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Full Length Research Paper

Anti-swing control of a double-pendulum-type overhead crane using parallel distributed fuzzy LQR controller

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One of the common industrial structures that are used widely in many harbors and factories, and buildings is overhead crane. Overhead cranes are usually operated manually or by some conventional control methods. In this paper, we propose a hybrid controller, that includes both position regulation and anti-swing control. According to Takagi-Sugeno fuzzy model of an overhead crane, a fuzzy controller designed with parallel distributed compensation and linear quadratic regulation. With the Takagi-Sugeno fuzzy modeling, the nonlinear system is approximated by the combination of several linear subsystems in the corresponding fuzzy state space region. Then by constructing a linear quadratic regulation sub-controller according to each linear subsystem, a parallel distributed fuzzy LQR controller is designed. Further, the stability of the overhead crane with the parallel distributed fuzzy LQR controller is discussed. Simulation results illustrated the validity of the proposed control algorithm and it is compared with a similar method parallel distributed fuzzy controller.

Key words: Pattern parallel distributed compensation, Takagi_Sugeno fuzzy modelling, overhead crane, linear matrix inequality, linear quadratic regulation.

INTRODUCTION

Overhead cranes are widely used in industry transportation system such as factories, harbors, work-shops, building and rubbish manipulation to transport all kinds of massive goods. The goal of controlling the overhead crane is transporting the payloads to the required position as accurately as possible without collision with other equipments. So, it is necessary to control the crane such that the swing angle of load, the position error of trolley and the control signal are minimized.

Different control methods have been developed for reducing position error of trolley and swing angle of load. Linear and nonlinear control solutions are developed in Wang et al. (2006), Karkoub and Zribi (2002), Goritov and Korikov (2001) and Yu et al. (1995) to design feed back and feed-forward controllers. Some fuzzy-based solutions are also developed in Benhidjeb and Gissinger (1995), Liu et al. (2003), Liu et al. (2005b), and Orbisaglia et al. (2008). Anti-swing control methods based on sliding mode control are proposed in Liu et al. (2003, 2005a, b), Orbisaglia et al. (2008), Bartolini et al. (2000), and Corradini and Orlando (2007). Furthermore, gain scheduling (Corriga et al., 1998), input shaping control (Singhose et al., 2000, 1994), neural network and state feed-back control (Moreno et al., 1998), and cerebellar model articulation controller (Rodriguez et al., 2007; Albus, 1975) have been used to control swing angle of an overhead crane. In some studies, it was assumed that the mass of hook is negligible to simplify the modeling and control of system. But, in this study we did not ignore the mass of the hook and the overhead crane was looked as a double-pendulum-type system.

In this study, an alternative approach to nonlinear optimal control based on fuzzy logic was proposed to control the trolley position and load swinging angle of an overhead crane. The proposed optimal fuzzy control is based on a quadratic performance function (Tanaka and

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Wang, 2001). In this method, the controller is designed by solving a minimization problem that minimizes the upper bound of a given quadratic performances function (Tanaka and Wang, 2001). One of the advantages of this method is that the design conditions are represented in terms of LMIs (Tanaka and Wang, 2001). The stability analysis and control design problems can be reduced to linear matrix inequality (LMI) problems (Tanaka and Wang, 2001).

In this study, the modeling of crane system is introduced, the related mathematical backgrounds are presented, a T-S fuzzy model is created to approximate the overhead crane, and also a parallel distributed fuzzy LQR controller is designed.

DYNAMIC MODEL OF OVERHEAD CRANE

A double-pendulum-type overhead crane with a suspended load is shown in Figure 1. It is made up of a trolley moving along a horizontal axis by applying a force, *F* (Chang, 2007). A cable hangs below the trolley and supports a hook and the rigging and the payload are modeled as another cable and a mass point.

