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Fuzzy programming and data envelopment analysis for improving redundancy-reliability allocation problems in series-parallel systems

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Redundancy-reliability allocation problems in multi-stage series-parallel systems under uncertain environments are addressed in this study. First, a multi-objective programming model is formulated for simultaneously maximizing system reliability and minimizing system total cost. Due to the nature of uncertainty in the problem, the fuzzy set theory and technique are used to convert the deterministic multi-objective programming model into a fuzzy nonlinear programming problem. A heuristic method is developed to get a set of satisfactory solutions for the fuzzy nonlinear programming problem. Then, a modified data envelopment analysis (DEA) model, is applied for completely ranking those satisfactory solutions considering some criteria of satisfactory, reliability, cost, volume and weight. A case study that is related to the electronic control unit installed on aircraft engine over-speed protection system is used to implement the developed approach. Results suggest that the developed fuzzy programming and DEA approach can effectively resolve the fuzzy and uncertain problem when design goals and constraints are not clearly confirmed at the initial conceptual design phase.

Key words: Fuzzy programming, data envelopment analysis, reliability allocation.

INTRODUCTION

For a system design with low reliability requirement, the designer can adopt series-parallel-systems techniques to improve system reliability and redundancy allocation. However, without further consideration, series-parallel-systems design techniques will increase system complexity, cost, weight, volume, and power dissipation. These constraint elements shall be considered when series-parallel-systems are applied.

In practice, solution methods for series-parallel-systems with redundancy-reliability allocation problems can be categorized into two methods: active redundant model and standby redundant model. An active redundant model adopts several parallel components, in which each component shall be actively operated. The entire system will operate well if a specific number of components within this system operate normally. A standby redundant model adopts several parallel components as well. In order to solve reliability redundant optimization problems (Misra and Sharma, 1991) launched the research of using bound search techniques, which were integer programming methods. Li (1996) proposed a bound dynamic programming (BDP) method that could solve reliability redundant optimization problems.

Li and Jia (1997) further used a partial bound enumeration (PBE) technique that could solve reliability redundant optimization problems. Baxter and Harche (1992) used exact algorithm to solve the optimal reliability

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allocation of series-parallel systems. Chern et al. (1991) also used exact algorithms and parametric non-linear integer programming methods to solve the application of the reliability optimization problems with multiple constraints for the series-parallel systems. Petrovic (1991) utilized heuristic method to improve decision support of system reliability by redundancy allocation. Sharma and Misra (1990) utilized heuristic method to optimize system reliability and developed an effective algorithm to solve integer programming problems of reliability optimization, respectively. Xu et al. (1990) utilized heuristic method by the allocation of reliability redundancy to solve optimal constraints of improving system reliability. Jiang and Chen (2003) presented a computational model of fuzzy reliability, focusing on solving the engineering problems with random general stress-fuzzy general strength. Kuo and Wan (2007) provided a comprehensive review on recent advances in optimal reliability allocation.

In order to make sure that the whole system can be operated functionally, certain numbers of components are required to be operated normally. Besides, when some of parallel components fail, components in standby will be operated using switch devices. If the failure rate of switching devices is excluded, the system reliability of standby redundant model is higher than that of active redundant model. Although the system reliability of both models is high, these two models could cause higher cost, higher weight, and higher volume. Especially, when some switch devices are adopted by the standby redundant model, an additional cost will increase in accordance with the number of switch devices used.

On the other hand, if the failure rate of switching devices is included, we need to further study the relationship among the whole system reliability, the contact reliability of switching devices, and the conditional dynamic/static system reliability. Ida et al. (1994) utilized meta-heuristic method by genetic algorithm (GA) to solve system reliability optimization with several failure modes. Yokota et al. (1996) utilized meta-heuristic method to solve system reliability optimization with several failure modes, mixed integer non-linear programming problems and its application, respectively. Levitin and Lisnianski (2001) presented a technique for solving a family of multi-state system reliability optimization problems, such as structure optimization, optimal expansion, maintenance optimization and optimal multistage modernization. Liang and Smith (2004) used an ant colony meta-heuristic optimization method to solve the redundancy allocation problem. Yun and Kim (2004) addressed the problem in which redundancy is available at all levels in a series system and presented a mixed integer programming model.

The objective of reliability maximization is always pursued while the system cost became higher, or the objective of system cost minimization is obtained while the system reliability was sacrificed for traditional single objective optimization method. If multi-objective optimization method is adopted to solve reliability allocation trade-off problems of series-parallel systems, one could consider the optimization of system reliability and total system cost at the same time. Also one could consider the constrained factors of weight and volume. The system reliability requirement with the above design disciplines could be achieved. When the product reliability demonstration is carried out, one do not need to spend too much cost and time. Several objectives could be conflicting. The decision procedures were also very complicated, as it involves different levels of uncertainties, such as characteristics of expert information, qualitative statements and fuzzy, etc.

Misra and Sharma (1991) utilized meta-heuristic method and an effective reliability design tool to solve integer programming problems; utilize multi-objective programming method to solve multi-objective redundancy optimization problems, respectively. Prasad and Raghavachari (1998) utilized heuristic method to solve optimal component allocation of series-parallel networks and utilized optimal allocation of inter-exchangeable component method to solve the optimal component allocation of series-parallel and parallel-series systems, respectively. Zadeh and Bellman (1990) designed a fuzzy-decision environment to provide different kinds of total solutions for design problems. Huang (1997) proposed fuzzy multi-objective optimization decision method, which could provide two or above goals of reliability optimization decisions. Elegbede et al. (2003) considered the allocation of reliability and redundancy to parallel-series systems, while minimizing the cost of the system. Coit et al. (2004) addressed system reliability optimization when component reliability estimates are treated as random variables with estimation uncertainty. Coit and Konak (2006) proposed a new multiple weighted objective heuristic for the redundancy allocation problem.

