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Full Length Research Paper

Generalized functional projective synchronization of Chen-Lee chaotic systems and its circuit implementation

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This paper investigates the problem of generalized functional projective synchronization of Chen-Lee systems. Based on Lyapunov stability theory, an effective control law is designed for asymptotic stability of the null solution of an error dynamics between master and slave chaotic systems. In order to verify the effectiveness of proposed control scheme, computer simulation via Matlab software is applied to a numerical example and then its circuit implementation is included as real applications.

Key words: Chen-Lee chaotic system, generalized function projective synchronization, Lyapunov method, simulation.

INTRODUCTION

Since Lorenz (1963) found the first classical chaotic attractor, chaos systems have been extensively studied and applied to various research fields in engineering and sciences. The system has unstable periodic orbits. The ergodicity causes the trajectory of the system to spread to the neighborhood of each of these periodic orbits. Also, the keen sensitivity to initial conditions is a major property of the system, which means small change of initial conditions leads to the unpredictability of the system.

Synchronization is a very hot topic of chaotic system, which has attracted much interest from scientists and engineers since Pecora and Carroll (1990) introduced the concept of synchronization. Up to date, chaos synchronization has been developed extensively due to its various applications (Li et al., 2005; Lü et al., 2004; Park et al., 2009; Vincent, 2005; Wu et al., 2003; Yassen, 2006). Originally, chaos synchronization refers to the state in which the master (or drive) and the slave (or response) systems have precisely identical trajectories as time goes by. We usually regard such the synchronization as complete synchronization or identical synchronization.

Over the last decade, various methods for chaos synchronization have been proposed, which include complete synchronization (Park, 2006), phase synchronization (Rosenblum et al., 1996), lag synchronization (Rosenblum et al., 1997), intermittent generalized synchronization (Hramov et al., 2005), projective synchronization (PS) (Mainieri et al., 1999), generalized synchronization (Park, 2007), and adaptive modified projective synchronization (Park et al., 2009).

Amongst all kinds of chaos synchronization, the functional projective synchronization (FPS) is the state of the art subject of synchronization study. As compared with PS, FPS means that the master and slave systems could be synchronized up to a scaling function, but not a constant.

In this paper, we consider the Chen-Lee chaotic systems as a model for applying our functional projective synchronization problem. In most of literatures, the numerical examples were only provided to verify their synchronization algorithms without circuit implementations. By the way, most of real practical chaos applications have been investigated by Chua's circuit which is a simple electronic circuit that exhibits chaotic behavior. However, it is well-known that another chaotic systems have some errors between theoretical system parameters and practical system parameters. Therefore, in this paper, the revised practical Chen-Lee master and slave systems will be applied to show our control scheme using NI Multisim 10.0 (Du et al., 2009; Sheu et al., 2008).

This paper is organized as follows: the problem statement

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and master-slave synchronization scheme are presented for Chen-Lee systems. In addition, a numerical simulation via Matlab is given to demonstrate the effectiveness of the proposed control method. Circuit implementations are presented to show real applications of the method. Finally, concluding remarks are given.

MAIN RESULTS

Consider the following master (drive) and slave (response) chaotic systems

$$\dot{x}(t) = f(t, x), \tag{1}$$

$$\dot{y}(t) = f(t, y) + u(t), \tag{2}$$

where $x(t)$ and $y(t)$ are n dimensional state vectors of master and slave systems, respectively, $f(x): \Re \times \Re^n \to \Re^n$ and $g(x): \Re \times \Re^n \to \Re^n$ are continuous nonlinear vector functions and $u(t, x, y) = (u_1, u_2, \ldots, u_n)^T \in \Re^n$ is the control input for synchro-nization between master (1) and slave (2).

For continuously differentiable nonzero scaling functions $\alpha_i(t)(i = 1, \ldots, n)$, let us define the error signal as

$$e_i(t) = y_i(t) - \alpha_i(t) x_i(t). \tag{3}$$

Remark 1

This synchronization scheme based on the error signal given in (3) is called generalized functional projective synchronization (GFPS).

Definition 1

It is said that GFPS occurs between master system (1) and slave system (2) if there exist scaling functions $\alpha_i(t)$ such that $\lim_{t\to\infty} \| y(t) - \alpha(t)x(t) \| = 0$ for all i.