Denoting with *m* the trolley mass, m_1 the hook mass, m_2 the load mass, l_1 the primal cable length, l_2 the cable length that have modeled the rigging, *g* the gravitational acceleration, *x* the position of the gantry along horizontal axis, θ_1 the swing angle of the primal cable, θ_2 the swing angle of the second cable respect the vertical and *F* the force applied at the gantry, the dynamic equation of the overhead crane system in the x-y plane will be derived as follows (Weiping et al., 2005):

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \quad (1)$$

where $q=[x,\theta_1,\theta_2]^T$, $\tau=[F,0,0]^T$

$$M(q)=\begin{bmatrix} m+m_1+m_2 & (m_1+m_2)l_1\cos\theta_1 & m_2l_2\cos\theta_2 \\ (m_1+m_2)l_1\cos\theta_1 & (m_1+m_2)l_1^2 & m_2l_1l_2\cos(\theta_1-\theta_2) \\ m_2l_2\cos\theta_2 & m_2l_1l_2\cos(\theta_1-\theta_2) & m_2l_2^2 \end{bmatrix},$$

$$C(q,\dot{q})=\begin{bmatrix} 0 & -(m_1+m_2)l_1\dot{\theta}_1\sin\theta_1 & -m_2l_2\dot{\theta}_2\sin\theta_2 \\ 0 & 0 & m_2l_1l_2\dot{\theta}_1\sin(\theta_1-\theta_2) \\ 0 & -m_2l_1l_2\dot{\theta}_1\sin(\theta_1-\theta_2) & 0 \end{bmatrix}$$

$$G(q)=\begin{bmatrix} 0 & (m_1+m_2)gl_1\sin\theta_1 & m_2gl_2\sin\theta_2 \end{bmatrix}^T$$

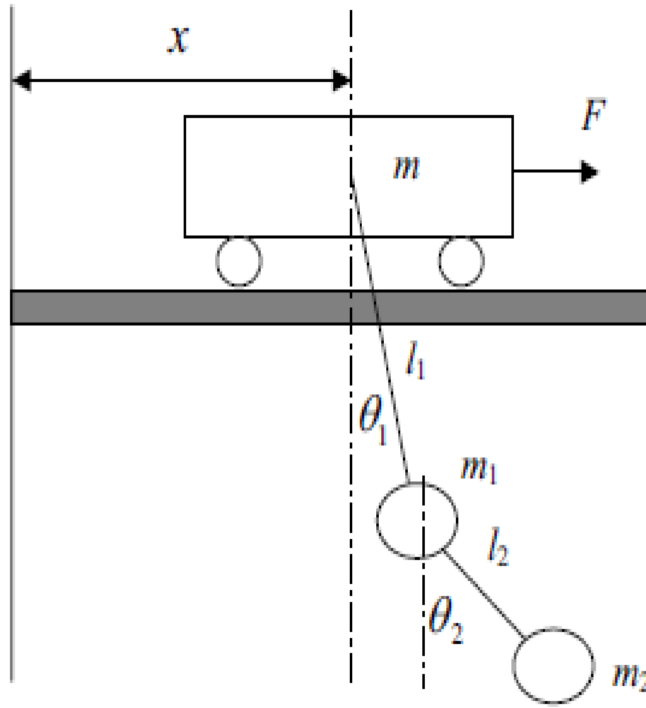

Figure 1. Model of a double-pendulum-type overhead crane.

MATHEMATICAL BACKGROUNDS

Parallel distributed compensation

T-S fuzzy modelling technique is a useful method to approximate complex nonlinear systems (Tao et al., 2010). The state space of the nonlinear system is fuzzily partitioned into fuzzy regions. In each fuzzy region, the nonlinear system is approximated by a linear subsystem to reach a T-S fuzzy model. Consider the nonlinear system, $\dot{x}=f(x,u)$, where f is a nonlinear function. The i^{th} rule of T-S fuzzy models is of the following forms (Tanaka and Wang, 2001):

Model rule i:

IF $z_1(t)$ is M_{i1} and … and $z_p(t)$ is M_{ip},

Then

$$\begin{cases} \dot{x}(t)=A_i^*x(t)+B_i^*u(t) \\ y(t)=C_ix(t) \end{cases}, i=1,2,\dots,r. \quad (2)$$

The state equation of the system with a T-S fuzzy model is inferred as follows:

$$\dot{x}(t)=\frac{\sum_{i=1}^{r} w_i(z(t))\{A_i^*x(t)+B_i^*u(t)\}}{\sum_{i=1}^{r} w_i(z(t))}, \quad (3)$$