In this study, an approach with fuzzy programming and DEA methods is developed for dealing with the design of engine protection systems under uncertain environments. First, a fuzzy multi-objective programming technique is utilized for solving series-parallel systems with redundant-design. A heuristic method is devised to generate a group of satisfactory solutions. Secondly, a design performance index is defined and a modified DEA model is applied to completely rank those satisfactory solutions. Finally, the proposed approach is implemented in the over-speed protection systems of turbo engine. Real-world data is collected and analyzed to display the benefit of this study.

Crisp reliability design model for series-parallel systems

Figure 1 displays a diagram for an N-stage series-parallel system with redundancy-reliability allocation problems. In this system, several parallel and identical components

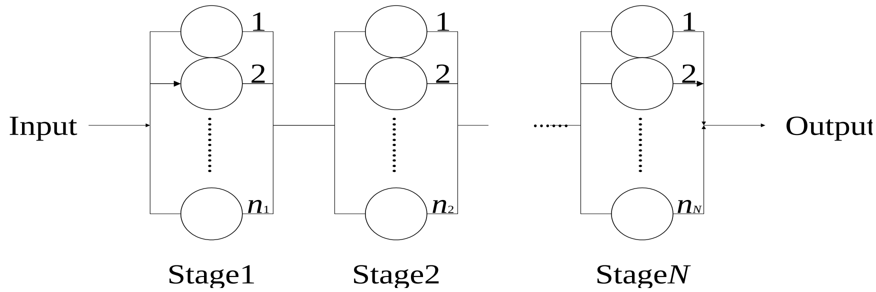


Figure 1. Diagram for N-stages series-parallel systems.

are arrayed in each stage. While the series-parallel system with redundancy can be applied to increase the system reliability, this technique may inevitably add more complexity, weight, volume, or cost to the design system. It would be better if a multi-objective programming model is developed in tackling this problem.

When modeling the target problem, the objectives are two-fold. One is to determine optimal design reliability for each component and the other to select an optimal number of components within each stage. That is, the overall system reliability is maximized and the overall system cost is minimized. In addition, several constraint design criteria, such as minimum requirements for system reliability, system cost, system volume, and system weight, are considered in this model. In order to develop a mathematical design model for the engine protection systems, we define the following decision variables and parameters.

Decision variable

R_i=represent the component reliability within the *i*-th stage; n_i=represent the number of components within the *i*-th stage; f_1=represent the overall system reliability; f_2=represent the overall system cost.

Parameter

$C_i(R_i)$=represent the component cost within the *i*-th stage; w_i=represent the component weight within the *i*-th stage; v_i=represent the component volume within the *i*-th stage; R=represent the lower limit for the overall system reliability; C=represent the upper limit for the overall system cost; W=represent the maximum limit for the system weight; V=represent the maximum limit for the system volume; N=represent the number of stages in the design system; N_{high}=represent the maximum number of components within each stage; N_{low}=represent the minimum number of components within each stage; R_{high}=represent the maximum limit of reliability within each stage; R_{low}=represent the minimum requirement of reliability within each stage.

Then a multi-objective mathematical reliability design model may be given as follows:

Maximize $$f_1 = \prod_{i=1}^{N} \left[1-(1-R_i)^{n_i}\right] \quad (1)$$

Minimize $$f_2 = \sum_{i=1}^{N} C_i(R_i) \quad (2)$$

Subject to

$$\prod_{i=1}^{N} \left[1-(1-R_i)^{n_i}\right] \geq R \quad (3)$$

$$\sum_{i=1}^{N} w_i\, n_i \exp(n_i / N) \leq W \quad (4)$$

$$\sum_{i=1}^{N} v_i n_i^2 \leq V \quad (5)$$

$$\sum_{i=1}^{N} C_i(R_i)\left[n_i + \exp(n_i / N)\right] \leq C \quad (6)$$

$$N_{low} \leqq n_i \leqq N_{high}, \quad i = 1,\ldots,N \quad (7)$$

$$R_{low} \leqq R_i \leqq R_{high}, \quad i = 1,\ldots,N \quad (8)$$

The objective Function (1) is used to maximize the overall system reliability for the engine protection systems, while the objective function (2) is used to minimize the overall system cost. Constraint (3) is used to set the minimum requirement for the system reliability. Constraint (4) is used to set the maximum limit for the system weight. Constraint (4) indicates that as the number of component increases the system weight is proportional to the multiplication of component weight within the *i*-th stage and the number of component factored by the term of $exp(n_i / N)$.

The term $exp(n_i / N)$ is used to describe the incremental effect of added components on the total system weight. Constraint (5) is used to set the maximum limit for the system volume. The incremental effect of added components on the system volume is quadratic in polynomial terms, while the incremental effect of added components on the system weight is assumed to be exponential. Constraint (6) is used to set the maximum limit for the system cost, where $C_i(R_i)$ is the unit cost for the component with R_i reliability in the *i-th* stage. Constraint (7) denotes the allowable range of component number for each stage, and Constraint (8) is used to specify the range of reliability for each component in each stage.