Remark 2

Chaos synchronization schemes such as complete synchronization, anti-synchronization, projective synchronization, and functional projective synchronization are special case of GFPS. When $\alpha_i(t) = 1$, $\alpha_i(t) = -1$, $\alpha_i(t) =$ constant and $\alpha_i(t) =$ function, then GFPS becomes complete synchronization, anti synchronization, projective synchronization and functional projective synchronization, respectively.

As previously stated, we consider the following Chen-Lee master and slave systems with $a, b, c \in \Re$:

$$\begin{aligned} Master: \dot{x}_1 &= ax_1 - x_2 x_3 \\ \dot{x}_2 &= bx_2 + x_1 x_3 \\ \dot{x}_3 &= cx_3 + (1/3) x_1 x_2 \\ Slave: \dot{y}_1 &= ay_1 - y_2 y_3 + u_1 \\ \dot{y}_2 &= by_2 + y_1 y_3 + u_2 \\ \dot{y}_3 &= cy_3 + (1/3) y_1 y_2 + u_3. \end{aligned} \tag{4}$$

Note that it is well-known that the system is chaotic when $a = 5, \quad b = -10, \quad c = -3.8$.

In order to show chaotic motion of the system (4), let us take an initial condition $x(0) = (1,\ 2,\ -3)^T$. Then Figure 1 shows actually chaotic behavior of Chen-Lee system as expected.

Now, for our synchronization scheme, let us define error signals for Chen-Lee chaotic system in the sense of Definition 1 as

$$\begin{aligned} e_1 &= y_1 - \alpha_1 x_1 \\ e_2 &= y_2 - \alpha_2 x_2 \\ e_3 &= y_3 - \alpha_3 x_3. \end{aligned}$$

The time derivative of this error signal is

$$\begin{aligned} \dot{e}_1 &= \dot{y}_1 - \dot{\alpha}_1 x_1 - \alpha_1 \dot{x}_1 \\ \dot{e}_2 &= \dot{y}_2 - \dot{\alpha}_2 x_2 - \alpha_2 \dot{x}_2 \\ \dot{e}_3 &= \dot{y}_3 - \dot{\alpha}_3 x_3 - \alpha_3 \dot{x}_3. \end{aligned} \tag{5}$$

By substituting (4) into (5), we have the following error dynamics

$$\begin{aligned} \dot{e}_1 &= ae_1 - y_2 y_3 + \alpha_1 x_2 x_3 - \dot{\alpha}_1 x_1 + u_1 \\ \dot{e}_2 &= be_2 + y_1 y_3 - \alpha_2 x_1 x_3 - \dot{\alpha}_2 x_2 + u_2 \\ \dot{e}_3 &= ce_3 + \frac{1}{3} y_1 y_2 - \frac{1}{3} \alpha_3 x_1 x_2 - \dot{\alpha}_3 x_3 + u_3. \end{aligned} \tag{6}$$

Here, our goal is to achieve generalized functional projective synchronization between two Chen-Lee systems with different initial conditions. For this end, the following control laws are designed with a negative constant, k:

$$\begin{aligned} u_1 &= (k - a)e_1 + y_2 y_3 - \alpha_1 x_2 x_3 + \dot{\alpha}_1 x_1 \\ u_2 &= -y_1 y_3 + \alpha_2 x_1 x_3 + \dot{\alpha}_2 x_2 \\ u_3 &= -\frac{1}{3} y_1 y_2 + \frac{1}{3} \alpha_3 x_1 x_2 + \dot{\alpha}_3 x_3. \end{aligned} \tag{7}$$

Substituting the control input (7) into Equation (6) gives that

$$\dot{e}_1 = ke_1,\ \dot{e}_2 = be_2,\ \dot{e}_3 = ce_3.$$

Then, we have the following theorem.

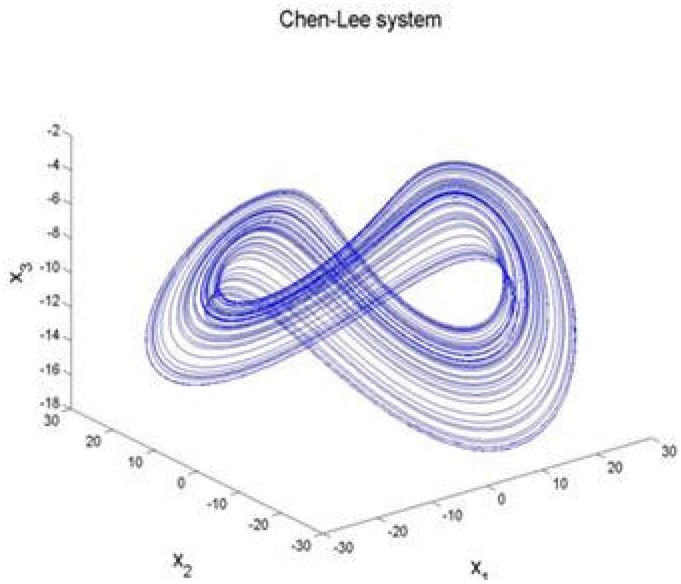

Figure 1. Chaotic motion of Chen-Lee system.