Where $z(t)=\lfloor z_1(t) \quad z_2(t) \quad \dots \quad z_p(t) \rfloor$,

$w_i(z(t))=\prod_{j=1}^{p} M_{ij}z_j(t)$ and the fuzzy controller via PDC is as follows:

Control rule i:

If $z_1(t)$ is M_{i1} and … and $z_p(t)$ is M_{ip}

Then

$$u(t)=-F_i x(t),\ i=1,2,\ldots,r. \tag{4}$$

The overall fuzzy controller is represented by

$$u(t)=-\frac{\sum_{i=1}^{r} w_i(z(t))F_i x(t)}{\sum_{i=1}^{r} w_i(z(t))}. \tag{5}$$

Parallel distributed fuzzy controller design

According to control law (Equation 5), the goal is to find F_i ($i=1,\ldots,r$). We can define F_i using LMIs.
Stable fuzzy controller design: Find $X>0$ and M_i ($i=1,\ldots,r$) satisfying (Tanaka and Wang, 2001).

$$-XA_i^T - A_i X + M_i^T B_i^T + B_i M_i > 0, \tag{6}$$

$$-XA_i^T - A_i X - XA_j^T - A_j X + M_j^T B_i^T + B_i M_j$$
$$+M_i^T B_j^T + B_j M_i \geq 0$$

$$i<j \quad s.t \quad h_i \cap h_j \neq \phi \tag{7}$$

Where $M_i = F_i X$ and $X = P^{-1}$ such that P is a common positive definite matrix that its existance guarantees the stability of controller. The above conditions are LMIs with respect to X and M_i. The feedback gains F_i can be obtained as $F_i = M_i X^{-1}$.

Quadratic performance function

The control objective of optimal fuzzy control is to minimize certain performance functions. A fuzzy controller design to minimize the upper bound of the quadratic performance function is presented as follows (Tanaka and Wang, 2001):

$$J=\int_0^{\infty}\{y^T(t)Wy(t)+u^T(t)Ru(t)\}dt \tag{8}$$

Where $y(t)=\sum_{i=1}^{r} h_i(z(t))C_i x(t)$, $h_i(z(t))=\dfrac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))}$, R is a positive definite weight matrix and W is a positive semi-definite weight matrix .

Optimal fuzzy controller design

In the course of this study, the optimal fuzzy controller was introduced. In the control design procedure, a "sub-optimal" controller since $x^T(0)Px(0)$ will be minimized instead of J. The following theorem summarizes the control design condition for such scheme (Tanaka and Wang, 2001):

Theorem 1. The feedback gain to minimize the upper bound of the performance function can be obtained by solving the following LMIs. From the solution of the LMIs, the feedback gains are obtained as $F_i = M_i X^{-1}$ for all i. Then the performance function satisfies $J < x^T(0)Px(0) < \lambda$ (Tanaka and Wang, 2001).

$$\underset{X, M_1, \ldots, M_r, Q_0}{\text{minimize}} \ \lambda$$

subject to

$X>0$, $Q_0 \geq 0$,

$$\begin{bmatrix} \lambda & x^T(0) \\ x(0) & X \end{bmatrix} > 0, \tag{9}$$

$$\hat{U}_{ii} + (s-1)Q_3 < 0, \tag{10}$$

$$\hat{V}_{ij} - 2Q_4 < 0, \quad i<j\ s.t.\ h_i \cap h_j \neq \phi, \tag{11}$$

where $s>1$,

$$\hat{U}_{ii}=\begin{bmatrix} \begin{pmatrix} XA_i^T + A_i X \\ -B_i M_i - M_i^T B_i^T \end{pmatrix} & XC_i^T & -M_i^T \\ C_i X & -W^{-1} & 0 \\ -M_i & 0 & -R^{-1} \end{bmatrix},$$

$$\hat{V}_{ij}=\begin{bmatrix} \begin{pmatrix} XA_i^T + A_i X \\ -B_i M_j - M_j^T B_i^T \\ +XA_j^T + A_j X \\ -B_j M_i - M_i^T B_j^T \end{pmatrix} & XC_i^T & -M_j^T & XC_j^T & -M_i^T \\ C_i X & -W^{-1} & 0 & 0 & 0 \\ -M_j & 0 & -R^{-1} & 0 & 0 \\ C_j X & 0 & 0 & -W^{-1} & 0 \\ -M_i & 0 & 0 & 0 & -R^{-1} \end{bmatrix},$$

$Q_3 = block-diag(Q_0 \quad 0 \quad 0)$,

$Q_4 = block-diag(Q_0 \quad 0 \quad 0 \quad 0 \quad 0)$.