In Constraint (6), the term $C_i(R_i)$ represents the component cost within the *i*-th stage and is a function of component failure rate and in turn a function of component reliability R_i. $C_i(R_i)$ can be formulated as: $C_i(R_i) = \alpha_i / \lambda_i^{\beta_i}$, where α_i and β_i are constant and characteristics factors for each component in the *i-th* stage and λ_i is the failure rate of component in the *i-th* stage. This formula can be found in Kumar et al. (2009). Furthermore, by the relationship of $R_i = \exp(-\lambda_i t)$, we can obtain $C_i(R_i) = \alpha_i \cdot [-t/\ln(R_i)]^{\beta_i}$, where t is the active operation time. Hence, Constraint (6) can be reformulated as:

$$\sum_{i=1}^{N} \alpha_i \times \left[-t / \ln(R_i)\right]^{\beta_i} \times [n_i + \exp(n_i / N)] \le C \tag{9}$$

The developed reliability design model is one type of multi-objective mixed integer nonlinear programming problem. This is a NP-hard problem. In addition, information about the reliability, cost, weight, and volume parameters in the model can be uncertain or incomplete in terms of data collection in the early design stage of system life cycle. It would be better to apply fuzzy set techniques to solve this problem.

Fuzzy nonlinear programming model for series-parallel systems

The fuzzy set theory (Zadeh, 1985) is applied to construct a fuzzy nonlinear programming for solving the series-parallel systems with redundancy-reliability allocation problems. First, those objective functions and constraints in the multi-objective programming model are treated as fuzzy objective functions and fuzzy constraints using membership functions to quantify uncertain parameters. The following notations for developing a fuzzy nonlinear model are provided.

Notation

$^{-}$ = fuzzy information; $\cap$ = fuzzy intersection; $\bar{F}_i$ = the *i-th* fuzzy objective function (*i*=1,…, *k*); $\bar{G}_j$ = the *j-th* fuzzy constraint (*j*=1,…, *m*); $\bar{D}$ = the fuzzy decision set; μ_i = the *i-th* fuzzy membership function; α_i = the degree of satisfaction for the *i-th* objective function; α_j = the degree of satisfaction for the *j-th* fuzzy constraint; Φ = the fuzzy set for the decision space; $A(\Phi)$ = the overall satisfaction; R_S = the reliability goal which is set up by designer; ∇R = difference between reliability goal and minimal reliability limit; C_S = the cost goal which is set up by designer; ∇C = difference between cost goal and maximal cost limit; w_s = the system weight; v_s = the system volume.

The fuzzy set used to describe the membership functions can be an *L-R* trapezoidal fuzzy number denoted by $(m_1, m_2, \alpha, \beta)_{LR}$ or an *L-R* triangular fuzzy number denoted by $(m, \alpha, \beta)_{LR}$ where α and β are the left and right spreads. The triangular fuzzy number is simple and commonly easy to describe the fuzzy nature for many attributes. The membership function for the fuzzy reliability objective function and the degree of satisfaction of reliability function may be given as follows: The operational range varies from minimal reliability limit to reliability goal.

$$\alpha_{i=1} = \mu_{\bar{F}_1}(R_S) = \begin{cases} 0 & R_S \le R \\ \dfrac{R_S - R}{\nabla R} & R < R_S < R + \nabla R \\ 1 & R_S \ge R + \nabla R \end{cases} \tag{10}$$

The membership function for fuzzy cost objective function and the degree of satisfaction of cost function may be expressed as follows: The operational range varies from cost goal to maximal cost limit.

$$\alpha_{i=2} = \mu_{\bar{F}_2}(C_S) = \begin{cases} 1 & C_S \le C \\ \dfrac{C + \nabla C - C_S}{\nabla C} & C < C_S < C + \nabla C \\ 0 & C_S \ge C + \nabla C \end{cases} \tag{11}$$

The membership function for the degree of satisfaction of weight constraint may be expressed as follows: The operational range varies from 0 to maximal weight limit.

$$\alpha_{j=1} = \mu_{\bar{G}_1}(w_S) = \begin{cases} 0 & w_S > W \\ 1 & w_S \le W \end{cases} \tag{12}$$

The membership function for the degree of satisfaction of volume constraint may be expressed as follows: The operational range varies from 0 to maximal volume limit.

$$\alpha_{j=2} = \mu_{\overline{G}_2}(v_S) = \begin{cases} 0 & v_S > V \\ 1 & v_S \leq V \end{cases} \quad (13)$$

Assume Φ is a fuzzy set of decision space and α(Φ) denotes the degree of overall satisfaction for the developed engine protection systems. The degree of overall satisfaction, α(Φ), within this decision space may be expressed as follows:

$$\alpha(\Phi) = \mu_{\overline{D}}(R^*) = \left[\mu_{\overline{F}_1}(R_s) \cap \mu_{\overline{F}_2}(C_s)\right] \cap \left[\mu_{\overline{G}_1}(w_s) \cap \mu_{\overline{G}_2}(v_s)\right] = \left[\mu_{\overline{F}_1}(R_S) \cap \mu_{\overline{F}_2}(C_S) \cap \mu_{\overline{G}_1}(w_S) \cap \mu_{\overline{G}_2}(v_S)\right] \quad (14)$$

If the fuzzy objective Functions (10) and (11) and the fuzzy Constraints (12) and (13) are known with certainty, the degree of satisfaction for those fuzzy functions will be available, and the overall satisfaction can be obtained via seeking the intersection area of those fuzzy functions. Mathematically, it is equivalent to find feasible solutions, optimal solution (R^*) and the maximal α(Φ) in the following fuzzy nonlinear programming problem:

Maximize α(Φ) (15)