Theorem 1

For given scaling functions $\alpha_i(\mathrm{t})$ $(\mathrm{i}=1,\,2,\,3)$ the GFPS between master and slave systems given in Equation (4) will occur by the control law (7). This implies that the error signals satisfy $\lim_{t\to\infty}\|e_i(t)\|=0\ (i=1,\,2,\,3)$.

Proof

Let us define the following Lyapunov function candidate

$$V=\frac{1}{2}(e_1{}^2+e_2{}^2+e_3{}^2)\,. \tag{8}$$

By differentiating Equation (8) and using (7), we obtain

$$\begin{aligned}\dot V &= e_1\dot e_1+e_2\dot e_2+e_3\dot e_3\\ &=-\begin{bmatrix}e_1\\e_2\\e_3\end{bmatrix}^T\begin{bmatrix}-k&0&0\\0&-b&0\\0&0&-c\end{bmatrix}\begin{bmatrix}e_1\\e_2\\e_3\end{bmatrix}\\ &=-e^TPe<0.\end{aligned}$$

Therefore, the error system is asymptotically stable by Lyapunov stability theory. This means that the slave system synchronizes the master system in the sense of GFPS. This completes the proof.

NUMERICAL SIMULATIONS

In order to demonstrate the validity of proposed ideas, a numerical simulation is presented. Fourth - order Runge-Kutta method with sampling time 0.001 [sec] is used to solve the system of differential equations (4).

The system parameters are used by $a=5,\ \ b=-10,$ $c=-3.8$ in numerical simulation. The initial conditions for master and slave system are given by $x(0)=(1,2,-3)^T$ and $y(0)=(-3,-2,2)^T$, respectively. Also, the control gain k is chosen by -100. The scaling functions for generalized functional synchronization are taken for a choice as

$$\begin{aligned}\alpha_1(t)&=2\\ \alpha_2(t)&=-1.5+\sin t\\ \alpha_3(t)&=1+0.5\sin 2t.\end{aligned} \tag{9}$$

Figure 2 shows that error signals of GFPS go to zero asymptotically. It means GFPS occurs between state of $\alpha_i(t)x_i(t)$ and state of $y_i(t)$, for all i.

CIRCUIT IMPLEMENTATION

In this section, we present circuit implementations for proposed synchronization scheme.

For the circuit implementation of mathematical dynamic model (4), we use transformed Chen-Lee chaotic system because of some problems. Based on electronic circuit of Equation (4), the range of state variables exceed the limit of power supply. So, the reasonable transformation is to multiply the scaling ratio 10 to nonlinear terms.

Consider the following transformed Chen-Lee equations

$$\begin{aligned}\dot x_1 &= ax_1-10x_2x_3\\ \dot x_2 &= bx_2+10x_1x_3\\ \dot x_3 &= cx_3+(10/3)x_1x_2.\end{aligned} \tag{10}$$

This system can be more easily operated with analog circuit because all the state variables gave reasonable dynamic ranges and circuit voltages remain well within the range of typical power supply limits. The analog circuit of transformed Chen-Lee Equation (10) is shown in Figure 3.

The electrical equations of the circuit are given by

$$\begin{aligned}\dot x_1 &= \frac{1}{R_4C_2}\left(\frac{R_{12}}{R_3}x_1-\frac{R_{12}}{R_2}x_2x_3\right)\\ \dot x_2 &= \frac{1}{R_{13}C_3}\left(-\frac{R_5}{R_7}x_2+\frac{R_5}{R_6}x_1x_3\right)\\ \dot x_3 &= \frac{1}{R_8C_4}\left(-\frac{R_{11}R_{17}}{R_{10}R_{16}}x_3+\frac{R_{11}}{R_8}x_1x_2\right).\end{aligned} \tag{11}$$