DESIGN OF A PARALLEL DISTRIBUTED FUZZY CONTROLLER FOR OVERHEAD CRANE

Using Equations 1 and 2 by choosing,

$$z_1 = \cos(X_3 - X_5)$$

$$z_2 = X_6 \sin(X_3 - X_5)$$

$$z_3 = X_6 \sin(X_5)$$

$$z_4 = \sin c(X_3)$$

$$z_5 = \cos(X_3)$$

$$z_6 = \cos(X_5)$$

$$z_7 = \text{sinc}(X_3 - X_5)$$

$$z_8 = X_4 \sin(X_3)$$

$$z_9 = \text{sinc}(X_5) \quad (12)$$

we have,

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \\ \dot{X}_5 \\ \dot{X}_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{23} & A_{24} & A_{25} & A_{26} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & A_{43} & A_{44} & A_{45} & A_{46} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & A_{63} & A_{64} & A_{65} & A_{66} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \\ 0 \\ B_4 \\ 0 \\ B_6 \end{bmatrix} \tau,$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{bmatrix}, \quad (13)$$

Where $X_1 = x$, $X_2 = \dot{x}$, $X_3 = \theta_1$, $X_4 = \dot{\theta}_1$, $X_5 = \theta_2$, $X_6 = \dot{\theta}_2$ and,

$$A_{23} = \frac{1}{den}(b_8 z_5 z_4 + b_{12} z_1 z_4 z_6 + b_9 z_1 z_5 z_7 + b_{13} z_6 z_7),$$

$$A_{24} = \frac{1}{den}(b_1 z_8 + b_2 z_1^2 z_8 + b_7 z_2 z_5 + b_{11} z_1 z_2 z_6),$$

$$A_{25} = \frac{1}{den}(b_{10} z_1 z_5 z_9 + b_{14} z_6 z_9 - b_9 z_1 z_5 z_7 - b_{13} z_6 z_7),$$

$$A_{26} = \frac{1}{den}(b_3 z_3 + b_4 z_1^2 z_3),$$

$$A_{43} = \frac{1}{den}(c_2 z_4 + c_{14} z_4 z_6^2 + c_3 z_1 z_7 + c_{11} z_5 z_6 z_7),$$

$$A_{44} = \frac{1}{den}(c_5 z_5 z_8 + c_8 z_1 z_6 z_8 + c_1 z_2 + c_{13} z_2 z_6^2),$$

$$A_{45} = \frac{1}{den}(c_4 z_1 z_9 + c_9 z_1 z_6 z_9 + c_{12} z_5 z_6 z_9 - c_3 z_1 z_7 - c_{11} z_5 z_6 z_7),$$

$$A_{46} = \frac{1}{den}(c_6 z_3 z_5),$$

$$A_{63} = \frac{1}{den}(d_4 z_1 z_4 + d_8 z_4 z_5 z_6 + d_1 z_7 + d_5 z_5^2 z_7),$$

$$A_{64} = \frac{1}{den}(d_7 z_2 z_5 z_6 + d_9 z_1 z_5 z_8 + d_{12} z_6 z_8 + d_3 z_1 z_2),$$