Subject to

$$\alpha(\Phi_{\overline{F}}) \leqq \mu_{\overline{F}_1}(R_S) \cap \mu_{\overline{F}_2}(C_S) \quad (16)$$

$$\alpha(\Phi_{\overline{G}}) \leqq \mu_{\overline{G}_1}(w_S) \cap \mu_{\overline{G}_2}(v_S) \quad (17)$$

$$\alpha_{i=1} = \mu_{\overline{F}_1}(R_S) = \begin{cases} 0 & R_S \leq R \\ \dfrac{R_S - R}{\nabla R} & R < R_S < R + \nabla R \\ 1 & R_S \geq R + \nabla R \end{cases} \quad (18)$$

$$\alpha_{i=2} = \mu_{\overline{F}_2}(C_S) = \begin{cases} 1 & C_S \leq C \\ \dfrac{C + \nabla C - C_S}{\nabla C} & C < C_S < C + \nabla C \\ 0 & C_S \geq C + \nabla C \end{cases} \quad (19)$$

$$\alpha_{j=1} = \mu_{\overline{G}_1}(w_S) = \begin{cases} 0 & w_S > W \\ 1 & w_S \leq W \end{cases} \quad (20)$$

$$\alpha_{j=2} = \mu_{\overline{G}_2}(v_S) = \begin{cases} 0 & v_S > V \\ 1 & v_S \leq V \end{cases} \quad (21)$$

$$0 \leqq \alpha(\Phi_{\overline{F}}) \leqq 1 \quad (22)$$

$$0 \leqq \alpha(\Phi_{\overline{G}}) \leqq 1 \quad (23)$$

The objective Function (15) is used to maximize the degree of overall satisfaction for the design system. Constraint (16) is used to set the minimum satisfaction requirement for the reliability and cost objectives. Constraint (17) is used to set the minimum satisfaction requirement for the weight and volume functions. Constrain (18) is used to specify the degree of satisfaction of reliability function. Constraint (19) is used to specify the degree of satisfaction of cost function. Constraint (20) is used to specify the degree of satisfaction of weight function. Constraint (21) is used to specify the degree of satisfaction of volume function. Constraint (22) provides the range between 0 and 1 for the degree of overall satisfaction about the objective functions. Constraint (23) provides the range between 0 and 1 for the degree of overall satisfaction. The developed model can allow one to achieve a maximum overall satisfaction value while satisfying multi-objective fuzzy objective functions and fuzzy constraints within a fuzzy decision space. The developed fuzzy programming model is one type of nonlinear problem, in which the fuzzy multi-objective programming problem is converted into a deterministic single objective problem. For the developed fuzzy programming model, an α–search heuristic method is devised to generate a group of satisfactory solutions. The procedure of the α-search heuristic method is given as follows:

Step 0. Initialization:

Set K=the maximal number of iterations, N= the number of stages, R_{low}=lower bounds for reliability, R_{high}=upper bounds for reliability, $\triangle R_i$=interval of increment for reliability, N_{low}=minimal components within each stage, N_{high}=maximal components within each stage, $\Phi(\ )$ = solution set, Q=maximal number of elements in solution set.

Step 1. Initialization:
Provide an initial solution: k=1, $R_i^k = R_{low}$, n_i^k=N_{low}, $i=1,\ldots,N$. Place them into the solution set $\Phi(\ )$.

Step 2. Validation:
Compute and compare the overall satisfaction $\alpha^k(\Phi)$ for the incumbent solution. If $\alpha^k(\Phi) > \alpha^{k-1}(\Phi)$, replace the incumbent solution with previous solution in the solution set. If $n_i^k = N_{high}$ and $R_i^k = R_{high}$, $i = 1,\ldots,N,$ go to Step 4. Otherwise, go to Step 3.

Step 3. Improvement:
If $R_i^k = R_{high}$, then $n_i^k = n_i^k+1$ and $R_i^k = R_{low}$. Otherwise, $R_i^k = R_i^{k+1} + \triangle R_i$, $k = k+1$.

Compute the values for:
1. Reliability fuzzy set and its degree of satisfaction;
2. Cost fuzzy set and its degree of satisfaction;
3. Weight fuzzy set and its degree of satisfaction;
4. Volume fuzzy set and its degree of satisfaction.
Return to Step 2

Step 4. Closing:
Generate a group of satisfactory solutions.
A computer programming language, Delphi 7.0, is used to code and compile the above procedure in the developed α–search heuristic. A graphic user-interface is also provided for simulating alternative solutions.

Data envelopment analysis for selecting an efficient solution

In this study, a complete ranking system for efficient decision making units is developed. The criteria considered include satisfactory, reliability, cots, volume, and weight. First, an efficient design indicator, *I*, is defined for the p^{th} decision-making unit (DMU$_p$) as follows:

$$I = \frac{u_1 y_{1p} + u_2 y_{2p}}{v_1 x_{1p} + v_2 x_{2p} + v_3 x_{3p}} \quad (24)$$

where y_{1p} and y_{2p} are output on the DMU$_p$ representing reliability and satisfactory, respectively; x_{1p} , x_{2p} and x_{3p} are input on the DMU$_j$, representing volume, weight, and cost respectively; u_1 and u_2 are weights associated with the output y_{1p} and y_{2p}; and v_i v_2 and v_3 are weights associated with input x_{1p} , x_{2p} and x_{3p}.