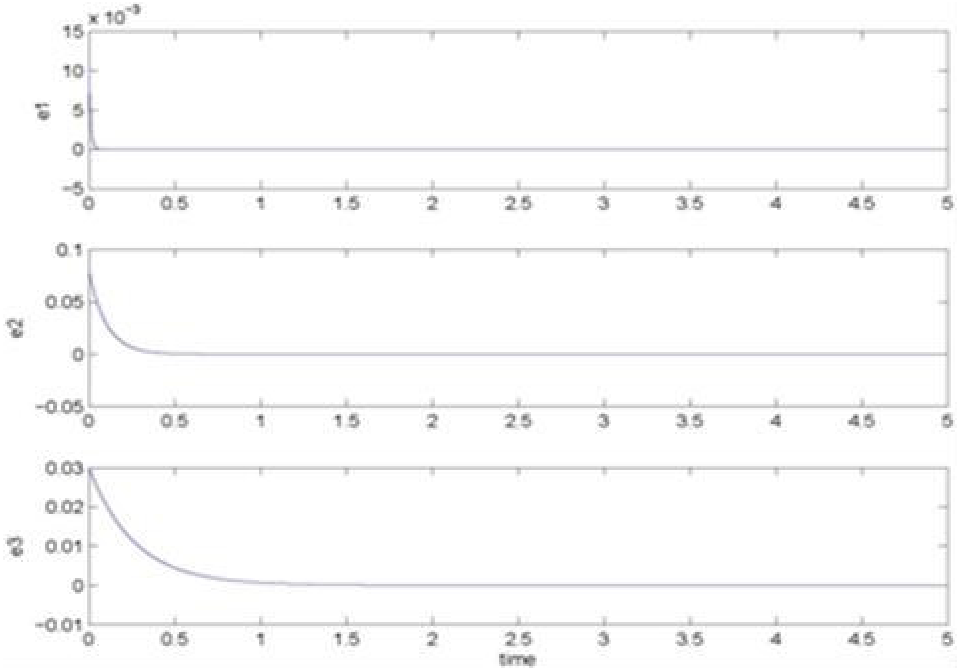

Figure 2. Error signals of numerical example.

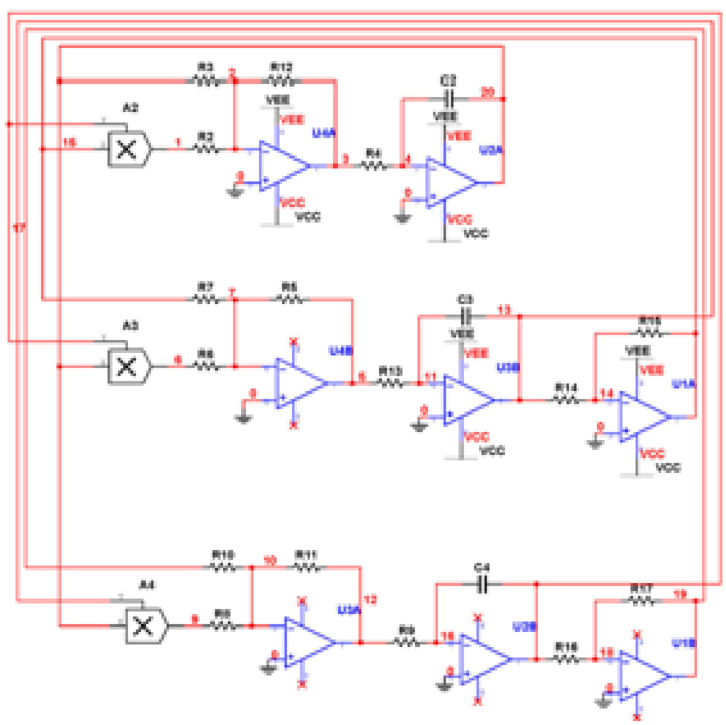

Figure 3. Chen-Lee circuit.

Note that Equation (11) is equivalent to Equation (10) after some calculation and applying the required electrical parameters such as R_2, R_6, $R_7 = 10k\Omega$; $R_3 = 20k\Omega$; R_4, R_9, $R_{13} = 1G\Omega$; R_5, R_{10}, R_{11}, R_{12}, R_{14}, R_{15}, R_{16} $= 100k\Omega$; $R_8 = 30k\Omega$; $R_{17} = 380k\Omega$; $C_i = 1nF$, $(i = 2, 3, 4)$. The operational amplifiers are considered to be ideal, the time step is 0.001 [s] and the initial condition of master circuit is $x(0) = (0.2, 0.2, 0.2)$ [V].