$$A_{65} = \frac{1}{den}(d_2 z_9 + d_6 z_5^2 z_9 - d_1 z_7 - d_5 z_5^2 z_7),$$

$$A_{66} = \frac{1}{den}(d_{10} z_1 z_3 z_5 + d_{13} z_3 z_6),$$

$$B_2 = \frac{1}{den}(b_5 + b_6 z_1^2),$$

$$B_4 = \frac{1}{den}(c_7 z_5 + c_{10} z_5 z_6),$$

$$B_6 = \frac{1}{den}(d_{11} z_1 z_5 + d_{14} z_6),$$

Where $den = a_1 - d_3 z_1^2 + c_5 z_5^2 + a_2 z_1 z_5 z_6 - d_{10} z_6^2$ and according to basic parameters $m = 5Kg$, $m_1 = 2Kg$, $m_2 = 5Kg$, $l_1 = 2m$ and $l_2 = 1m$, the coefficient values are, $a_1 = 1680$, $a_2 = -c_3 = d_3 = 1200$, $b_1 = -d_5 = d_9 = -d_{12} = 1960$, $b_2 = -b_9 = -1400$, $b_3 = b_7 = c_8 = c_{11} = -d_7 = d_{10} = -d_{13} = 700$, $b_4 = b_{11} = -500$, $b_5 = d_{11} = -d_{14} = 140$, $b_6 = -100$, $b_8 = 48020$, $b_{10} = b_{12} = -b_{14} = 2c_{12} = -2c_{14} = -6860$, $b_{13} = -2000$, $c_1 = -600$, $c_2 = -8232$, $c_4 = 5880$, $c_5 = -980$, $c_6 = -350$, $c_7 = -70$, $c_9 = c_{13} = 250$, $c_{10} = 50$, $d_1 = 3360$, $d_2 = -d_4 = -16464$, $d_6 = 9604$, $d_8 = -1372$.

Without impressive loss of accuracy, we can assume that z_4, z_5, z_6, z_7, z_8, z_9 and den are constant because their variations are low. So, according to z_1, z_2 and z_3 and their maximum and minimum values we will have $2^3 = 8$ rules. The fuzzy model has the following 8 rules,

Model rule 1:
If z_1 is "minimum" and z_2 is "minimum" and z_3 is "minimum",

Then $\dot{x}(t) = A_1^* x(t) + B_1^* u(t)$

Model rule 2:
If z_1 is "minimum" and z_2 is "minimum" and z_3 is "maximum",
Then $\dot{x}(t) = A_2^* x(t) + B_2^* u(t)$

Model rule 3:
If z_1 is "minimum" and z_2 is "maximum" and z_3 is "minimum",

Then $\dot{x}(t) = A_3^* x(t) + B_3^* u(t)$

Model rule 4:
If z_1 is "minimum" and z_2 is "maximum" and z_3 is "maximum",
Then $\dot{x}(t) = A_4^* x(t) + B_4^* u(t)$

Model rule 5:
If z_1 is "maximum" and z_2 is "minimum" and z_3 is "minimum",
Then $\dot{x}(t) = A_5^* x(t) + B_5^* u(t)$

Model rule 6:
If z_1 is "maximum" and z_2 is "minimum" and z_3 is "maximum",
Then $\dot{x}(t) = A_6^* x(t) + B_6^* u(t)$

Model rule 7:
If z_1 is "maximum" and z_2 is "maximum" and z_3 is "minimum",
Then $\dot{x}(t) = A_7^* x(t) + B_7^* u(t)$

Model rule 8:
If z_1 is "maximum" and z_2 is "maximum" and z_3 is "maximum",
Then $\dot{x}(t) = A_8^* x(t) + B_8^* u(t)$.

SIMULATION RESULTS

In the simulations, the basic initial state is: $x = -30m$, $\dot{x} = 0\,\text{m/sec}$, $\theta_1 = pi/18\,rad$, $\dot{\theta}_1 = 0\,rad/\text{sec}$, $\theta_2 = pi/15\,rad$, $\dot{\theta}_2 = 0\,rad/\text{sec}$. According to 8 rules, 8 subsystems are as follows:

$$A_1^* = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 34.3069 & -0.1030 & 0.6710 & -0.0772 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -4.8908 & 0.1124 & 2.1871 & 0.0830 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 12.0535 & -0.1036 & -6.9105 & 0.0062 \end{bmatrix},$$

$$A_2^* = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 34.3069 & -0.1030 & 0.6710 & 0.0772 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -4.8908 & 0.1124 & 2.1871 & -0.0830 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 12.0535 & -0.1036 & -6.9105 & -0.0062 \end{bmatrix},$$