For the p^{th} decision-making unit (DMU$_p$), one can find the weights that maximize the ratio output per input through the following mathematical model:

Maximize $$I = \frac{u_1 y_{1p} + u_2 y_{2p}}{v_1 x_{1p} + v_2 x_{2p} + v_3 x_{3p}} \quad (25)$$

Subject to

$$\frac{u_1 y_{1j} + u_2 y_{2j}}{v_1 x_{1j} + v_2 x_{2j} + v_3 x_{3j}} \le 1, \quad j = 1,..,P \quad (26)$$

$$u_r \ge \varepsilon > 0, \quad r = 1, 2, \qquad v_i \ge \varepsilon > 0, \quad i = 1, 2, 3 \quad (27)$$

where *P* is the number of DMU to be evaluated and ε is a non-Archimedean small, positive number. This is a fractional programming problem. By linearization, the fractional programming problem can be transformed into a linear programming problem as follows:

Maximize $$z = \sum_{r=1}^{2} u_r y_{rj} \quad (28)$$

Subject to

$$\sum_{r=1}^{2} u_r y_{rj} - \sum_{i=1}^{3} v_i x_{ij} \le 0, \quad j = 1,\ldots,P, \quad (29)$$

$$\sum_{i=1}^{3} v_i x_{ip} = 1, \quad (30)$$

$$u_r \ge \varepsilon > 0, \quad r = 1, 2, \quad v_i \ge \varepsilon > 0, \quad i = 1, 2, 3.$$

The dual form is given as follows:

Minimize $$\mu = \theta - \varepsilon\left(\sum_{i=1}^{3} s_i^- + \sum_{r=1}^{2} s_r^+\right) \quad (31)$$

Subject to

$$\sum_{j=1}^{P} \lambda_j x_{ij} + s_i^- = \theta x_{ip}, \quad i = 1, 2, 3. \quad (32)$$

$$\sum_{j=1}^{P} \lambda_j y_{rj} - s_r^+ = y_{rp}, \quad r = 1, 2. \quad (33)$$

$\lambda_j \ge 0$, $j = 1,\ldots, P$.
$s_i^- \ge 0$, $i = 1, 2, 3$.
$s_r^+ \ge 0$, $r = 1, 2$.
θ free,

where μ is an efficiency measure.
If $\theta^* = 1, s^{-*} = 0$ and $s^{+*} = 0$, DMU$_p$ is efficient. Otherwise, DMU$_p$ is inefficient. A DMU is denoted by strong efficient (SE) if its efficiency score equals 1 by the DEA model. Very often, the DEA model is weak in complete ranking of all DMUs because more SE DMUs can be generated from the DEA model. If SE DMUs are excluded from the reference set of all the other DMUs and allow the efficient frontier to be closer in relation to the inefficient DMUs, then we can find the most efficient SE DMU (Jahanshahloo et al., 2007).

Hence, in order to obtain complete ranking for the entire DMUs, the non-SE DMUs should be re-evaluated through the following model:

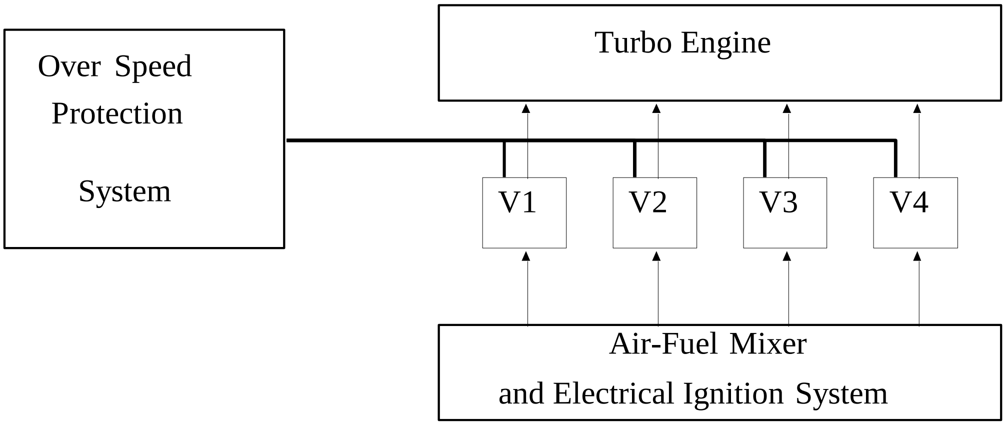


Figure 2. Diagram for over-speed protection system of Turbo Engine.

Minimize $$\partial_{a,b} = \theta - \varepsilon\left(\sum_{i=1}^{3} s_i^- + \sum_{r=1}^{2} s_r^+\right) \quad (34)$$

Subject to

$$-\sum_{j\in J-\{b\}} \lambda_j x_{ij} + \theta x_{ia} - s_i^- = 0, \quad i = 1, 2, 3. \quad (35)$$

$$\sum_{j\in J-\{b\}}^{n} \lambda_j y_{rj} - s_r^+ = y_{ra}, \quad r = 1, 2. \quad (36)$$

$\lambda_j \geq 0, \quad j \in J - \{b\},$

$s_i^- \geq 0, \quad i = 1, 2, 3.$

$s_r^+ \geq 0, \quad r = 1, 2.$

θ free,

where $J = \{1,2,\ldots,P\}$, $a \in J_n, b \in J_e$, J_n is the set of non-SE DMUs and Je is the set of SE DMUs.

After calculating the measure $\partial_{a,b}$ for all the non-SE DMUs, the efficiency of SE DMUs will be denoted by Ω and will be calculated by:

$$\Omega_b = \frac{\sum_{a\in Jn} \partial_{a,b}}{\tilde{n}} \quad (37)$$

where b is the evaluated SE DMU and $\tilde{n}$ is the number of non-SE DMUs. The most efficient SE DMU can be found by examining the calculated Ω.