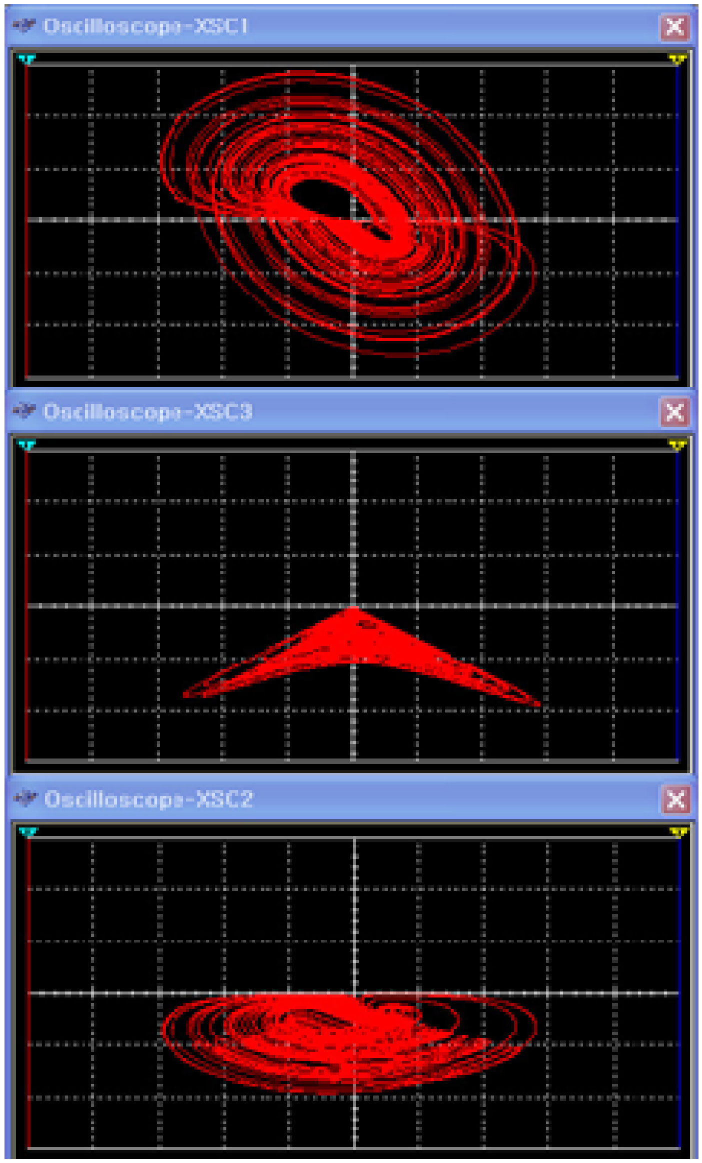

Figure 4. Chaotic phase of Chen-Lee system.

Figure 4 displays phase to phase of master system of $x_1 - x_2$, $x_1 - x_3$, $x_2 - x_3$, respectively.

For our synchronization scheme, the circuit of slave system is described by Figure 5. And the required electrical parameters are: R_1, R_{20}, $R_{21} = 10k\Omega$; $R_{18} = 20k\Omega$; R_{19}, R_{23}, $R_{28} = 1G\Omega$; $R_{22} = 30k\Omega$; R_{24}, R_{25}, R_{26}, R_{27}, R_{29}, R_{30}, $R_{31}, R_{45}, R_{46}, R_{47}, R_{48}, R_{53}$, R_{54}, R_{55}, R_{56}, R_{57}, R_{58}, R_{59}, $R_{60} = 100k\Omega$; $R_{32} = 380k\Omega$; $C_i = 1nF$, $(i = 1, 5, 6)$. The initial condition of slave circuit is $y(0) = (0.5, 0.5, 0.5)$ [v].

To show the effect of control input, first of all, we run the circuit without control inputs. Then circuit simulation result is obtained for generalized functional projective synchronization ($\alpha_1(t) = 2,$ $\alpha_2(t) = -1.5 + \sin t,$ $\alpha_3(t) = 1 + 0.5\sin 2t$) which is the same scaling factor given in Equation (9). The Figure 6 displays phase-phase and time-phase portraits of master and slave systems for this case. One can see that the errors do not approach to zero as expected since the control inputs are not applied. Next, let us consider the circuit of the whole synchronizing system given in Figure 7. The whole circuit is structured as three parts: master systems, slave systems, and controllers. In the control part, scaling functions $\alpha_i(\mathrm{t})$ $(\mathrm{i} = 1,\ 2,\ 3)$ are constituted by function generator and the initial condition of C_7, C_8, C_9 of differentiator are $0, 0,$ and 0 respectively. Then, Figure 8