$$A_3^* = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 34.3069 & 0.0451 & 0.6710 & -0.0772 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -4.8908 & -0.0853 & 2.1871 & 0.0830 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 12.0535 & 0.1083 & -6.9105 & 0.0062 \end{bmatrix},$$

$$A_4^* = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 34.3069 & 0.0451 & 0.6710 & 0.0772 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -4.8908 & -0.0853 & 2.1871 & -0.0830 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 12.0535 & 0.1083 & -6.9105 & -0.0062 \end{bmatrix},$$

$$A_5^* = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 33.7932 & -0.0779 & -0.1699 & -0.0482 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -5.0141 & 0.1097 & 2.9422 & 0.0830 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 13.8115 & -0.1498 & -6.9105 & -0.0152 \end{bmatrix},$$

$$A_6^* = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 33.7932 & -0.0779 & -0.1699 & 0.0482 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -5.0141 & 0.1097 & 2.9422 & -0.0830 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 13.8115 & -0.1498 & -6.9105 & 0.0152 \end{bmatrix},$$

$$A_7^* = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 33.7932 & 0.0418 & -0.1699 & -0.0482 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -5.0141 & -0.0879 & 2.9422 & 0.0830 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 13.8115 & 0.1385 & -6.9105 & -0.0152 \end{bmatrix},$$

$$A_8^* = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 33.7932 & 0.0418 & -0.1699 & 0.0482 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -5.0141 & -0.0879 & 2.9422 & -0.0830 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 13.8115 & 0.1385 & -6.9105 & 0.0152 \end{bmatrix},$$

$$B_1^* = B_2^* = B_3^* = B_4^* = \begin{bmatrix} 0 \\ 0.0534 \\ 0 \\ -0.0207 \\ 0 \\ -0.0043 \end{bmatrix}, \quad B_5^* = B_6^* = B_7^* = B_8^* = \begin{bmatrix} 0 \\ 0.0333 \\ 0 \\ -0.0207 \\ 0 \\ 0.0105 \end{bmatrix}.$$

Using LMI optimization algorithm, with $W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $R = 1$ and $s = 7$, we obtain

$$P = 10^3 \times \begin{bmatrix} 0.0068 & 0.0170 & 0.9246 & 0.3071 & -0.0732 & 0.1103 \\ 0.0170 & 0.0603 & 3.7459 & 1.2548 & -0.3520 & 0.4576 \\ 0.9246 & 3.7459 & 412.7916 & 125.4248 & -83.3167 & 42.6671 \\ 0.3071 & 1.2548 & 125.4248 & 48.5454 & -20.6997 & 12.7607 \\ -0.0732 & -0.3520 & -83.3167 & -20.6997 & 27.8266 & -5.6266 \\ 0.1103 & 0.4576 & 42.6671 & 12.7607 & -5.6266 & 7.3337 \end{bmatrix},$$

$$Q_0 = 10^{-5} \times \begin{bmatrix} 260 & -150 & 2.4447 & -1.4436 & 4.6078 & -2.7709 \\ -150 & 100 & -1.9016 & 1.2698 & -3.5770 & 2.4131 \\ 2.4447 & -1.9016 & 0.2561 & -0.0354 & 0.0836 & -0.0664 \\ -1.4436 & 1.2698 & -0.0354 & 0.244 & -0.0662 & 0.0592 \\ 4.6078 & -3.5770 & 0.0836 & -0.0662 & 0.3686 & -0.1243 \\ -2.7709 & 2.4131 & -0.0664 & 0.0592 & -0.1243 & 0.3232 \end{bmatrix},$$

and

$$F_1 = 10 \times \begin{bmatrix} -0.5368 \\ -2.2292 \\ -232.4659 \\ -90.4946 \\ 40.3859 \\ -23.1816 \end{bmatrix}^T, \quad F_2 = 10 \times \begin{bmatrix} -0.5126 \\ -2.1313 \\ -222.0268 \\ -86.2196 \\ 38.1553 \\ -22.4833 \end{bmatrix}^T,$$