IMPLEMENTATION

In this study, the developed approach is implemented to a design problem of over-speed protection system in turbo engines. The mission of this problem is to design a protection system during the over-speed operation of turbo engines. Figure 2 displays a functional block diagram for an over-speed protection system that is installed in turbo engines. This protection system consists of one electronic control valve and three mechanical valves, which provide over-speed protection for the turbo engine in a continuous way. Due to the incomplete or uncertain information about the design parameters during the early design phase, the proposed fuzzy programming model combined with DEA technique is utilized to provide solution methods.

Table 1 provides the design data for this case study, which includes number of stages, reliability and cost goal, limits for reliability, cost, weight, and volume, and operational time. Table 2 provides the physical characteristics of redundant components for different stages. Figures 3, 4, 5, and 6 provide the membership function for reliability, cost, weight, and volume, respectively. The intersection area of these four membership functions and its individual fuzzy set can lead us to find out feasible solutions as long as we maximize the degree of overall satisfaction, α(Φ), which is shown in Figure 7.

A computer programming language, Delphi 7.0, is used to code and compile the algorithm. A graphic user-interface is illustrated in Figure 8. Table 3 then provides some results obtained from the application to the fuzzy nonlinear programming problem. If we consider all the enumerations of α(Φ) which is just greater than 0.995, then we can find out eight combinations of system reliability, system cost, system weight, and system volume. Each combination is associated with one value for α(Φ). When the information about the design

Table 1. Design data of case study.

Number of stages	N = 4
Reliability goal which is set up by designer	R_s = 0.99
Cost goal which is set up by designer	C_s = 300
Lower limit of reliability	R = 0.90
Upper limit of cost	C = 400
Upper limit of weight	W = 500
Upper limit of volume	V = 250
Operation time	T = 1000 h

Table 2. Physical characteristics of redundant components for each stage.

Stage	α_i	β_i	v_i	w_i
1^{st}	1.0×10^{-5}	1.5	1	6
2^{nd}	2.3×10^{-5}	1.5	2	6
3^{rd}	0.3×10^{-5}	1.5	3	8
4^{th}	2.3×10^{-5}	1.5	2	7

$$\alpha_1 = \mu_{\tilde{F}_1}(R_S) = \begin{cases} 0 & R_S \le 0.9 \\ \dfrac{R_S - 0.9}{0.09} & 0.9 < R_S < 0.99 \\ 1 & R_S = 0.99 \end{cases}$$

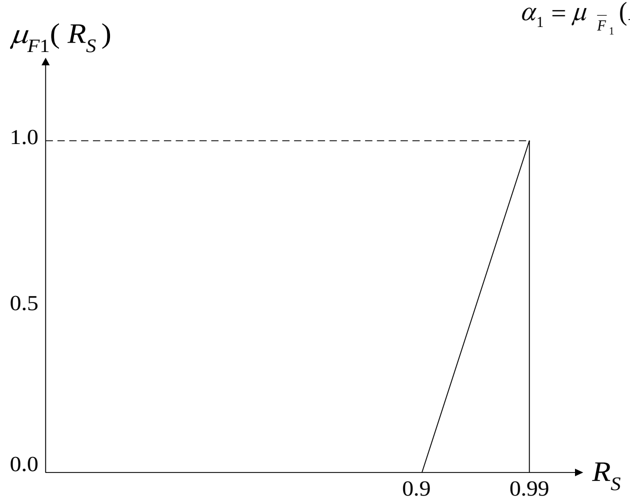
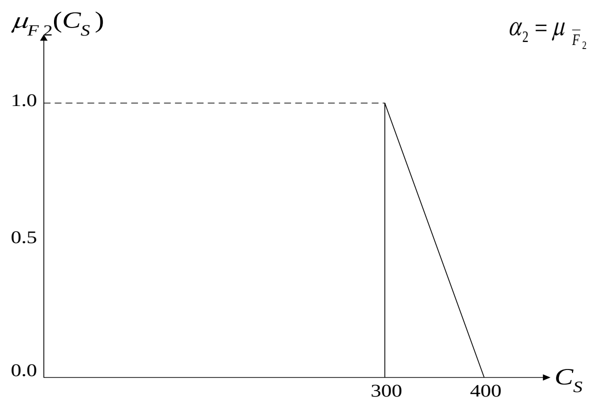


Figure 3. Membership function for reliability.

parameters is uncertain or incomplete for the series-parallel systems with redundancy, the developed fuzzy goal programming technique can be applied to provide satisfactory solutions for decision makers. The results also suggest that fuzzy multi-objective programming can effectively resolve the fuzzy and uncertain problem when design goals and constraints are not still clearly confirmed at the initial conceptual design phase.

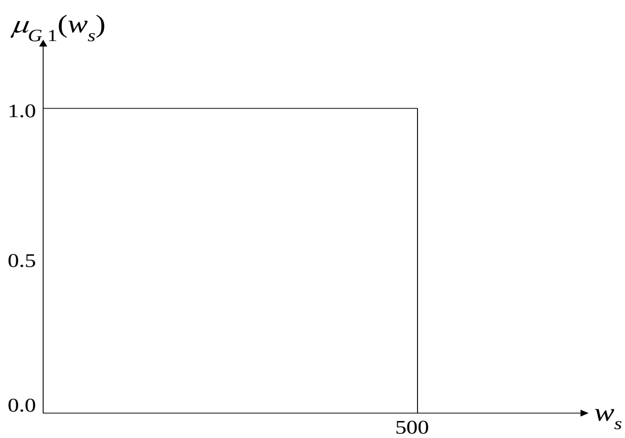
Moreso, the DEA method is applied to rank those satisfactory solutions, with satisfactory and reliability as output; and cost, weight, and volume as input. Table 4 shows the list of ranking from the efficient DMU to the least. There are two strong efficiency DMUs in this list, so the developed ranking DEA model is applied further to show the complete ranking list. Table 5 displays the complete ranking results. The most efficiency solution appears when α(Φ) equals 0.997084 with R_s=0.989816, C=300.087, W=224.753, and V=89.0.