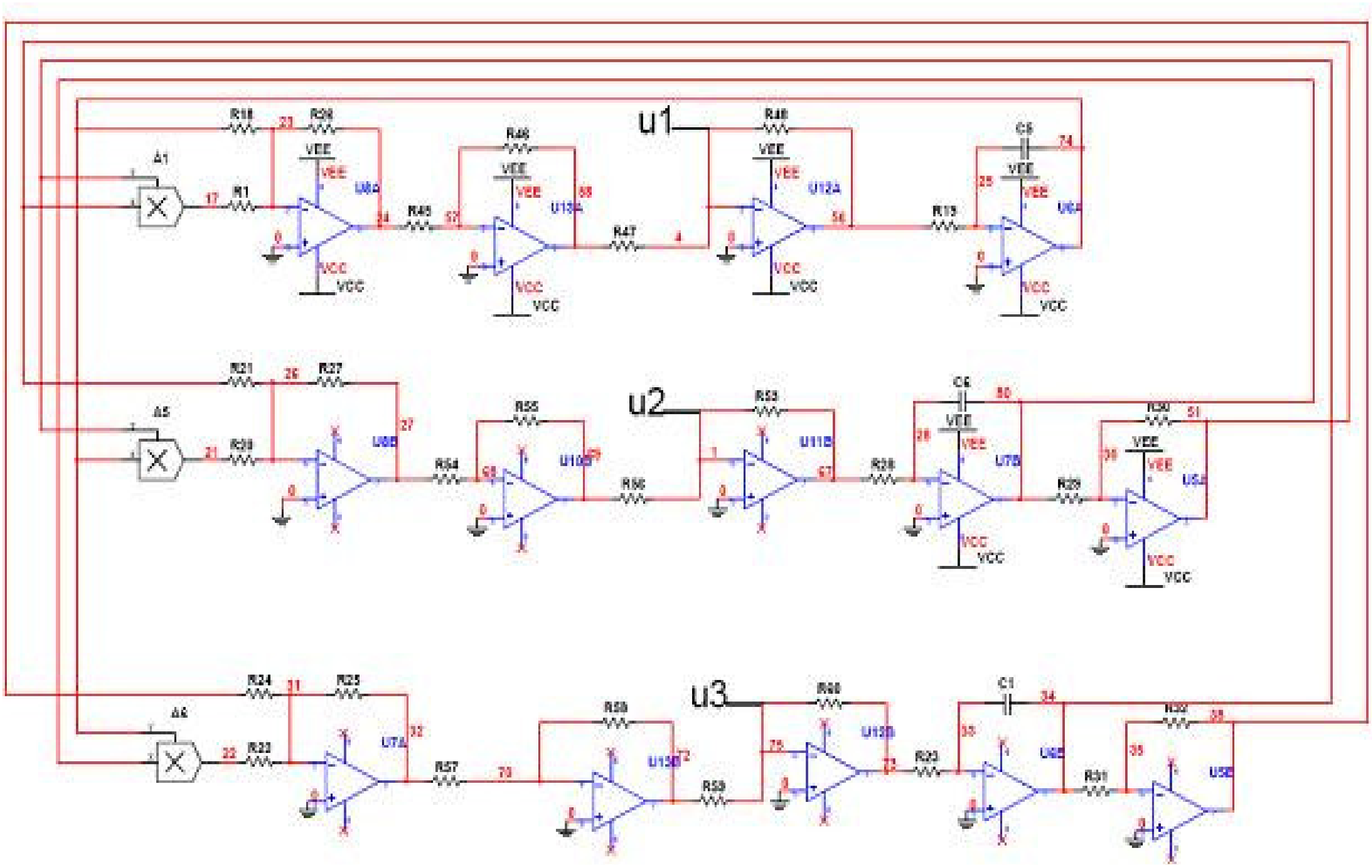

Figure 5. Slave system circuit.