$$F_3 = 10 \times \begin{bmatrix} -0.5336 \\ -2.2160 \\ -230.8056 \\ -90.0873 \\ 40.0714 \\ -23.0187 \end{bmatrix}^T, \quad F_4 = 10 \times \begin{bmatrix} -0.5073 \\ -2.1101 \\ -219.5230 \\ -86.1333 \\ 37.6575 \\ -22.0479 \end{bmatrix}^T,$$

$$F_5 = 10 \times \begin{bmatrix} -0.4365 \\ -1.8159 \\ -188.8217 \\ -76.7448 \\ 32.6407 \\ -17.8495 \end{bmatrix}^T, \quad F_6 = 10 \times \begin{bmatrix} -0.4567 \\ -1.8992 \\ -196.9510 \\ -79.7415 \\ 33.6704 \\ -18.8658 \end{bmatrix}^T,$$

$$F_7 = 10 \times \begin{bmatrix} -0.4390 \\ -1.8261 \\ -189.8098 \\ -77.0761 \\ 32.8210 \\ -17.9641 \end{bmatrix}^T, \quad F_8 = 10 \times \begin{bmatrix} -0.4643 \\ -1.9298 \\ -200.3477 \\ -80.7152 \\ 34.3683 \\ -19.2134 \end{bmatrix}^T,$$

and $\lambda = 6.6532 \times 10^8$, it can be shown that stability conditions of theorem 1 are satisfied.

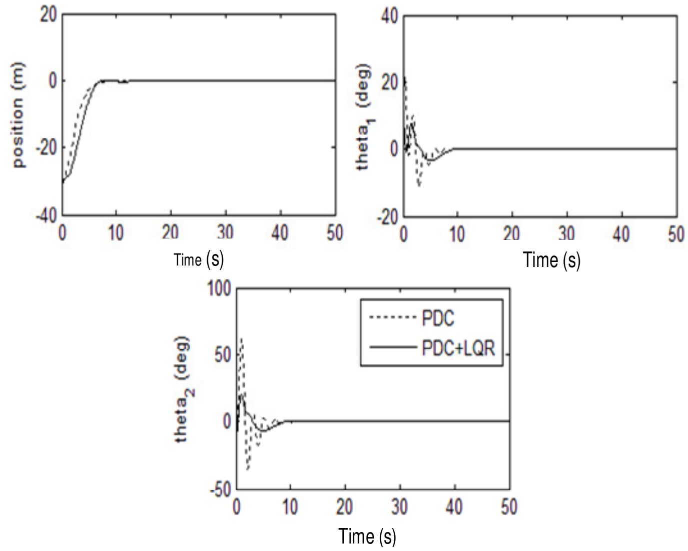

Figure 2. System dynamics of the overhead crane with PDC and parallel distributed fuzzy LQR controller.

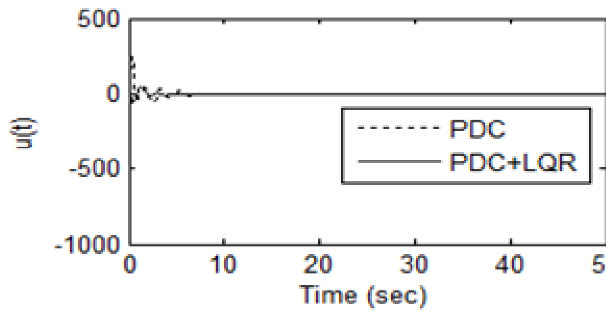

Figure 3. Control signals of PDC and parallel distributed fuzzy LQR controller.

Figure 2 shows the double-pendulum-type overhead crane dynamics using parallel distributed fuzzy LQR controller compared with parallel distributed fuzzy controller and the control signals *u(t)* are shown in Figure 3. As it can be seen from Figure 2 in comparison with the other studies on double-pendulum-type overhead crane (Weiping et al., 2005; Liu et al., 2005a) that the system dynamics converge to desired values in about 30 s, in our proposed method this time has been decreased to less than 15 s and it shows that by using this method we will reach faster performance.

CONCLUSION

In this paper, T-S fuzzy modeling of a double-pendulum-type overhead crane is used and a parallel distributed fuzzy LQR controller is designed to control position regulation and swing angle. With the proposed controller, we reach the desired values of position error and swing angle. The simulation results indicates the effectiveness of the designed stable controller.

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