The highest value of α(Φ) does not necessarily imply the best combination of reliability, cost, weight, and



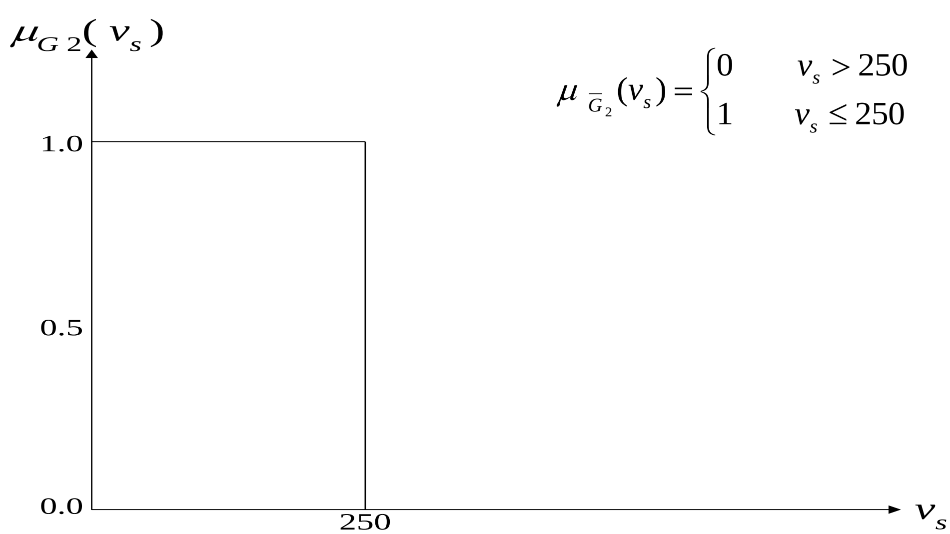
$$\alpha_2 = \mu_{\bar{F}_2}(C_S) = \begin{cases} 1 & C_S = 300 \\ \dfrac{400 - C_S}{100} & 300 < C_S < 400 \\ 0 & C_S \geq 400 \end{cases}$$

Figure 4. Membership function for cost.



$$\mu_{\bar{G}_1}(w_s) = \begin{cases} 0 & w_s > 500 \\ 1 & w_s \leq 500 \end{cases}$$

Figure 5. Membership function for weight.



$$\mu_{\bar{G}_2}(v_s) = \begin{cases} 0 & v_s > 250 \\ 1 & v_s \leq 250 \end{cases}$$

Figure 6. Membership function for volume.

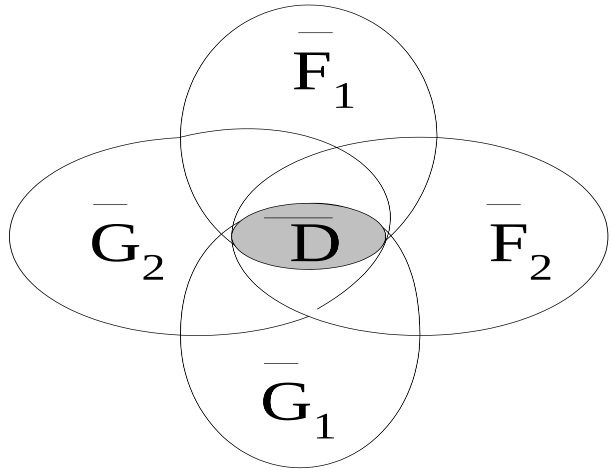


Figure 7. Decision space $\overline{D}$.

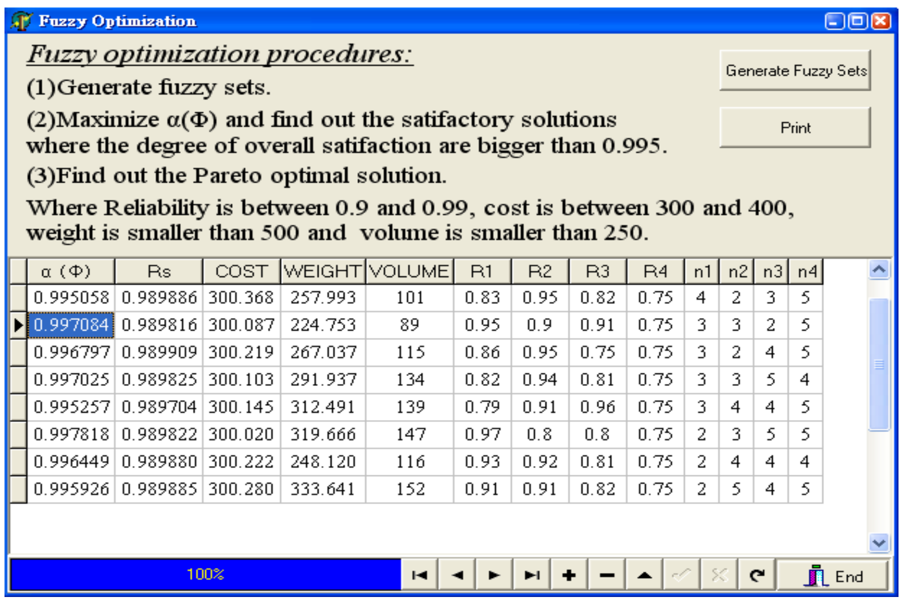


Figure 8. A graphic user-interface for the case study.