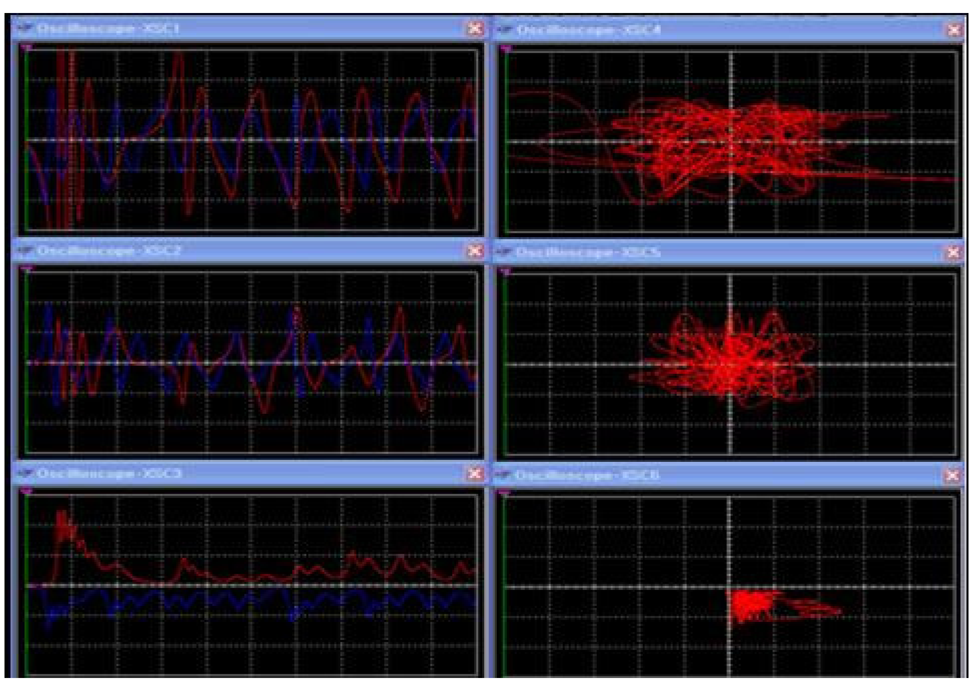

Figure 6. Simulation results for generalized functional projective synchronization without control.

