements are the same. In each frame, location of the connection points are varied and only two eccentricity cases are considered that is $e_1 = 0.2$ and $e_1 = 0.3$. The results as depicted in Figure 11 show that the frame stiffness decreases as the height/span ratio increases. However, this trend is not consistent with increasing values of e_2 , where opposite pattern is observed. Again, for all frame ratio, the stiffness decreases as the

connection point moves towards the two ends of the frame diagonal.

For a higher frame ratio, the value of e_2 particularly in the range of 0.3 to 0.7 is less significant as the stiffness of the frame in this range is almost consistent. Hence, designer has more freedom to locate the opening. For a small frame ratio, the value of e_2 is very sensitive, where the frame stiffness is significantly varies.

CONCLUSION

Off-centre braced system which is used in seismic areas is more desirable than some other conventional seismic resistance designs. For this particular system, location of the brace elements connection point is very important because of its significant effect on the frame stiffness. This will lead to improvement of the energy dissipation due to earthquake as well as its eccentricity allows architects to have more openings in the panel areas. The analysis and parametric studies show that by increasing the eccentricity that is the braced elements connection point moves closer to the frame corner, the system's stiffness decreases. By increasing the cross-section area of the third member, the loss of stiffness due to eccentricity can be recovered in some extent.

It is suggested that the designer should decrease the value of e_1 in their designs and try to manage the coefficient of e_2 to be in the range of 0.3 to 0.7 to optimize the stiffness.

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