Table 3. Satisfactory solutions obtained by the heuristic.

Satisfactory solution with α(Φ) > 0.995	Characteristics for each satisfactory solution			
	Reliability	**Cost**	**Weight**	**Volume**
0.995058	0.989886	300.368	257.993	101.0
0.995257	0.989704	300.145	312.491	139.0
0.995926	0.989885	300.280	333.641	152.0
0.996449	0.989880	300.222	248.120	116.0
0.996797	0.989909	300.219	267.037	115.0
0.997025	0.989825	300.103	291.937	134.0
0.997084	0.989816	300.087	224.753	89.0
0.997818	0.989822	300.020	319.666	147.0

Table 4. Ranking list from the DEA model.

DMU	Efficiency	Cost	Weight	Volume	Reliability	Satisfactory
1	0.999088	300.368	257.993	101	0.989886	0.995058
2	0.999496	300.145	312.491	139	0.989704	0.995257
3	0.999198	300.280	333.641	152	0.989885	0.995926
4	0.999559	300.222	248.120	116	0.989880	0.996449
5	0.999552	300.219	267.037	115	0.989909	0.996797
6	0.999794	300.103	291.937	134	0.989825	0.997025
7	1	300.087	224.753	89	0.989816	0.997084
8	1	300.020	319.666	147	0.989822	0.997818

Table 5. Complete ranking list for the DEA model.

J_n	μ	DMU 7	DMU 8
DMU 1	0.999088	1	0.999135
DMU 2	0.999496	0.999609	0.999694
DMU 3	0.999198	0.999198	0.999427
DMU 4	0.999559	1	0.999615
DMU 5	0.999552	1	0.999654
DMU 6	0.999794	0.999978	0.999956
Ω_b		0.9997938	0.9995802

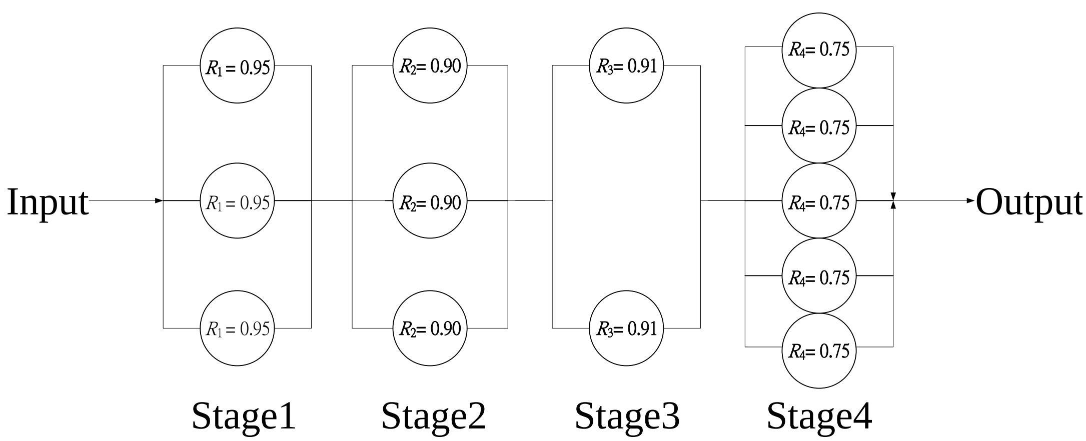


Figure 9. A reliability block diagram for the case study.

volume for the design system. The reason for this is that the values for reliability and cost did not vary much with the change of α(Φ), while the values of weight and volume, changed significantly. Hence, system weight and system volume play the key role in determining the most efficient solution. In other words, the most satisfactory solution for the fuzzy programming problem using the α–search heuristic method does not necessarily equal the most efficiency solution for the DEA model.

Finally, a reliability block diagram for this case based on the outcomes of this study is shown in Figure 9. The results suggest that the developed two-stage technique provide higher quality solutions regardless of size and complexity of problems. When the information about the design parameters is uncertain or incomplete for the series-parallel systems with redundancy problem, the developed fuzzy-based DEA technique can be applied to provide an efficiency solution for the decision maker.

CONCLUDING REMARKS

When design-in system reliability is low, series-parallel systems are adopted as a design guideline for improving system reliability. However, this design guideline will increase the total system cost and weight. Hence this design guideline seldom meets the practical requirement. In this study, fuzzy goal programming techniques are applied to deal with multi-stage series-parallel systems with redundancy problem. The developed fuzzy goal programming model can provide the most satisfactory solutions for determination of system/component reliability and number of components at each stage. A heuristic search method and the associated graphical user-interface are devised. Also, a DEA method is applied for completely ranking those selected satisfactory solutions.

A case study that relates to the electronic control unit installed on aircraft's engine over-speed protection system was used in implementing the developed approach. Results from this study suggests that the developed fuzzy multi-objective programming and DEA approach can effectively resolve the fuzzy and uncertain problem when design goals and constraints are not still clearly confirmed at the initial conceptual design phase. These models can also be applied efficiently and effectively for proper decision-making procedures when ill-structured situations occur.

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