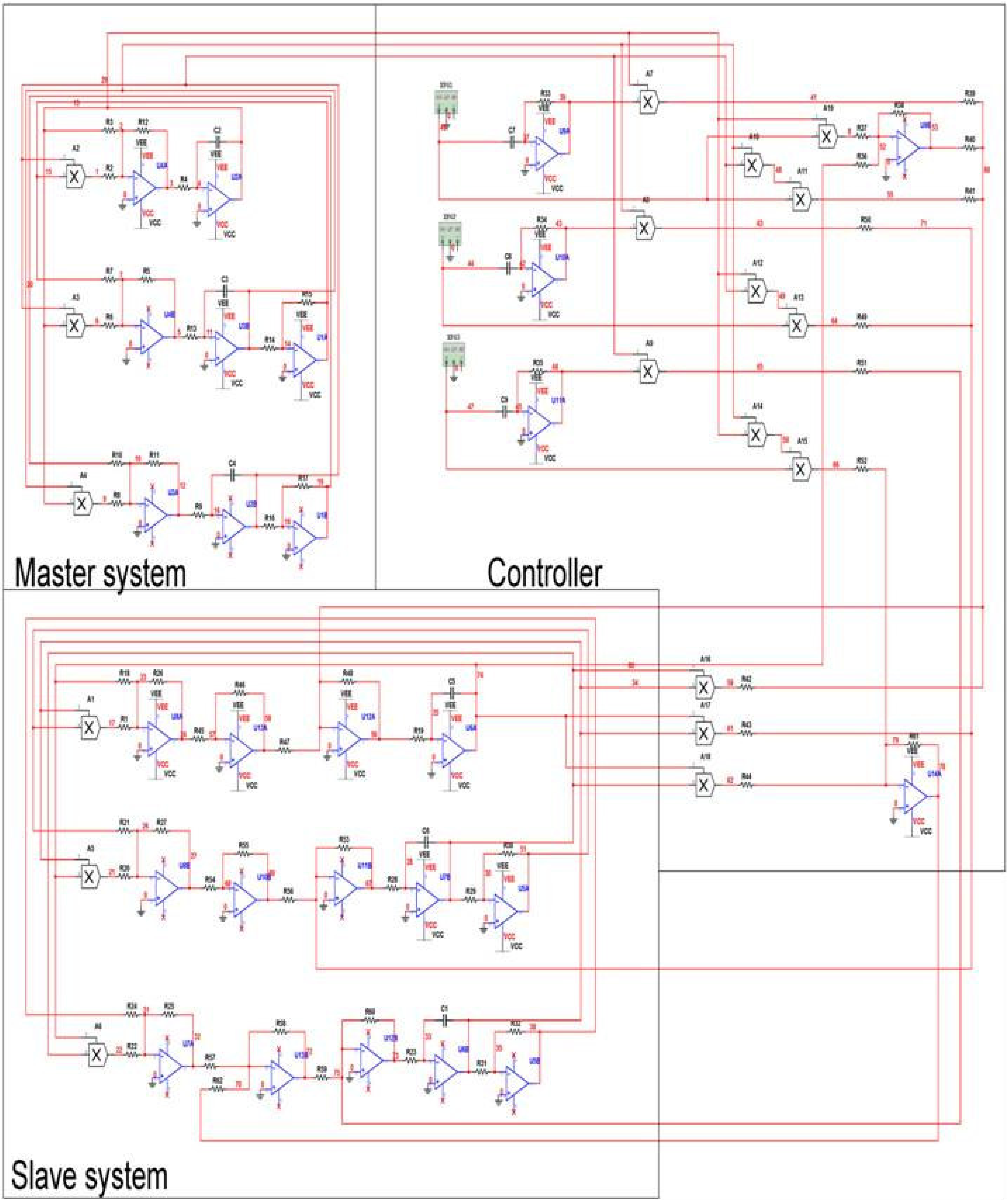

Figure 7. Circuit for controlled systems.

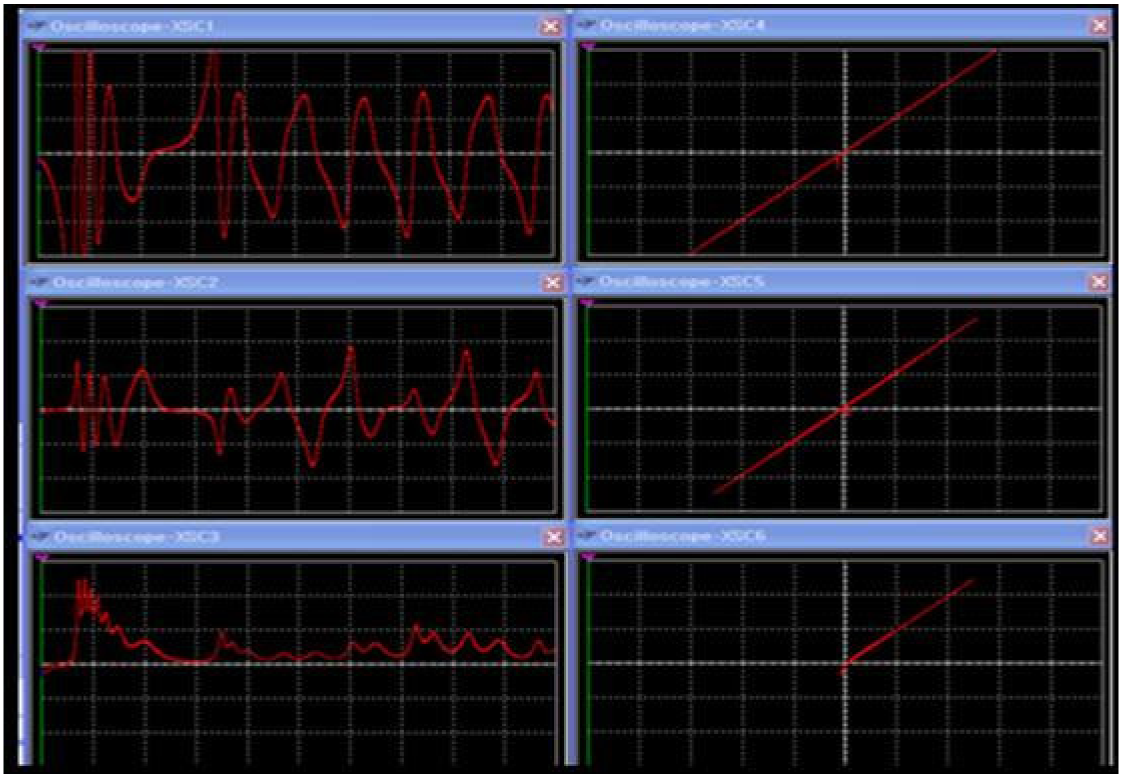

Figure 8. Simulation results for functional projective synchronization with control.

displays that GFPS of Chen-Lee system is achieved by control inputs as expected.

CONCLUSION

In this paper, we have investigated the generalized functional projective synchronization problem for Chen-Lee systems. The proposed control scheme is by computer and circuit simulations of the system. The final remark is that the proposed method is applicable to any chaotic systems.

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